University of California – Berkeley
Department of Economics
ECON 201A Economic Theory
Choice Theory
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Revealed preference
(Rubinstein Ch. 5 and Kreps Ch. 4)

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"I can tell you of an important new result I got recently. I have what I suppose to be a completely general treatment of the revealed preference problem, which will give a fresh setting for the related work of Samuelson-Houthakker-Uzawa. Calculus methods are unavailable. The methods are set-theoretic or algebraical."

—A letter from Sydney Afriat to Oskar Morgenstern, 1964—

ECONOMETRIC SOCIETY MONOGRAPHS

Revealed Preference Theory

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CAMBRIDGE

In ‘classic’ consumer behavior we have taken $\succsim$ as the primitive concept and derived the restrictions that the u-maximization model imposes on observed demand.

- These are the Slutsky restrictions — the substitution terms matrix is symmetric and negative semidefinite.

- These restrictions are (in principle) testable, but require assumptions on the parametric form/shape of the demand function.

- The standard approach is postulating some (semi)parametric family of functions and error structure.

The most basic question to ask about choice data $(\mathbf{p}^t, \mathbf{x}^t)$:

$$\text{prices } \mathbf{p}^t \text{ and associated chosen bundles } \mathbf{x}^t \text{ for } t = \mathbf{1}, ..., T$$

is whether it is consistent with utility maximization. Classical revealed preference theory provides a direct test:

> $(\mathbf{p}^t, \mathbf{x}^t)$ are consistent with maximizing a (well-behaved) u-function if and only if they satisfy the Generalized Axiom of Revealed Preference (GARP).

We say that u-function rationalizes the observed behavior $(\mathbf{p}^t, \mathbf{x}^t)$ if for all for $t = 1, ..., T$

$$u(\mathbf{x}^t) \geq u(\mathbf{x}) \text{ for all } \mathbf{x} \text{ such that } \mathbf{p}^t\mathbf{x}^t \geq \mathbf{p}^t\mathbf{x},$$

that is, $u(\cdot)$ achieves its maximum value on the budget set at the chosen bundles.

Question Suppose that the data $(\mathbf{p}^t, \mathbf{x}^t)$ were generated by such a maximization process. What observable restrictions must the observed choices satisfy?

Answer None! Without any assumptions about $u(\cdot)$ there are no restrictions: $u(\cdot)$ can be a constant function (so the consumer was indifferent to all observed bundles...).

– We rule out this trivial case: what are the observable restrictions imposed by the maximization of a locally non-satiated u-function?

– Now, when $\mathbf{x}^t$ was chosen when $\mathbf{x}$ could have been strictly chosen $\mathbf{p}^t\mathbf{x}^t > \mathbf{p}^t\mathbf{x}$, the utility of $u(\mathbf{x}^t)$ must be strictly larger than the utility of $u(\mathbf{x})$.

We will say that $\mathbf{x}^t$ is

- $\mathbf{x}^t R^D \mathbf{x}$: *directly revealed preferred* to $\mathbf{x}$ if $\mathbf{p}^t \mathbf{x}^t \geq \mathbf{p}^t \mathbf{x}$.

- $\mathbf{x}^t P^D \mathbf{x}$: *strictly directly revealed preferred* to $\mathbf{x}$ if $\mathbf{p}^t \mathbf{x}^t > \mathbf{p}^t \mathbf{x}$.

- $\mathbf{x}^t R \mathbf{x}$: *revealed preferred* to $\mathbf{x}$ if there exists a sequence $\{\mathbf{x}\}_{k=1}^{K}$ with

$$\mathbf{x}^1 = \mathbf{x}^t \text{ and } \mathbf{x}^K = \mathbf{x} \text{ such that } \mathbf{x}^k R^D \mathbf{x}^{k+1}$$

for every $k = 1, ..., K-1$ (R is the transitive closure of R^D).

It is clear that if the data $(\mathbf{p}^t, \mathbf{x}^t)$ were generated by a non-satiated u-function then

$$\mathbf{x}^t R \mathbf{x} \Longrightarrow u(\mathbf{x}^t) \geq u(\mathbf{x}).$$

Consider any two observations $\mathbf{x}^t$ and $\mathbf{x}^s$: we now have a condition to determine whether $u(\mathbf{x}^t) \geq u(\mathbf{x}^s)$ and an (observable) condition to determine whether $u(\mathbf{x}^s) > u(\mathbf{x}^t)$.

Obviously, these two conditions should not both be satisfied. This condition (GARP) can be stated in the notation introduced above.

Generalized Axiom of Revealed Preference (GARP)

$$\mathbf{x}^t R \mathbf{x}^s \text{ implies not } \mathbf{x}^s P^D \mathbf{x}^t.$$

In words, if $\mathbf{x}^t$ is indirectly revealed preferred to $\mathbf{x}^s$, then $\mathbf{x}^s$ is not strictly directly revealed preferred to $\mathbf{x}^t$.

- GARP requires that if $\mathbf{x}^t R \mathbf{x}^s$ then $\mathbf{p}^s \mathbf{x}^s \leq \mathbf{p}^s \mathbf{x}^t$ ($\mathbf{x}^t$ must cost at least as much as $\mathbf{x}^s$ at the prices prevailing when $\mathbf{x}^s$ is chosen).

- As the name implies, GARP is a generalization of various other revealed preference tests...

<u>Weak Axiom of Revealed Preference (WARP)</u>

$$\mathbf{x}^t R^D \mathbf{x}^s \text{ and } \mathbf{x}^t \neq \mathbf{x}^s \text{implies not } \mathbf{x}^s R^D \mathbf{x}^t.$$

<u>Strong Axiom of Revealed Preference (SARP)</u>

$$\mathbf{x}^t R \mathbf{x}^s \text{ and } \mathbf{x}^t \neq \mathbf{x}^s \text{implies not } \mathbf{x}^s R \mathbf{x}^t.$$

! WARP and SARP require that there be a unique demand bundle at each budget, while GARP allows for multiple demanded bundles (flat spots in the indifference curves).

!! Afriat's (1967) Theorem tells us that if a (finite) data set generated by an individual's choices satisfies GARP, then the data can be rationalized by a well-behaved utility function.

Afriat's Theorem: The following conditions are equivalent:

(i) The data satisfy GARP.

(ii) There exists a non-satiated u-function that rationalizes the data.

(iii) There exists a concave, monotonic, continuous, non-satiated u-function that rationalizes the data.

(iii) trivially implies (ii) and we have already seen that (ii) implies (i). All that is left is the proof that (i) implies (iii). Not exactly...

(iv) There exist positive numbers (u^t, λ^t) for $t = \mathbf{1}, ..., T$ that satisfy the so-called Afriat inequalities:

$$u^s \leq u^t + \lambda^t \mathbf{p}^t(\mathbf{x}^s - \mathbf{x}^t) \text{ for all } t, s.$$

The Afriat numbers u^t and λ^t can be interpreted as utility levels (u^t) and marginal utilities (λ^t) that are consistent with the observed choices (more below).

We will show that (iv) implies (iii). The proof that (i) implies (iv) is omitted. See Chambers and Echenique (2016) for the argument, at your own risk...

Proof: consider the following u-function

$$u(\mathbf{x}) = \min_{t=1,\ldots,T} \{u^t + \lambda^t \mathbf{p}^t(\mathbf{x} - \mathbf{x}^t)\}$$

(which is the lower envelope of a finite number of hyperplanes). This function is continuous, locally non-satiated and monotonic (as long as $\mathbf{p}^t > 0$), and concave (trust me on this).

We will show that this function rationalizes the data—achieves its constrained maximum at $\mathbf{x}^t$ when prices are $\mathbf{p}^t$.

– Note that $u(\mathbf{x}^t) = u^t$. If this is not the case, we have

$$u(\mathbf{x}^t) = u^m + \lambda^m \mathbf{p}^m(\mathbf{x} - \mathbf{x}^m) < u^t$$

which violates one of the Afriat inequalities and thus $u(\mathbf{x}^t) = u^t$.

– Now suppose that $\mathbf{p}^s\mathbf{x}^s \geq \mathbf{p}^s\mathbf{x}$. It follows that

$$\begin{aligned} u(\mathbf{x}) &= \min_{t=1,\ldots,T}\{u^t + \lambda^t \mathbf{p}^t(\mathbf{x} - \mathbf{x}^t)\} \\ &\leq u^s + \lambda^s \mathbf{p}^s(\mathbf{x} - \mathbf{x}^s) \\ &\leq u^s \\ &= u(\mathbf{x}^s) \end{aligned}$$

Therefore $u(\mathbf{x}^s) \geq u(\mathbf{x})$ for any $\mathbf{x}$ such that $\mathbf{p}^s\mathbf{x}^s \geq \mathbf{p}^s\mathbf{x}$.■

A note on concave functions:

– A function $f : A \to \mathbb{R}$ defined over a convex set $A \subset \mathbb{R}^N$ is concave if for all $x, x' \in A$ and any $\alpha \in [0, 1]$

$$f(\alpha x + (1-\alpha)x') \geq \alpha f(x) + (1-\alpha) f x'.$$

– Letting $z = x' - x$, this condition can be rewritten as

$$f(x+z) \leq f(x) + \frac{f(x+\alpha z) - f(x)}{\alpha},$$

and for continuously differentiable function

$$f(x+z) \leq f(x) + \nabla f(x) \cdot z \text{ as } \alpha \to 0.$$

The u-function defined in the proof of Afriat's theorem

$$u(\mathbf{x}) = \min_t \{u^t + \lambda^t \mathbf{p}^t(\mathbf{x} - \mathbf{x}^t)\}$$

has a natural interpretation. If $u(\mathbf{x})$ is also differentiable then it must satisfy the T FOCs (the gradient vector):

$$\nabla u(\mathbf{x}^t) = \lambda^t \mathbf{p}^t. \qquad (*)$$

And since it also concave it must satisfy the standard concavity conditions

$$u(\mathbf{x}^t) \leq u(\mathbf{x}^s) + \nabla u(\mathbf{x}^s)(\mathbf{x}^t - \mathbf{x}^s) \qquad (**)$$

Substituting from $(*)$ into $(**)$, we have

$$u(\mathbf{x}^t) \leq u(\mathbf{x}^s) + \lambda^t \mathbf{p}^t(\mathbf{x}^t - \mathbf{x}^s).$$

The Afriat numbers can thus be interpreted as utility levels (u^t) and marginal utilities (λ^t) that are consistent with the observed choices.

1. Using similar methods there are (finite) tests for: homotheticity, weak and additive separability, expected utility, and more.

2. They involve checking to see whether a solution exists to a particular set of linear Afriat inequalities.

3. Well known graph theory algorithms can be used to verify whether or not these conditions are satisfied.

Afriat's theorem has (at least) two remarkable implications:

- If there is a locally non-satiated u that rationalizes the data then there must exist a continuous, monotonic, and concave u that rationalizes the data.

- If the underlying u had the "wrong" curvature at some points, we would never observe choices being made at such points (do not satisfy the right 2^{nd}-order conditions).

(1) Market data do not allow us to reject the hypotheses of convexity and monotonicity of preferences.

- Since GARP is a necessary and sufficient condition for u-maximization, it must imply conditions analogous to comparative statics results of classic demand theory.

- These include the Slutsky decomposition of a price change into the income and the substitution effects (for finite changes in a price rather than just infinitesimal changes).

(2) Since revealed preferences provide a complete set of the restrictions imposed by u-maximization, they must contain all information available about preferences.

The critical cost efficiency index (CCEI)

An obvious difficulty: GARP provides an exact test of u-maximization—either the data satisfy GARP or they do not.

– But choices involve at least some errors: compute incorrectly, execute intended choices incorrectly, err in other less obvious ways...

– Afriat (1972) suggested the following approach: for any number $0 \leq e \leq 1$, define the direct revealed preference relation $R^D(e)$ as

$$\mathbf{x}^t R^D(e)\mathbf{x} \text{ if } e\mathbf{p}^t\mathbf{x}^t \geq \mathbf{p}^t\mathbf{x}$$

and define $P^D(e)$ and $R(e)$ accordingly.

- Let e^* be the largest value of e such that the data $(\mathbf{p}^t, \mathbf{x}^t)$ satisfies GARP. Afriat's CCEI is the value of e^* associated with the data.

- e^* can be interpreted as saying that the consumer is 'wasting' as much as $1 - e^*$ of his income by making inefficient choices.

- The closer e^* to one, the smaller the 'perturbation' required to remove all violations and thus the closer the data are to satisfying GARP.

Recovering preferences and forecasting behavior: a brief outline

The tightest possible bounds on indifference curves through an allocation $\mathbf{x}^0$ not observed in the data $(\mathbf{p}^t, \mathbf{x}^t)$ for $t = 1, ..., T$.

- Consider the set of prices at which $\mathbf{x}^0$ could be chosen and be consistent—does not add violations of GARP—with the previously observed data.

- This set of prices is the solution to the system of linear inequalities constructed from the data and revealed preference relations. Call this set $S(\mathbf{x}^0)$.

- Use $S(\mathbf{x}^0)$ to generate set of observations—$RP(\mathbf{x}^0)$ and $RW(\mathbf{x}^0)$—revealed preferred/worse than $\mathbf{x}^0$.

- $RP(\mathbf{x}^0)$ is simply the convex monotonic hull of all allocations revealed preferred to $\mathbf{x}^0$ and $RW(\mathbf{x}^0)$ is constructed as follows:

$$\mathbf{x}^0 R^D \mathbf{x}^t \text{ for all prices } \mathbf{p}^0 \in S(\mathbf{x}^0)$$
$$\Downarrow$$
$$\mathbf{x}^0 R \mathbf{x}^s \text{ for any } \mathbf{x}^s \text{ such that } \mathbf{x}^t R \mathbf{x}^s.$$

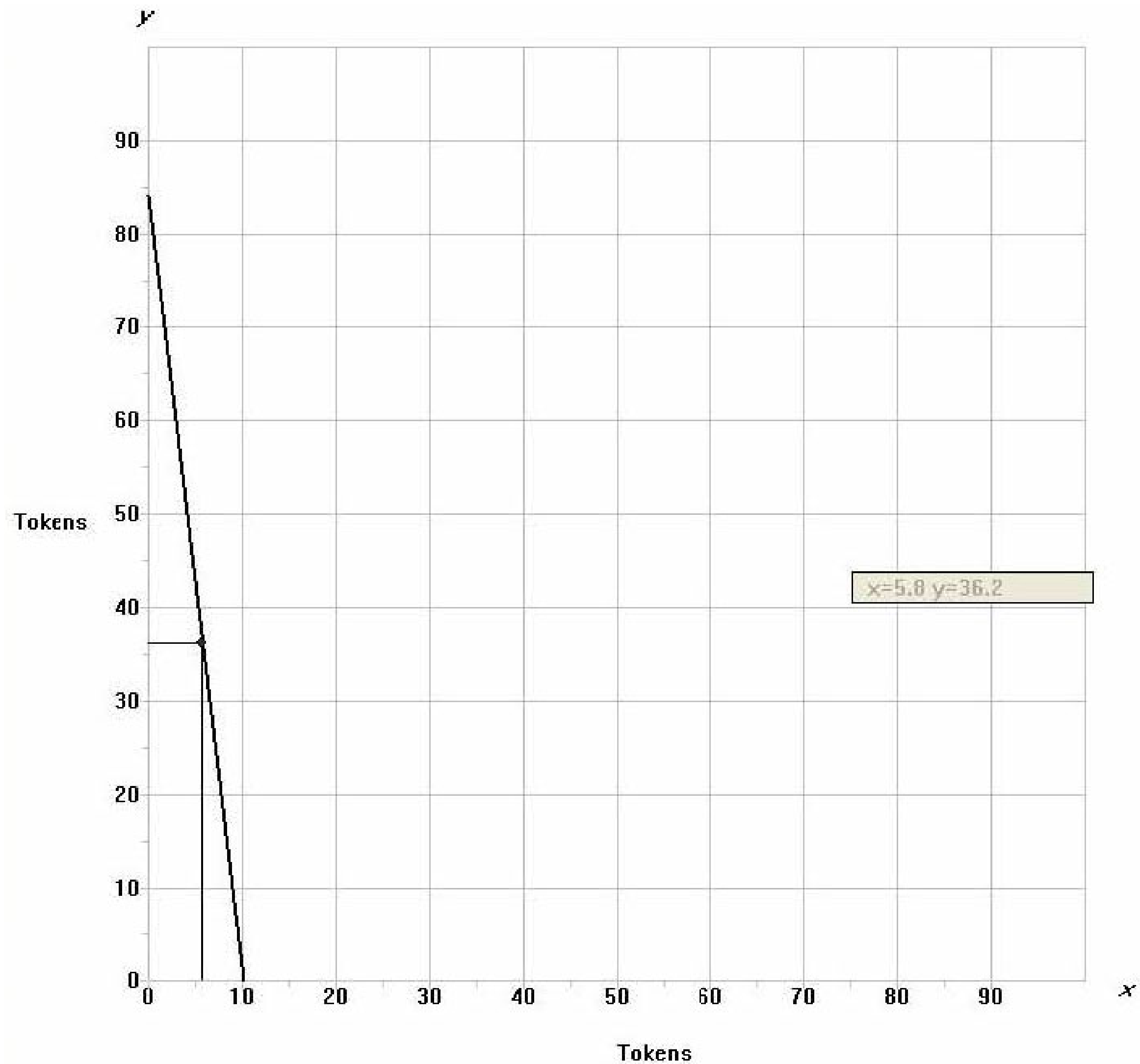
- $RP(\mathbf{x}^0)$ and the complement of $RW(\mathbf{x}^0)$ form the tightest inner and outer bounds on the set of allocations preferred to $\mathbf{x}^0$.

- $RW(\mathbf{x}^0)$ and the complement of $RP(\mathbf{x}^0)$ form the tightest inner and outer bounds on the set of allocations worse than $\mathbf{x}^0$.

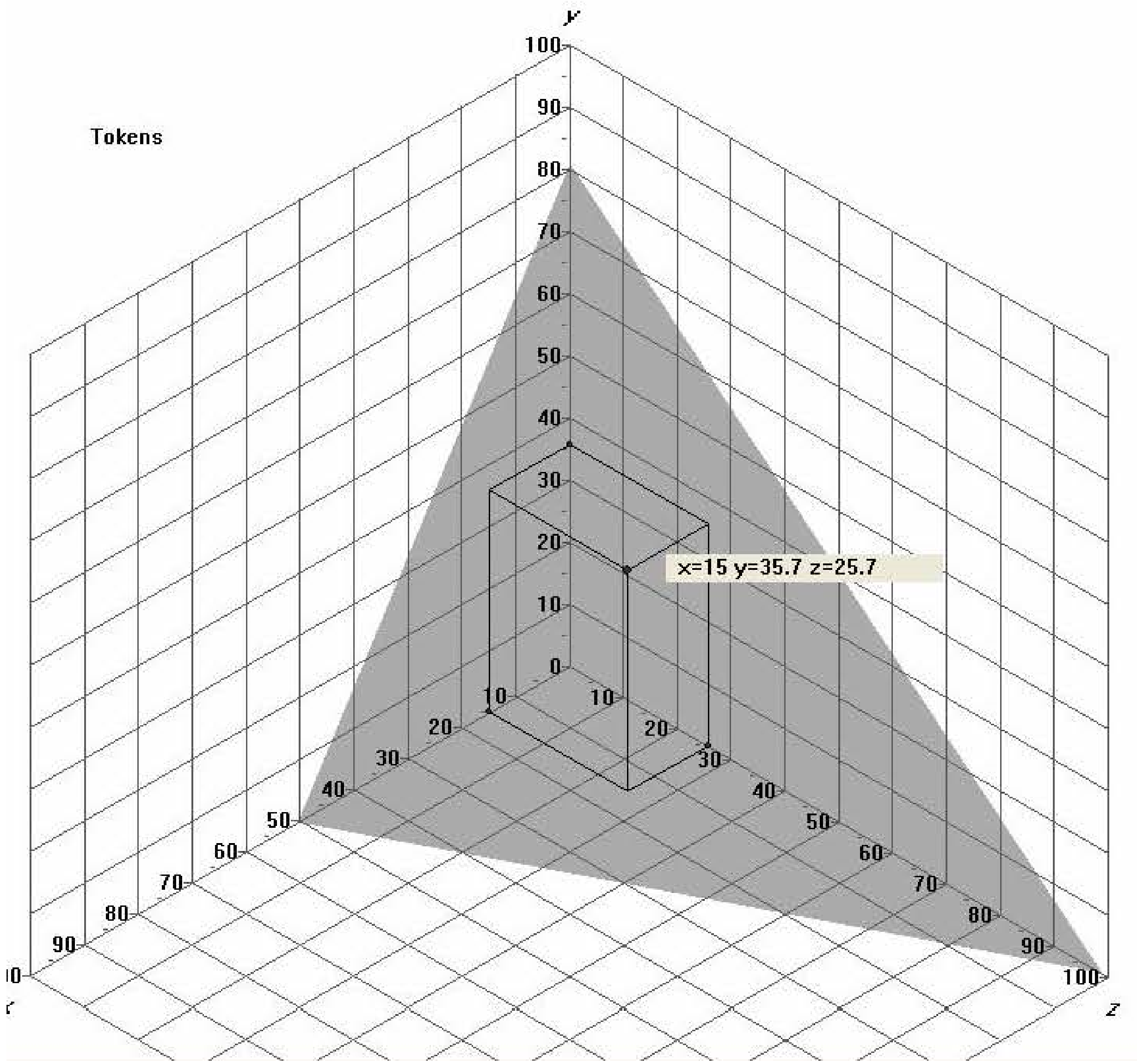
Forecasting: $\mathbf{x}^0$ chosen from budget set $\mathbf{p}^0$? Use the same algorithm!!!

Appendix
Who is (more) homo economicus?

2D experimental interface



3D experimental interface



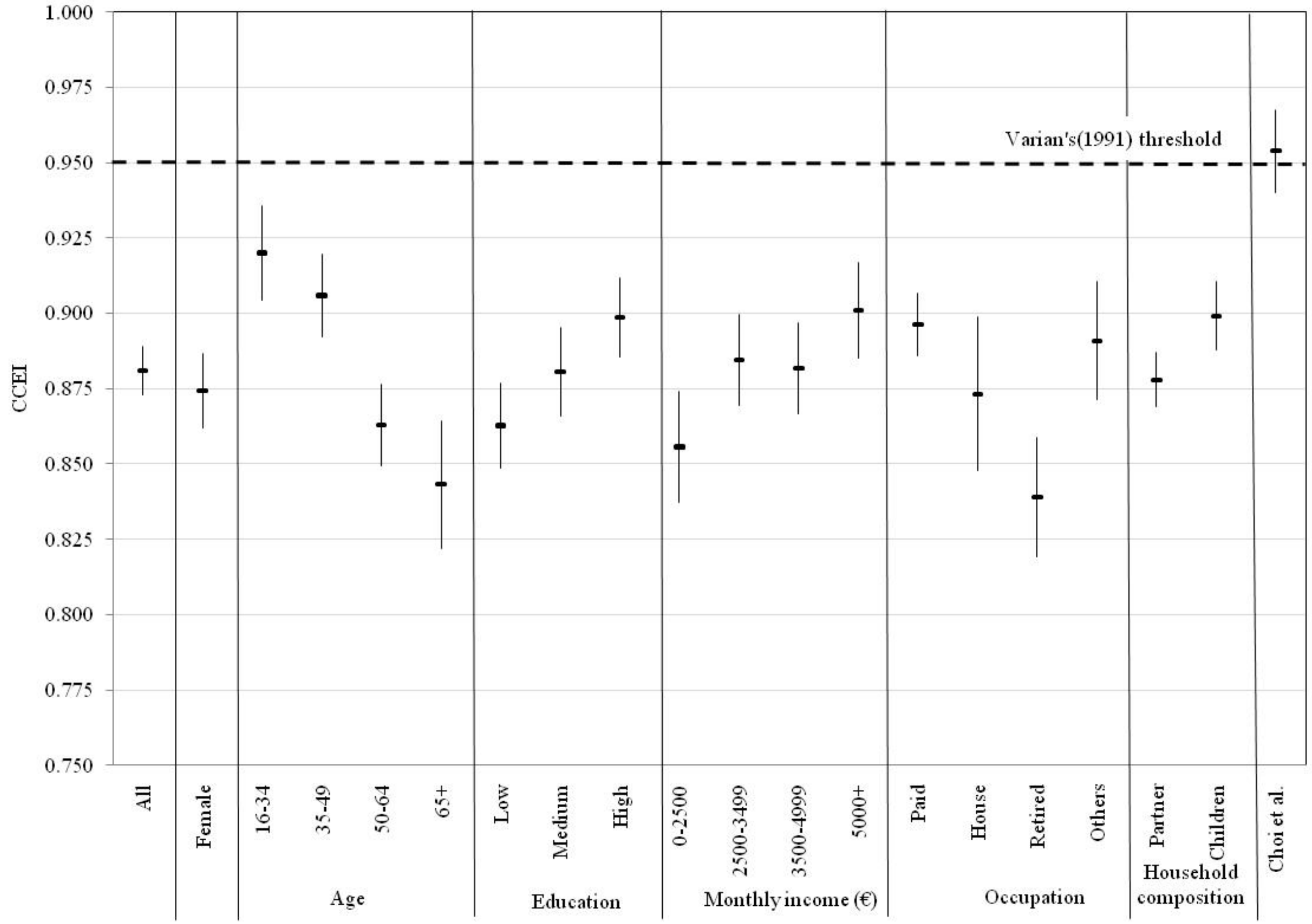
Judgment about the quality of decision-making is generally made difficult by twin problems of *identification* and *measurement*:

- **The identification problem** Distinguishing differences in decision-making quality from unobserved differences in preferences, information, beliefs or constraints.

Identification is important because welfare conclusions and thus (constrained) optimal policy will depend on the sources of any systematic differences in choices.

- **The measurement problem** Defining (and implementing) a *portable*, *practical*, *autonomous*, *quantifiable*, and *economically interpretable* measure of decision-making quality.

Mean CCEI scores



Wealth differentials

$\Longrightarrow$ The heterogeneity in wealth is not well-explained either by standard observables (income, education, family structure) or by standard unobservables (intertemporal substitution, risk tolerance).

$\Longrightarrow$ If consistency with utility maximization in the experiment were a good proxy for (financial) decision-making quality then the degree to which consistency differ across subjects should help explain wealth differentials.

	Ln(hhld wealth)			
CCEI	1.425**	1.348*	1.781**	1.728**
	(0.565)	(0.714)	(0.746)	(0.750)
Combined CCEI		0.078	-0.091	-0.038
		(0.381)	(0.381)	(0.384)
Risk attitude			-1.361	-1.366
			(0.838)	(0.840)
Conscientiousness				0.103
				(0.072)
Ln(hhld income '08)	0.601***	0.602***	0.520***	0.514***
	(0.127)	(0.127)	(0.121)	(0.121)
Female	-0.228	-0.229	-0.299	-0.321
	(0.164)	(0.164)	(0.168)	(0.169)
Age	-0.286	-0.284	-0.310	-0.282
	(0.316)	(0.316)	(0.319)	(0.316)
Age^2	0.006	0.006	0.007	0.006
	(0.005)	(0.005)	(0.005)	(0.005)
Age^3	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Partner	0.682***	0.682***	0.733***	0.714***
	(0.183)	(0.183)	(0.191)	(0.191)
# of children	0.103	0.103	0.095	0.090
	(0.092)	(0.093)	(0.095)	(0.095)
Education controls	Y	Y	Y	Y
Constant	5.932	5.888	7.797	7.371
	(5.862)	(5.879)	(5.880)	(5.841)
R^2	0.1794	0.1778	0.1801	0.1819
# of obs.	517	517	494	494

	Ln(hhld wealth)		Fraction of hhld wealth			
			Checking	Saving	Stocks	House
CCEI	1.907**	1.792**	-0.100*	-0.179*	-0.003	0.336***
	(0.751)	(0.705)	(0.057)	(0.096)	(0.050)	(0.129)
Ln(hhld income '08)	0.686***	0.25	-0.030**	-0.062***	0.011	0.098***
	(0.133)	(0.184)	(0.013)	(0.021)	(0.010)	(0.023)
Ln(hhld income '06)		0.467*				
		(0.262)				
Ln(hhld income '04)		0.312*				
		(0.176)				
Female	-0.144	-0.037	0.019	0.017	0.005	-0.039
	(0.188)	(0.184)	(0.018)	(0.029)	(0.012)	(0.039)
partner	0.790***	0.740***	-0.033	-0.058*	-0.008	0.136***
	(0.222)	(0.222)	(0.021)	(0.033)	(0.014)	(0.044)
# of children	0.111	0.123	-0.004	-0.043***	0	0.069***
	(0.107)	(0.101)	(0.009)	(0.013)	(0.007)	(0.019)
Constant	4.282	-0.415	0.045	1.42	-0.287	-0.777
	(5.947)	(6.032)	(0.750)	(1.197)	(0.388)	(1.289)
R^2	0.236	0.269	0.051	0.104	0.029	0.146
# of obs.	377	377	512	502	514	479

A comprehensive nonparametric test

Test complete representations of preferences rather than focusing on individual axiom(s) (comprehensive) and make no auxiliary functional form assumptions (nonparametric):

- utility maximization (rationalizability)
- stochastically monotone utility maximization (FOSD-rationalizability)
- expected utility maximization (EU-rationalizability)

A not-so-new experimental design

An experimental design that has a couple of innovations:

- A selection of a bundle of contingent commodities from a budget set (a portfolio choice problem).

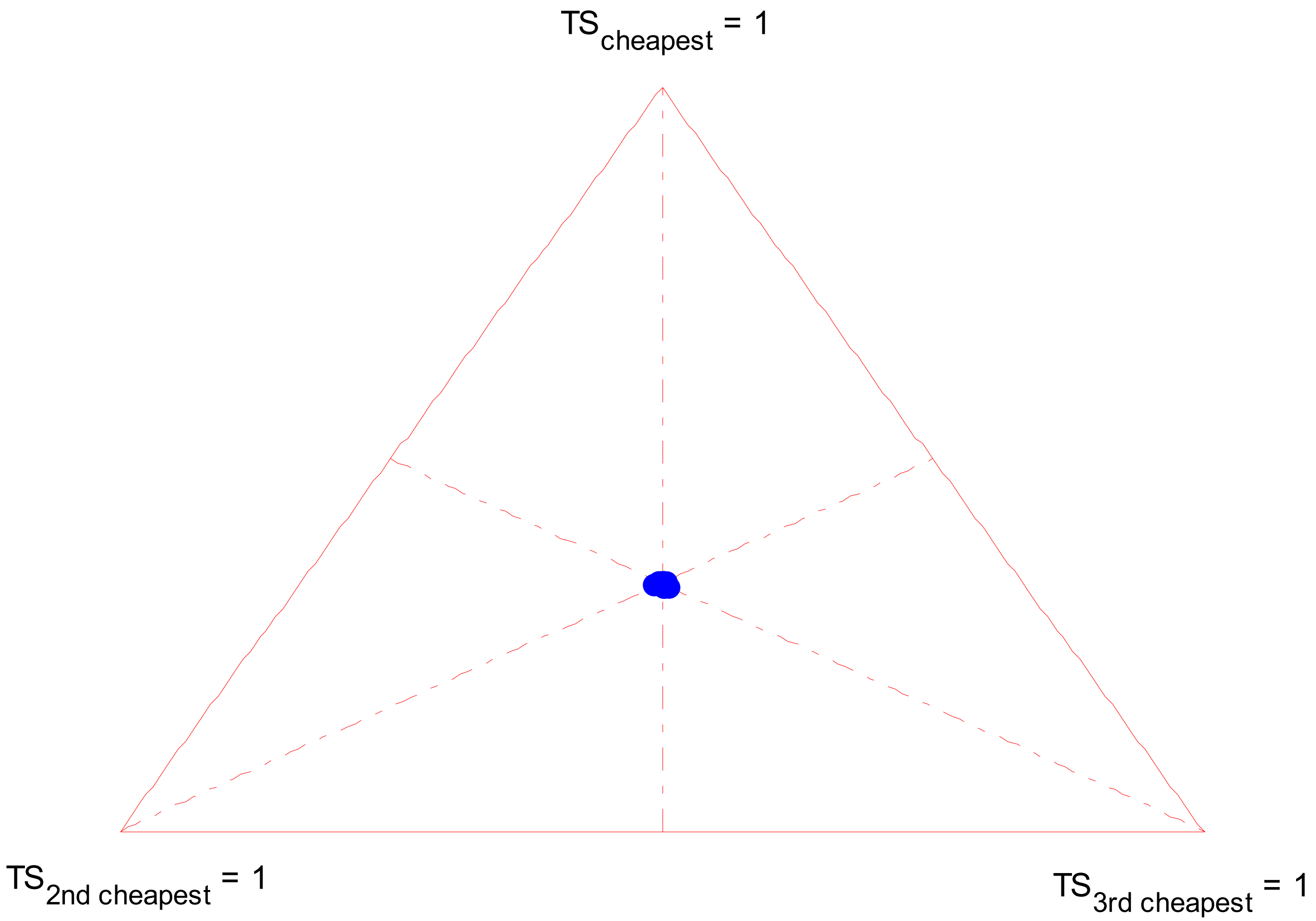
- A large menu of decision problems that are representative, in the statistical sense and in the economic sense.

- A graphical experimental interface that allows for the collection of a rich individual-level data set.

$\Rightarrow$ Build on Nishimura, Ok and Quah (2017), and Polisson, Quah and Renou (2020) and (1) allow subjects to make choices over three-dimensional budget sets, and (2) study choice under ambiguity.

Individual behaviors

Token Shares for Subject ID 35



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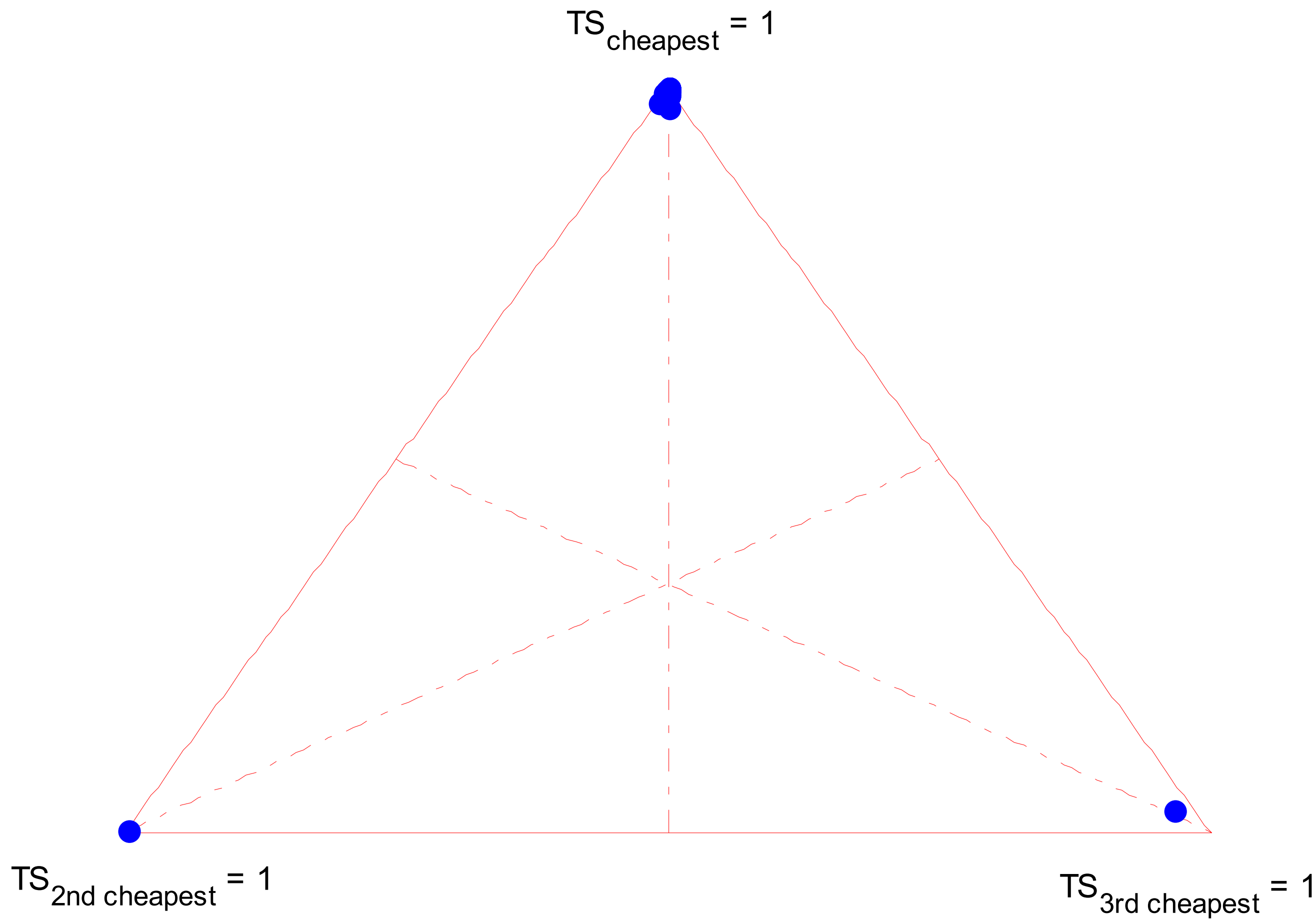

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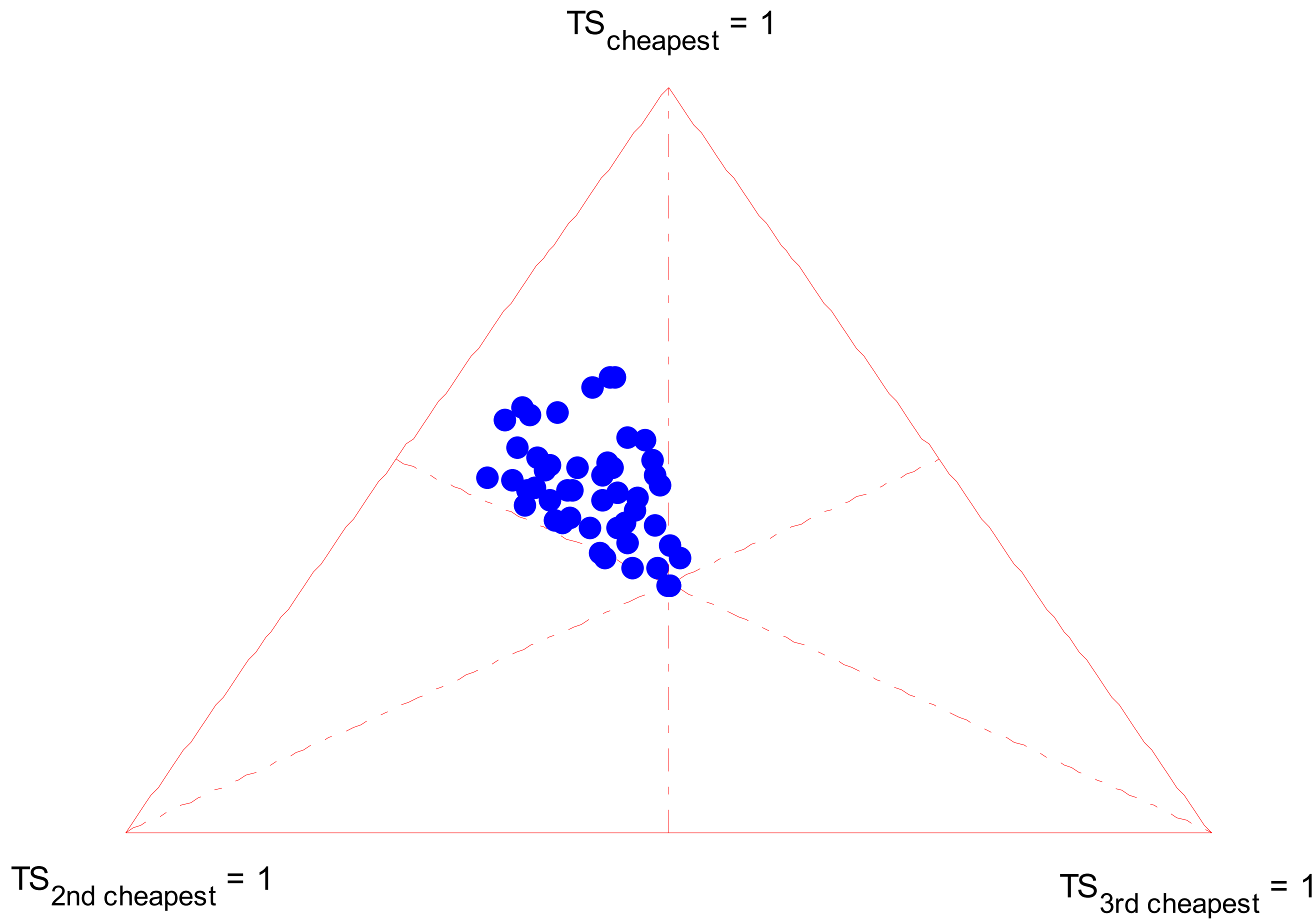
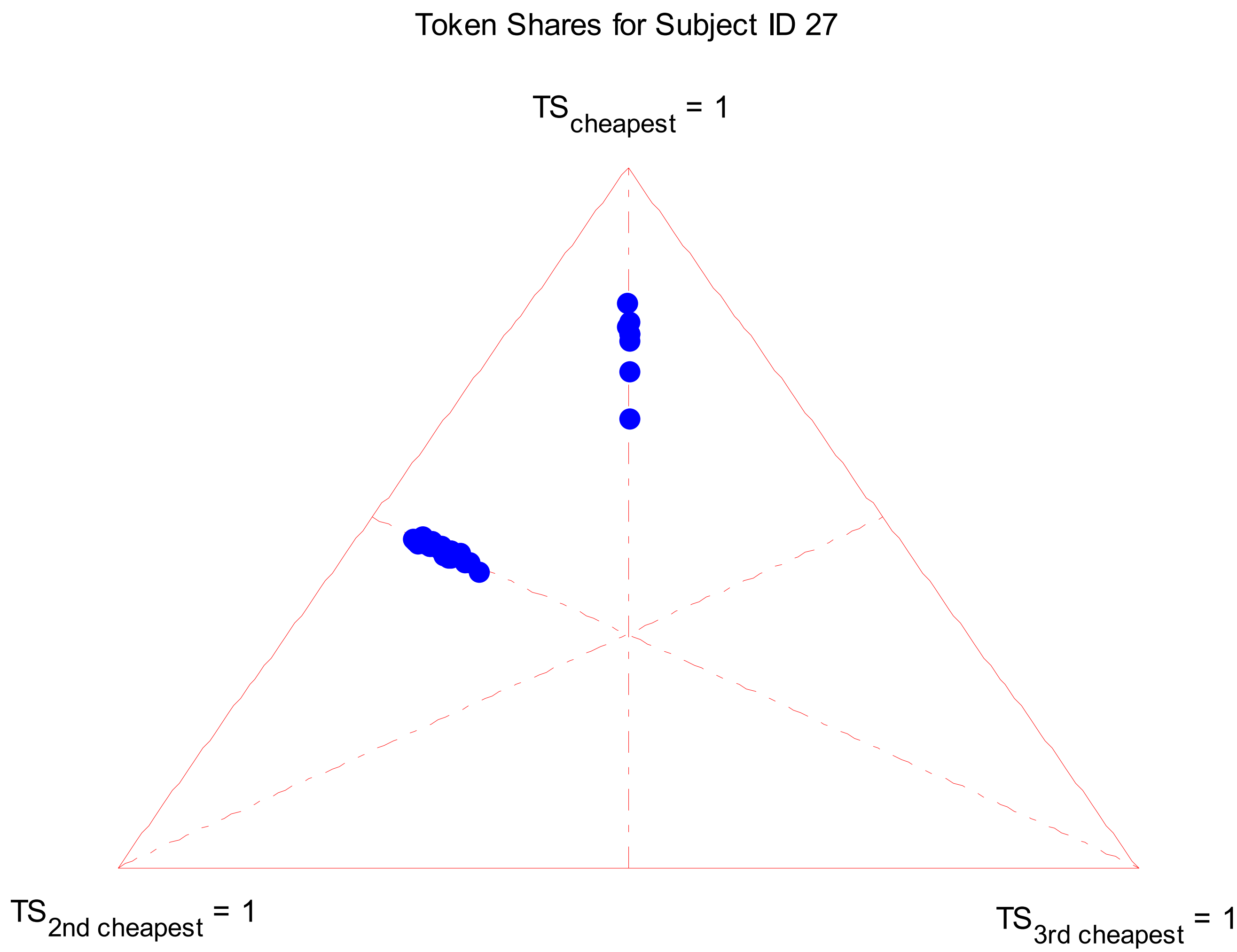
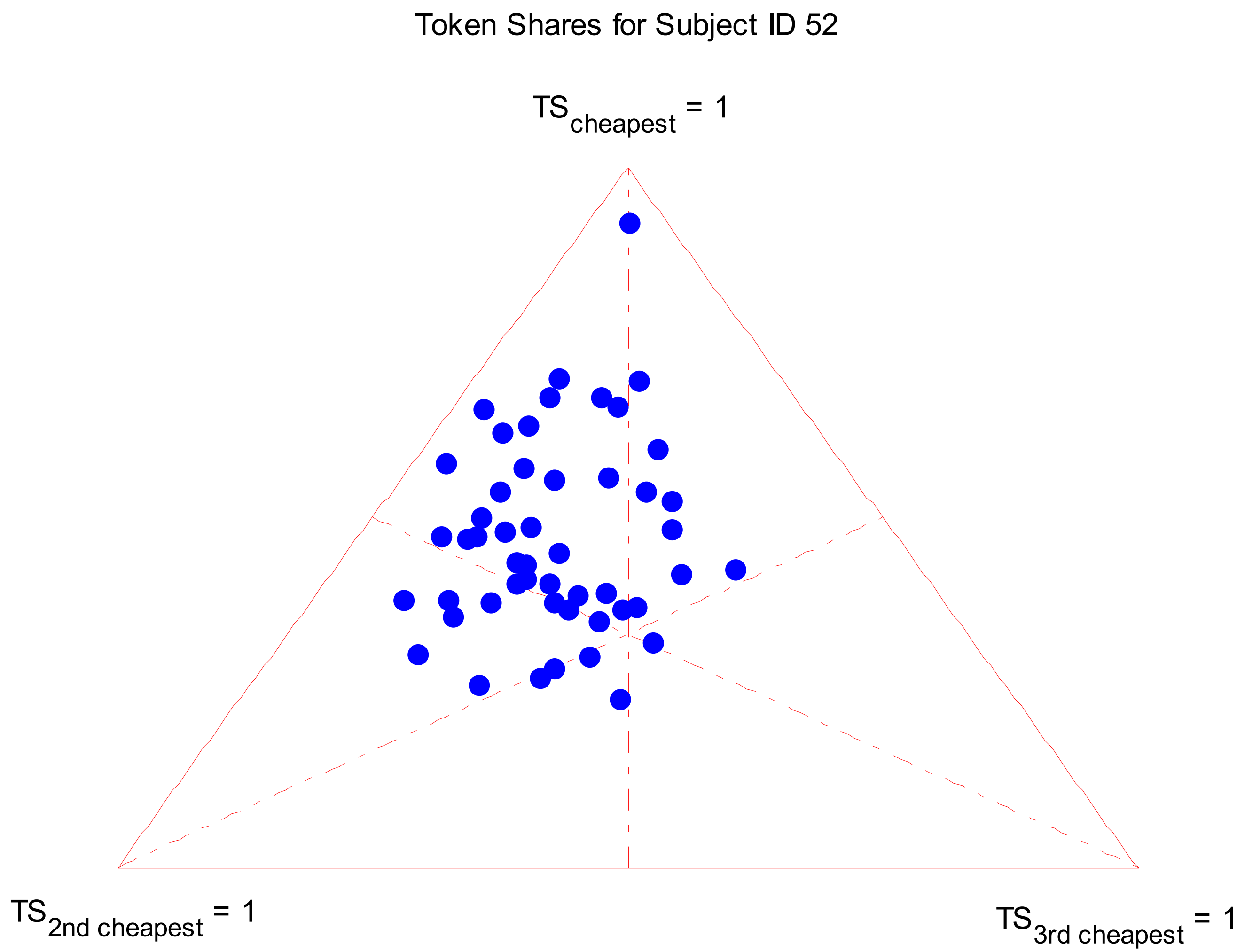



Token Shares for Subject ID 27
TS cheapest = 1
TS 2nd cheapest = 1
TS 3rd cheapest = 1


Token Shares for Subject ID 52
$TS_{cheapest} = 1$
$TS_{2nd\ cheapest} = 1$
$TS_{3rd\ cheapest} = 1$

Rationalizability

Let $\left\{(\mathbf{p}^i, \mathbf{x}^i)\right\}_{i=1}^{50}$ be the data generated by some individual's choices: $\mathbf{p}^i$ is the i-th observation of the price vector and $\mathbf{x}^i$ is the associated allocation.

The Generalized Axiom of Revealed Preference (GARP)

If $\mathbf{x}^i$ is indirectly revealed preferred to $\mathbf{x}^j$, denoted $\mathbf{x}^i R \mathbf{x}^j$, then $\mathbf{x}^j$ is not *strictly* directly revealed preferred to $\mathbf{x}^i$, denoted $\mathbf{x}^i R^D \mathbf{x}^j$.

Consistency with GARP thus implies consistent preferences, but any consistent preference ordering over lotteries is admissible.

FOSD-rationalizability

- Choices can be consistent with GARP and yet fail to be reconciled with any utility function that is normatively appealing given the decision problem at hand.

- The experiment is *symmetric* (each state had an equal probability), choice behavior should respond symmetrically to *permutations* in prices.

- Compute the CCEI obtained by augmenting the set of revealed preference comparisons at each observation.

EU-rationalizability

(The GRID method of Polisson, Quah and Renou, 2020)

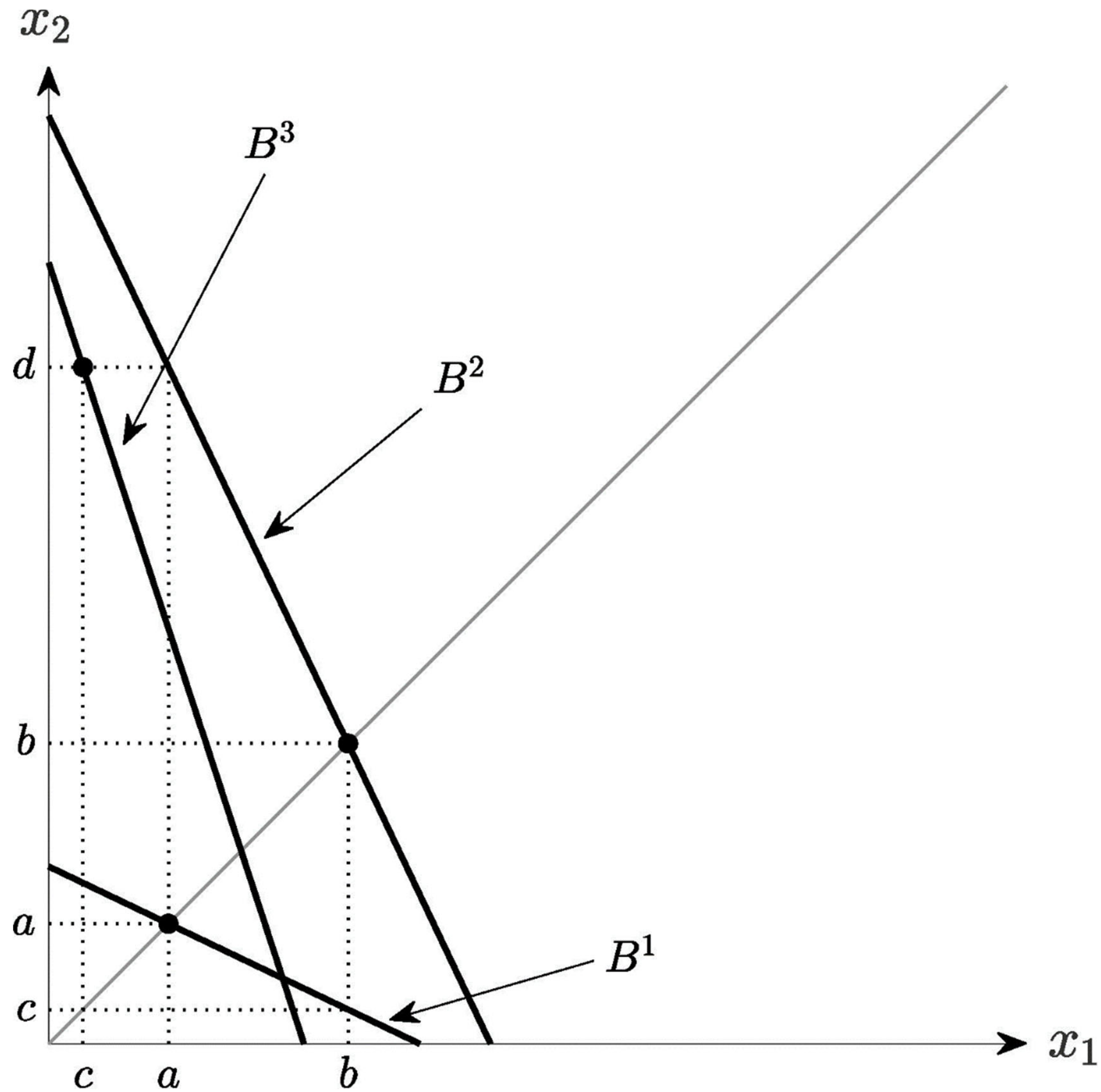
An example of a preference ordering that is FOSD-rationalizable but not EU-rationalizable, is rank-dependent utility function (Quiggin, 1993):

$$U(\mathbf{x}) = \omega_L u(\text{min}\{\mathbf{x}\}) + \omega_M u(\text{med}\{\mathbf{x}\}) + \omega_H u(\text{max}\{\mathbf{x}\})$$

When weights $w_H < w_M < w_L$, the indifference curves have "kinks" where $x_s = x_{s'}$

$\Longrightarrow$ Allocations that satisfy $x_s = x_{s'}$ will be chosen for a non-negligible set of price vectors, which is not consistent with EU.

A simple violation of EU-rationalizability



EU requires that

$$\begin{aligned} U(a,a) &= 2u(a) \geq u(b) + u(c) \\ U(b,b) &= 2u(b) \geq u(a) + u(d) \end{aligned}$$

b/c $(a,a)R^D(b,c)$ and $(b,b)R^D(a,d)$.

But rearranging yields

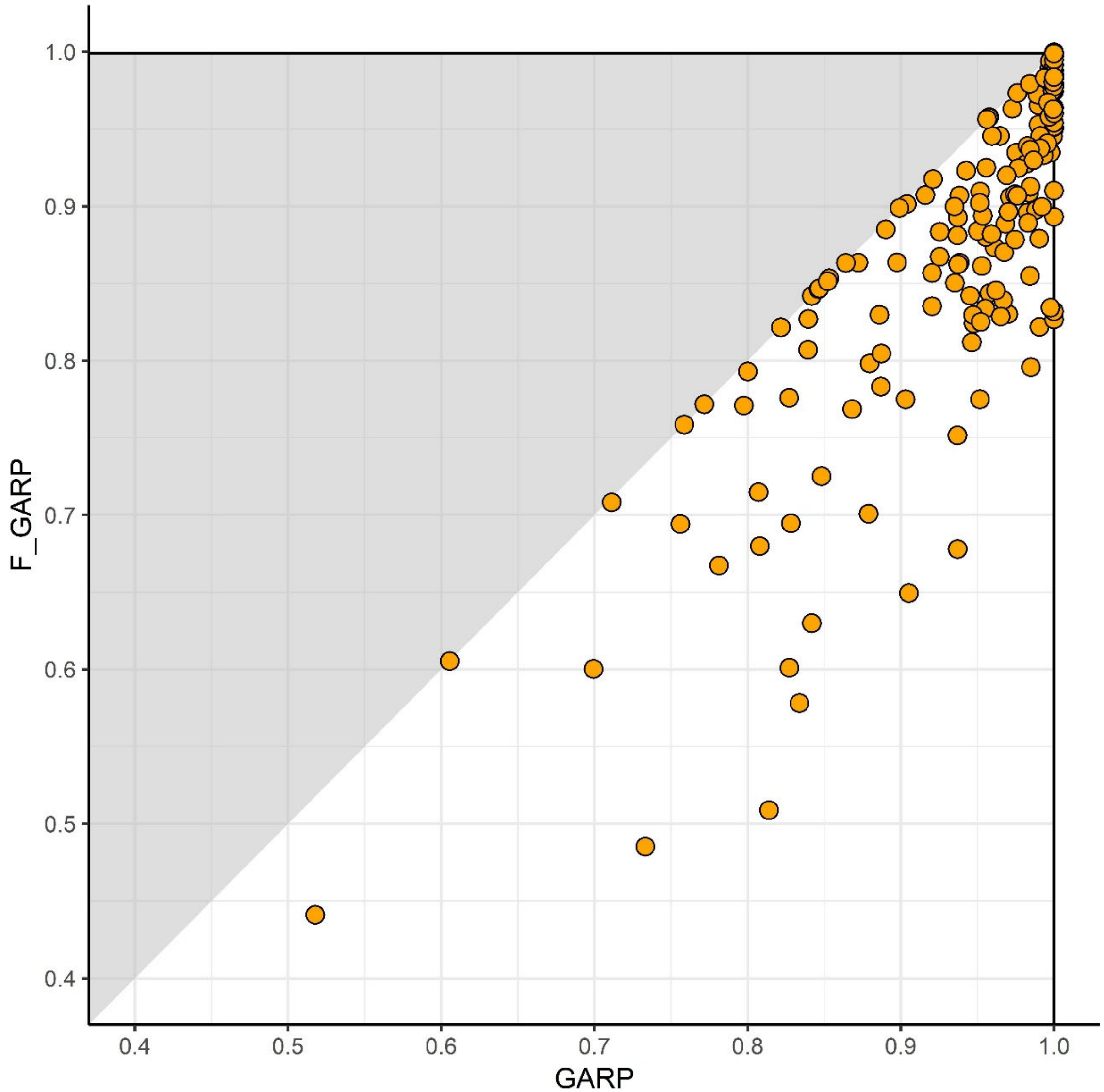
$$u(a) + u(b) \geq u(c) + u(d)$$

which contradicts that $(c,d)R^D(a,b)$.

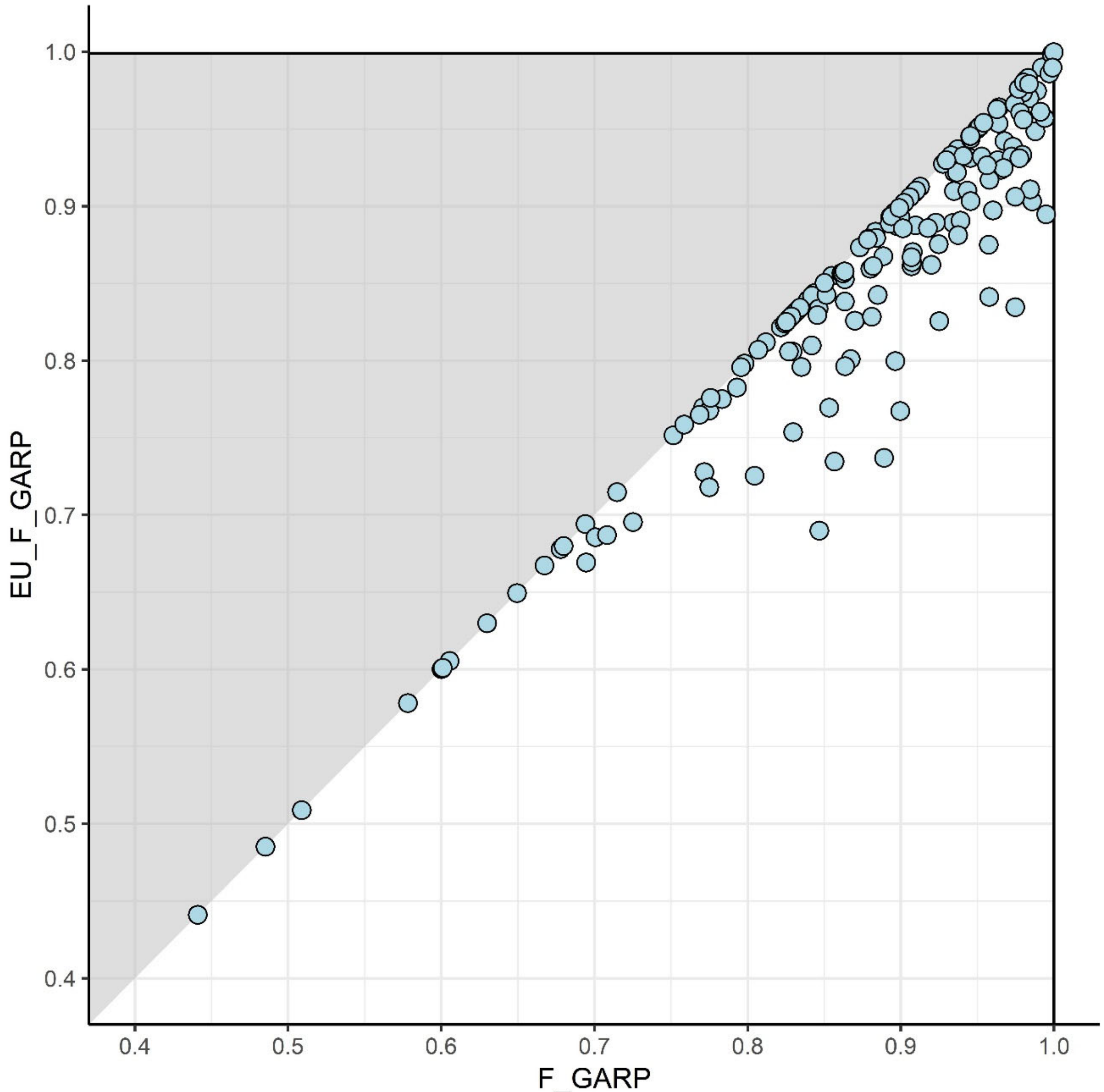
- e^* – maximizing any utility function (GARP).
- $e^{**} \leq e^*$ – maximizing a monotonic utility function (GARP+FOSD).
- $e^{***} \leq e^{**}$ – maximizing an expected utility function (GARP+FOSD+EU).

⇒ For all non-EU theories, which number well into double figures (Starmer, 2000), including stochastic reference dependence (Kőszegi and Rabin, 2006 & 2007):

$$e^{***} < e^{**} = e^* = 1.$$



F_GARP
GARP
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.4
0.5
0.6
0.7
0.8
0.9
1.0


EU_F_GARP
F_GARP
1.0
0.9
0.8
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0.4
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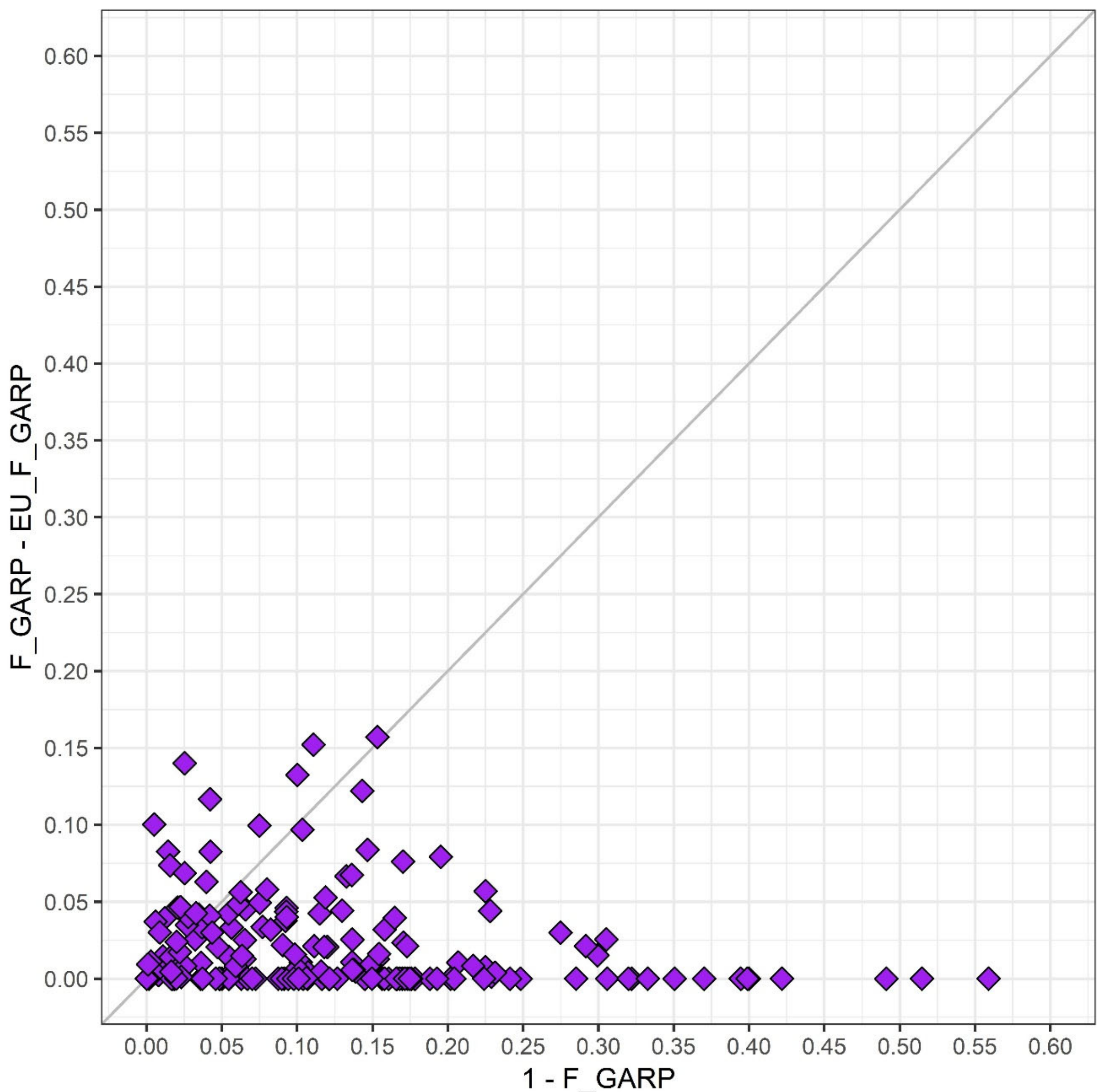
0.8
0.9
1.0



0.60
0.55
0.50
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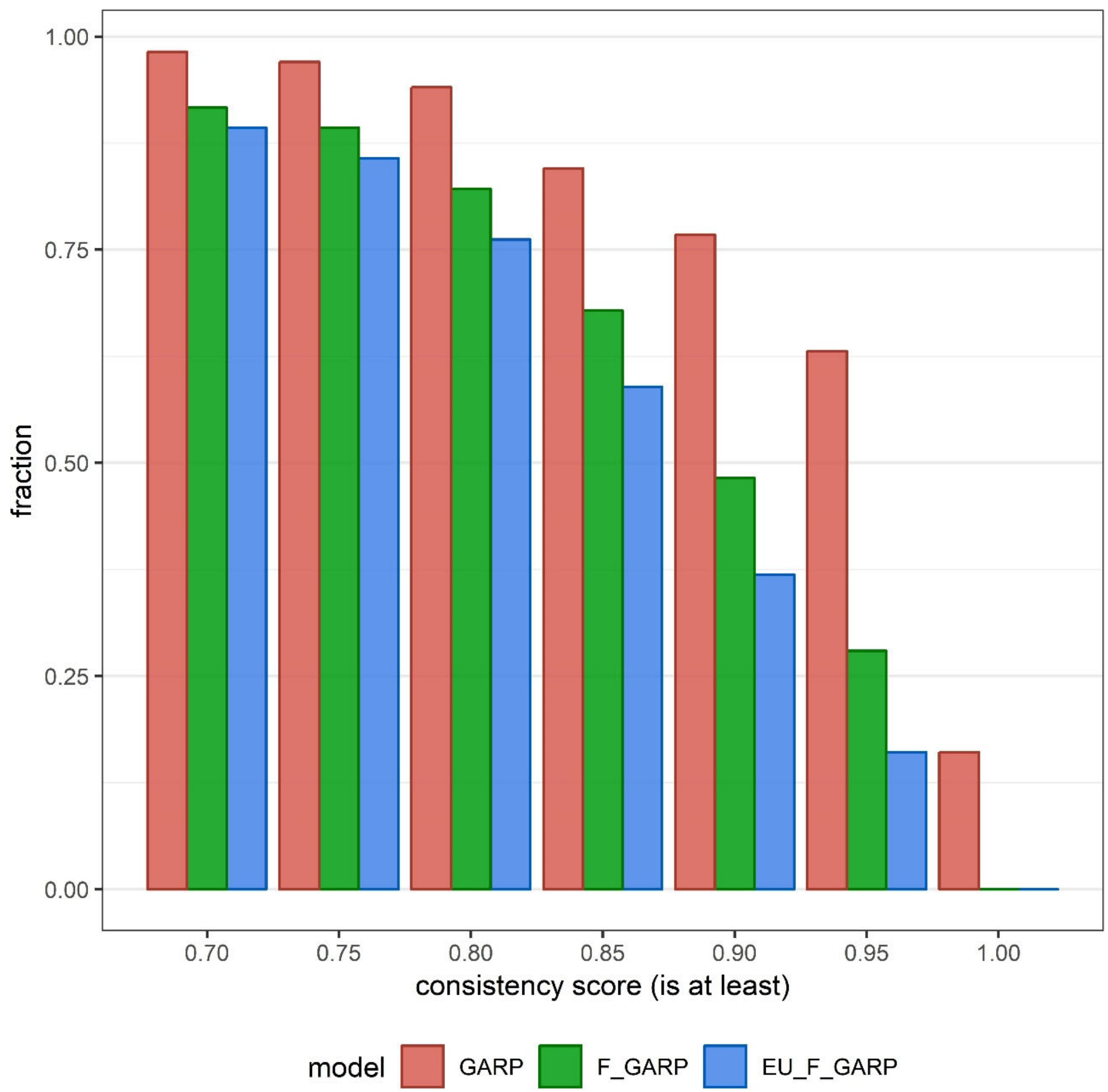
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
F_GARP - EU_F_GARP
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60
1 - F_GARP


1.00
0.75
0.50
0.25
0.00
fraction
0.70
0.75
0.80
0.85
0.90
0.95
1.00
consistency score (is at least)
model
GARP
F_GARP
EU_F_GARP

Stationarity, time invariance, and time consistency

- Time discount rates decline as tradeoffs are pushed into the temporal distance.
  - Subjects often choose the larger and later of two rewards when both are distant in time, but prefer the smaller and earlier one as both rewards draw nearer to the present.

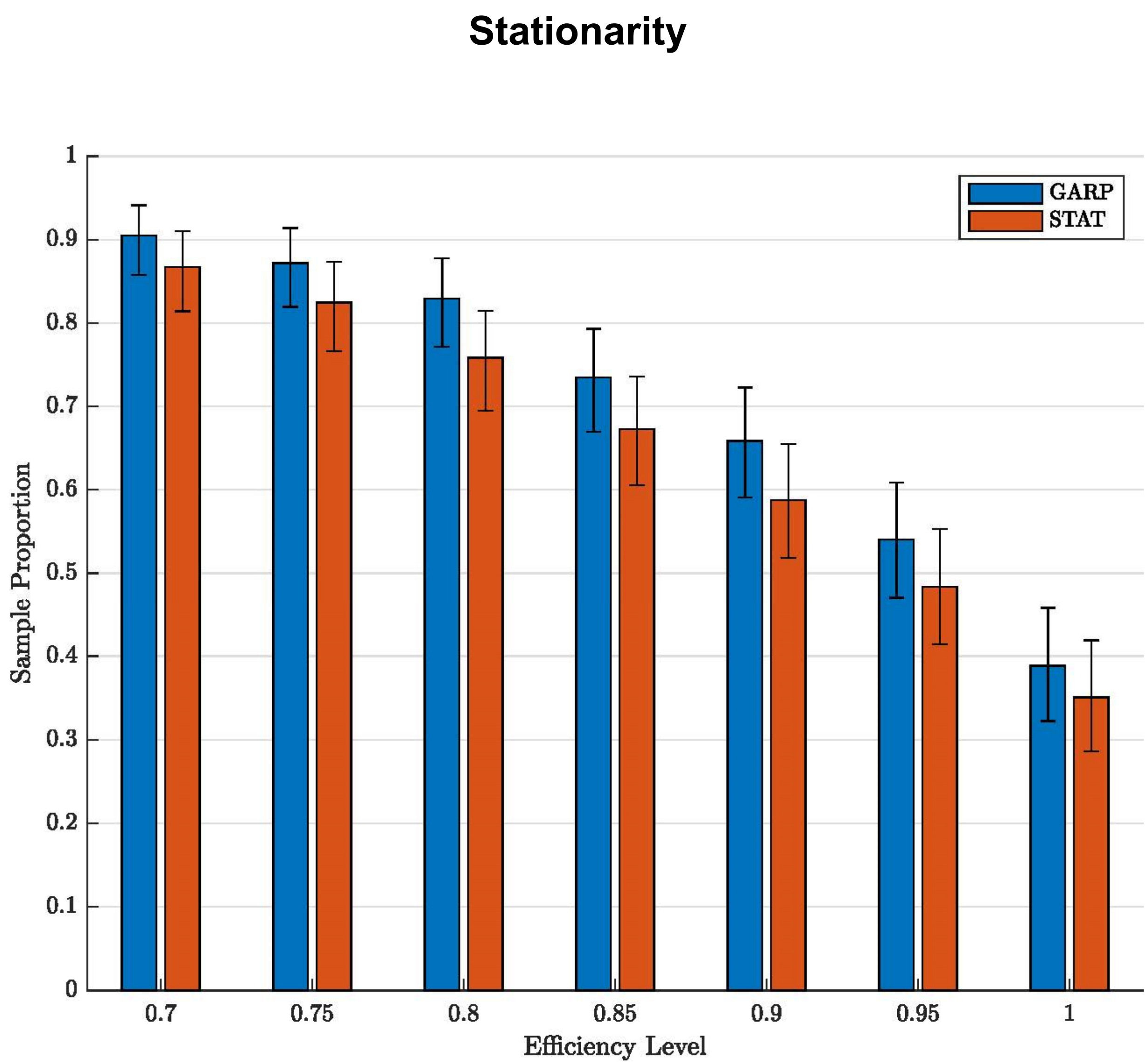
- Interpreted as non-constant time discounting, these preference reversals have important implications.
  - Under standard assumptions, non-constant time discounting implies time-inconsistency – self-control problems and a demand for commitment thus emerge.

Stationarity

$\succsim_t$ is stationary if for every $t, t' \geq 0$ and $\Delta_1, \Delta_2 \geq 0$

$$(x, t + \Delta_1) \sim_t (x', t + \Delta_2) \iff (x, t' + \Delta_1) \sim_t (x', t' + \Delta_2).$$

Ranking does not depend on the distance from t. Tested in the standard static experiment.


Stationarity
GARP
STAT
Sample Proportion
Efficiency Level
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.7
0.75
0.8
0.85
0.9
0.95
1

Exponential vs. quasi-hyperbolic