Economics 201A
Economic Theory
(Fall 2009)
Bayesian Games

Topics: normal form games (OR 2.6), extensive form games (OR 12.3).

Bayesian equilibrium (OR 2.6)

A Bayesian game consists of a finite set N of players, a finite set Ω of decision-relevant states (characteristics of players), and for each player $i \in N$

- a set A_i of actions
- a finite set T_i of types and a signal function $\tau_i : \Omega \to T_i$
- a probability measure p_i on Ω (prior belief) for which $p_i(\tau_i^{-1}(t_i)) > 0$ for all $t_i \in T_i$.
- a preference relation $\succsim_i$on the set of probability measure over $A \times \Omega$.

$a^* \in \times_{(i,t_i)} A_i$ is a Bayes-Nash equilibrium of a Bayesian game

$$\langle N, \Omega, (A_i), (T_i), (\tau_i), (p_i), (\gtrsim_i) \rangle$$

if it is a NE in which the set of players is the set of all pairs (i, t_i) for all $i \in N$ and $t_i \in T_i$, and for each player (i, t_i)

$$a^* \gtrsim_{(i,t_i)} b^* \Leftrightarrow L_i(a^*, t_i) \gtrsim_i L_i(b^*, t_i)$$

where $L_i(a^*, t_i)$ is a *lottery* over $A \times \Omega$ that assigns a probability $\frac{p_i(\omega)}{p_i(\tau_i^{-1}(t_i))}$ to

$$(a^*(j, \tau_j(\omega)))_{j \in N, \omega} \text{ if } \omega \in p_i(\tau_i^{-1}(t_i))$$

and zero otherwise.

Example: BoS with one-side imperfect information

$\omega = y$

	B	S
B	2, 1	0, 0
S	0, 0	1, 2

$\omega = n$

	B	S
B	2, 0	0, 2
S	0, 1	1, 0

Then, the expected payoffs of player 1 are given by

	(B,B)	(B,S)	(S,B)	(S,S)
B	2	$2p$	$2(1-p)$	0
S	0	$1-p$	$1-p$	1

For any belief $p \in (0,1)$, $(B,(B,S))$ is an equilibrium (B is optimal for player 1 given the actions of the two types of player 2 and his beliefs).

Perfect Bayesian equilibrium (OR 12.3)

A Bayesian extensive game $\langle \Gamma, (\Theta_i), (p_i), (u_i) \rangle$ is a game with observable actions where

- Γ is an extensive game of perfect information and simultaneous moves,
- Θ_i is a finite set of possible types of player i,
- p_i is a probability distribution on Θ_i for which $p_i(\theta_i) > 0$ for all $\theta_i \in \Theta_i$, and
- $u_i : \Theta_i \times Z \rightarrow \mathbb{R}$ is a vNM utility function.

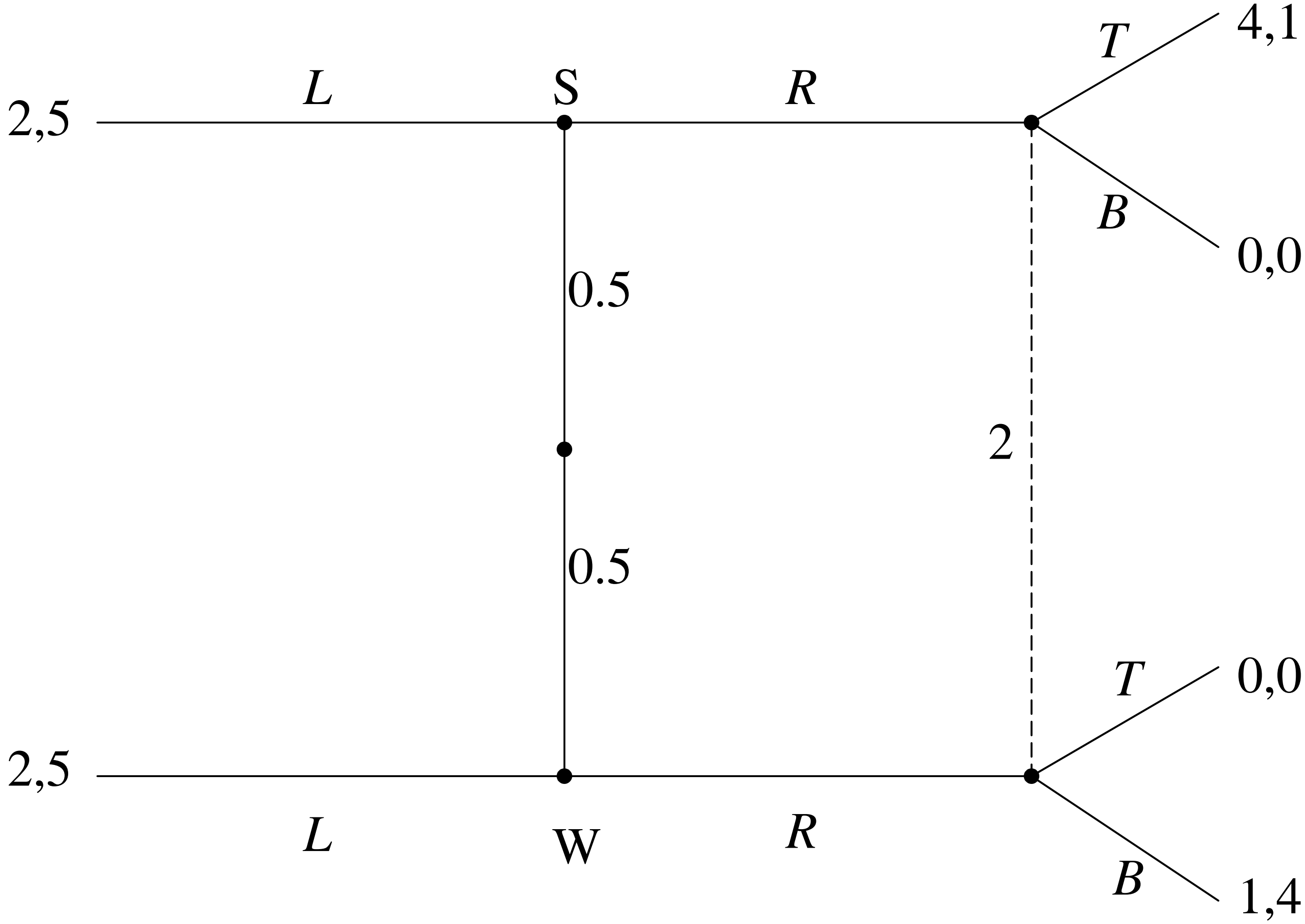
Let $\sigma_i(\theta_i)$ be a behavioral statuary of player i of type θ_i and $\mu_{-i}(h)$ be a probability measure over Θ_i.

(σ, μ) is sequentially rational if for every $h \in H \backslash Z$, $i \in P(h)$ and $\theta_i \in \Theta_i$

$$O(\sigma, \mu_{-i}\,|h) \succsim_i O((\sigma_i', \sigma_{-i}), \mu_{-i}\,|h)\ \forall \sigma_i'$$

(σ, μ) is PB-consistent if for each $i \in N$ $\mu_{-i}(\emptyset) = p_i$ (correct initial beliefs) and μ_{-i} is derived from p_i and $a_i \in A(h)$ via Bayes' rule (action-determined beliefs) when possible.

(σ, μ) is a perfect Bayesian equilibrium (PBE) if it is sequentially rational and PB-consistent.


2,5
L
S
R
T
4,1
B
0,0
0.5
2
0.5
T
0,0
2,5
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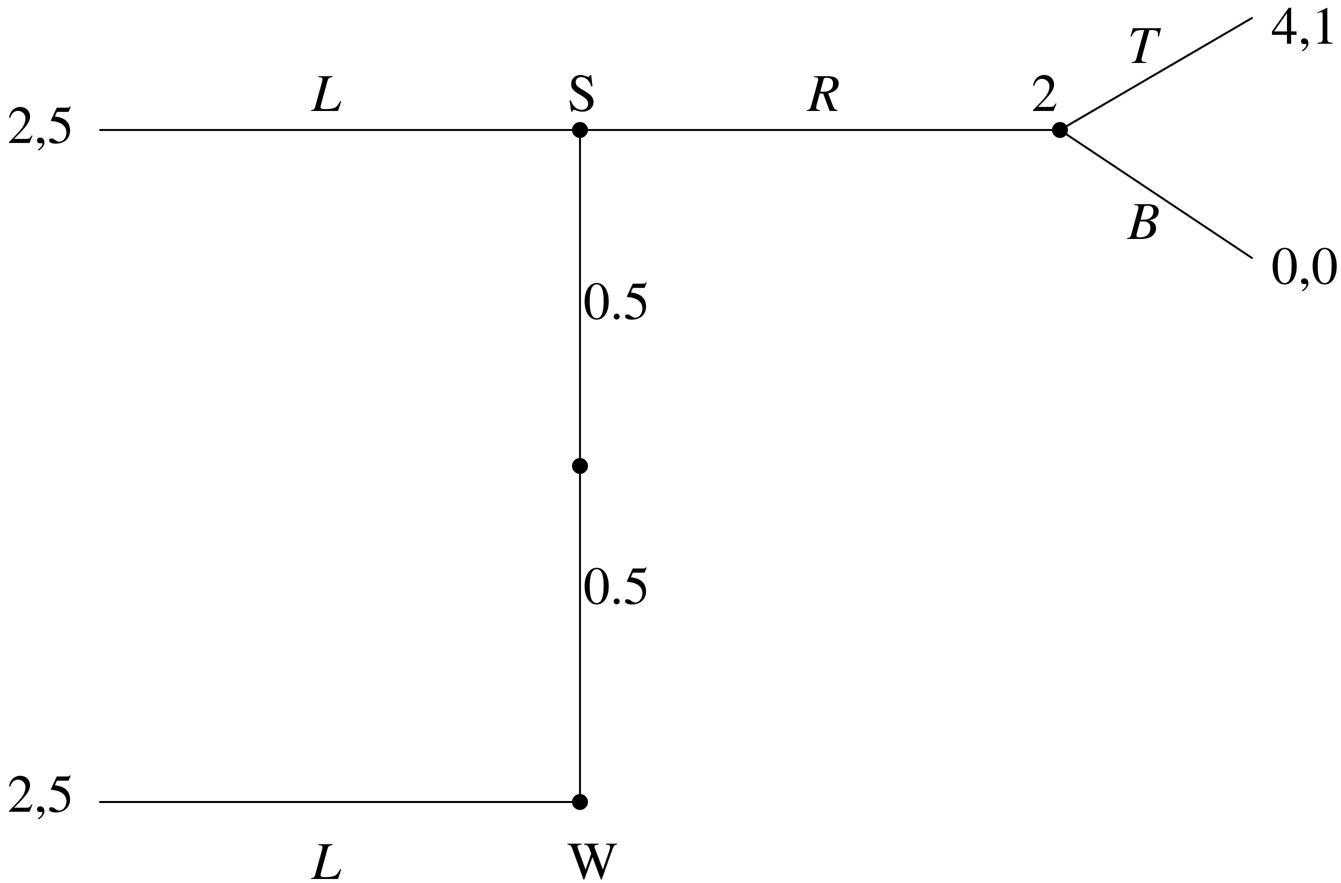
W
R
B
1,4



4,1
T
2,5
L
S
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2
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0,0
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2,5
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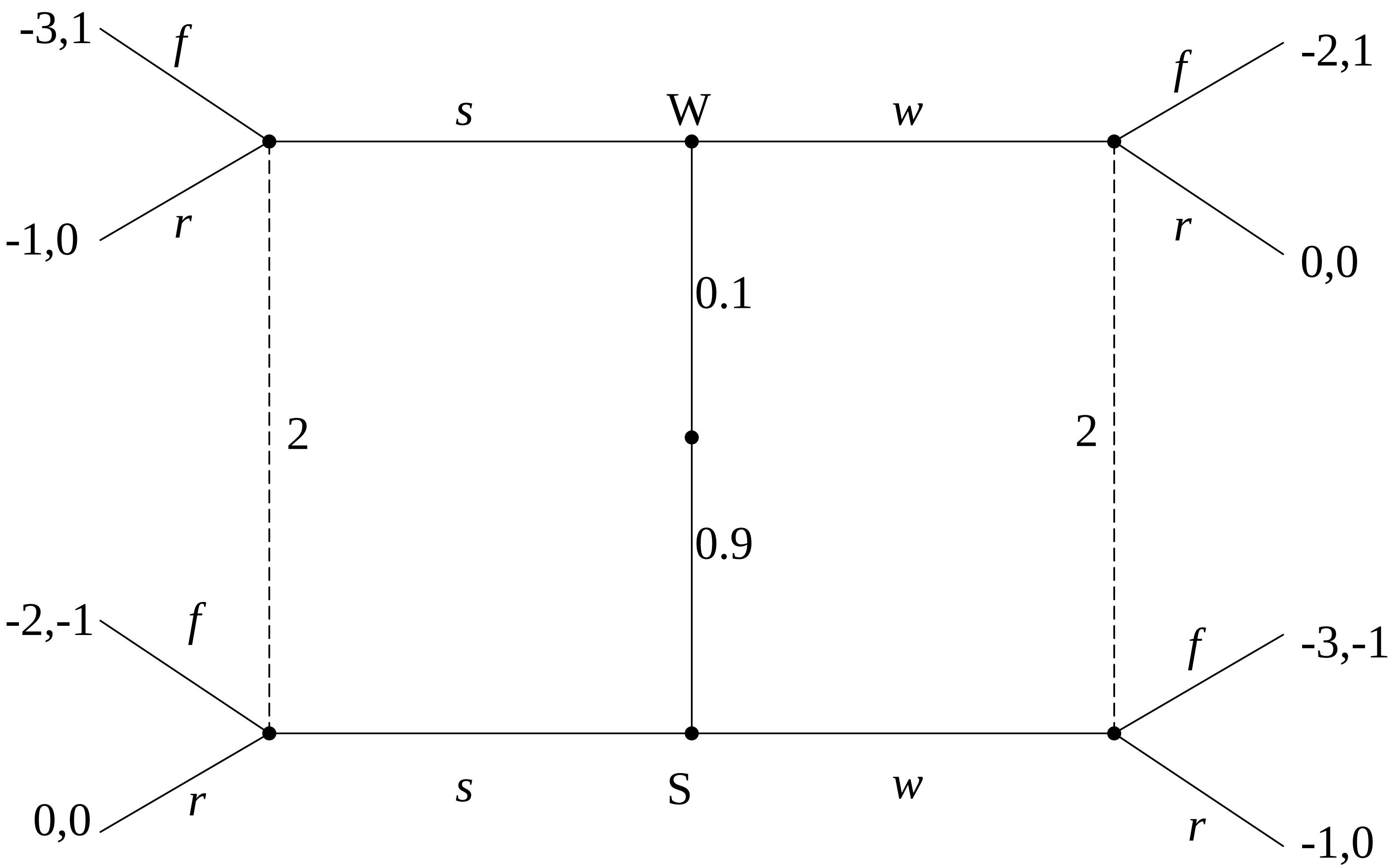
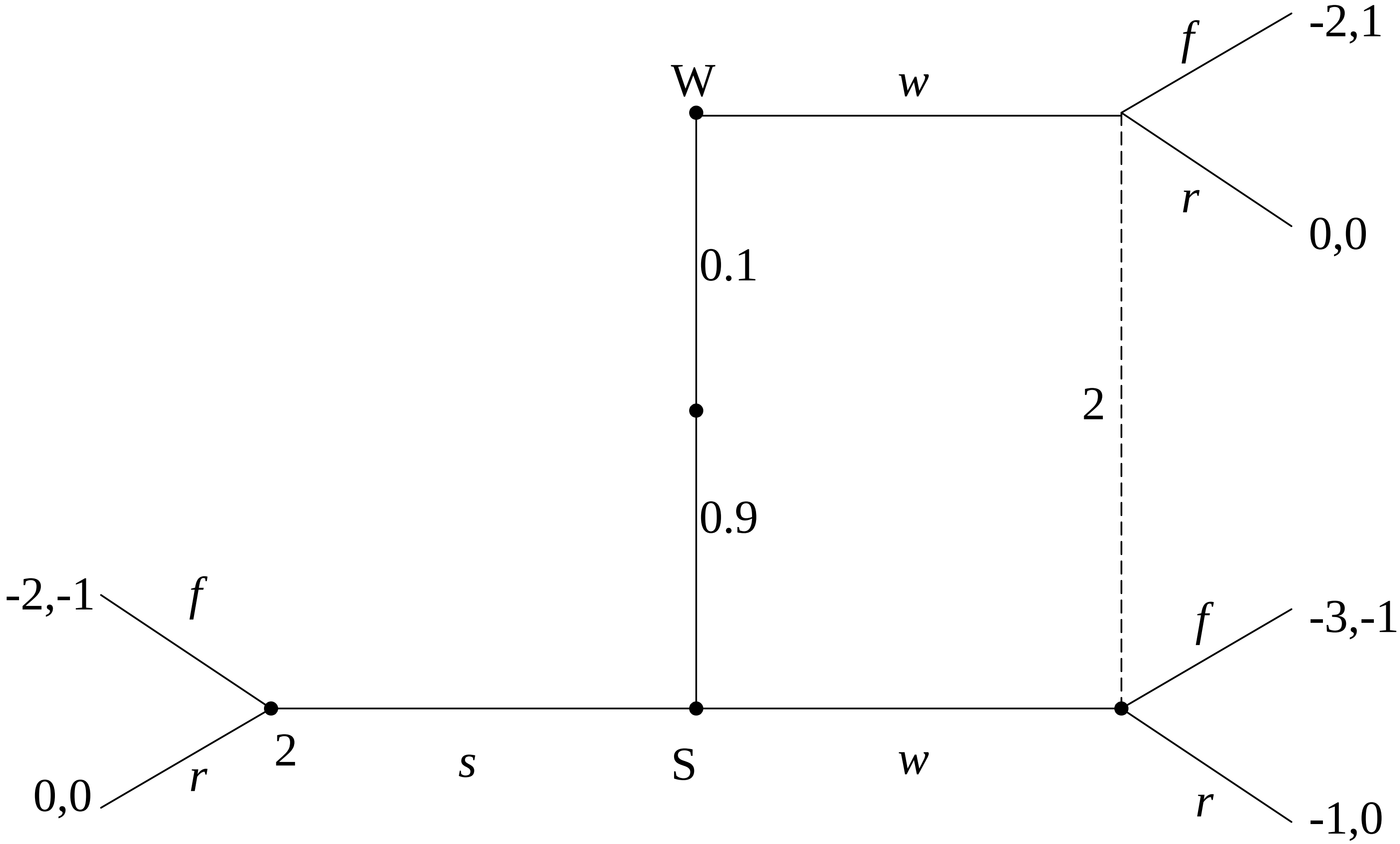
Beer-Quiche





W
w
f
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r
0,0
0.1
2
0.9
-2,-1
f
f
-3,-1
2
s
S
w
r
r
0,0
-1,0