Economics 209A
Theory and Application of Non-Cooperative Games
(Fall 2013)

Extensive games with imperfect information
OR 11 and 12, FT 8

Imperfect information

An extensive game with imperfect information

$$\Gamma = \langle N, H, P, f_c, (\mathcal{I}_i)_{i \in N}, (\succsim_i) \rangle$$

consists of

- a probability measure $f_c(\cdot\,|h)$ on $A(h)$ for all h such that $P(h) = c$ (chance determines the action taken after the history h), and

- an information partition $\mathcal{I}_i$ of $\{h \in H : P(h) = i\}$ for every $i \in N$ such that

$$A(h) = A(h')$$

whenever $h, h' \in I_i$ (an information set).

Perfect and imperfect recall

Let $X_i(h)$ be player i's experience along the history h:

- all I_i encountered,
- actions $a_i \in A(I_i)$ taken at them, and
- the order that these events occur.

An extensive game with imperfect information has perfect recall if for each $i \in N$

$$X_i(h) = X_i(h')$$

whenever $h, h' \in I_i$.

Pure, mixed and behavioral strategies

In an extensive game $\langle N, H, P, f_c, (\mathcal{I}_i)_{i \in N}, (\succsim_i) \rangle$, for player $i \in N$

- a pure strategy assigns and action $a_i \in A(I_i)$ to each information set $I_i \in \mathcal{I}_i$,
- a mixed strategy is a probability measure over the set of pure strategies, and
- a behavioral strategy is a collection of independent probability measures $(\beta_i(I_i))_{I_i \in \mathcal{I}_i}$.

For any $\sigma = (\sigma_i)_{i \in N}$ (mixed or behavioral) an outcome $O(\sigma)$ is a probability distribution over z that results from σ.

Outcome-equivalent strategies

Two strategies (mixed or behavioral) of player i, σ_i and σ_i', are outcome equivalent if

$$O(\sigma_i, s_{-i}) = O(\sigma_i', s_{-i})$$

for every collection s_{-i} of pure strategies.

In any finite game with perfect recall, any mixed strategy of a player has an outcome-equivalent behavioral strategy (the converse is true for a set of games that includes all those with perfect recall).

Strategies and beliefs

- Under imperfect information, an equilibrium should specify actions and beliefs about the history that occurred (an assessment).

- An assessment thus consists of a profile of behavioral strategies and a belief system (a probability measure for each information set).

- An assessment is sequentially rational if for each information set, the strategy is a best response given the beliefs.

Consistency of the players' beliefs:

(i) derived from strategies using Bayes' rule

(ii) derived from some <u>alternative</u> strategy profile using Bayes' rule at information sets that need not be reached

(iii) all players share the same beliefs about the cause of any unexpected event.

Sequential equilibrium

An assessment (β, μ) is sequentially rational if for each $i \in N$ and every $I_i \in \mathcal{I}_i$

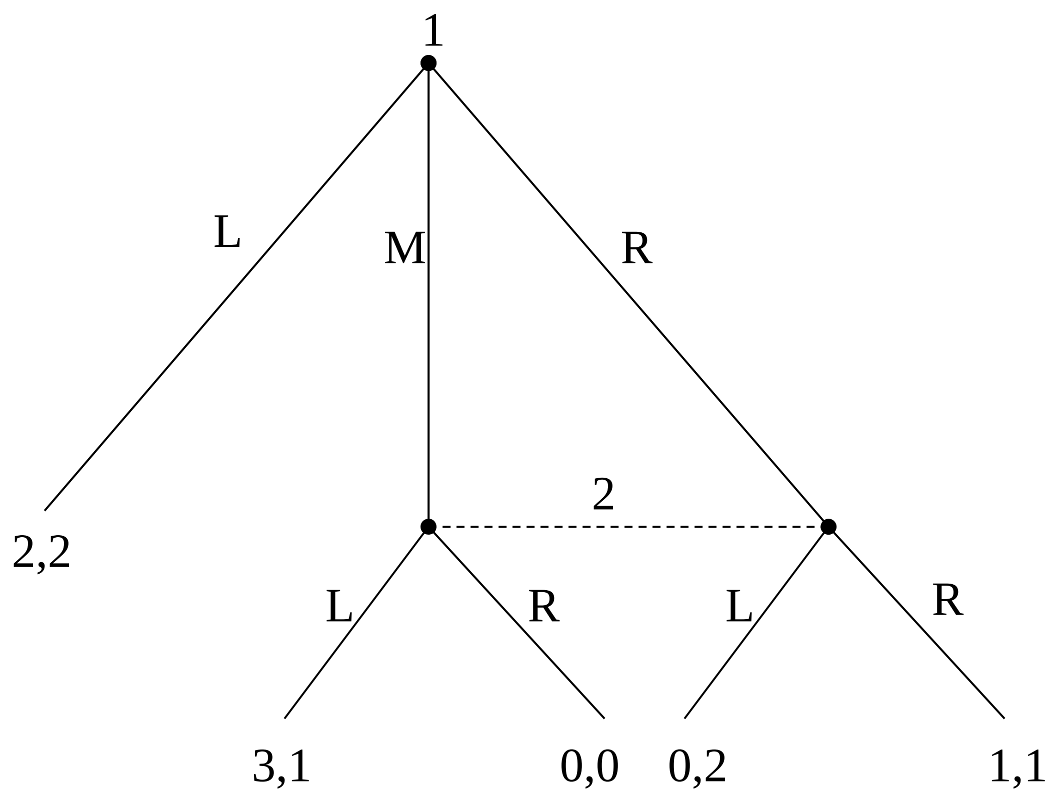
$$O(\beta, \mu \,|\, I_i) \succsim_i O((\beta_i', \beta_{-i}), \mu \,|\, I_i) \text{ for all } \beta_i'.$$

(β, μ) is consistent if there is a sequence $((\beta^n, \mu^n))_{n=1}^{\infty} \to (\beta, \mu)$ such that for each n:

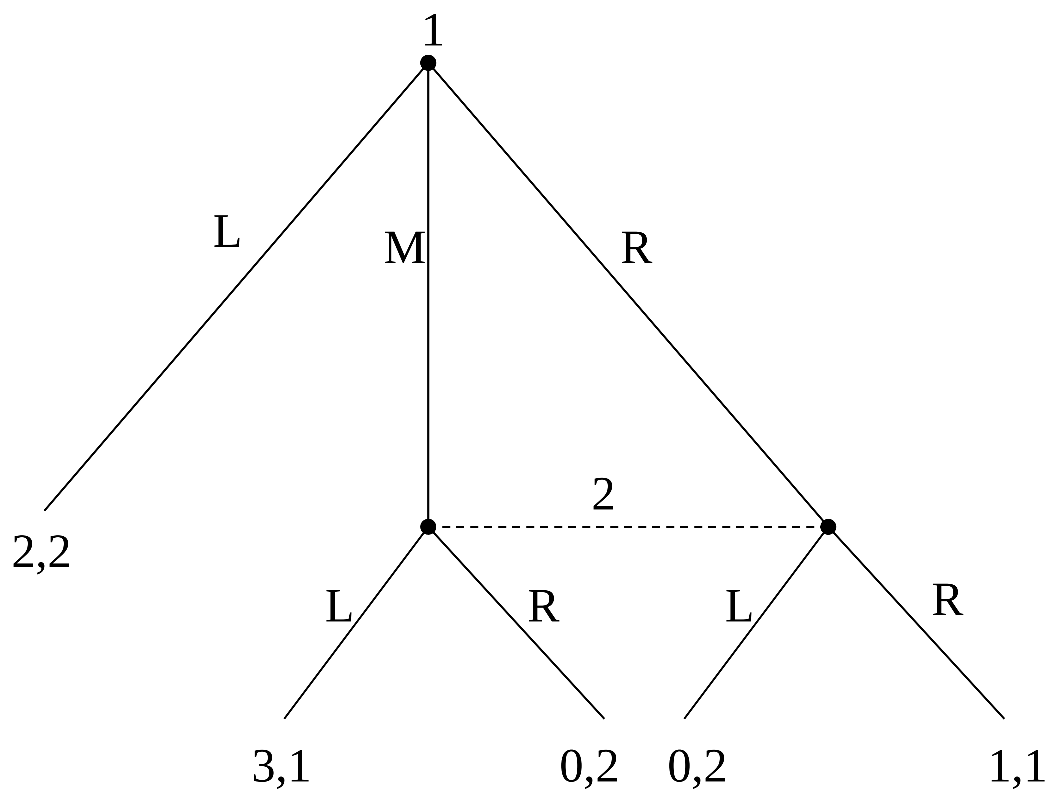
– β^n is completely (strictly) mixed and μ^n is derived from β^n using Bayes' rule.

(β, μ) is a sequential equilibrium if it is sequentially rational and consistent (Kreps and Wilson, 1982).

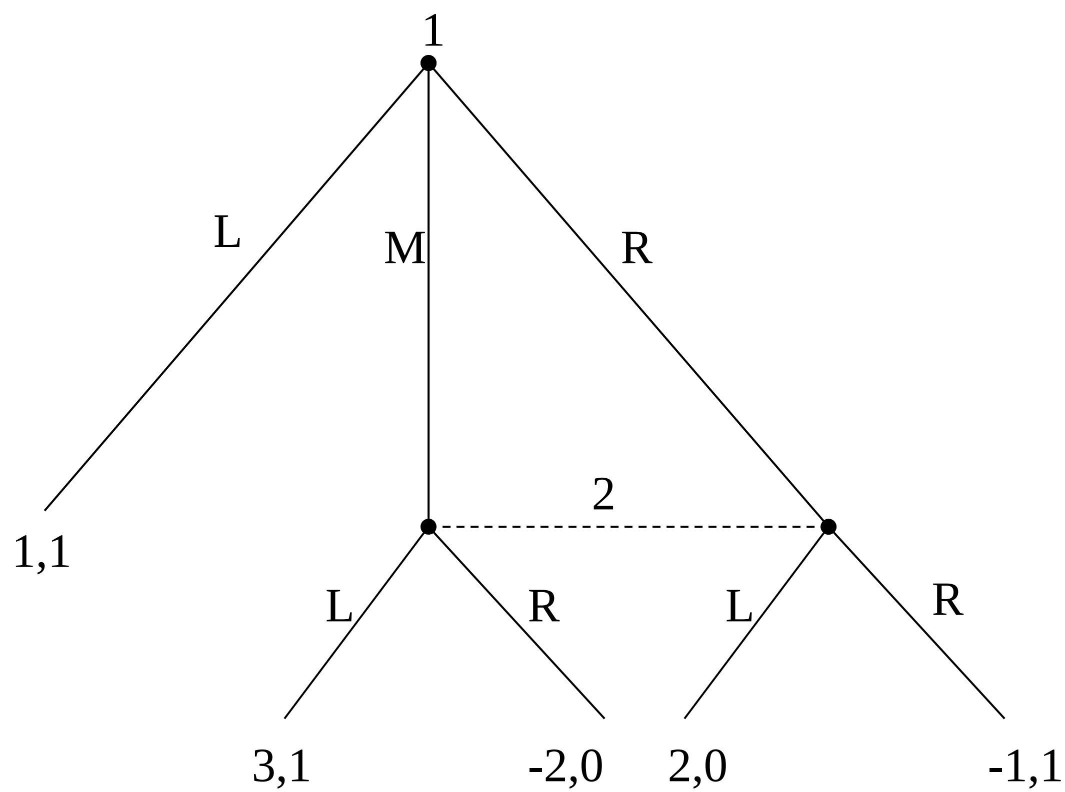
OR 219.1



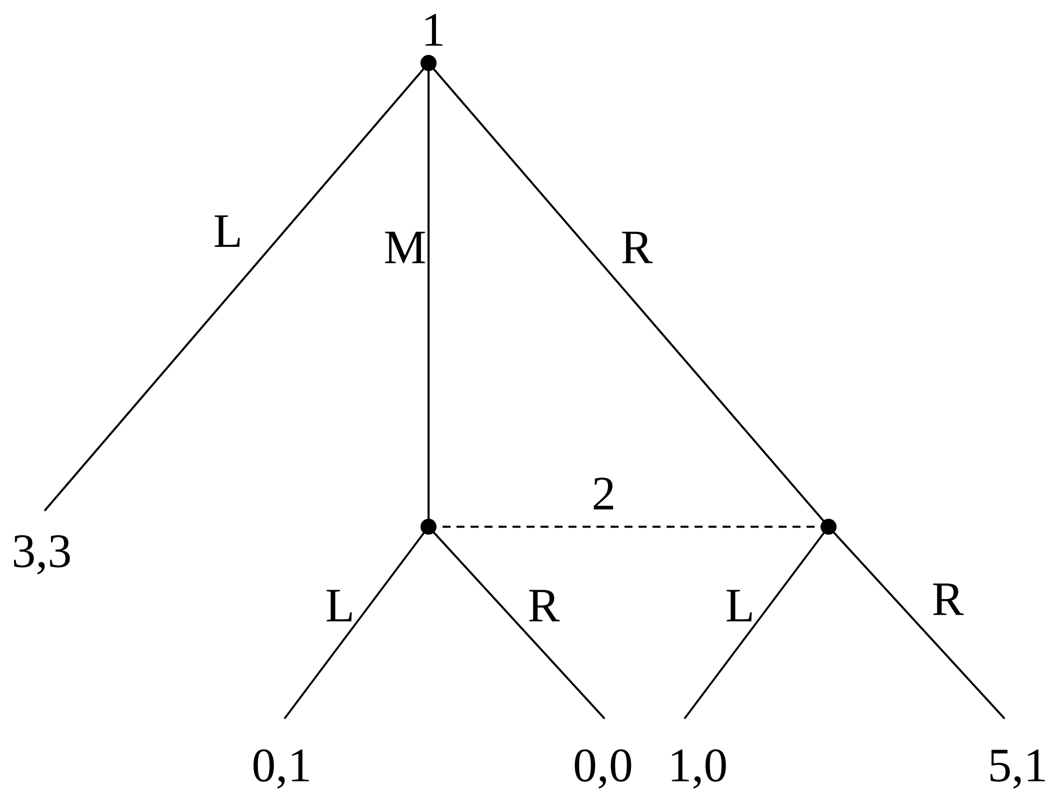
OR 220.1



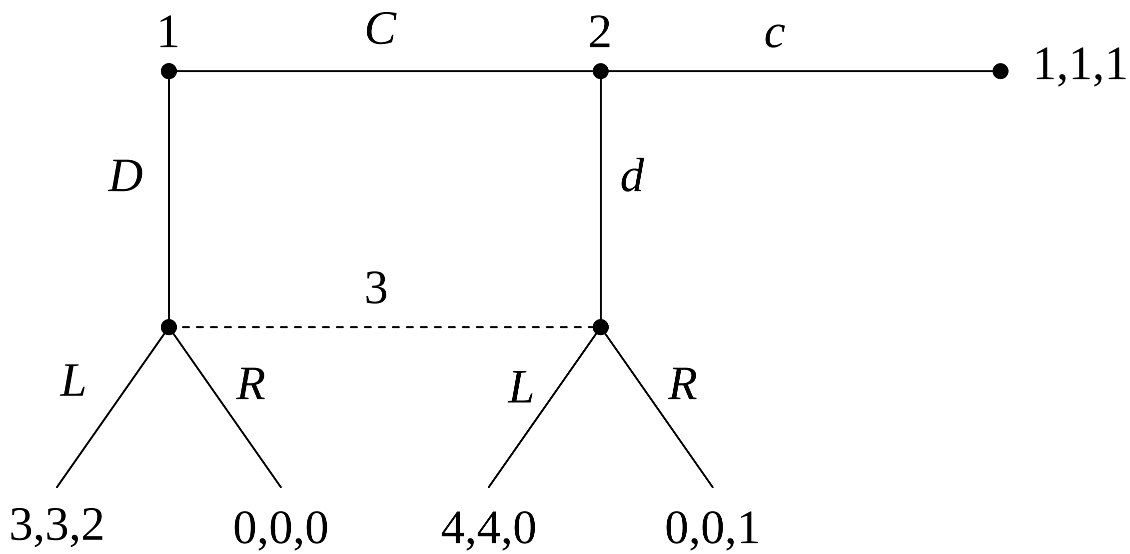
OR 226.1

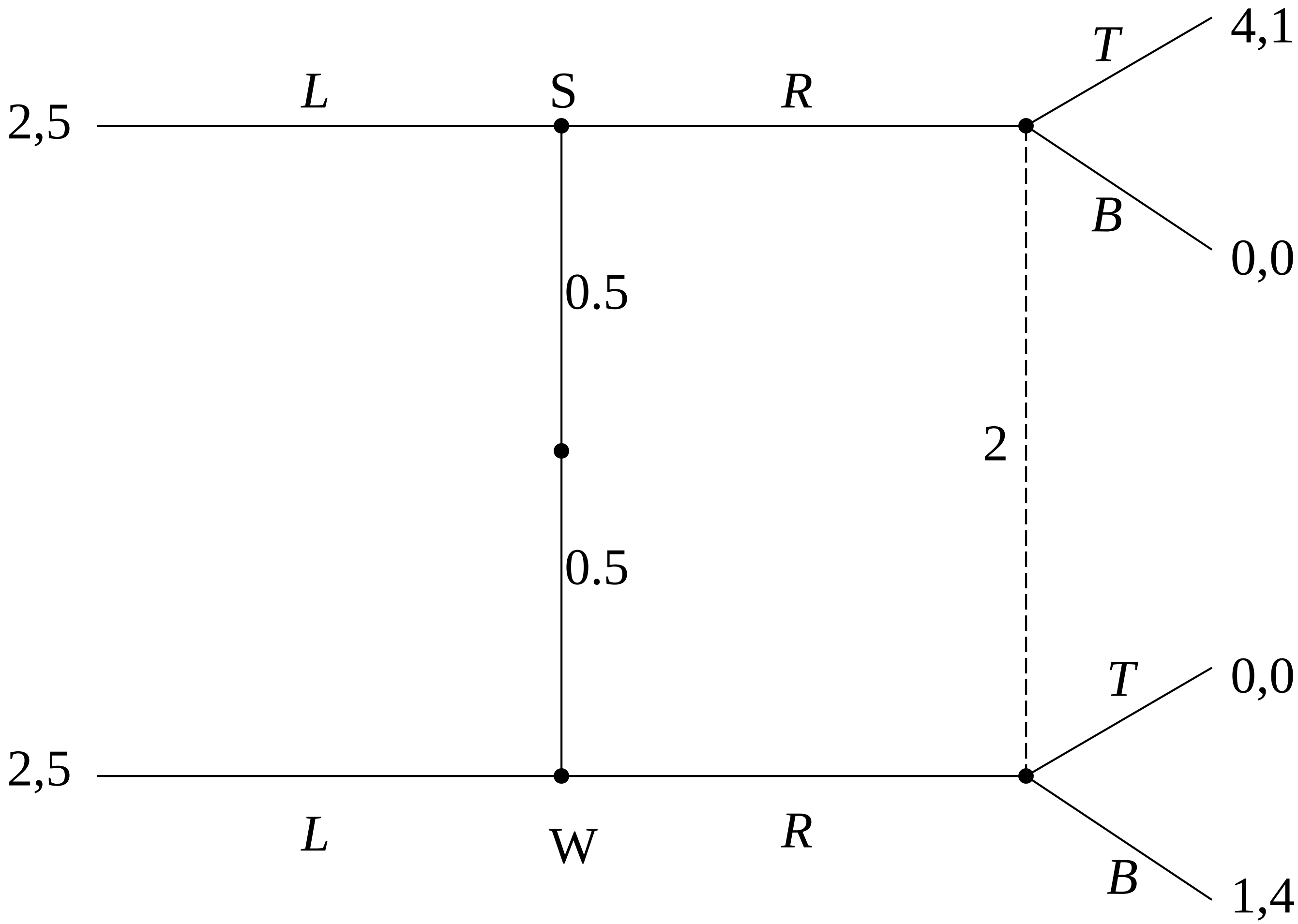


OR 227.1

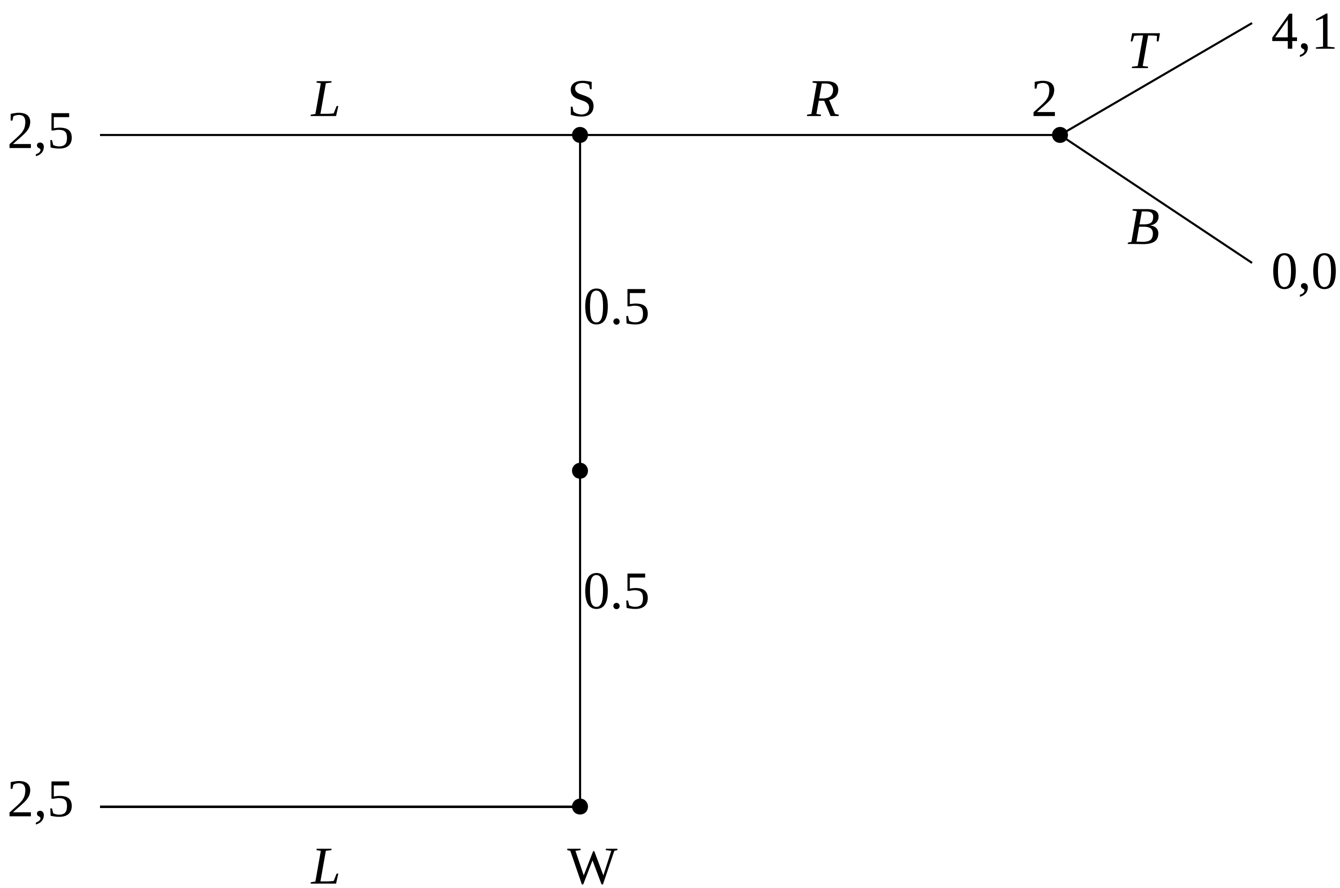


OR 225.1 (Selten's horse)

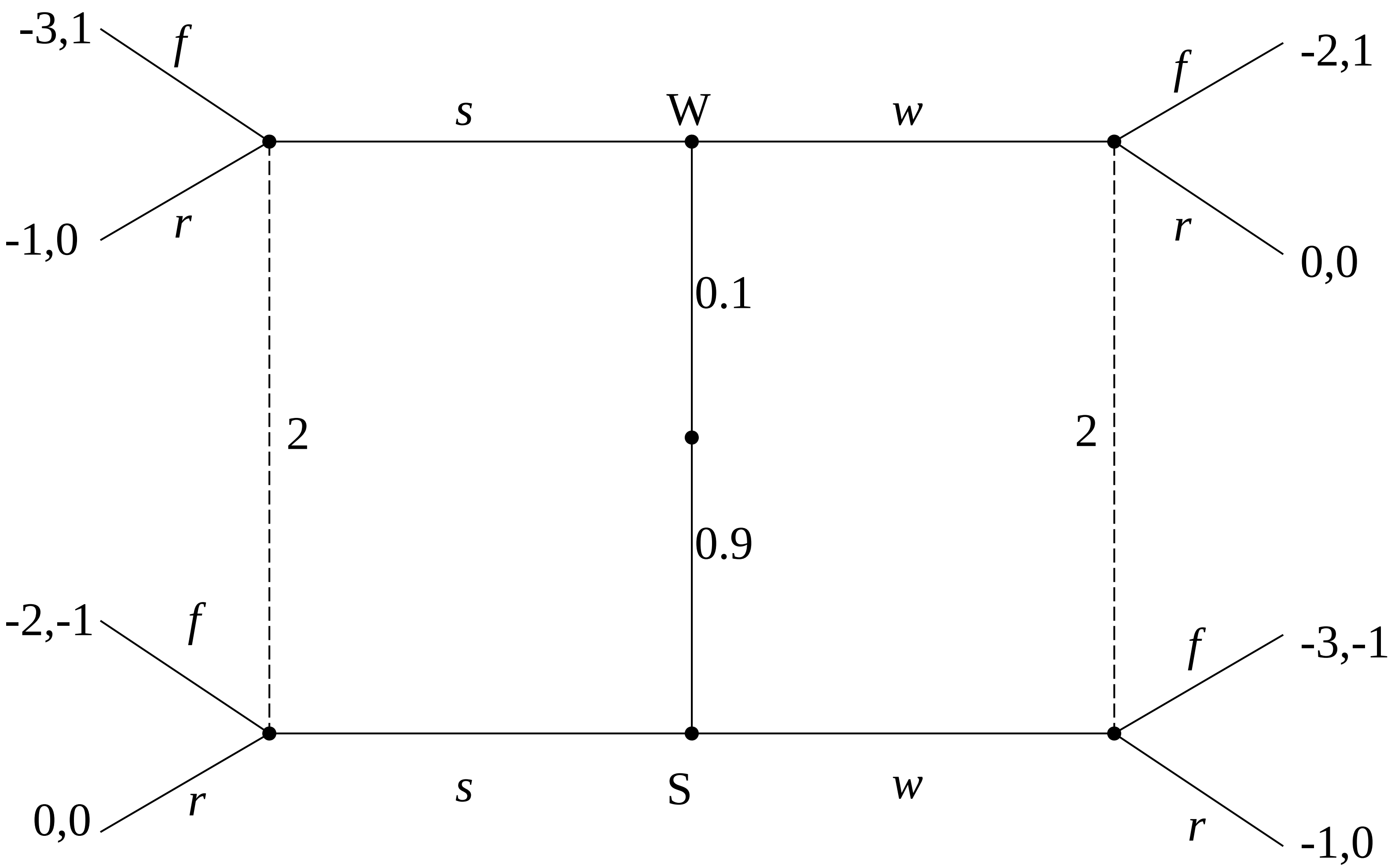


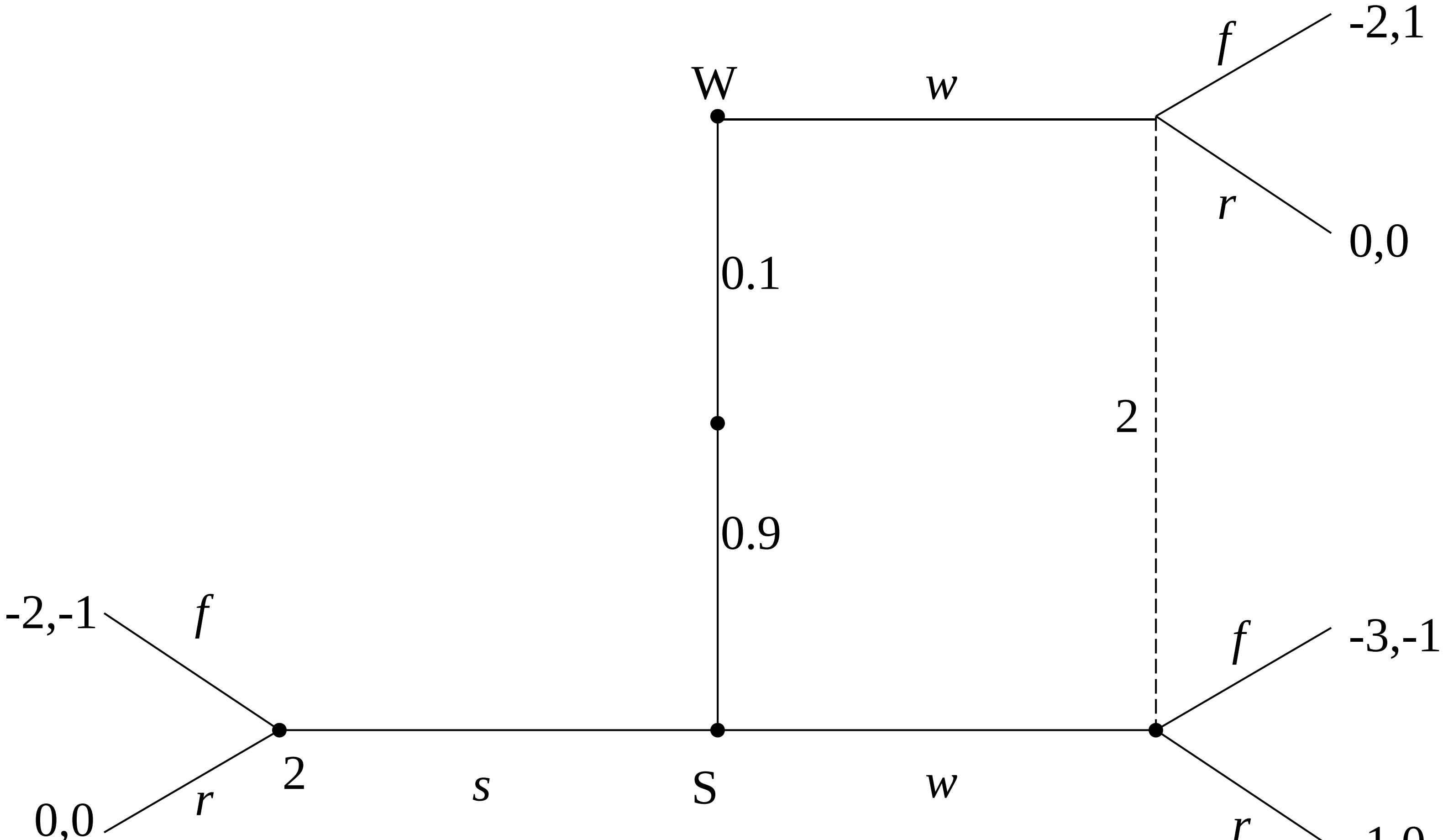


4,1
T
L
S
R
2,5
B
0,0
0.5
2
0.5
T
0,0
2,5
L
W
R
B
1,4


4,1
T
2,5
L
S
R
2
B
0,0
0.5
0.5
2,5
L
W

OR 245.1 (Beer-Quiche)




W
w
f
-2,1
r
0,0
0.1
2
0.9
-2,-1
f
f
-3,-1
2
s
S
w
r
-1,0
0,0
r

Trembling hand perfection

A trembling hand perfect equilibrium (THP) of a finite strategic game is a mixed strategy profile α such that there exists $(\alpha^k)_{k=1}^{\infty}$ of completely mixed strategy profiles such that

- $(\alpha^k)_{k=1}^{\infty}$ converges to α, and
- $\alpha_i \in BR_i(\alpha_{-i}^k)$ for each player i and all k.

A strategy profile α^* in a two-player game is a THP equilibrium iff it is a mixed strategy NE and the strategy of neither player is weakly dominated.

A THP of a finite extensive game is a behavioral strategy profile β that corresponds to a THP of the agent strategic form of the game.

Example (OR 248.1)

	A	B	C
A	0, 0	0, 0	0, 0
B	0, 0	1, 1	2, 0
C	0, 0	0, 2	2, 2

The Nash equilibria (A, A) and (C, C) are not trembling hand perfect equilibria.

Example 1 (OR 249.1)

l:

	L	R
T	$1,1,1$	$1,0,1$
B	$1,1,1$	$0,0,1$

r:

	L	R
T	$1,1,0$	$0,0,0$
B	$0,1,0$	$1,0,0$

The Nash equilibrium (B, L, l) is not a trembling hand perfect equilibrium.