Ever Since Allais*

Aluma Dembo, Shachar Kariv, Matthew Polisson, and John K.-H. Quah†

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The Allais critique of expected utility theory (EUT) has led to the development of theories of choice under risk that relax the independence axiom, but which adhere to the conventional axioms of ordering and monotonicity. Unlike many existing laboratory experiments designed to test independence, our experiment systematically tests the entire set of axioms, providing much richer evidence against which EUT can be judged. Our within-subjects analysis is nonparametric, using only information about revealed preference relations in the individual-level data. For most subjects we find that departures from independence are statistically significant but minor relative to departures from ordering and/or monotonicity.

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†Dembo: Interdisciplinary Center Herzliya (aluma.dembo@idc.ac.il); Kariv: University of California, Berkeley (kariv@berkeley.edu); Polisson: University of Bristol (matthew.polisson@bristol.ac.uk); Quah: Johns Hopkins University and National University of Singapore (john.quah@jhu.edu).

> "I can tell you of an important new result I got recently. I have what I suppose to be a completely general treatment of the revealed preference problem, which will give a fresh setting for the related work of Samuelson-Houthakker-Uzawa. Calculus methods are unavailable. The methods are set-theoretic or algebraical."
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> — A letter from Sydney Afriat to Oskar Morgenstern, 1964

1 Introduction

Canonical decision-theoretic models of choice under risk consider a decision-maker who has a *complete* and *transitive* preference relation over the set of lotteries (probability measures) on a set of consequences (outcomes). By Debreu's (1954, 1960) theorem, any *continuous* preference relation can be represented by a continuous utility function, but any such continuous utility representation is admissible. For the utility function to have an expected utility representation, the preference relation must also satisfy the familiar von Neumann and Morgenstern (1947) *independence* axiom.

Expected utility theory (EUT) lies at the very heart of economics, and so it is natural that experimentalists would want to empirically test the axioms which characterize the EUT model. Empirical violations of these axioms generate intriguing questions about the rationality of individual behavior, and specifically raise criticisms of the independence axiom and its status as the touchstone for rational decision-making in the context of risk. In response to these criticisms, various generalizations of EUT have been formulated, and the experimental scrutiny of these theories has led to new empirical regularities in the laboratory.

Considerable effort has been put towards developing alternatives to EUT. Almost all of these models embody ordering (completeness and transitivity) and generalize EUT by weakening the independence axiom, while generally staying within the class of utility functions that are monotone (in other words, increasing) with respect to first-order stochastic dominance (FOSD); this is true, for example, of weighted expected utility (Dekel, 1986; Chew, 1989), rank-dependent utility (Quiggin, 1982, 1993), cumulative prospect theory (Tversky and Kahneman, 1992), and (under certain restrictions) reference-dependent risk preferences

(Kőszegi and Rabin, 2007).[1,2] The accompanying experimental investigations for the most part use pairwise choices, à la Allais, to test EUT and its generalizations, presuming that subjects have well-defined preferences.

Given that EUT is part of the core of economics—and not something that one can or should abandon lightly—we wish to provide a comprehensive assessment of all the axioms on which EUT is based, and not just the independence axiom. Our overall objective is to provide a better, positive account of choice behavior under risk by evaluating the performance of EUT (as well as non-EUT models) in a choice environment where all axioms underpinning these model(s) can be evaluated. Our experiment and analysis draw upon our prior work (in particular, Choi *et al.* (2007a,b) and Polisson, Quah, and Renou (2020)). Importantly, in the experiment reported here, subjects choose—through a "point-and-click" design—an allocation of contingent commodities from three-dimensional budget sets instead of two-dimensional budget lines.

The experiment involving three states and three associated securities has a number of important advantages in testing rationality over earlier experiments involving two states and two associated securities (as collected by Choi *et al.* (2007a), Choi *et al.* (2014), and Halevy, Persitz, and Zrill (2018), and analyzed by Polisson, Quah, and Renou (2020), de Clippel and Rozen (2021), and Echenique, Imai, and Saito (2021), among others):

- First, a well-known result attributed to Rose (1958) states that with only two goods, the Weak Axiom of Revealed Preference (WARP) and the Generalized Axiom of Revealed Preference (GARP) are observationally equivalent—so choices from two-dimensional budget lines can only violate GARP if they violate WARP, whereas with three-dimensional budget sets, a subject whose choices are pairwise consistent (satisfy WARP) can nonetheless display inconsistency across three or more choices (violate GARP). Thus having three goods allows subjects to display a wider range

[1]For an exception to this rule see, for example, Manzini and Mariotti (2008), where monotonicity can be violated. The original formulation of prospect theory (Kahneman and Tversky, 1979) also allows for violations of monotonicity but, partly for this reason, it was reformulated as cumulative prospect theory (Tversky and Kahneman, 1992) to exclude such behavior. However, for the most part, monotonicity with respect to first-order stochastic dominance is a widely accepted principle in decision theory, as pointed out by Quiggin (1990), Wakker (1993), and Starmer (2000), among others.

[2]In choice acclimating personal equilibrium (Kőszegi and Rabin, 2007), monotonicity holds if the coefficient of loss aversion is within a certain range (see Masatlioglu and Raymond (2016)).

of inconsistent behavior. This is a crucial point, because the experimental design should first provide a rigorous test of consistency with utility maximization before jointly testing additional structural properties on the rationalizing utility function, and specifically those properties which arise under EUT.

- Second, within the context of choices from three-dimensional budget sets, prominent non-EUT models give rise to distinct utility specifications, thereby yielding a set of empirically testable restrictions on observed behavior. These specific differences in functional form are no longer prominent within the context of choices from two-dimensional budget lines. The greater empirical separation among non-EUT models in three-dimensional choice data allows for a more rigorous test of EUT (by testing it against a richer set of alternatives).

- Third, as our power analysis shows, data from three-dimensional budget sets also provide a much stronger test in terms of power—especially of EUT versus non-EUT alternatives—than data from two-dimensional budget lines. For instance, comparing simulated subjects who maximize *any* non-EUT utility function across the two- and three-dimensional experiments shows that the EUT model is significantly more likely to be rejected in three-dimensional data.

Our empirical analysis is in the revealed preference tradition of Afriat (1967, 1973), Diewert (1973), and Varian (1982, 1983a, 1990). Afriat's (1967) theorem tells us that if a finite dataset generated by an individual's choices from linear budget sets satisfies GARP, then the data can be rationalized by a well-behaved (by which we mean a continuous and increasing) utility function. This result provides a practical way of checking whether a dataset is *rationalizable* in this minimal/basic sense. There are also extensions of Afriat's theorem that allow us to test whether a dataset can be rationalized by a utility function with stronger properties. In particular, we could test whether a dataset is *FOSD-rationalizable*, in the sense that it is consistent with the maximization of a utility function that is monotone with respect to FOSD, and whether a dataset is *EUT-rationalizable*, in the sense that it is consistent with the maximization of an expected utility function.

For datasets that do not satisfy GARP exactly, Afriat (1973) introduces the notion of the Critical Cost Efficiency Index (CCEI), which measures the extent to which budget sets need to be reduced in order to rationalize the data. The CCEI, denoted by e^*, is bounded between 0 and 1; the closer it is to 1, the smaller are the budgetary adjustments required for rationalizability. There are also known procedures to measure the extent to which budget sets need to be adjusted in order for a dataset to be FOSD-rationalizable and EUT-rationalizable. Thus, for any dataset collected from an individual subject's choices, three CCEI-type scores can be calculated: e^* for (basic) rationalizability, e^{**} for FOSD-rationalizability (which can be no greater than e^* since FOSD-rationalizability is the more stringent requirement) and e^{***} for EUT-rationalizability (which can be no greater than e^{**} since EUT-rationalizability is the more stringent requirement).

While other measures of violations of rationalizability are available, we adopt the CCEI since it is straightforward to calculate and interpret and, partly for those reasons, the most commonly used measure in empirical work. The use of the same measure for all three models we consider has the very important advantage that we can decompose violations of EUT and compare the magnitudes of violations of the different axioms from which EUT can be derived. Perfect consistency with EUT implies that $1 = e^* = e^{**} = e^{***}$, whereas perfect consistency with any of the familiar non-EUT alternatives (such as rank-dependent utility) that respect FOSD but not EUT itself implies that $1 = e^* = e^{**} > e^{***}$. Our rich individual-level data also allow us to make statistical comparisons of rationalizability (e^*), FOSD-rationalizability (e^{**}), and EUT-rationalizability (e^{***}) *for each subject*, using a purely nonparametric econometric approach.

Figure 1 depicts the distributions of the e^*, e^{**}, and e^{***} rationalizability scores. The horizontal axis presents score values; the vertical axis indicates the percent of subjects whose score is above each value. Only a small fraction of our subjects are perfectly rationalizable (have no violations of GARP), but none are perfectly FOSD-rationalizable and thus EUT-rationalizable. More importantly, the difference between *perfect* rationalizability and FOSD-rationalizability ($1 - e^{**}$) is much larger at all score values than the difference between FOSD-rationalizability and EUT-rationalizability ($e^{**} - e^{***}$). This difference in differences is statistically significant for nearly all subjects. Violations of EUT thus run deeper than violations of independence, challenging the most prominent non-EUT alternatives.

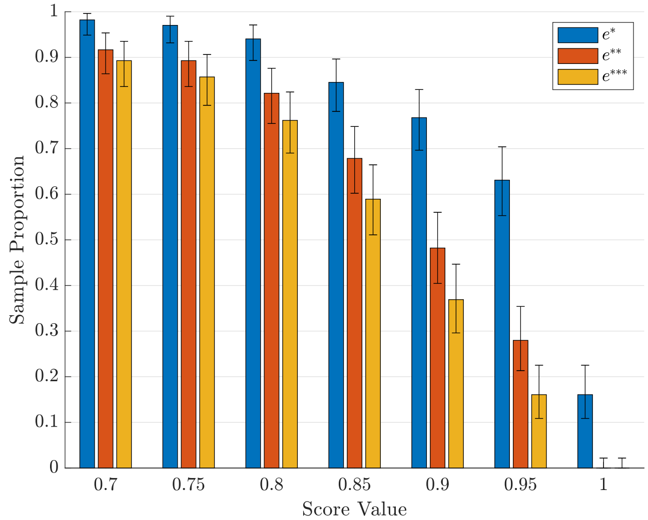


Figure 1: Distributions of Rationalizability Scores

To interpret the bars, consider the score value 0.9. The proportion of subjects in the sample with $e^* \geqslant 0.9$ is 76.8 percent, the proportion with $e^{**} \geqslant 0.9$ is 48.2 percent, and the proportion with $e^{***} \geqslant 0.9$ is 36.9 percent. The braces represent exact 95 percent confidence intervals on the proportions.

The emphasis in our paper is to provide a *comprehensive* and *nonparametric* test of complete representations of preferences under risk rather than focusing on individual axioms. Our main result—that violations of EUT are relatively minor after accounting for violations of ordering and monotonicity—is what Quiggin (1982) calls an "undesirable result" as ordering and monotonicity are more fundamental principles than the standard independence axiom, and they are embodied in the most prominent non-EUT theories of choice under risk. As Starmer (2000) notes, economists have taken the view that the independence axiom needs to be weakened on the grounds of predictive validity and psychological realism, but have generally left ordering and monotonicity unchallenged.

Our rich individual-level experimental data involving three states and three associated securities could also be used, in principle, to test each non-EUT theory against the others—in particular because the different (weaker) alternatives deliver more empirically testable restrictions on observed behavior in the case of three states than in the case of two states. However, for most subjects there is only a small (or no) difference between FOSD-rationalizability (e^{**}) and EUT-rationalizability (e^{***}), which implies that there is little scope for existing non-EUT alternatives to explain observed behavior.

The rest of the paper is organized as follows. The next section provides more background and motivation. Section 3 describes our tests of rationalizability, experimental procedures, and the power of the experiment. Section 4 summarizes the experimental results. Section 5 describes how the paper is related to the literature, focusing on recent revealed preference papers on choice under risk. Section 6 outlines what we think theorists, experimentalists, and other economists should take away from the paper. In the interests of brevity, all technical details that are not essential for understanding the results are relegated to appendices.

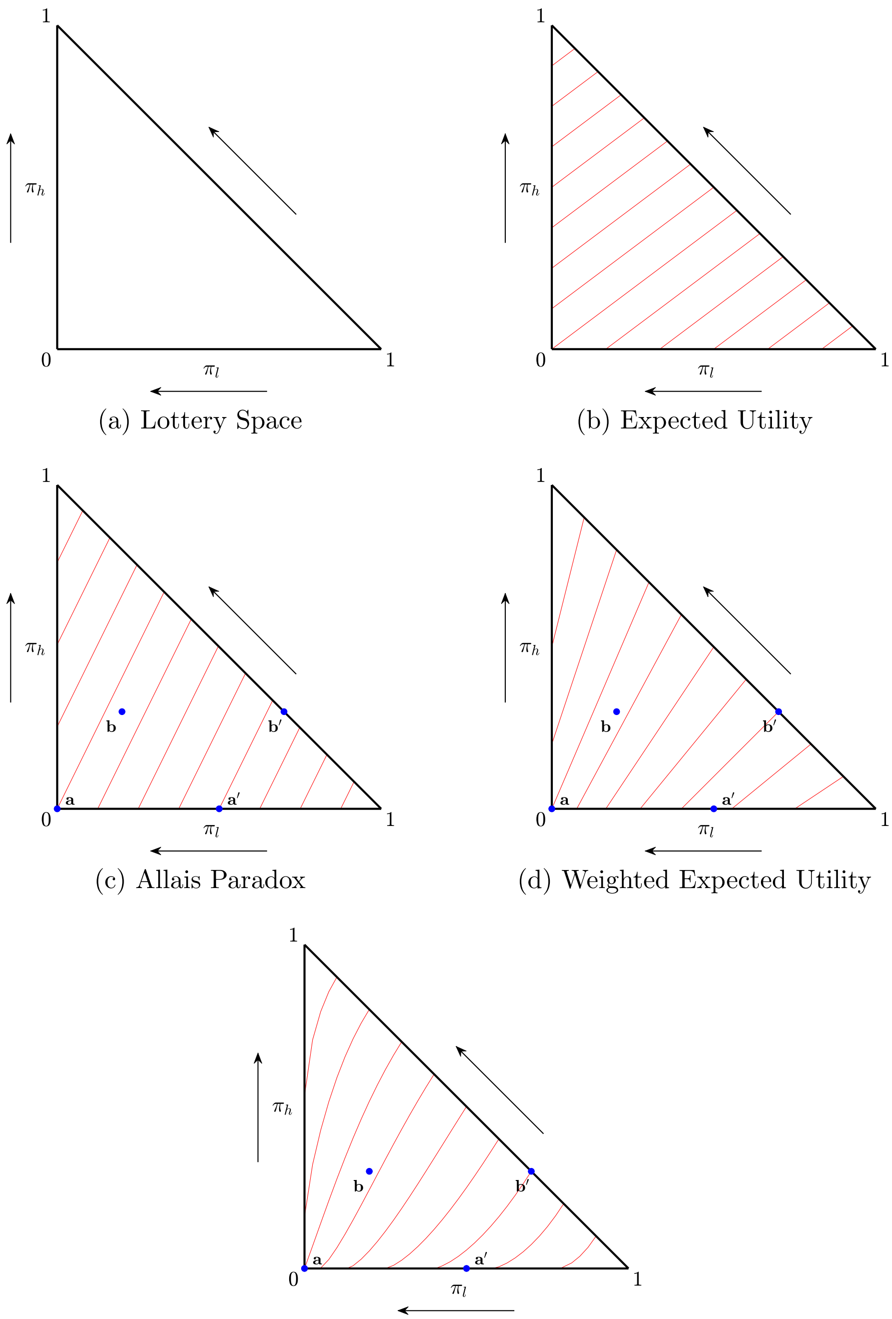
2 Background and Motivation

Much of the experimental literature on choice under risk is directed towards finding violations of EUT. To understand the role of each of the axioms on which EUT is based, suppose that there are three mutually non-indifferent outcomes $x_h \succ x_m \succ x_l$ and consider the probability triangle depicted in Figure 2. Each point in the triangle represents a lottery (π_h, π_m, π_l) over the outcomes (x_h, x_m, x_l), where $\pi_h = 0$ on the horizontal edge, $\pi_m = 0$ on the hypotenuse (because $\pi_h + \pi_l = 1$), and $\pi_l = 0$ on the vertical edge.[3]

Monotonicity with respect to FOSD implies that preferences are increasing from right to left along horizontal lines, from bottom to top along vertical lines, and from bottom-right to top-left along lines parallel to the hypotenuse (Figure 2a). Ordering (completeness and transitivity) plus continuity imply that there exists a map of (non-intersecting) indifference curves. Assuming that these axioms hold, independence then implies that preferences admit an expected utility representation, so that the indifference curves in the triangle are parallel straight lines (Figure 2b). Viewed within the context of the triangle, independence is a strong requirement, leaving only the slope of the indifference lines undetermined (steeper lines imply higher risk aversion).

An example of the famous Allais (1953) paradox can be illustrated by a pair of binary choices—between lotteries $\mathbf{a}$ and $\mathbf{b}$ and between lotteries $\mathbf{a}'$ and $\mathbf{b}'$ (Figure 2c). The imaginary straight lines connecting lotteries $\mathbf{a}$ and $\mathbf{b}$ and lotteries $\mathbf{a}'$ and $\mathbf{b}'$ are parallel to each other and flatter than the indifference curves so $\mathbf{a} \succ \mathbf{b}$ and $\mathbf{a}' \succ \mathbf{b}'$. But experimental subjects often make choices revealing that $\mathbf{a} \succ \mathbf{b}$ and $\mathbf{b}' \succ \mathbf{a}'$ (or $\mathbf{b} \succ \mathbf{a}$ and $\mathbf{a}' \succ \mathbf{b}'$), which is

[3] The probability triangle was introduced by Marschak (1950) and popularized by Machina (1982) as a way of representing the choice space over lotteries.



(a) Lottery Space (b) Expected Utility

(c) Allais Paradox (d) Weighted Expected Utility

(e) Rank-Dependent Utility

Figure 2: Probability Triangles

The probability triangle depicts the lottery space as a set of probability weights (π_h, π_m, π_l) over three fixed outcomes (x_h, x_m, x_l). (a) Ordering (completeness and transitivity) plus continuity guarantee non-intersecting indifference curves; monotonicity (with respect to FOSD) guarantees that preferences are increasing as shown (see arrows). (b) Adding independence gives rise to EUT, characterized by indifference curves that are parallel straight lines. (c) The Allais paradox arises because EUT requires $\mathbf{a} \succ \mathbf{b}$ and $\mathbf{a'} \succ \mathbf{b'}$, but experimental subjects often make choices revealing that $\mathbf{a} \succ \mathbf{b}$ but $\mathbf{b'} \succ \mathbf{a'}$. Alternatives to EUT like (d) weighted expected utility and (e) rank-dependent utility often avoid the Allais paradox by relaxing independence while adhering to ordering and monotonicity.

commonly taken as evidence against independence. This persistent finding has led to a large literature with the objective of developing new models of choice under risk that weaken the independence axiom.[4]

In weighted expected utility (Dekel, 1986; Chew, 1989), for example, all indifference curves are again straight lines but they typically "fan out"—that is, they become steeper (corresponding to higher risk aversion) when moving northwest in the triangle (Figure 2d).[5] Or in rank-dependent utility (Quiggin, 1982, 1993) and prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) the indifference curves are not straight lines and they can "fan out" or "fan in", especially near the triangle boundaries (Figure 2e). Each of the conventional alternatives to EUT gives rise to indifference curves with distinctive shapes in the triangle, but with the common feature that they avoid the Allais paradox.

In many experimental studies, the main criterion used to evaluate a given theory is the fraction of choices that it correctly predicts. A few studies have also estimated parametric utility functions for individual subjects. Generally speaking, these experiments involve collecting a small number of decisions from each subject, with the decisions involving very specific choices that are narrowly tailored to discover violations of independence and its various generalizations. There is less emphasis on ensuring that these decision problems are representative, both in the statistical sense and in the economic sense. As a result, the accumulated experimental evidence against independence that has prompted theorists to develop formal alternatives to EUT consists primarily of Allais-type behaviors—choices inconsistent with linear indifference curves in the probability triangle. Such an approach is unsurprising, given the focus on the independence axiom and that, apart from a few notable exceptions,[6] non-EUT models have relaxed the independence axiom while maintaining ordering and monotonicity with respect to FOSD. However, our basic contention is that we ought to have a wider view of the performance (or underperformance) of EUT and therefore

[4]Interestingly, while violations of the independence axiom appear to be widespread, in a recent survey on the experimental robustness of the Allais paradox across 83 experiments and 30 studies, Blavatskyy, Ortmann, and Panchenko (2021) concludes that the Allais paradox is a somewhat fragile empirical finding. This survey's conclusion is compatible with our main message.

[5]The indifference curves corresponding to loss/disappointment aversion (Gul, 1991) are also straight lines but "fan in" for lotteries better than x_m (top part of the triangle) and "fan out" for lotteries worse than x_m (bottom part of the triangle). See Gul (1991), Figure 2 (p. 679).

[6]For generalizations of EUT that allow for nontransitivity, see, for example, Bell (1982), Fishburn (1982), and Loomes and Sugden (1982).

that all of the assumptions which underpin the model deserve closer scrutiny.

In this paper, we develop tests of rationalizability that are *comprehensive*, in the sense that we check whether a given model—taken as a whole—succeeds or fails in explaining the data, rather than focusing on specific individual axioms. Furthermore, by evaluating the performances of progressively restrictive models using a common measure of model performance, we can compare the relative impact of the different axioms which make up EUT. Another important feature of our tests is that they are *nonparametric*, in the sense that we make no auxiliary functional form assumptions on the utility function. The overall objective of our experiment and analysis is to provide a positive account of choice under risk in natural economic environments.

3 Framework for Analysis

In this section, we describe the theory on which the experimental design is based, the design itself, and the power of the experiment. All technical details that are not essential for understanding the experimental results are relegated to appendices.

3.1 Theory

We consider a portfolio choice framework with S states of nature, each state denoted by $s = 1, \ldots, S$. For each state s, there is an Arrow security that pays one in state s and zero in the other state(s). Let $x_s \geqslant 0$ denote the demand for the security that pays off in state s and $p_s > 0$ denote the corresponding price, so that $\mathbf{x} = (x_1, \ldots, x_S)$ is a demand allocation and $\mathbf{p} = (p_1, \ldots, p_S)$ is a price vector. Let $\mathcal{D} := (\mathbf{p}^i, \mathbf{x}^i)$ be the data generated by a subject's choices from linear budget sets, where $\mathbf{p}^i$ denotes the i-th observation of the price vector and $\mathbf{x}^i$ denotes the associated allocation.

3.1.1 Rationalizability

We say that a data set $\mathcal{D}$ is *rationalizable* if there is a utility function $U : \mathbb{R}^S_+ \to \mathbb{R}$ such that $U(\mathbf{x}^i) \geqslant U(\mathbf{x})$ for all

$$\mathbf{x} \in \mathcal{B}^i = \{\mathbf{x} \in \mathbb{R}^S_+ : \mathbf{p}^i \cdot \mathbf{x} \leqslant \mathbf{p}^i \cdot \mathbf{x}^i\}.$$

In other words, the utility of $\mathbf{x}^i$ is weakly higher than that of any alternative that is weakly cheaper at the price vector $\mathbf{p}^i$.

Note that rationalizability, as defined, has no empirical content, since any dataset $\mathcal{D}$ can be rationalized by a constant utility function. For this concept to be meaningful, some restriction has to be imposed on U. A well-known result, due to Afriat (1967), tells us that $\mathcal{D}$ can be rationalized by a *well-behaved* (in the sense of being continuous and increasing) utility function if and only if the data satisfy the Generalized Axiom of Revealed Preference (GARP). GARP is an intuitive and (more importantly from the perspective of empirical application) easy-to-check condition on $\mathcal{D}$.

> Note 1 The Weak Axiom of Revealed Preference (WARP) is a weakening of GARP since it only involves *pairwise* comparisons. As shown by Rose (1958), with only two goods, WARP and GARP are observationally equivalent—in the sense that a violation of one axiom implies a violation of the other. However, the Rose (1958) equivalence no longer holds in the context of three or more goods, where a subject who displays consistent pairwise rankings could still display inconsistent (cyclic) rankings involving three or more choices. Thus choices from three-dimensional budget sets provide a stronger test of utility maximization than choices from two-dimensional budget lines because they allow for a greater variety of inconsistent behavior. This is a crucial step before jointly testing the additional axioms underpinning EUT.

To account for data that are not exactly rationalizable, Afriat (1972, 1973) proposes the notion of the Critical Cost Efficiency Index (CCEI). Given a number $e \in (0, 1]$, a dataset $\mathcal{D}$ is said to be *rationalizable at cost efficiency* e if there is a well-behaved utility function U such that $U(\mathbf{x}^i) \geqslant U(\mathbf{x})$ for all

$$\mathbf{x} \in \mathcal{B}^i(e) = \{\mathbf{x} \in \mathbb{R}^S_+ : \mathbf{p}^i \cdot \mathbf{x} \leqslant e\, \mathbf{p}^i \cdot \mathbf{x}^i\}.$$

Clearly, approximate rationalizability weakens the notion of rationalizability since $\mathcal{B}^i(e)$ is a subset of $\mathcal{B}^i$. As Afriat (1973) notes, this definition captures the idea that while the consumer "has a definite structure of wants," she "programs at a level of cost-efficiency e." The approach is otherwise agnostic about the deeper nature of the "errors" which may arise in individual choices.

It is not difficult to see that *every* dataset $\mathcal{D}$ could be rationalized by a well-behaved utility function at an efficiency level e for some $e \in (0,1]$ that is sufficiently close to zero. The CCEI, denoted by e^*, of a dataset $\mathcal{D}$ is *the greatest e for which $\mathcal{D}$ is rationalizable.* For example, if $e^* = 0.95$, then we can find U such that $U(\mathbf{x}^i)$ is greater than $U(\mathbf{x})$ for any bundle $\mathbf{x}$ that is more than 5 percent cheaper than $\mathbf{x}^i$ at the prevailing prices $\mathbf{p}^i$. Alternatively, the decision maker is effectively "wasting" as much as 5 percent of his income by making "irrational" choices. Just as GARP characterizes rationalizabilty by a well-behaved utility function, so too is there a modified version of GARP that can be used to check whether a dataset is rationalizable by a well-behaved utility function at some efficiency level e. It follows that one could easily obtain e^*.

Afriat's Theorem is just the first of a long list of results developed by various authors with the following pattern: $\mathcal{D}$ is rationalizable by a well-behaved utility function belonging to some family if and only if $\mathcal{D}$ obeys some property. For our purposes, two families are particularly important.

3.1.2 FOSD-Rationalizability

The first is the family of well-behaved utility functions that are monotone with respect to FOSD. In our framework, the probability of state s is commonly known to be $\pi_s > 0$, so that $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_S)$ is a vector of probability weights with $\pi_1 + \cdots + \pi_S = 1$. Then we say that U is monotone with respect to FOSD if $U(\mathbf{x}'') \geqslant U(\mathbf{x}')$ whenever $\mathbf{x}''$ (considered as a distribution through $\boldsymbol{\pi}$) first-order stochastically dominates $\mathbf{x}'$ (with the inequality being strict if the dominance is strict).[7] It is straightforward to check that, in the case where the states are equiprobable (as in our experiment), a well-behaved utility function is monotone with respect to FOSD if and only if it is symmetric.

A dataset $\mathcal{D}$ is said to be *FOSD-rationalizable* (with respect to a given $\boldsymbol{\pi}$) if it can be rationalized by a utility function that is well-behaved and monotone with respect to FOSD.

[7] A utility function U that is monotone with respect to FOSD is increasing (in the sense that $U(\mathbf{x}'') > U(\mathbf{x}')$ whenever $\mathbf{x}'' > \mathbf{x}'$) but the converse is not true. Suppose that there are just two equiprobable states. Then $U(1,3) > U(2,1)$ if U is monotone with respect to FOSD because $(1,3)$ first-order stochastically dominates $(2,1)$, but no relationship between $U(1,3)$ and $U(2,1)$ is implied by U being increasing.

Relying on Nishimura, Ok, and Quah (2017), we provide an easy-to-implement (necessary and sufficient) test of whether $\mathcal{D}$ is FOSD-rationalizable; furthermore, one could also check whether $\mathcal{D}$ can be rationalized at cost efficiency e by a utility function in this family and thus the corresponding CCEI, denoted by e^{**}, can easily be calculated. Since this family of utility functions is contained within the family of well-behaved utility functions, it must be the case that $e^{**} \leqslant e^{*}$.

3.1.3 EUT-Rationalizability

The second important family is the family of well-behaved utility functions that satisfy expected utility. These are utility functions U taking the form

$$U(\mathbf{x}) = \pi_1 u(x_1) + \cdots + \pi_S u(x_S),$$

where the Bernoulli index $u : \mathbb{R}_+ \to \mathbb{R}$ is continuous and increasing. Recently, Polisson, Quah, and Renou (2020) have developed a procedure called the Generalized Restriction of Infinite Domains (or GRID) method that could be employed to test whether a dataset is rationalizable (at cost efficiency e) by a well-behaved expected utility function, or *EUT-rationalizable*. Using this method, one could also calculate e^{***}, the CCEI corresponding to EUT-rationalizability. Since this family of utility functions is contained within the family of well-behaved utility functions which respect FOSD, it must be the case that $e^{***} \leqslant e^{**}$.

3.1.4 Summary

To recap, given any dataset $\mathcal{D}$ we could calculate three rationalizability scores corresponding to three nested models, with

$$1 \geqslant e^{*} \geqslant e^{**} \geqslant e^{***} > 0.$$

There are, of course, other families of utility functions besides these three, and there will be rationalizability scores corresponding to those families as well. In particular, specific families of utility functions (such as rank-dependent utility) which generalize expected utility and respect FOSD will *necessarily* have rationalizability scores between e^{**} and e^{***}.

The great advantage of measuring—on the same scale—a dataset's consistency with three increasingly stringent models is that it allows us to determine the *source* of the departure from EUT. A subject who is perfectly EUT-rationalizable will have $1 = e^{*} = e^{**} = e^{***}$. More generally, e^{***} will be strictly less than one, and the corresponding values of e^{*} and e^{**} will then allow us to say something about why that has occurred. For example, if $1 = e^{*} = e^{**} > e^{***}$, then it would be plausible to believe that the subject is indeed violating the independence axiom and her behavior could potentially be explained by a utility model that relaxes the independence axiom, while retaining monotonicity with respect to FOSD. On the other hand, a subject for whom $1 = e^{*} > e^{**} = e^{***}$ could be utility-maximizing, but her choices could only be explained by a model that departs from monotonicity with respect to FOSD. Last but not least, the choice behavior of a subject with $1 > e^{*}$ is not consistent with the maximization *any* utility function; she may or may not also be violating the independence axiom, but understanding her behavior would require a more radical departure from the classical framework.

In an appendix, we provide more details on GARP and the other conditions for checking rationalizability (or rationalizability at a given cost efficiency) with respect to specific families of utility functions.

3.2 Experiment

In this paper, we employ the same experimental methodology as in Choi *et al.* (2007a, 2014) and Halevy, Persitz, and Zrill (2018), except that instead of having just two states of nature ($S = 2$) and two associated securities, the new experiment incorporates three states ($S = 3$) and three associated securities, with a price for each security. As we have explained, choices from three-dimensional budget sets provide more rigorous tests of rationalizability than choices from two-dimensional budget lines, in particular when it comes to testing EUT (see more on this below in our discussion of the power of the experiment).

We conducted the experiment at UC Berkeley and UCLA. The subjects in the experiment were recruited from undergraduate classes at these institutions. In the experiment, subjects choose an allocation from a three-dimensional budget set presented using the graphical interface introduced by Choi *et al.* (2007b). Subjects make choices by using the computer mouse to move the pointer on the computer screen to the desired point, and are restricted to allocations on the budget constraint. The full experimental instructions, including the computer program dialog windows, are reproduced in an appendix.[8]

The experimental procedures described below are identical to those described by Choi *et al.* (2007b) and used by Choi *et al.* (2007a) to study a portfolio choice problem with two risky assets, except that each choice involved choosing a point on a three-dimensional (instead of two-dimensional) graph representing the set of possible allocations. In the experimental task, there are three *equally likely* states denoted by $s = 1, 2, 3$ and three associated securities, each of which promises a payoff of one token (the experimental currency) in one state and nothing in the others. Recall that $x_s \geqslant 0$ denotes the demand for the security that pays off in state s and $p_s > 0$ denotes the corresponding price. Without loss of generality, we assume that the budget is normalized to 1. The budget set is then given by $\mathcal{B} = \{\mathbf{x} : \mathbf{p} \cdot \mathbf{x} = 1\}$, where $\mathbf{x} = (x_1, x_2, x_3)$ denotes the portfolio of securities and $\mathbf{p} = (p_1, p_2, p_3)$ denotes the vector of security prices.

Each experimental subject faced 50 independent decision rounds. For each subject, the computer selected 50 budget sets randomly from the set of planes that intersect at least one axis at or above the 50 token level and intersect all axes at or below the 100 token level. The budget sets selected for each subject were independent of one another and of the budget sets selected for other subjects. Subjects were not informed of any state that was actually realized until the end of the experiment. This procedure was repeated until all 50 rounds were completed. At the end of the experiment, the computer randomly selected one of the 50 decision rounds to carry out for payoffs, and token allocations were converted into dollars. The round selected depended solely on chance.

[8]We are building on the expertise that we have acquired in previous work using the experimental method across different types of individual choice problems. Choi *et al.* (2014) introduces the graphical interface of Choi *et al.* (2007b) into a nationally representative sample. The datasets of Choi *et al.* (2007a, 2014) have been analyzed in many papers, including Halevy, Persitz, and Zrill (2018), Polisson, Quah, and Renou (2020), de Clippel and Rozen (2021), and Echenique, Imai, and Saito (2021). Fisman, Kariv, and Markovits (2007), Fisman *et al.* (2015), Fisman, Jakiela, and Kariv (2015, 2017), and Li, Dow, and Kariv (2017) employ a similar experimental methodology to study social preferences across a number of different samples, including a nationally representative sample. Three-dimensional budget sets have been used by Fisman, Kariv, and Markovits (2007) to study preferences for giving, and also by Ahn *et al.* (2014) to study ambiguity aversion, but so far have not been used to study risk. Other related work by Zame *et al.* (2020) develops theoretical tools and experimental methods for testing the linkages between preferences for personal and social consumption and attitudes toward risk and inequality.

3.3 Power

To illustrate that the three-dimensional experiment is more powerful than the two-dimensional experiments previously used in the literature—and specifically that it is sufficiently powerful to detect whether or not EUT is the right model of choice under risk—we start by building on the test designed by Bronars (1987) which employs as a benchmark the choices of a simulated subject who randomizes uniformly among all allocations on each budget set. The simulated subject makes 50 choices from randomly generated budget sets, in the same way as do the human subjects.

To focus on EUT-rationalizability, each choice is drawn independently from the uniform distribution over all allocations on the budget set, subject to keeping the data *perfectly* compatible with FOSD-rationalizability, that is $e^{**} = 1$. Figure 3 provides a clear graphical illustration by comparing the distributions of e^{***} generated by such simulated subjects in the two- and three-dimensional experiments. The horizontal axis shows the value of e^{***} and the vertical axis measures the fraction of simulated subjects whose scores are above each value. If we choose $e^{***} = 0.9$ as our critical value, we find that more than 80 percent of simulated subjects have e^{***} above 0.9 in the two-dimensional experiment, while just over 20 percent have e^{***} above 0.9 in the three-dimensional experiment.

Another benchmark against which to compare the power of the two- and three-dimensional designs involves the choices of a simulated subject who maximizes a non-EUT utility function. To illustrate such preferences when there are three states ($S = 3$), consider the rank-dependent utility function:

$$U(\tilde{\mathbf{x}}) = \beta_L u(x_L) + \beta_M u(x_M) + \beta_H u(x_H),$$

where $\beta_L, \beta_M, \beta_H > 0$ are decision weights that sum to unity, $\tilde{\mathbf{x}} = (x_L, x_M, x_H)$ is a *rank-ordered* portfolio with payoffs $x_L \leqslant x_M \leqslant x_H$, and u is the Bernoulli index. This formulation encompasses a number of non-EUT models and reduces to EUT when $\beta_L = \beta_M = \beta_H$ (since each state has an equal likelihood of occurring).[9] When there are two states of nature

[9] As Starmer (2000) points out, although the number of so-called non-EUT models "is well into double figures," the preferences generated by rank-dependent utility Quiggin (1982, 1993) is the leading contender. Machina (1994) concludes that rank-dependent utility is "the most natural and useful modification of the

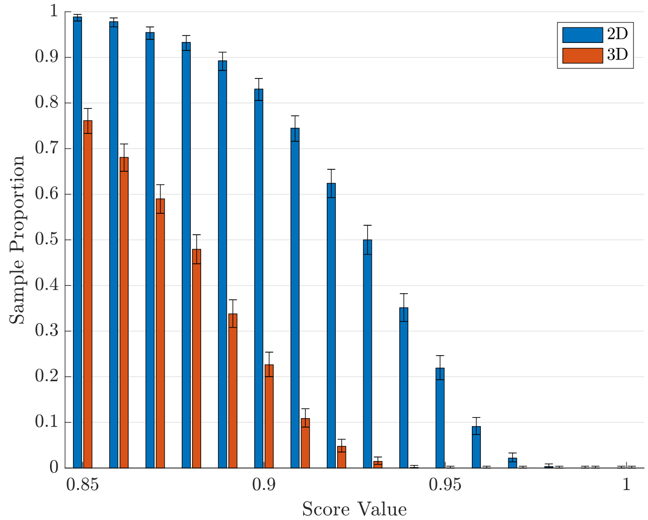


Figure 3: Power of EUT-Rationalizability

The three-dimensional (3D) experiment is more powerful than the two-dimensional (2D) experiment in detecting violations of EUT. We compare the distributions of EUT-rationalizability scores (e^{***}) in 2D and 3D for simulated subjects who choose randomly conditional on perfect FOSD-rationalizability ($e^{**} = 1$). The proportion of simulated subjects that have e^{***} above 0.9 (conditional on $e^{**} = 1$) is over 80 percent in the 2D experiment but just over 20 percent in the 3D experiment.

($S = 2$), the rank-dependent utility function takes the simpler form

$$U(\tilde{\mathbf{x}}) = \beta_L u(x_L) + \beta_H u(x_H),$$

where β_L, β_H are the decision weights and $\tilde{\mathbf{x}} = (x_L, x_H)$ is the rank-ordered portfolio with payoffs $x_L \leqslant x_H$. The rank-dependent formula for the rank-ordered portfolio $\tilde{\mathbf{x}}$ can be expressed in terms of the *probability weighting function* w (see more on this below) as follows:

$$\begin{aligned} \beta_L &= 1 - w\left(\tfrac{2}{3}\right), \\ \beta_M &= w\left(\tfrac{2}{3}\right) - w\left(\tfrac{1}{3}\right), \\ \beta_H &= w\left(\tfrac{1}{3}\right), \end{aligned}$$

classical expected utility formula," and Starmer (2000) argues that "if one is looking to organize the data from the large number of triangle experiments, then the decision-weighting models are probably the best bet." Yaari (1987), Segal (1990), Wakker (1994), and Abdellaoui (2002), among others, provide axiomatizations of rank-dependent utility, and Diecidue and Wakker (2001) discusses its underlying intuition.

for three states ($S = 3$), and

$$\beta_L = 1 - w\left(\tfrac{1}{2}\right),$$
$$\beta_H = w\left(\tfrac{1}{2}\right),$$

for two states ($S = 2$). That is, the cumulative distribution function of the induced lottery assigns to each monetary payoff the probability of receiving that payoff or anything less.[10]

NOTE 2 In the case of two states ($S = 2$), if the rank-dependent utility parameters satisfy $\beta_H < \beta_L$, then the indifference curves have a "kink" at the safe allocation where $x_1 = x_2$, and agents choose this allocation for a non-negligible set of price vectors. When the Bernoulli index is smooth, this behavior is inconsistent with EUT. Indifference curves that have a kink at the 45-degree line, which corresponds to the allocation with a certain payoff, can also be generated by other classes of non-EUT preferences such as the theory of loss/disappointment aversion (Gul, 1991). In this interpretation, the safe allocation $x_1 = x_2$ is taken to be the reference point.

In the case of three states ($S = 3$), on the other hand, if the rank-dependent utility parameters satisfy $\beta_H < \beta_M < \beta_L$, then the indifference curves have kinks wherever $x_s = x_{s'}$, and agents choose allocations that satisfy $x_s = x_{s'}$ for some $s \neq s'$ for a non-negligible set of price vectors. In contrast, the indifference curves generated by loss/disappointment aversion *only* have a kink at the safe allocation where $x_1 = x_2 = x_3$. These observations suggest that three-dimensional budget sets could allow for a more rigorous test of EUT versus non-EUT alternatives compared to two-dimensional budget lines, because the scope for non-EUT models to explain choice behavior (unaccounted for by EUT) could be greater when there are three states of nature compared to two.

In order to draw a comparison across the two- and three-dimensional experiments using simulated subjects maximizing a rank-dependent utility function, we hold the weighting fixed using the weighting function suggested by Tversky and Kahneman (1992), which distorts

[10] The weighting function w, which is increasing and satisfies $w(0) = 0$ and $w(1) = 1$, transforms the distribution function into decision weights. By definition, the decision weight β_H is equal to $w\left(\frac{1}{3}\right)$ in the case of three states and to $w\left(\frac{1}{2}\right)$ in the case of two states.

each probability $\pi \in (0,1)$ according to

$$w(\pi) = \frac{\pi^\gamma}{[\pi^\gamma + (1-\pi)^\gamma]^{1/\gamma}}.$$

This formulation takes the familiar (inverted) s-shaped form for $0 < \gamma < 1$, and any $\gamma > 0.279$ guarantees that w is increasing.[11] When $\gamma = 1$ we have $w(\pi) = \pi$, and so we get the standard EUT representation. In our numerical simulation, we set $\gamma = 0.5$ (in order to generate sufficient "pessimism") and we specify $u(x) = \log(x)$. Clearly, for these simulated subjects $1 = e^* = e^{**}$ since their choices are FOSD-rationalizable by construction. However, as a simple indication, while *all* of the simulated subjects have e^{***} above 0.95 in the two-dimensional experiment, *none* have e^{***} above 0.95 in the three-dimensional experiment.

Despite the advantages of the three-dimensional design, we nevertheless complement our analysis of these data by analyzing observations collected from a further 956 subjects, each making 50 choices over two-dimensional budget lines. (These experiments are identical to the (symmetric) risk experiment of Choi *et al.* (2007a).) We discuss these results in Section 4.3; the bottom line is that the major findings in the three-dimensional experiment are replicated across the two-dimensional experiments.

4 Experimental Results

In this section, we present the experimental results. The data from the experiment contain observations on 168 individual subjects. For each subject, we have a set of 50 observations $\mathcal{D} := (\mathbf{p}^i, \mathbf{x}^i)_{i=1}^{50}$, where $\mathbf{p}^i = (p_1^i, p_2^i, p_3^i)$ denotes the i-th observation of the price vector and $\mathbf{x}^i = (x_1^i, x_2^i, x_3^i)$ denotes the associated allocation. The experiment provides a large set of data consisting of many individual decisions over a wide range of three-dimensional budget sets. This is an important point, because as our power analysis shows, a large number of individual decisions over three-dimensional instead of two-dimensional budget sets is crucial in order to provide a sufficiently powerful test of the entire set of axioms underlying EUT.

[11] The other widely-used (single parameter) probability weighting function was proposed by Prelec (1998).

4.1 Illustrative Subjects

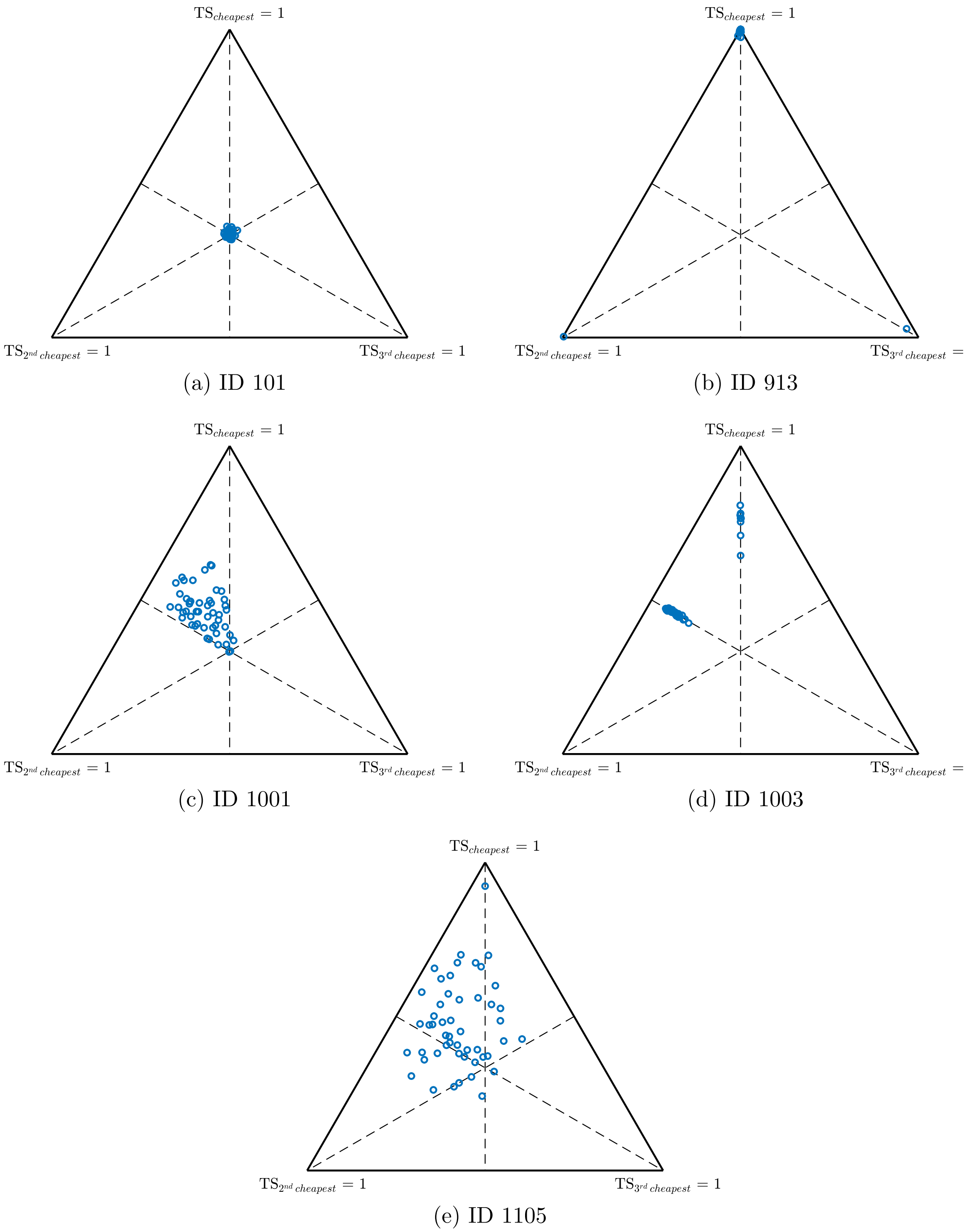
In the Introduction, we provide an overview of the important aggregate features of our experimental data, which we summarize by reporting the distributions of our indices of rationalizability (e^*), FOSD-rationalizability (e^{**}), and EUT-rationalizability (e^{***}). But the aggregate data tell us little about the choice behavior of individual subjects. To get some idea of the wide range of observed behaviors, we present in Figure 4 scatterplots depicting all 50 choices for five illustrative subjects. We have chosen subjects whose behavior corresponds to one of several prototypical choices and illustrates the striking regularity within subjects and heterogeneity across subjects that is characteristic of our data.

Figure 4 depicts the choices in terms of token shares for the three securities as points in the unit simplex. For each allocation $\mathbf{x}^i = (x_1^i, x_2^i, x_3^i)$, we relabel the states $s = 1, 2, 3$ so that $p_1^i < p_2^i < p_3^i$ and define the *token share* of the security that pays off in state s to be the number of tokens payable in state s as a fraction of the sum of tokens payable across states

$$\bar{x}_s^i = \frac{x_s^i}{x_1^i + x_2^i + x_3^i},$$

and $\bar{\mathbf{x}}^i = (\bar{x}_1^i, \bar{x}_2^i, \bar{x}_3^i)$ is the vector of token shares corresponding to the allocation $\mathbf{x}^i$. Each panel of Figure 4 contains a scatterplot of the token share vectors corresponding to the 50 allocations chosen by one of the five illustrative subjects. The vertices of the unit simplex correspond to allocations consisting of one of the three securities, and each point in the simplex represents an allocation as a convex combination of the extreme points.

The behaviors of the first three subjects are roughly EUT-rationalizable. In the scatterplot for subject ID 101 (Figure 4a), all of the vectors of token shares lie near the *center* of the simplex where $\bar{\mathbf{x}}^i = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$; this behavior is consistent with infinite risk aversion. In the scatterplot for subject ID 913 (Figure 4b), the token shares are all concentrated on (or, in a few cases, adjacent to) the top *vertex* of the simplex where $\bar{\mathbf{x}}^i = (1, 0, 0)$; this behavior is consistent with risk neutrality. A more interesting behavior is illustrated in the scatterplot for subject ID 1001 (Figure 4c). The choices of this subject roughly equalize expenditures $p_1^i x_1^i = p_2^i x_2^i = p_3^i x_3^i$, rather than tokens, across the three securities; this behavior is consistent with maximizing a logarithmic von Neumann-Morgenstern expected utility function.



(a) ID 101

(b) ID 913

(c) ID 1001

(d) ID 1003

(e) ID 1105

Figure 4: Subject Behavior

Each plot shows all 50 choices for a single subject in terms of token shares. Each vertex of the unit simplex corresponds to a full allocation to one of the three securities. Some subjects are roughly EUT-rationalizable: (a) ID 101 is consistent with infinite risk aversion; (b) ID 913 is consistent with risk neutrality; (c) ID 1001 is consistent with the maximization of logarithmic von Neumann-Morgenstern expected utility. Some subjects are distinctly *not* EUT-rationalizable: (d) ID 1003 is FOSD-rationalizable and could be explained by rank-dependent utility; and (e) ID 1105 is not FOSD-rationalizable.

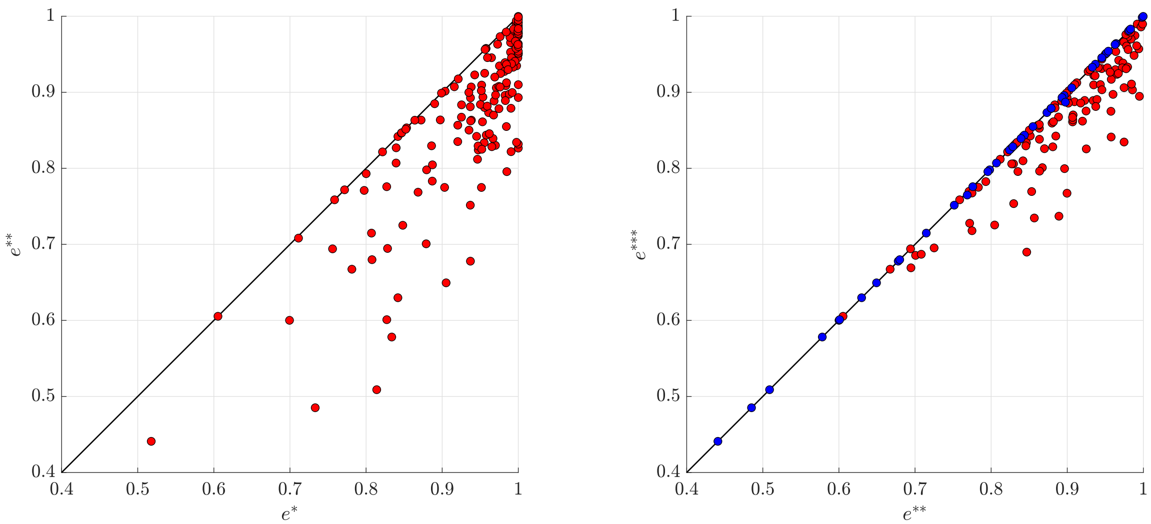
The next two subjects are *not* EUT-rationalizable. In the scatterplot for subject ID 1003 (Figure 4d), all token shares lie roughly along the *bisectors* of the angles of the simplex where $\bar{x}_1^i = \bar{x}_2^i$ or $\bar{x}_2^i = \bar{x}_3^i$; this behavior—equalizing the demands for two out of the three securities for a non-negligible set of price vectors—is FOSD-rationalizable (because $\bar{x}_1^i \geqslant \bar{x}_2^i \geqslant \bar{x}_3^i$ where $p_1^i < p_2^i < p_3^i$) but not EUT-rationalizable. As we explain in an appendix, preferences generated by rank-dependent utility (Quiggin, 1982, 1993) could give rise to such choices. Finally, in the scatterplot for subject ID 1105 (Figure 4e), the token shares are not confined to the top left subset of the simplex where $\bar{x}_1^i \geqslant \bar{x}_2^i \geqslant \bar{x}_3^i$; this behavior is not FOSD-rationalizable (and thus also not EUT-rationalizable). We have obviously shown just a small subset of our full set of subjects, and these are of course special cases where regularities in the data are very clear.[12]

4.2 Rationalizability Scores

As a first basic check on the rationalizability (e^*), FOSD-rationalizability (e^{**}), and EUT-rationalizability (e^{***}) of individual subjects, Figure 5 shows scatterplots of e^* against e^{**} (Figure 5a) and of e^{**} against e^{***} (Figure 5b). By definition, $e^* \geqslant e^{**} \geqslant e^{***}$ so all points in both scatterplots must lie on or below the 45-degree lines. An individual subject who is perfectly EUT-rationalizable will have $1 = e^* = e^{**} = e^{***}$. When e^{***} is strictly less than one, the corresponding values of e^* and e^{**} will then allow us to isolate the source of the subject's departure from EUT.

Out of our 168 subjects, the choices of only 27 subjects (16.1 percent) are perfectly rationalizable ($e^* = 1$), but the choices of *none* of our subjects are perfectly FOSD-rationalizable ($e^{**} = 1$), and hence perfectly EUT-rationalizable ($e^{***} = 1$). Most interestingly, only 11 subjects (6.5 percent) fall along the 45-degree line in the scatterplot of e^* against e^{**} (Figure 5a); the choices of these subjects are not necessarily perfectly rationalizable but they are not less FOSD-rationalizable than they are rationalizable ($e^* = e^{**}$). By contrast, 65 subjects (38.7 percent) fall along the 45-degree line in the scatterplot of e^{**} against e^{***} (Figure 5b); the choices of these subjects are not perfectly FOSD-rationalizable but they are not less

[12]For most subjects, the behavioral regularities are much less clear. However, a full review of the data reveals both regularities within subjects and heterogeneity across subjects. The scatterplots for the full set of subjects are available upon request.



(a) e^* versus e^{**} (b) e^{**} versus e^{***}

Figure 5: Scatterplots of Rationalizability Scores

The plots depict rationalizability scores for individual subjects. By definition, $e^* \geqslant e^{**} \geqslant e^{***}$ so all points in both scatterplots must lie on or below the 45-degree lines. (a) All individual-level differences between e^* and e^{**} are statistically significant at the 1 percent significance level (red). (b) The individual-level differences between e^{**} and e^{***} are statistically significant for 75.0 percent of the sample (red), but there is also a sizeable minority of subjects for whom this is not the case (blue).

EUT-rationalizable than they are FOSD-rationalizable ($e^{**} = e^{***}$). Only 3 subjects (1.8 percent), fall along the 45-degree line in both scatterplots; the choices of these subjects are not less EUT-rationalizable than they are rationalizable ($e^* = e^{**} = e^{***}$).

Our rich individual-level data also allow us to make statistical comparisons of rationalizability (e^*) versus FOSD-rationalizability (e^{**}) and of FOSD-rationalizability (e^{**}) versus EUT-rationalizability (e^{***}) using a purely nonparametric econometric approach. To this end, for each subject, we split the 50 observations into two *non-overlapping* partitions of 25 observations, generating paired subsamples of observations. Clearly, we cannot examine all $\binom{50}{25} > 10^{14}$ possible paired subsamples of the observed individual-level data; instead we draw 1,000 such paired subsamples at random for each subject and construct the sampling distributions of e^* and e^{***} on one subsample and the sampling distribution of e^{**} on the other. Note that given the non-overlapping partitions, the orderings $e^* \geqslant e^{**}$ and $e^{**} \geqslant e^{***}$ are no longer guaranteed. We can then straightforwardly test whether the mean difference between the pairs of e^* and e^{**} and of e^{**} and e^{***} are zero (or not) using a paired t-test.

In Figure 5, individual subjects are depicted in red if the two scores—either e^* and e^{**}

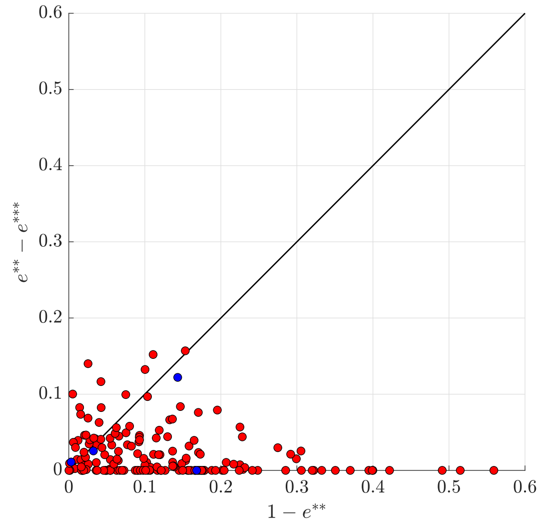


Figure 6: Scatterplot of Score Differences

The plot depicts rationalizability score differences for individual subjects. For the vast majority of subjects, the difference between FOSD-rationalizability and EUT-rationalizability ($e^{**} - e^{***}$) is small (or non-existent), while the difference between perfect rationalizability and FOSD rationalizability ($1 - e^{**}$) is much larger: 85.1 percent of subjects fall below the 45-degree line, and of those 45.5 percent fall along the horizontal axis ($e^{**} = e^{***}$). This difference in differences is statistically significant for 97.6 percent of subjects (red) at both the 1 and 5 percent significance levels.

(Figure 5a) or e^{**} and e^{***} (Figure 5b)—are statistically distinguishable at the 1 percent significance level and depicted in blue otherwise. All individual-level differences between e^* and e^{**} (Figure 5a) are statistically significant, including for those 11 subjects (6.5 percent) falling along the 45-degree line (for whom $e^* = e^{**}$ across all 50 observations). The individual-level differences between e^{**} and e^{***} (Figure 5b) are statistically significant for 126 subjects (75.0 percent), including for 25 of the 65 subjects (38.5 percent) falling along the 45-degree line (for whom $e^{**} = e^{***}$ across all 50 observations). If instead we evaluate at the 5 percent significance level, the individual-level differences between e^{**} and e^{***} are statistically significant for 134 subjects (79.8 percent). Hence, for the majority of subjects the difference between FOSD-rationalizability and EUT-rationalizability ($e^{**} - e^{***}$) is statistically significant, but there is also a sizeable minority for whom this is not the case.

Furthermore, we compare the *magnitudes* of differences between scores. Figure 6 shows a scatterplot of the difference between *perfect* rationalizability and FOSD-rationalizability ($1 - e^{**}$) against the difference between FOSD-rationalizability and EUT-rationalizability ($e^{**} - e^{***}$). Out of our 168 subjects, 143 (85.1 percent) fall below the 45-degree line in the scatterplot ($1 - e^{**} > e^{**} - e^{***}$), and of those 65 subjects (45.5 percent) fall along the horizontal axis ($e^{**} = e^{***}$). Hence, for the vast majority of our subjects there is only a small (or no) difference between FOSD-rationalizability and EUT-rationalizability ($e^{**} - e^{***}$), whereas the difference between perfect rationalizability and FOSD-rationalizability ($1 - e^{**}$) is much larger. For these subjects, there is little scope for the most prominent non-EUT alternatives, such as weighted expected utility, rank-dependent utility, or reference-dependent risk preferences, that relax the independence axiom to explain observed behavior, as they all postulate FOSD-rationalizability ($1 = e^{*} = e^{**} > e^{***}$).[13]

To provide a statistical test of the difference between $1 - e^{**}$ and $e^{**} - e^{***}$, we again draw 1,000 paired subsamples of observations for each subject and construct the sampling distribution of $1 - e^{**}$ on one subsample and the sampling distribution of $e^{**} - e^{***}$ on the other. We then test whether the mean difference in differences is statistically significant using a paired t-test. We find that it is significant for 164 subjects (97.6 percent) at both the 1 and 5 percent significance levels. These subjects are depicted in red in Figure 6; the other subjects are depicted in blue.

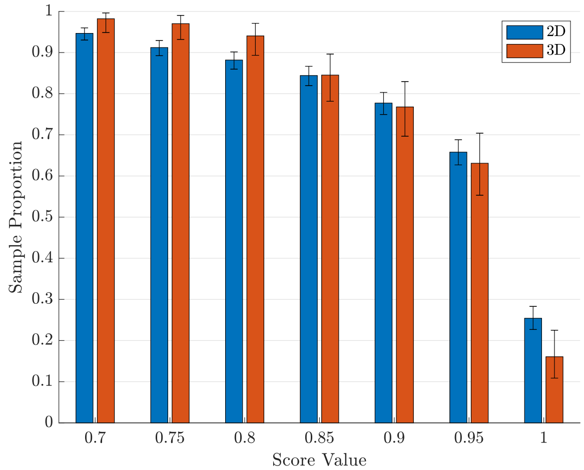
The broad conclusion from our analysis is clear: even for a single subject, the sources of violation of EUT are variegated; furthermore, for many subjects, violations of ordering and monotonicity are more prominent and much larger in magnitude than departures from the independence axiom.

4.3 Two- Versus Three-Dimensional Data

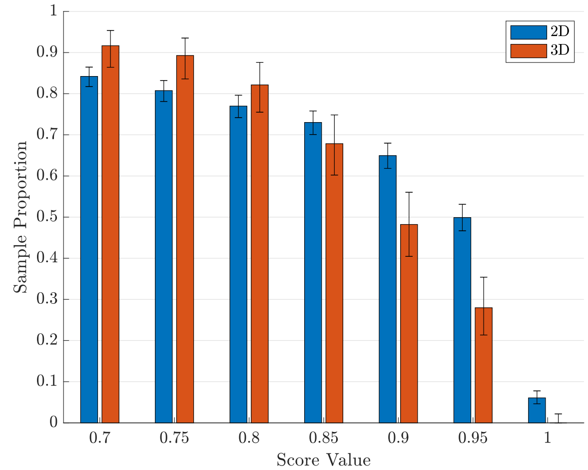
For comparison purposes, in an appendix we replicate our entire analysis with observations on 956 subjects making choices from two-dimensional budget lines. For each subject, we again have a set of 50 observations $\mathcal{D} := (\mathbf{p}^i, \mathbf{x}^i)_{i=1}^{50}$ where $\mathbf{p}^i = (p_1^i, p_2^i)$ denotes the i-th observation of the price vector and $\mathbf{x}^i = (x_1^i, x_2^i)$ denotes the associated allocation.[14] Figure

[13] Utility functions representing reference-dependent risk preferences (specifically the choice acclimating personal equilibrium model of Kőszegi and Rabin (2007)) can fail to be increasing if loss aversion is sufficiently high (see Masatlioglu and Raymond (2016)); however, these preferences are always locally locally nonsatiated and, in our experimental setting, symmetric. For reasons explained in greater detail in an appendix, utility functions that are symmetric and locally nonsatiated cannot rationalize any behavior that cannot also be rationalized by a symmetric and increasing utility function. Thus the rationalizability score for such preferences cannot improve on e^{**}.

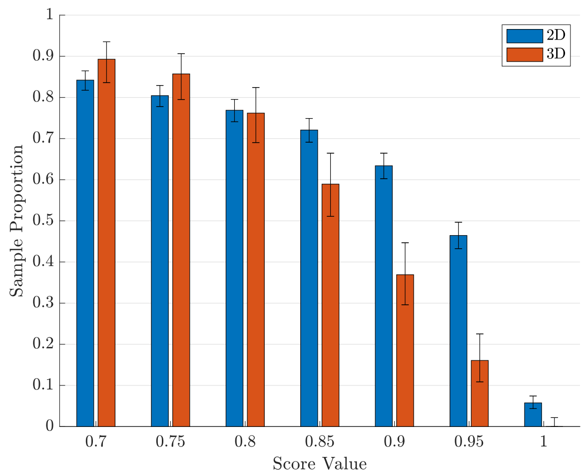
[14] The data include the (symmetric) data collected by Choi *et al.* (2007a) and similar data with different subject pools collected by Zame *et al.* (2020) and Cappelen *et al.* (2021) as well as new data. In all of these experiments, the individual-level data consist of 50 decision problems. We do not include the data of



(a) Distributions of e^*



(b) Distributions of e^{**}



(c) Distributions of e^{***}

Figure 7: Distributions of Rationalizability Scores

The plots depict distributions of rationalizability scores across the two-dimensional (2D) and three-dimensional (3D) experiments for (a) e^*, (b) e^{**}, and (c) e^{***}.

7 compares the rationalizability scores across the two- and three-dimensional experiments for e^* (Figure 7a), e^{**} (Figure 7b), and e^{***} (Figure 7c).

Note that choices from three-dimensional budget sets are at least as rationalizable (e^*) as choices from two-dimensional budget lines (Figure 7a), which is an interesting result in its own right. In the three-dimensional (resp. two-dimensional) experiment, 63.1 (resp.

Choi *et al.* (2014) which consist of 25, rather than 50, decision problems. Note that 25 individual decisions provide a rich enough data set to provide a powerful test of utility maximization (GARP). But as our power analysis shows, choices from two-dimensional budget lines provide a much weaker test of EUT, so we omit two-dimensional datasets with only 25 individual decisions, though this number is still higher than is usual in the literature. See, for examples, Cox (1997), Sippel (1997), Mattei (2000), Harbaugh, Krause, and Berry (2001), and Andreoni and Miller (2002), among others.

65.8) percent of the subjects have e^* scores above the 0.95 threshold, and 76.8 (resp. 77.7) percent have scores above the 0.9 threshold. Since choices from two-dimensional budget lines cannot satisfy WARP but violate GARP—but this is, of course, possible for choices from three-dimensional budget sets—the presence of three states could cause not just more departures from EUT, but also more fundamental departures from rationality. Our analysis suggests otherwise. At the very least, choices from three-dimensional budget sets are at least as rationalizable as choices from two-dimensional budget lines. As a practical note, this also suggests that subjects did not have any difficulties in understanding the three-dimensional experimental procedures or using the computer interface.

On the other hand, choices from three-dimensional budget sets are distinctly less FOSD-rationalizable (e^{**}) and EUT-rationalizable (e^{***}) than choices from two-dimensional budget lines (Figures 7b and 7c). In the three-dimensional experiment, 28.0 (resp. 16.1) percent of the subjects have e^{**} (resp. e^{***}) scores above the 0.95 threshold, and 48.2 (resp. 36.9) percent have scores above the 0.9 threshold. In the two-dimensional experiment (also with 50 choices), the corresponding percentages are 49.9 (resp. 46.4) and 65.0 (resp. 63.4).

Statistical tests on the two-dimensional data show that the individual-level differences between e^* and e^{**} are statistically significant for 859 (89.9 percent) and 866 (90.6 percent) at the 1 and 5 significant levels, respectively. In contrast, the individual-level differences between e^{**} and e^{***} are statistically significant for only 215 (22.5 percent) and 268 subjects (28.0 percent). This comparison suggests that three-dimensional budget sets (relative to two-dimensional budget lines) considerably improve the power of revealed preference tests of EUT-rationalizability. Finally, in the two-dimensional data, as in the three-dimensional data, the loss of consistency arising from EUT specifically is small, once we account for ordering and monotonicity. Indeed, $1 - e^{**} > e^{**} - e^{***}$ for 827 out of 956 subjects (86.5 percent). These differences in differences are statistically significant for 888 subjects (92.9 percent) and 890 subjects (93.1 percent) at the 1 and 5 percent significance levels, respectively.

5 Related Literature

There is a vast amount of research on decision making under risk and under uncertainty, and laboratory experiments have provided some key empirical guideposts for the development of new ideas in these areas. We will not attempt to review the large and growing experimental literature. Though now somewhat dated, an overview of experimental and theoretical work can be found in Camerer (1995), while Starmer (2000) provides a review of the risk literature that focuses on evaluating non-EUT theories.[15] Following the seminal work of Hey and Orme (1994) and Harless and Camerer (1994), a number of papers have estimated parametric utility functions. Harless and Camerer (1994) fits aggregate data, while Hey and Orme (1994) estimates functional forms on microdata (decisions from a large menu of binary choices) at the level of the individual subject.

More recently, Choi *et al.* (2007a) employs graphical representations of budget sets containing bundles of state-contingent commodities in order to elicit preferences; this experimental approach allows for the collection of a very rich individual-level dataset. For each subject in their experiment, Choi *et al.* (2007a) tests the data for consistency with GARP and estimates preferences in a parametric model with loss/disappointment aversion (Gul, 1991). This formulation encompasses a number of different theories and embeds EUT as a parsimonious and tractable special case. But testing EUT as a restriction on a non-EUT utility function has an obvious drawback—it depends on assumptions over functional form and the specification of the error structure. Indeed, Halevy, Persitz, and Zrill (2018) highlights the distinction between the non-parametric and parametric recoverability of preferences.

The most basic question that one could ask about individual-level choice data is whether they are compatible with utility maximization, and classical revealed preference theory (Samuelson, 1938, 1948, 1950; Houthakker, 1950; Afriat, 1967; Diewert, 1973; Varian, 1982) provides GARP as a direct test.[16] Consistency with GARP is implied by—and guarantees—choice from a coherent preference over all possible alternatives, but *any* consistent preference ordering that is locally nonsatiated is admissible. In particular, choices can be compatible with GARP and yet fail to be reconciled with the maximization of a utility function that is monotonic with respect to FOSD, which is not normatively appealing. One is thus naturally

[15]Camerer and Weber (1992) and Harless and Camerer (1994) also summarize the experimental evidence from testing the various utility theories of choice under risk and under uncertainty. Kahneman and Tversky (2000) collects many theoretical and empirical papers that have emerged from their pioneering work on prospect theory.

[16]For overviews of the revealed preference literature, see Crawford and De Rock (2014) and Chambers and Echenique (2016), as well as the papers by Afriat (2012), Diewert (2012), Varian (2012), and Vermeulen (2012), published in a special issue of the *Economic Journal* on the foundations of revealed preference.

led to go beyond consistency and to ask whether the choices made by a subject are compatible with a utility function that has some special structure, in particular one which is monotonic with respect to FOSD and/or adheres to EUT. To answer these questions properly requires the development of new revealed preference tests.

Originating in the works of Varian (1983a,b, 1988) and Green and Srivastava (1986), some more recent papers which pursue these questions include Diewert (2012), Bayer *et al.* (2013), Kubler, Selden, and Wei (2014, 2017), Echenique and Saito (2015), Chambers, Liu, and Martinez (2016), Chambers, Echenique, and Saito (2016), Nishimura, Ok, and Quah (2017), Echenique, Imai, and Saito (2019, 2021), Polisson, Quah, and Renou (2020), and de Clippel and Rozen (2021). We compare our approach and contribution to existing work along four dimensions—methods, measures, tests, and power.

Methods. With the exception of the GRID method, all other tests of EUT involve a *concave* Bernoulli index. The GRID method, by contrast, neither assumes nor guarantees concavity. This distinction is by no means cosmetic, since it has *empirical* implications. Although concavity of the Bernoulli index, which is equivalent to risk aversion under EUT, is widely assumed in empirical applications, we avoid imposing any further requirements that are not, strictly speaking, a part of EUT in our test of the model.[17] This feature of our analysis is an important part of our claim that our tests are purely *nonparametric*, with no extraneous assumptions on the parametric form or shape of the utility function.

Measures. Revealed preference relations generate exact tests while choice data almost always contain some violations. Given this, any serious empirical investigation requires an index to measure a model's goodness-of-fit, or (in other words) the extent to which a subject's choices are (in)compatible with the model. In this paper, we use Afriat's (1973) CCEI to measure a subject's consistency with (basic) rationalizability (e^{*}), FOSD-rationalizability (e^{**}), and EUT-rationalizability (e^{***}). Since the models

[17]For further discussion of this issue, see Polisson, Quah, and Renou (2020). A subject who maximizes expected utility will pass our test and be classified as EUT-rationalizable, even if that subject is *not* globally risk averse. For an example of choice data that are EUT-rationalizable but only with a non-concave Bernoulli index, see Section A4 of the online appendix in Polisson, Quah, and Renou (2020). (Note that Polisson, Quah, and Renou (2020) also develops a test for the case where the Bernoulli index is required to be concave.) This empirical distinction runs in contrast with the Afriat (1967) result on basic rationalizability, where concavity of the utility function (not necessarily of the expected utility form) is without loss of generality.

are nested, the indices must be ordered for any given subject, with $1 \geqslant e^{*} \geqslant e^{**} \geqslant e^{***} > 0$, where an index of 1 implies exact agreement with a given model.

The use of a common index across different models means that we can perform a comprehensive test of each relevant model (in which all the axioms of a model are tested *in combination*) and at the same time cleanly identify the incremental impact of additional axioms. We employ the CCEI (rather than some other index) for several related reasons: we know how to compute it for the three models under consideration; these computations can be implemented efficiently; and it is the most commonly used measure of goodness-of-fit.[18,19]

de Clippel and Rozen (2021) proposes a different index to measure goodness-of-fit which is applicable to different families of utility functions; roughly speaking, the index is based on the size of the departures from the first-order conditions. Building on the methodology in Echenique, Imai, and Saito (2020) within the context of intertemporal choice, Echenique, Imai, and Saito (2021) proposes essentially the same index as de Clippel and Rozen (2021) for expected utility, albeit with a somewhat different motivation. This index (or collection of indices) relies on a first-order (condition) approach, so they are only applicable to models representable by quasiconcave utility functions (defined on the space of contingent consumption). As such, it is not ideal for our purposes since we want to avoid imposing a concave Bernoulli index (or, more generally, a quasiconcave utility function) as a rationality requirement.

Tests. We create individual-level non-parametric *permutation* (randomization)

[18] A small subset of the many studies using the CCEI includes Harbaugh, Krause, and Berry (2001) on children's preferences, Andreoni and Miller (2002) and Fisman, Kariv, and Markovits (2007) on social preferences, and Choi *et al.* (2007a, 2014) and Carvalho, Meier, and Wang (2016) on risk preferences. Recently, Dziewulski (2020) provides a further behavioral interpretation for the CCEI based on a decision maker's cognitive inability to distinguish between bundles that are sufficiently similar.

[19] The index proposed by Varian (1990) is closely related to the CCEI and has been used in some important work (see, for example, Halevy, Persitz, and Zrill (2018)). There are known methods for calculating this index for the different models that we consider, but its calculation is much more computationally demanding than the CCEI (especially in the case of the EUT model) and therefore it is not practically implementable for us, given the size of our datasets and the scope of our empirical exercise. (For more on the computation of the Varian index to measure rationalizability, FOSD-rationalizability, and EUT-rationalizability, see Polisson, Quah, and Renou (2020).) One advantage of Varian's index is that it is generally less sensitive to a single errant observation compared to the CCEI. We address this sensitivity issue through our subsampling procedure (see Section 4.2), which allows us to test (at the level of an individual subject) whether the difference between, for example, $1 - e^{**}$ and $e^{**} - e^{***}$ is statistically significant.

tests. The approach builds only on revealed preference techniques and it is purely *nonparametric*, making no assumptions about the form of the subject's underlying utility function or on the error structure. That is, we obtain the (empirical) distribution functions for the test statistics under the null hypotheses—that choices are as FOSD-rationalizable as they are rationalizable ($e^{**} = e^{*}$) and as EUT-rationalizable as they are FOSD-rationalizable ($e^{***} = e^{**}$)—directly from the individual-level data. We are not aware of similar statistical tests performed in other work.

Power. A number of recent papers—including Polisson, Quah, and Renou (2020), de Clippel and Rozen (2021), and Echenique, Imai, and Saito (2021)—analyze the experimental data from Choi *et al.* (2014). This experiment is identical to Choi *et al.* (2007a), except that it consists of 25, rather than 50, decision problems involving two (equiprobable) states of nature and two associated Arrow securities. Echenique, Imai, and Saito (2021) also analyzes the experimental data from Carvalho, Meier, and Wang (2016) and Carvalho and Silverman (2019), which also consist of 25 problems. The Choi *et al.* (2007a) data have also been extensively analyzed, including by Halevy, Persitz, and Zrill (2018) and Polisson, Quah, and Renou (2020). The common thread in all these experiments is that there are two states and two securities.

The experiment reported in this paper consists of 50 decision problems involving *three* (equiprobable) states with *three* associated Arrow securities. Collecting 50, or even 25, individual decisions is more than is usual in the experimental literature on choice under risk and, as Choi *et al.* (2014) show, it does provide a rich enough individual-level dataset for a powerful test of (basic) rationalizability. However, our power analysis indicates that having three states significantly enhances the discriminatory power of the experiment, especially with respect to EUT-rationalizability, when compared to experiments with two states (and 25, or indeed 50, observations). Given that the primary purpose of this paper to reach a robust empirical conclusion on the sources of departure from EUT, our use of a more discriminating choice environment is crucial.

To conclude, Polisson, Quah, and Renou (2020), de Clippel and Rozen (2021), and Echenique, Imai, and Saito (2021) all develop new methodologies and apply their techniques to existing experimental data. Echenique, Imai, and Saito (2021) finds that subjects who are more rationalizable (as measured by the CCEI) are not necessarily more EUT-rationalizable (as measured by their index). However, these two rationalizability measures are not formally comparable, so the analysis cannot separate the empirical validity of each of the axioms on which EUT is based. More closely related to our theme, Polisson, Quah, and Renou (2020) observes a relatively small gap between FOSD-rationalizability and EUT-rationalizability; notwithstanding the use of a different measure, de Clippel and Rozen (2021) draws a similar conclusion. The focus of both Polisson, Quah, and Renou (2020) and de Clippel and Rozen (2021), however, is methodological rather than empirical and both also rely on existing two-dimensional datasets in their empirical analyses; as acknowledged by de Clippel and Rozen (2021), power issues cast doubts on the robustness of their empirical conclusions. In this paper, our findings rely on new experimental data with three-dimensional budget sets and 50 observations per subject. A thorough analysis of these data allows us to establish conclusively that subjects have multiple sources of EUT violations and, for the vast majority, violations of ordering and/or monotonicity rather than violations of independence are the main sources of departure from EUT.

6 Concluding Remarks

The standard model of choice under risk is based on von Neumann and Morgenstern's (1947) EUT. It is meant to serve as a normative guide for choice and also as a descriptive model of how individuals choose. However, much of the experimental and empirical evidence of "anomalies" in choice behavior suggests that EUT may not the right model. While EUT embodies three important axioms—ordering, monotonicity (with respect to FOSD), and independence—independence is the only axiom which the seminal alternatives to EUT relax.

It is thus natural that experimentalists should want to test the empirical validity of the independence axiom, and the overwhelming body of evidence against independence has raised criticisms about its status as the touchstone of rationality in the context of decision-making under risk. In response to these criticisms, various generalizations of EUT have been developed, and the experimental examination of these theories has led to new empirical regularities in the laboratory. Starmer (2000) calls this the "conventional strategy"—

theories/experiments designed to permit/test violations of independence (and weakened forms of independence) while retaining the more basic axioms of ordering and monotonicity.[20]

Combining theoretical tools, experimental methods, and non-parametric econometric techniques, our study confronts all of the axioms of EUT with individual-level experimental data that is richer than anything that has heretofore been used. The data are well-suited to purely nonparametric revealed preference tests which allow for the reality that individual behavior is not perfectly consistent with well-behaved preferences.

Why does this matter? It matters because choice data cannot be treated as being generated by a utility function, or by a utility function that is monotone with respect to FOSD, if there are large deviations from rationalizability or FOSD-rationalizability. In these cases, the standard approach of postulating some parametric family of utility functions (typically respecting FOSD), and estimating its parameters leads to model misspecification. As a result, the estimated preference will not be the true underlying preference, if such a preference ordering even exists, and positive predictions and welfare conclusions based on these models will be misleading.[21] Our findings also have implications for public policy; for example, in the practice of light paternalism, which is aimed at steering people toward better choices (Camerer *et al.*, 2003; Thaler and Sunstein, 2003; Loewenstein and Haisley, 2008). Clearly, decision-makers that only violate independence merit greater deference from policy-makers than the more boundedly rational ones that violate ordering and monotonicity because the choices of the former, unlike the latter, maximize a well-defined utility function and are thus of a higher quality (Kariv and Silverman, 2013).

To conclude, by applying the latest revealed preference techniques to an experiment involving three states with three associated securities, we provide strong *comprehensive* and *nonparametric* tests of complete representations of preferences under risk. Our main result is that while the vast majority of our subjects have statistically significant violations of

[20] Bell (1982), Fishburn (1982), and Loomes and Sugden (1982) (simultaneously) propose a model of nontransitive risk preference. Loomes and Sugden (1987) develop a version of this model that involves regret with pairwise choice. Starmer (2000) provides an overview of these models and relates them to other non-EUT alternatives.

[21] Halevy, Persitz, and Zrill (2018) parametrically estimates preferences for the dataset collected by Choi *et al.* (2007a) involving two states and two associated securities. They find *significant quantitative and qualitative differences* between the preferences induced by parametric estimation and the revealed preferences implied by choices, due to model misspecification.

independence, for many subjects these violations are minor when compared against violations of ordering and monotonicity. As EUT lies at the very heart of economics, these results have important implications for both economic theory and economic policy.

The experimental platform and analytical techniques that we have used are applicable to many other types of individual choice problems. One important direction is to study choice under ambiguity. In a separate paper, we apply the GRID method and other revealed preference techniques to the analogous data of Ahn *et al.* (2014) which similarly allow for a rigorous test of individual-level decision-making under ambiguity.

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