The Interdisciplinary Center, Herzliya
School of Economics
Advanced Microeconomics
Fall 2017

Confronting Theory with Experimental Data and vice versa

Risk and time preferences

The fundamental tradeoffs in life

People's attitudes towards risk, time and other people enter every realm of (financial) decision-making:

$$\begin{array}{rcl} \text{risk} & \Longleftrightarrow & \text{return} \\ \text{today} & \Longleftrightarrow & \text{tomorrow} \\ \text{self} & \Longleftrightarrow & \text{others} \end{array}$$

Risk, time and social preferences are thus important inputs into any broader measure of welfare and enter virtually every field of economics.

The touchstones of (financial) decision-making

Rational choice 'simply' requires consistent preferences over all possible alternatives, and choices that correspond to the most preferred alternative from the feasible set.

$$\begin{tikzcd} & \text{goals} & \\ \text{preferences} \arrow[ur] & & \text{constraints} \arrow[ul] \end{tikzcd}$$

Insofar as preferences are rational, then the techniques of economic analysis may be brought to bear on modeling the decisions governed by these preferences.

Research questions

Consistency

– Is behavior under uncertainty consistent with the utility maximization model?

Structure

– Is behavior consistent with a utility function with some special structural properties?

Recoverability

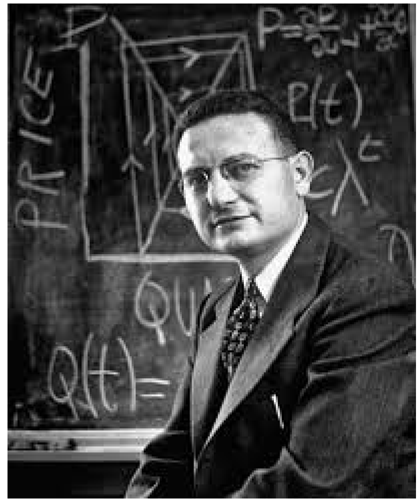
– Can the underlying utility function be recovered from observed choices?

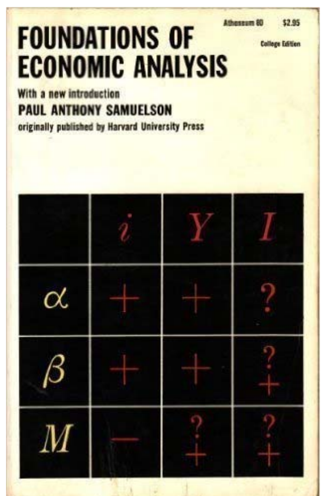
Extrapolation

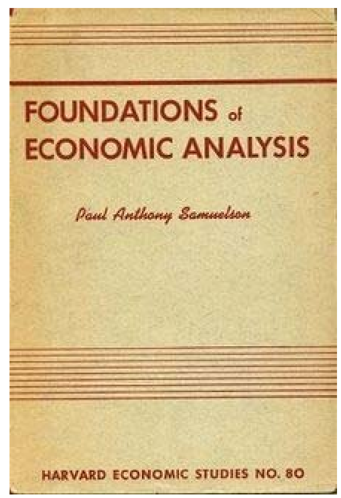
– Given behavior in the laboratory, can we forecast behavior in other environments?

Foundations of Economic Analysis (1947)

Paul A. Samuelson (1915-2009) – the first American Nobel laureate in economics and the foremost (academic) economist of the 20th century (and the uncle of Larry Summers…).


PRICE
P(t)
Q(t)=


Atheneum 80 $2.95
College Edition
FOUNDATIONS OF
ECONOMIC ANALYSIS
With a new introduction
PAUL ANTHONY SAMUELSON
originally published by Harvard University Press


FOUNDATIONS of
ECONOMIC ANALYSIS
Paul Anthony Samuelson
HARVARD ECONOMIC STUDIES NO. 80

Formally, we represent the consumer's preferences by a binary relation $\succsim$ defined on the set of consumption bundles.

For any pair of bundles x and y, if the consumer says that x is at least as good as y, we write

$$x \succsim y$$

and say that x is *weakly preferred* to y.

Bear in mind: economic theory often seeks to convince you with simple examples and then gets you to extrapolate. This simple construction works in wider (and wilder circumstances).

From the weak preference relation $\succsim$ we derive two other relations on the set of alternatives:

- Strict performance relation

$$x \succ y \quad \text{if and only if } x \succsim y \text{ and not } y \succsim x.$$

  The phrase $x \succ y$ is read x is *strictly preferred* to y.

- Indifference relation

$$x \sim y \quad \text{if and only if } x \succsim y \text{ and } y \succsim x.$$

  The phrase $x \sim y$ is read x is *indifferent* to y.

The basic assumptions about preferences

The theory begins with three assumptions about preferences. These assumptions are so fundamental that we can refer to them as "axioms" of decision theory.

[1] Completeness

$$x \succsim y \text{ or } y \succsim x$$

for any pair of bundles x and y.

[2] Transitivity

$$\text{if } x \succsim y \text{ and } y \succsim z \text{ then } x \succsim z$$

for any three bundles x, y and z.

Together, completeness and transitivity constitute the formal definition of *rationality* as the term is used in economics. Rational economic agents are ones who

> have the ability to make choices [1], and whose choices display a logical consistency [2].

(Only) the preferences of a rational agent can be represented, or summarized, by a *utility function* (more later).

The third axiom about consumer's preferences for one bundle versus another is that "more is better" (goods are desirable).

[3] Monotonicity

$$\text{if } x_1 \geq y_1 \text{ and } x_2 \geq y_2 \text{ then } x \succsim y$$

for any pair of bundles x and y.

Decision making under uncertainty

- Uncertainty is a fact of life so people's attitudes towards risk enter every realm of economic decision-making.

- We *must* study individual behavior with respect to choice involving uncertainty.

- Models of decision making under uncertainty play a key role in every field of economics.

Objectives

- Illustrate that agents (consumers and managers) frequently make decisions with uncertain consequences.

- Facing uncertain choices, maximizing the Expected Utility is how agents ought to choose.

- Individual behavior is often contrary to the assumptions of Expected Utility Theory.

Life is full of lotteries :-(

$$x := \begin{array}{l} \overset{p}{\nearrow} \quad \$A \\[1em] \underset{1-p}{\searrow} \quad \$B \end{array} \qquad\qquad y := \begin{array}{l} \overset{p}{\nearrow} \quad \$A \\ \xrightarrow{q} \quad \$B \\ \underset{1-p-q}{\searrow} \quad \$C \end{array}$$

A risky lottery (left) and an ambiguous lottery (right)

$$x := \begin{array}{cc} \overset{1/2}{\nearrow} & \$A \\ & \\ \underset{1/2}{\searrow} & \$B \end{array} \qquad\qquad y := \begin{array}{cc} \overset{?}{\nearrow} & \$A \\ & \\ \underset{1-?}{\searrow} & \$B \end{array}$$

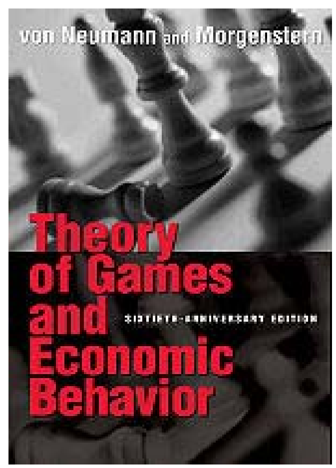
A compounded lottery

$$
x := \begin{array}{cccc}
 & & \overset{q}{\nearrow} & \$A \\
 & \overset{p}{\nearrow} & & \\
 & & \underset{1-q}{\searrow} & \$B \\
 & & & \\
 & & \overset{l}{\nearrow} & \$C \\
 & \underset{1-p}{\searrow} & & \\
 & & \underset{1-l}{\searrow} & \$D
\end{array}
$$

The reduction of a compounded lottery

$$
x := \begin{array}{ccccc}
 & & & & | \\
 & & \overset{q}{\nearrow} & \$A & | \quad pq \\
 & \overset{p}{\nearrow} & & & \\
 & & \underset{1-q}{\searrow} & \$B & | \quad p(1-q) \\
 & & & & | \\
 & & \overset{l}{\nearrow} & \$C & | \quad (1-p)l \\
 & \underset{1-p}{\searrow} & & & \\
 & & \underset{1-l}{\searrow} & \$D & | \quad (1-p)(1-l) \\
 & & & & |
\end{array}
$$

The paternity of decision theory and game theory (1944)



$$x := \begin{array}{ll} \overset{p_x}{\nearrow} & \$A \\ & \\ \underset{1-p_x}{\searrow} & \$B \end{array} \qquad \succ \qquad y := \begin{array}{ll} \overset{p_y}{\nearrow} & \$C \\ & \\ \underset{1-p_y}{\searrow} & \$D \end{array}$$

$$
x+z := \begin{array}{ccc}
 & \overset{p_x}{\nearrow} & \$A \\
 \overset{p}{\nearrow} & \underset{1-p_x}{\searrow} & \$B \\
 & & \\
 \underset{1-p}{\searrow} & \overset{p_z}{\nearrow} & \$E \\
 & \underset{1-p_z}{\searrow} & \$F
\end{array}
\qquad \succ \qquad
y+z := \begin{array}{ccc}
 & \overset{p_y}{\nearrow} & \$C \\
 \overset{p}{\nearrow} & \underset{1-p_y}{\searrow} & \$D \\
 & & \\
 \underset{1-p}{\searrow} & \overset{p_z}{\nearrow} & \$E \\
 & \underset{1-p_z}{\searrow} & \$F
\end{array}
$$

von Neumann and Morgenstern Expected Utility Theory (EUT)

Allais (1953) I

– Choose between the two gambles:

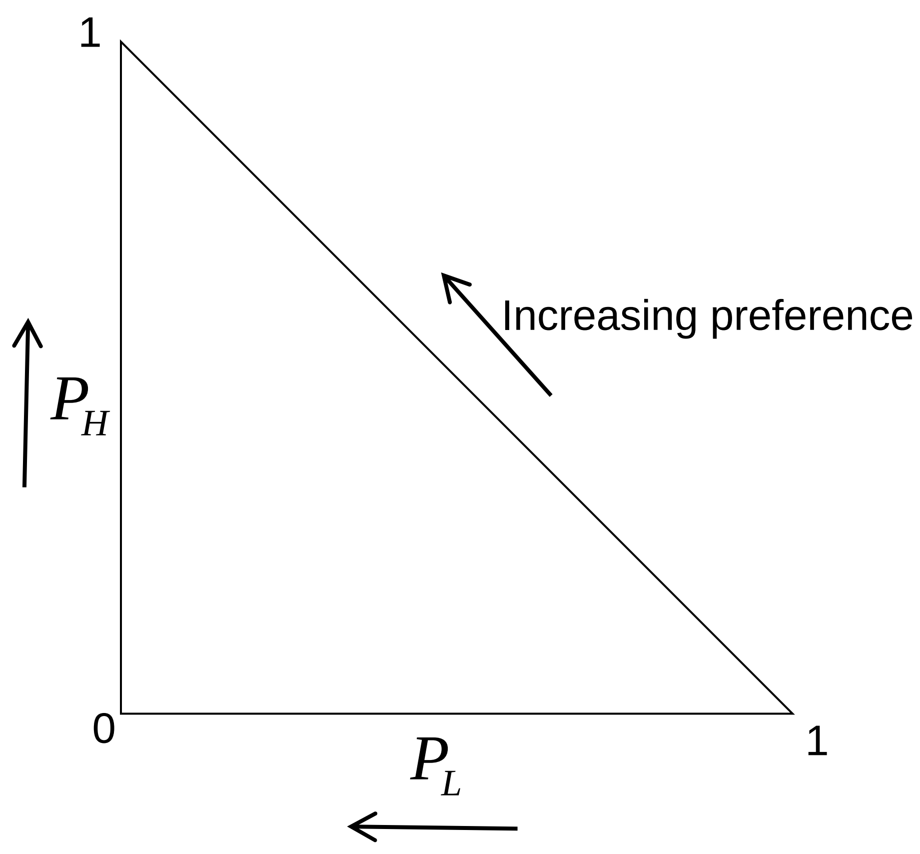
$$x := \begin{array}{l} \overset{.33}{\nearrow} \quad \$25,000 \\ \xrightarrow{.66} \quad \$24,000 \\ \underset{.01}{\searrow} \quad \$0 \end{array} \qquad y := \xrightarrow{1} \$24,000$$

Allais (1953) II

– Choose between the two gambles:

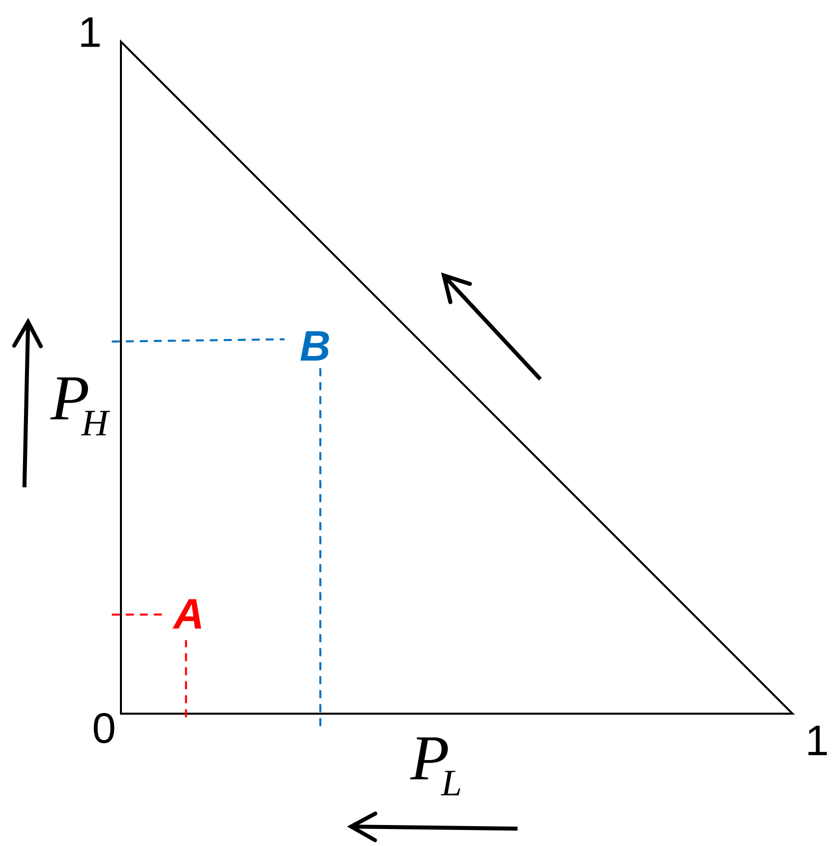
$$z := \begin{array}{ccc} & & \$25,000 \\ & \overset{.33}{\nearrow} & \\ & & \\ & \underset{.67}{\searrow} & \\ & & \$0 \end{array} \qquad w := \begin{array}{ccc} & & \$24,000 \\ & \overset{.34}{\nearrow} & \\ & & \\ & \underset{.66}{\searrow} & \\ & & \$0 \end{array}$$

The (Marschak-Machina) probability triangle



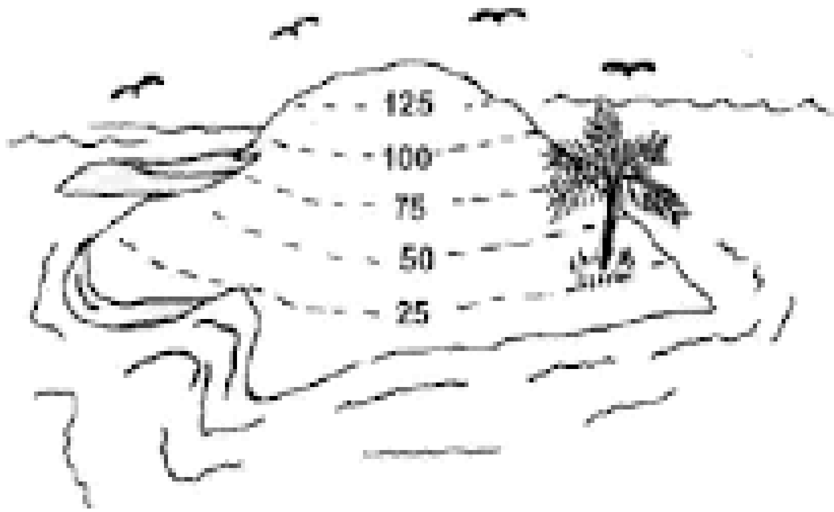
Consider three monetary payouts H, *M*, and *L* where *H>M>L*

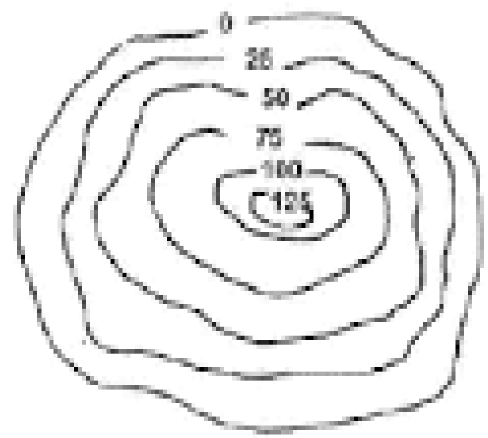
Risk profiling



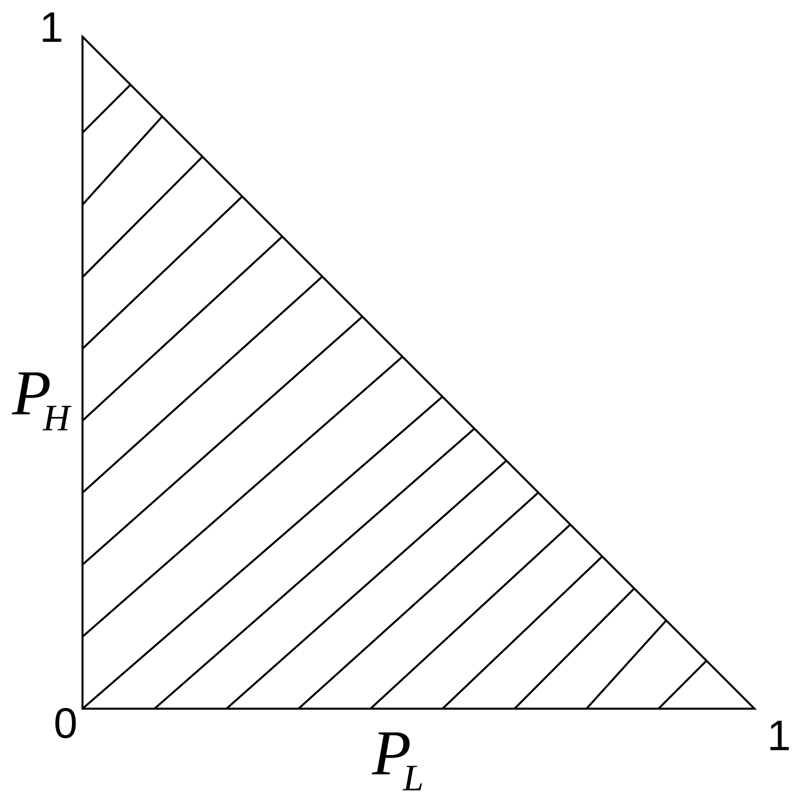
A “complete” risk profiling requires knowing all possible comparisons like between *A* and *B*.

A topographic map



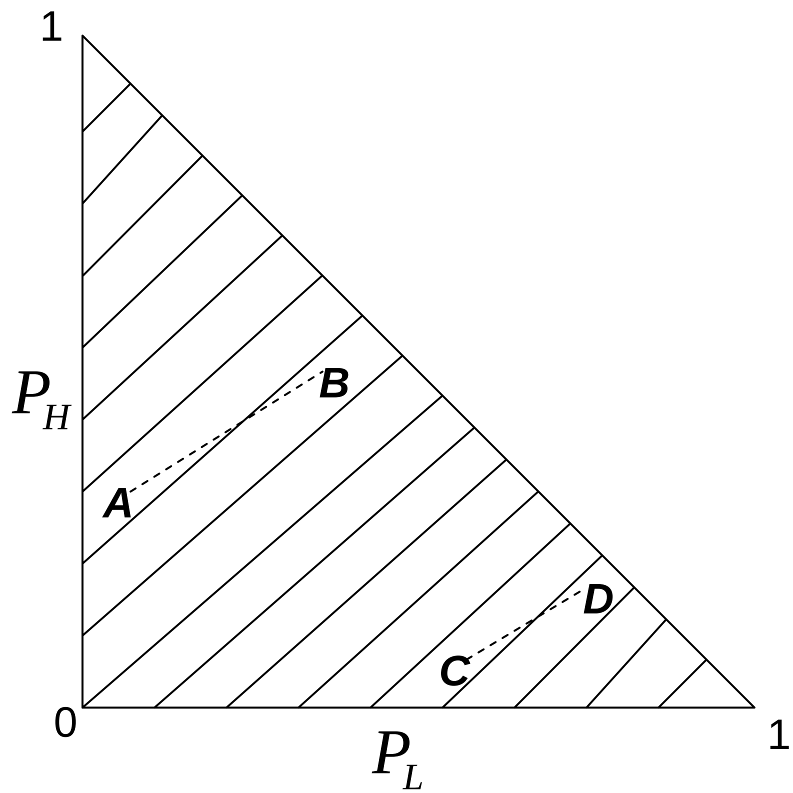


An indifference map of a loss-neutral (expected utility) individual



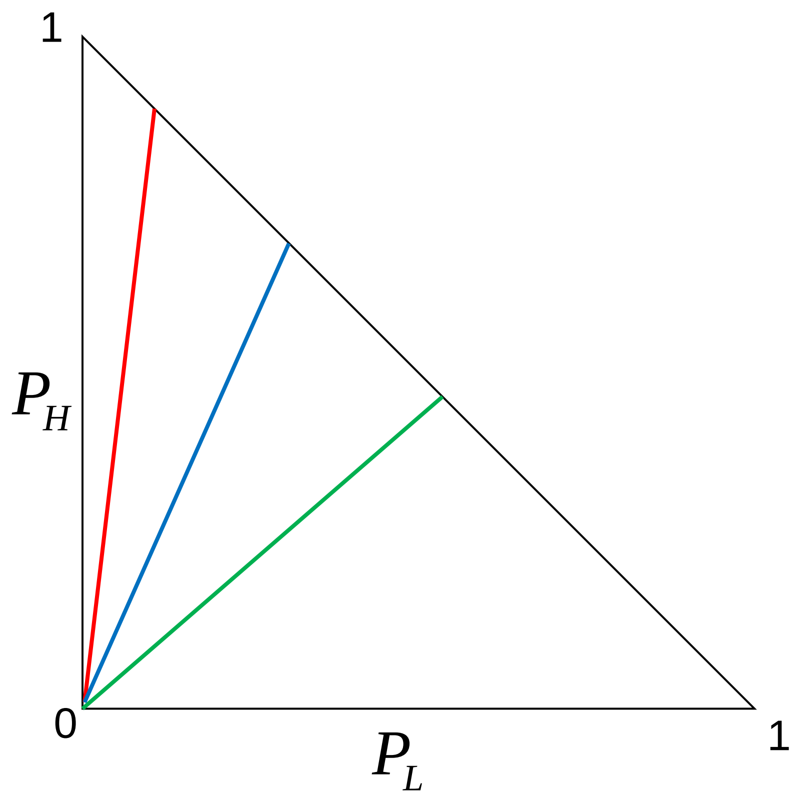
Expected Utility Theory (EUT) requires that indifference lines are parallel

A test of Expected Utility Theory (EUT)



EUT requires that indifference lines are parallel so one must choose either ***A*** and ***C***, or ***B*** and ***D***.

Loss neutral and more risk tolerant



Mr. Green is more risk tolerant than Mr. Blue who is more risk tolerant than Mr. Red. The gentlemen are loss neutral.

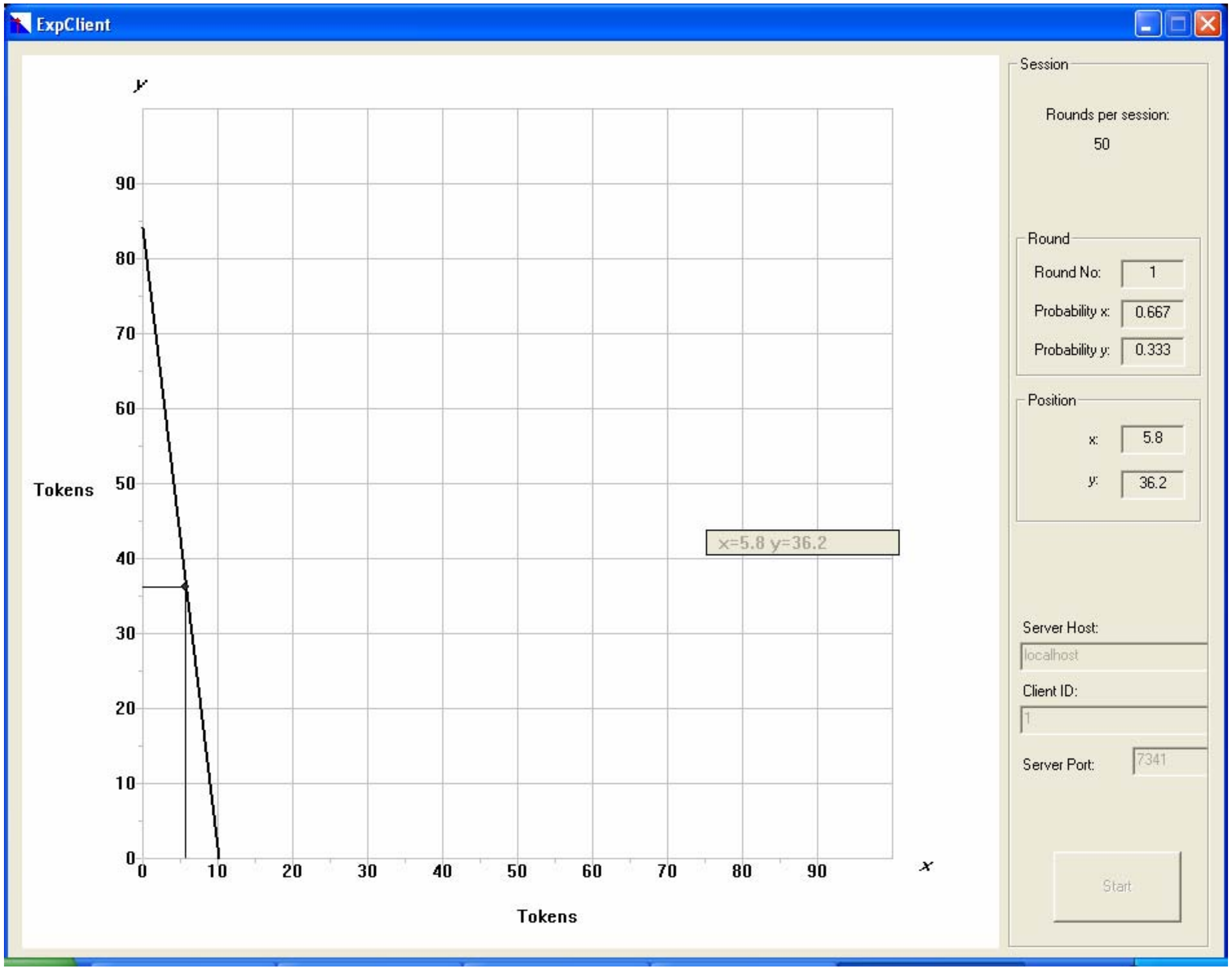
A new experimental design

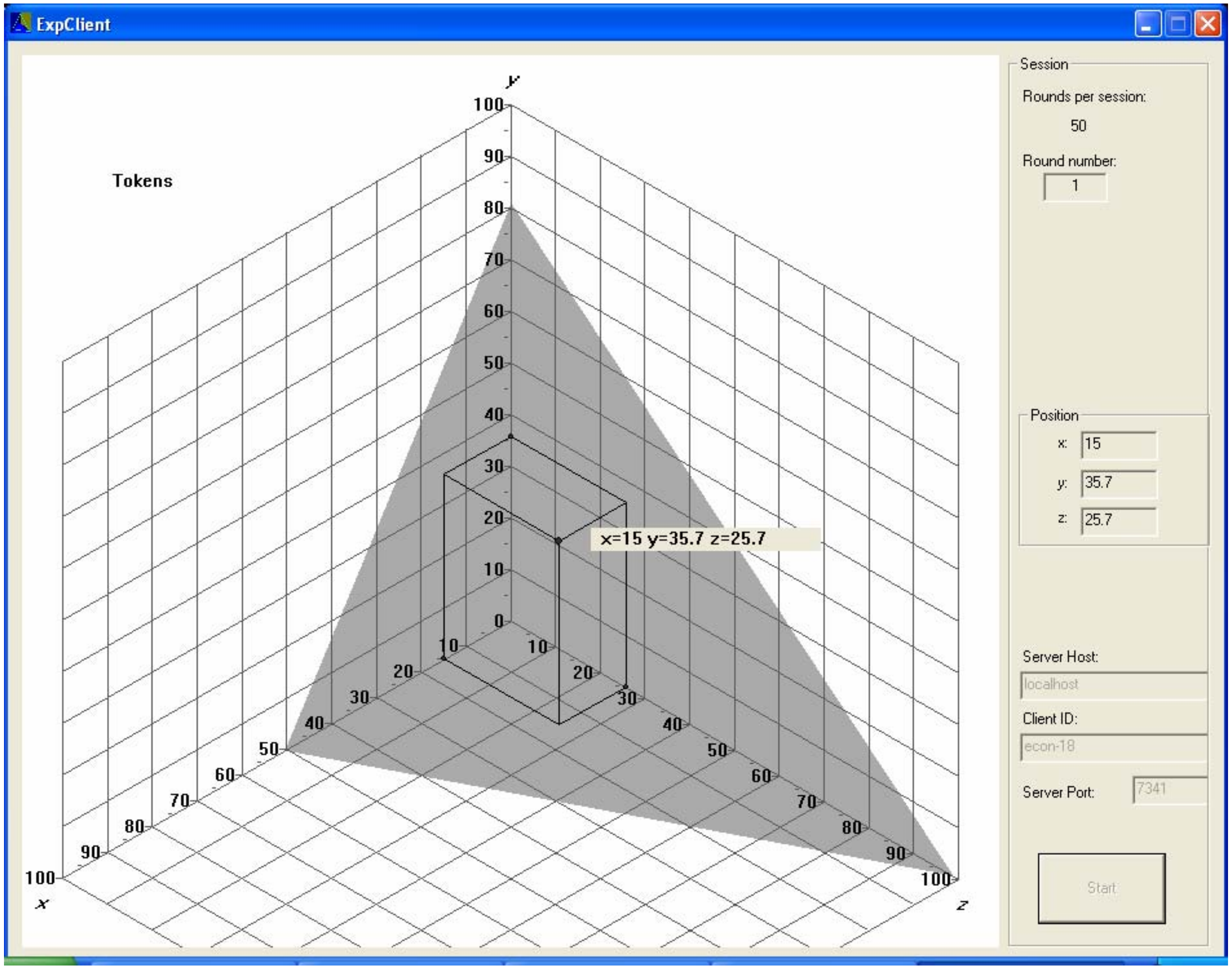
An experimental design that has a couple of fundamental innovations over previous work:

- A selection of a bundle of contingent commodities from a budget set (a portfolio choice problem).

- A graphical experimental interface that allows for the collection of a rich individual-level data set.

The experimental computer program dialog windows




ExpClient
Tokens
x=15 y=35.7 z=25.7
Session
Rounds per session:
50
Round number:
1
Position
x: 15
y: 35.7
z: 25.7
Server Host:
localhost
Client ID:
econ-18
Server Port:
7341
Start

Rationality

Let $\{(p^i, x^i)\}_{i=1}^{50}$ be some observed individual data (p^i denotes the i-th observation of the price vector and x^i denotes the associated portfolio).

A utility function $u(x)$ *rationalizes* the observed behavior if it achieves the maximum on the budget set at the chosen portfolio

$$u(x^i) \geq u(x) \text{ for all } x \text{ s.t. } p^i \cdot x^i \geq p^i \cdot x.$$

Revealed preference

A portfolio x^i is *directly revealed preferred* to a portfolio x^j if $p^i \cdot x^i \geq p^i \cdot x^j$, and x^i is *strictly directly revealed preferred* to x^j if the inequality is strict.

The relation *indirectly revealed preferred* is the transitive closure of the directly revealed preferred relation.

Generalized Axiom of Revealed Preference (GARP) *If* x^i *is indirectly revealed preferred to* x^j*, then* x^j *is not strictly directly revealed preferred (i.e.* $p^j \cdot x^j \leq p^j \cdot x^i$*) to* x^i*.*

GARP is tied to utility representation through a theorem, which was first proved by Afriat (1967).

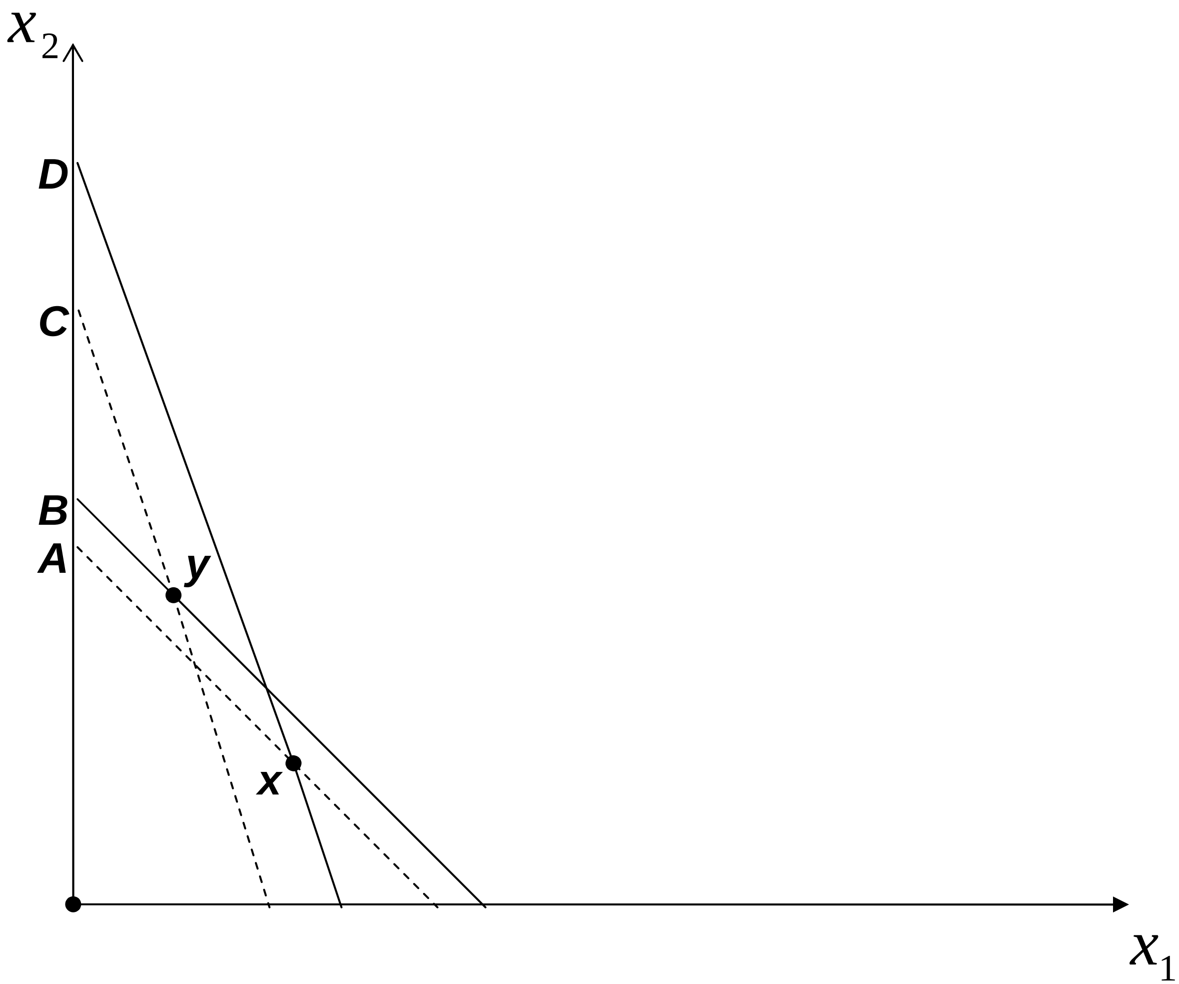
Afriat's Theorem *The following conditions are equivalent:*

- *The data satisfy GARP.*
- *There exists a non-satiated utility function that rationalizes the data.*
- *There exists a concave, monotonic, continuous, non-satiated utility function that rationalizes the data.*

Afriat's critical cost efficiency index (CCEI) *The amount by which each budget constraint must be relaxed in order to remove all violations of GARP.*

The CCEI is bounded between zero and one. The closer it is to one, the smaller the perturbation required to remove all violations and thus the closer the data are to satisfying GARP.

The construction of the CCEI for a simple violation of GARP



The agent is ‘wasting' as much as *A/B*<*C/D* of his income by making inefficient choices.

A benchmark level of consistency

A random sample of hypothetical subjects who implement the power utility function

$$u(x) = \frac{x^{1-\rho}}{1-\rho},$$

commonly employed in the empirical analysis of choice under uncertainty, with error.

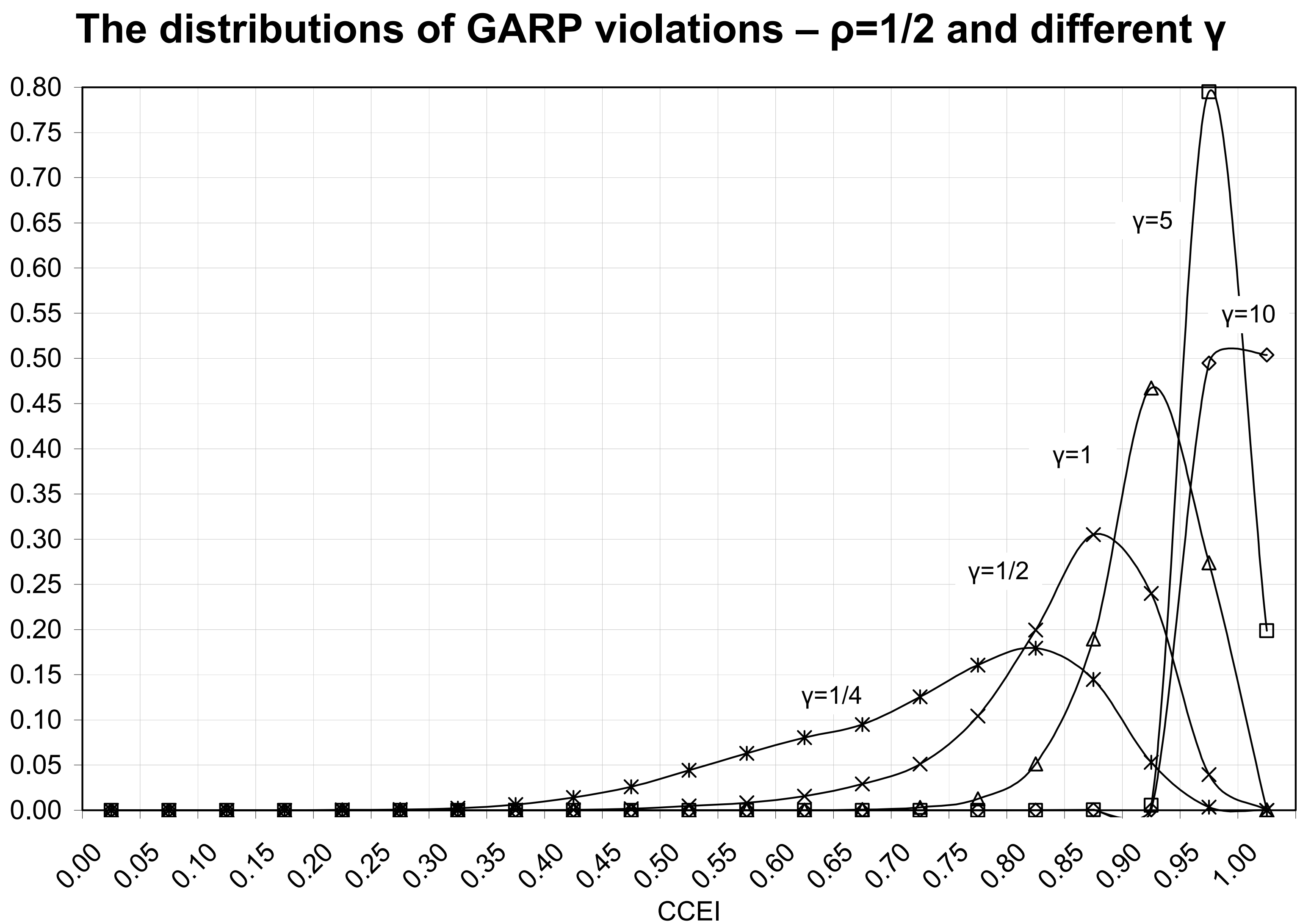
The likelihood of error is assumed to be a decreasing function of the utility cost of an error.

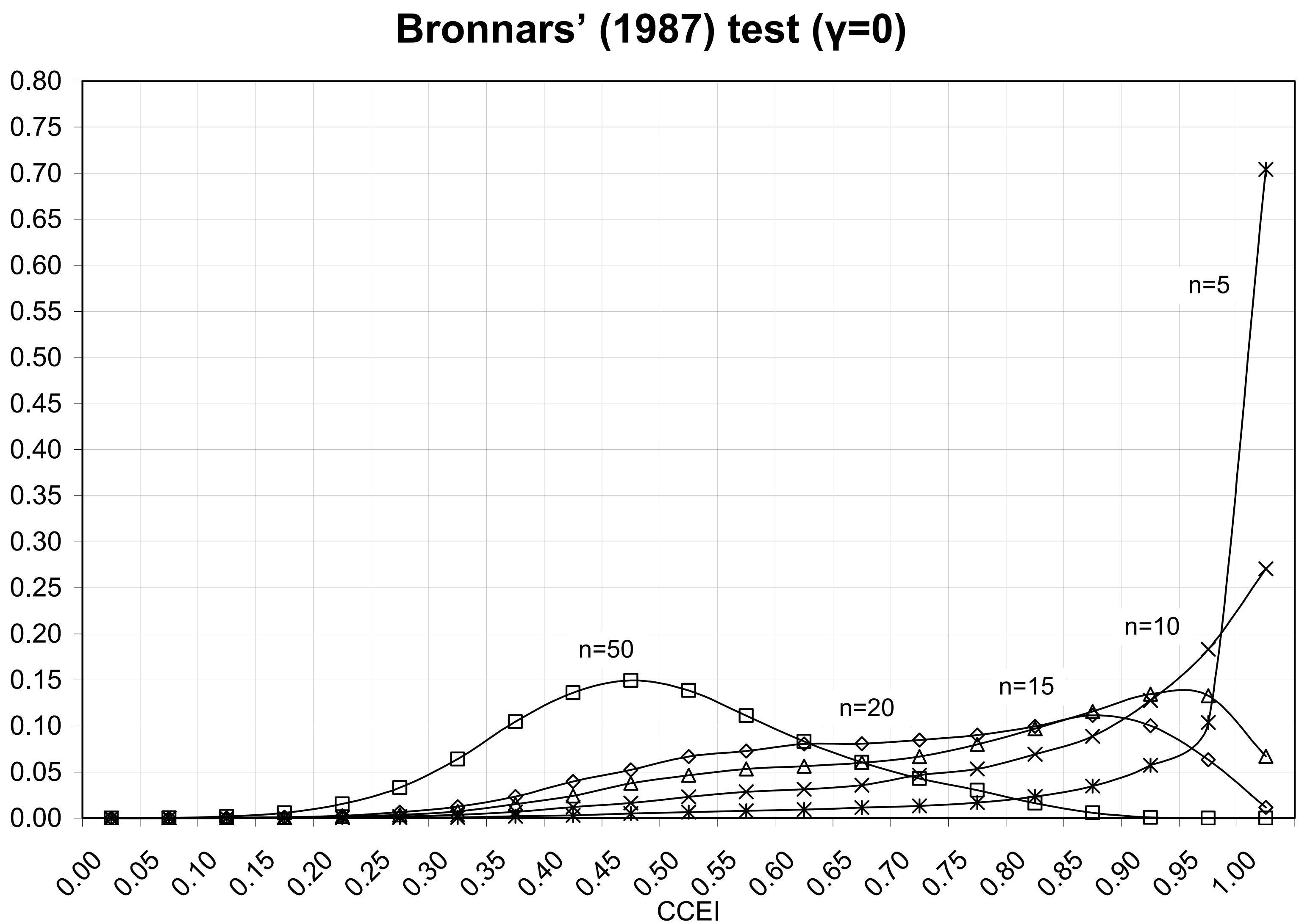
More precisely, we assume an idiosyncratic preference shock that has a logistic distribution

$$\mathsf{Pr}(x^*) = \frac{e^{\gamma \cdot u(x^*)}}{\int\limits_{x:p\cdot x=1} e^{\gamma \cdot u(x)}},$$

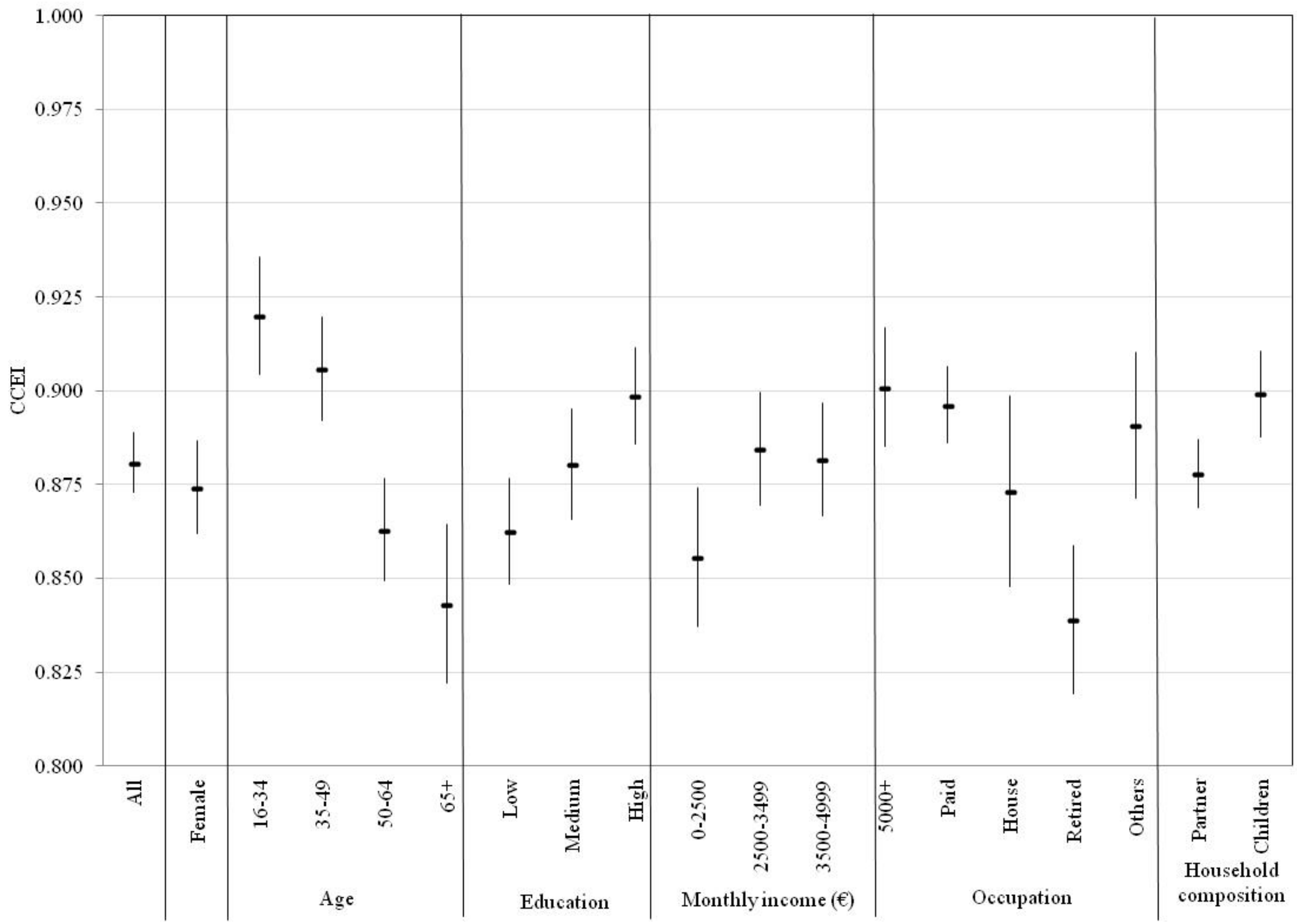
where the precision parameter γ reflects sensitivity to differences in utility.

If utility maximization is not the correct model, is our experiment sufficiently powerful to detect it?


The distributions of GARP violations – ρ=1/2 and different γ
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
γ=5
γ=10
γ=1
γ=1/2
γ=1/4
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
CCEI


Bronnars' (1987) test (γ=0)
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
n=5
n=10
n=15
n=20
n=50
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
CCEI

Homo Economicus: equiprobable lotteries



Wealth differentials

$\Longrightarrow$ The heterogeneity in wealth is not well-explained either by standard observables (income, education, family structure) or by standard unobservables (intertemporal substitution, risk tolerance).

$\Longrightarrow$ If consistency with utility maximization in the experiment is a good proxy for (financial) DMQ then the degree to which consistency differ across subjects should help explain wealth differentials.

The relationship between CCEI scores and wealth

	(1)	(2)	(3)
CCEI	1.351**	1.109**	101888.0*
	(0.566)	(0.534)	(52691.9)
Log 2008 household income	0.584***	0.606***	
	(0.132)	(0.126)	
2008 household income			1.776***
			(0.4)
Female	-0.313*	-0.356**	-32484.3*
	(0.177)	(0.164)	(17523.9)
Partnered	0.652***	0.595***	46201.9***
	(0.181)	(0.171)	(17173.7)
# of children	0.090	0.109	14078.6*
	(0.093)	(0.086)	(8351.5)
Age	Y	Y	Y
Education	Y	Y	Y
Occupation	Y	Y	Y
Constant	6.292	0.469	76214.4
	(6.419)	(3.598)	(559677.5)
R^2	0.179	0.217	0.188
# of obs.	517	566	568

The robustness of the correlation -- controls for constraints

	(1)	(2)	(3)	(4)	(5)
CCEI	1.322**	1.318**	1.925***	1.888***	1.441**
	(0.570)	(0.574)	(0.672)	(0.652)	(0.578)
Log household income					
2008	19.770	1.000	0.544***	0.285*	0.616***
	(14.629)	.	(0.137)	(0.165)	(0.128)
2008^2	-2.194				
	(1.533)				
2008^3	0.082				
	(0.053)				
2006				0.232	
				(0.231)	
2004				0.215	
				(0.174)	
Female	-0.291	-0.201	-0.337*	-0.296	-0.321*
	(0.181)	(0.173)	(0.185)	(0.186)	(0.176)
Partnered	0.598***	0.561***	0.734***	0.707***	0.641***
	(0.181)	(0.178)	(0.192)	(0.193)	(0.179)
# of children	0.091	0.101	0.018	0.031	0.088
	(0.092)	(0.096)	(0.099)	(0.095)	(0.093)
Age	Y	Y	Y	Y	Y
Education	Y	Y	Y	Y	N
Occupation	Y	Y	Y	Y	Y
Constant	-47.059	0.864	5.354	3.016	6.398
	(46.275)	(6.545)	(6.93)	(7.109)	(6.484)
R^2	0.187		0.205	0.217	0.177
# of obs.	517	517	449	449	517

The robustness of the correlation -- controls for preferences and beliefs

	(1)	(2)	(3)	(4)	(5)
CCEI	1.379**	1.396**	1.404**	1.214*	1.237**
	(0.568)	(0.568)	(0.569)	(0.625)	(0.623)
Risk tolerance					
Quantitative (experiment)	-0.768	-0.808	-0.766		
	(0.714)	(0.711)	(0.718)		
Qualitative (survey)		0.017	0.023		
		(0.074)	(0.076)		
Qualitative (survey) missing		-0.190	-0.162		
		(0.335)	(0.482)		
Conscientiousness			0.089		
			(0.072)		
Conscientiousness missing			-0.040		
			(0.668)		
Longevity expectations					-0.034
					(0.040)
Log 2008 household income	0.589***	0.578***	0.572***	0.443***	0.434***
	(0.132)	(0.131)	(0.133)	(0.123)	(0.123)
Female	-0.316*	-0.310*	-0.323*	-0.415**	-0.417**
	(0.177)	(0.181)	(0.181)	(0.186)	(0.186)
Partnered	0.655***	0.658***	0.642***	0.686***	0.687***
	(0.181)	(0.181)	(0.182)	(0.204)	(0.205)
# of children	0.086	0.087	0.083	0.075	0.083
	(0.093)	(0.093)	(0.093)	(0.102)	(0.102)
Age	Y	Y	Y	Y	Y
Education	Y	Y	Y	Y	Y
Occupation	Y	Y	Y	Y	Y
Constant	6.840	6.883	6.496	3.777	4.411
	(6.361)	(6.357)	(6.395)	(15.258)	(15.256)
R^2	0.179	0.176	0.176	0.163	0.163
# of obs.	517	517	517	414	414

Evaluating alternative measures of *DMQ*

	(1)	(2)	(3)	(4)
CCEI	1.253*	1.401*	1.269*	1.177**
	(0.712)	(0.729)	(0.729)	(0.583)
CCEI (combined dataset)	0.099			
	-0.38			
von Gaudecker et al. (2011)			0.927*	
			(0.485)	
Cognitive Reflection Test (CRT)				0.120*
				(0.071)
CRT missing				-0.203
				(0.237)
Log 2008 household income	0.586***	0.388*	0.383*	0.577***
	(0.132)	(0.155)	(0.154)	(0.132)
Female	-0.314*	-0.218	-0.207	-0.292*
	(0.177)	(0.212)	(0.211)	(0.176)
Partnered	0.653***	0.907***	0.926***	0.690***
	(0.181)	(0.230)	(0.228)	(0.181)
# of children	0.089	0.105	0.096	0.091
	(0.093)	(0.114)	(0.113)	(0.092)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	6.237	10.056	8.355	6.855
	(6.424)	(6.976)	(6.990)	(6.464)
R^2	0.177	0.225	0.232	0.181
# of obs.	517	326	326	517

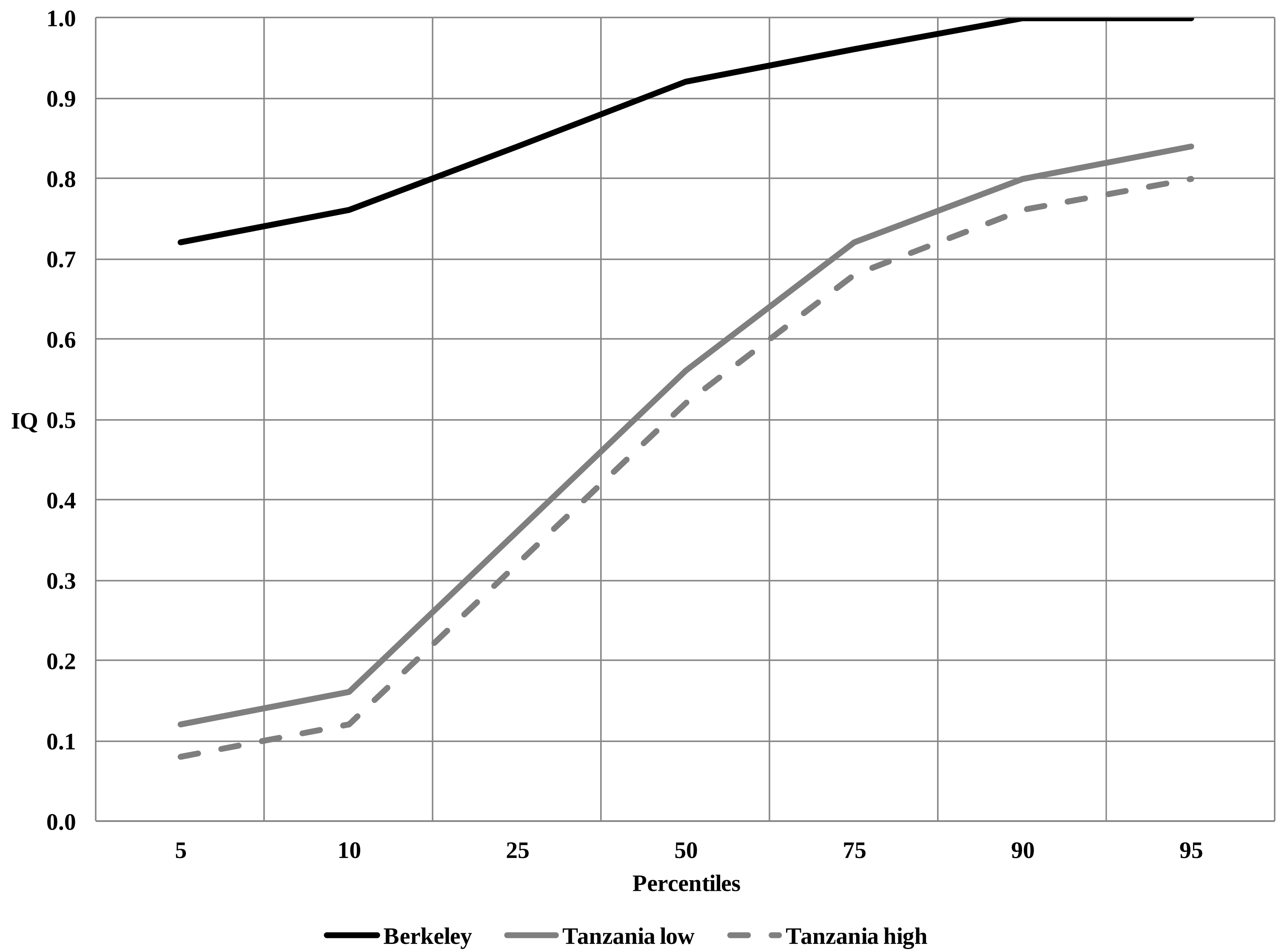
The sources of the relationship

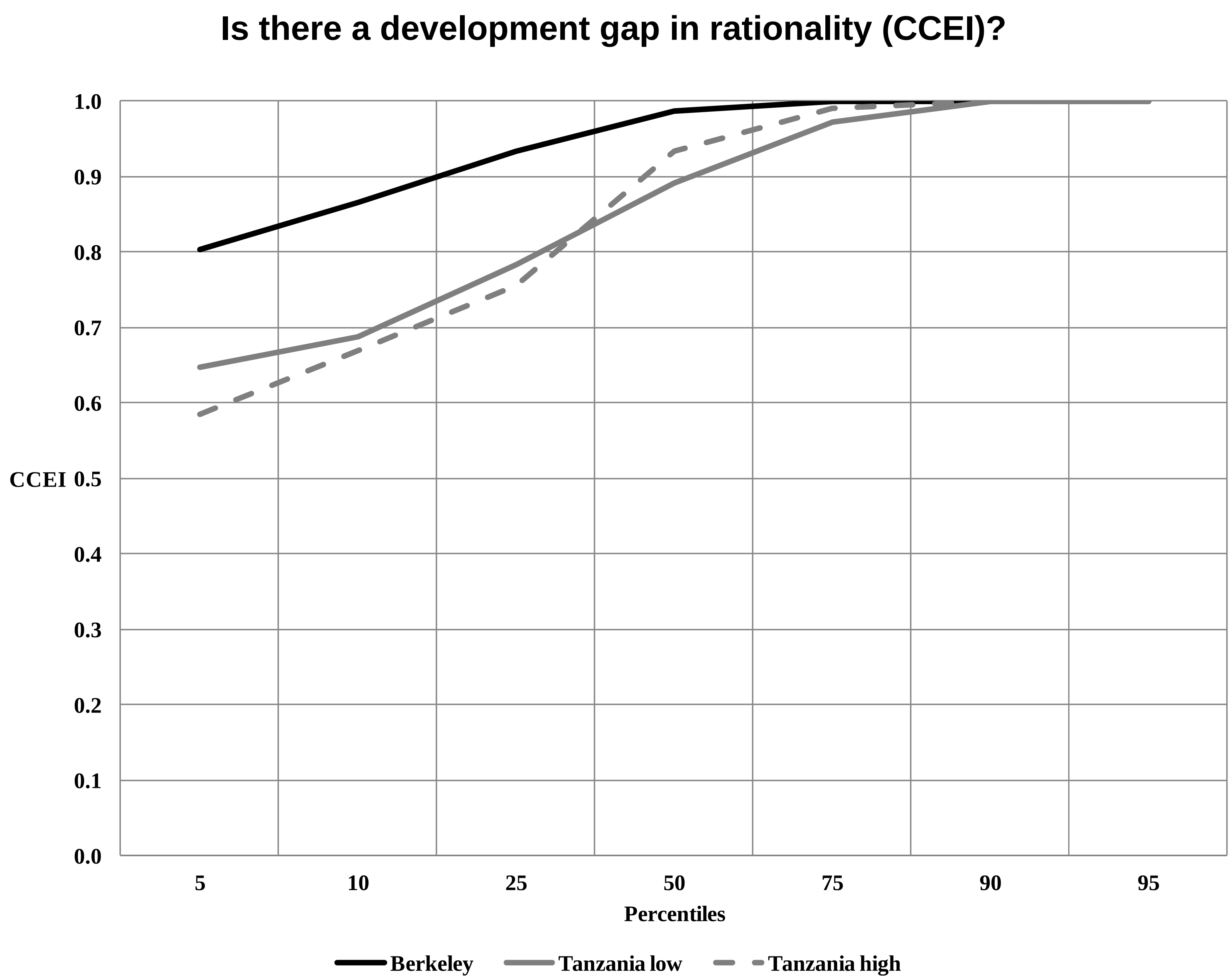
	(1)	(2)	(3)	(4)
	Have checking	Fraction in checking	Have saving	Fraction in saving
CCEI	0.03	-0.098*	-0.047	-0.162*
	(0.032)	(0.057)	(0.053)	(0.097)
Log 2008 household income	0.001	-0.029**	0.003	-0.068***
	(0.002)	(0.013)	(0.010)	(0.021)
Female	0.007	0.023	0.014	0.038
	(0.005)	(0.020)	(0.019)	(0.033)
Partnered	-0.005	-0.031	0.017	-0.054
	(0.004)	(0.020)	(0.022)	(0.033)
# of children	0.000	-0.004	-0.025*	-0.043***
	(0.001)	(0.010)	(0.014)	(0.013)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	0.998***	0.106	1.126	1.448
	(0.172)	(0.822)	(0.848)	(1.288)
R^2	-0.007	0.021	-0.011	0.083
# of obs.	512	512	502	502

The sources of the relationship (cont.)

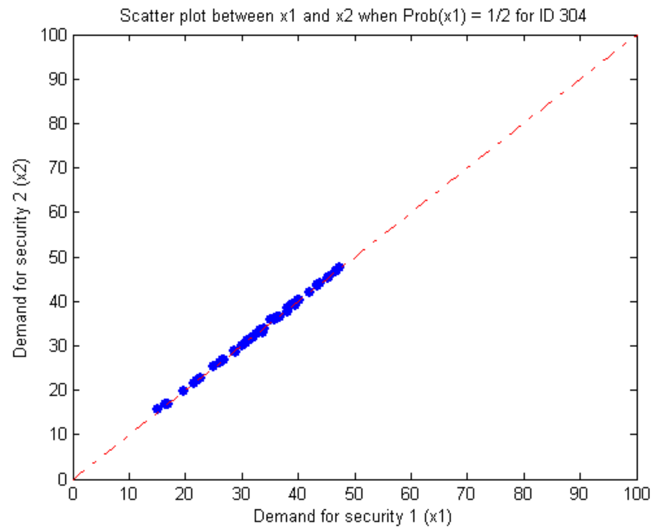
	(5)	(6)	(7)	(8)
	Have stocks	Fraction in stocks	Have a house	Fraction in house
CCEI	0.167	0.001	0.352**	0.324**
	(0.163)	(0.050)	(0.152)	(0.129)
Log 2008 household income	0.148***	0.013	0.134***	0.096***
	(0.031)	(0.009)	(0.029)	(0.024)
Female	0.007	0.009	-0.038	-0.066
	(0.050)	(0.013)	(0.050)	(0.043)
Partnered	0.005	-0.007	0.207***	0.127***
	(0.049)	(0.014)	(0.051)	(0.044)
# of children	0.003	0.000	0.048**	0.063***
	(0.026)	(0.007)	(0.020)	(0.019)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	-3.152*	-0.317	-1.047	-1.151
	(1.856)	(0.398)	(1.760)	(1.419)
R^2	0.079	0.002	0.148	0.123
# of obs.	514	514	479	479

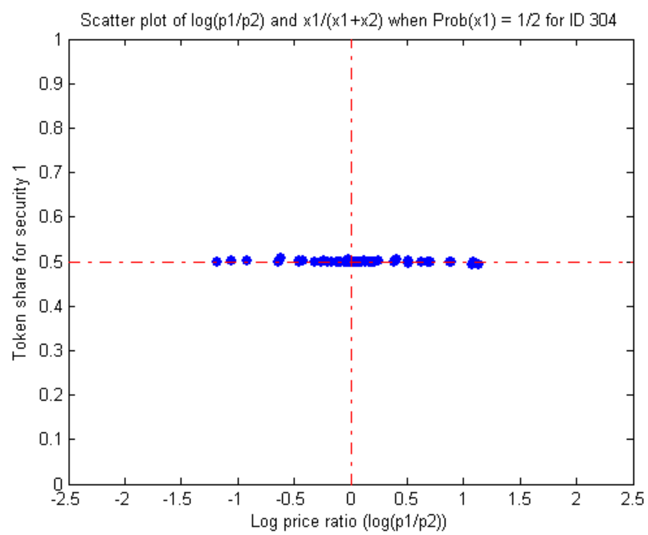
Is there a development gap in rationality (IQ)?

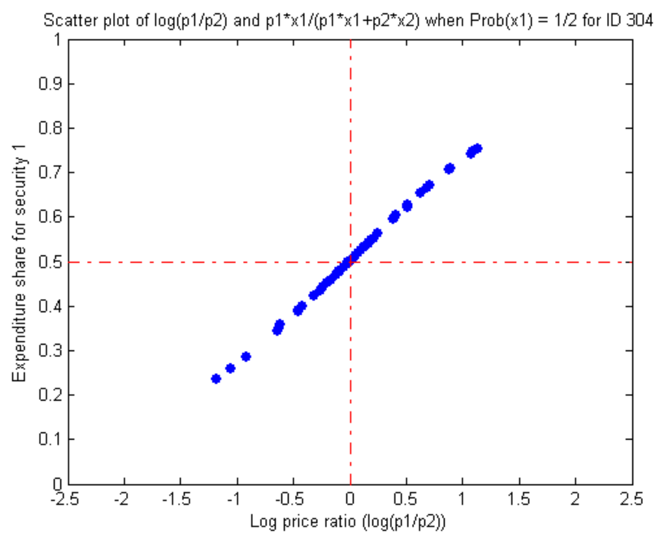


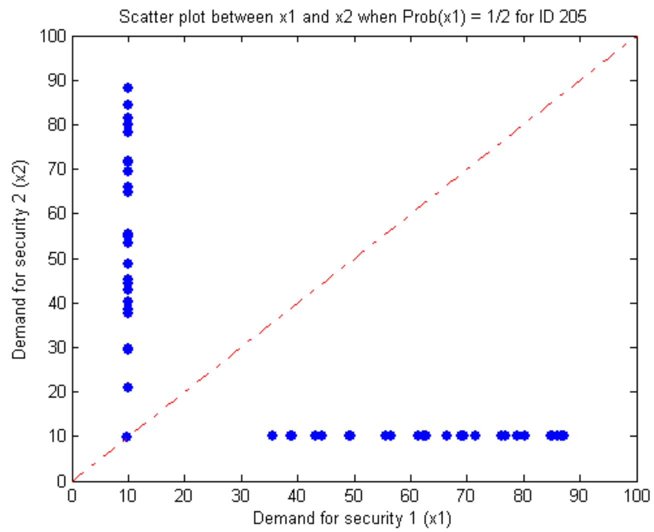

Is there a development gap in rationality (CCEI)?
CCEI
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
5
10
25
50
75
90
95
Percentiles
Berkeley
Tanzania low
Tanzania high

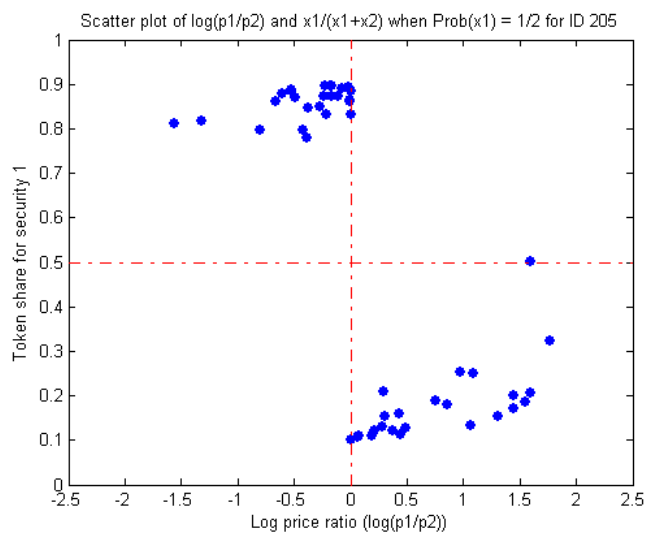
Individual-level data

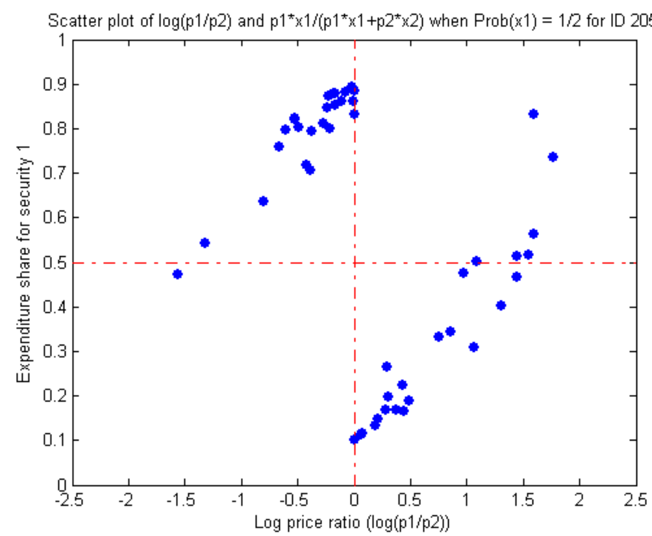


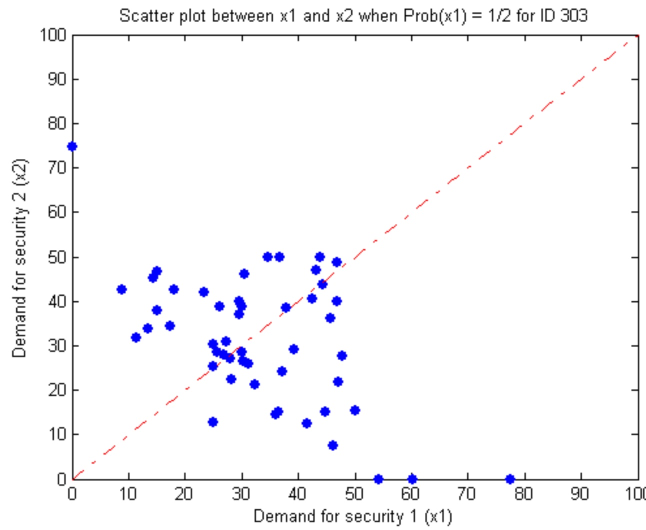


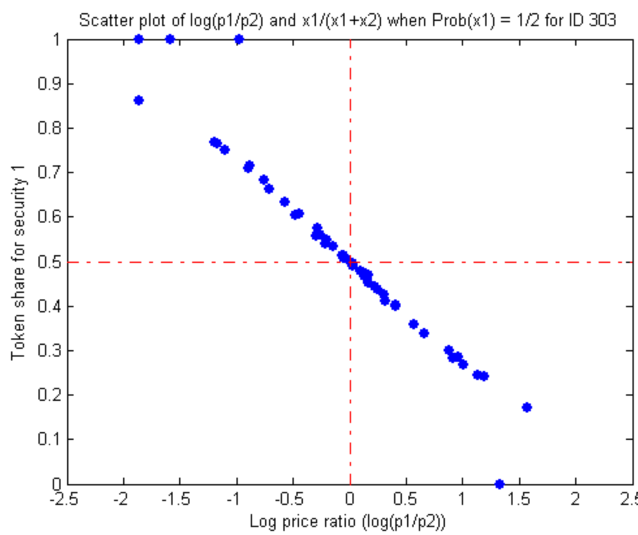


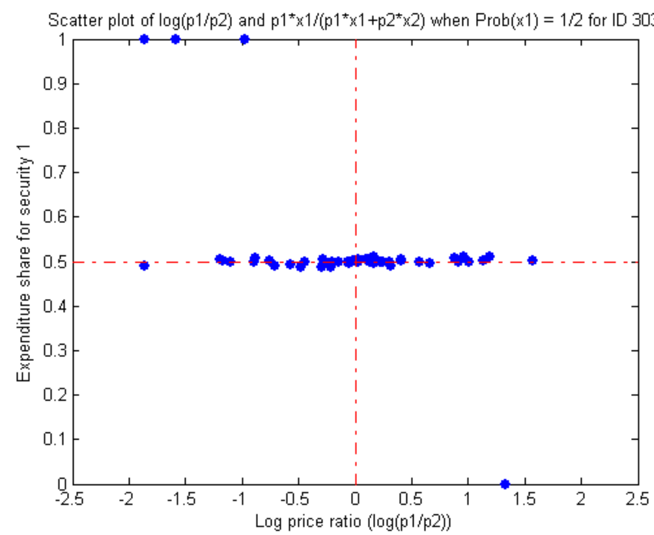

Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 205
Demand for security 2 (x2)
Demand for security 1 (x1)

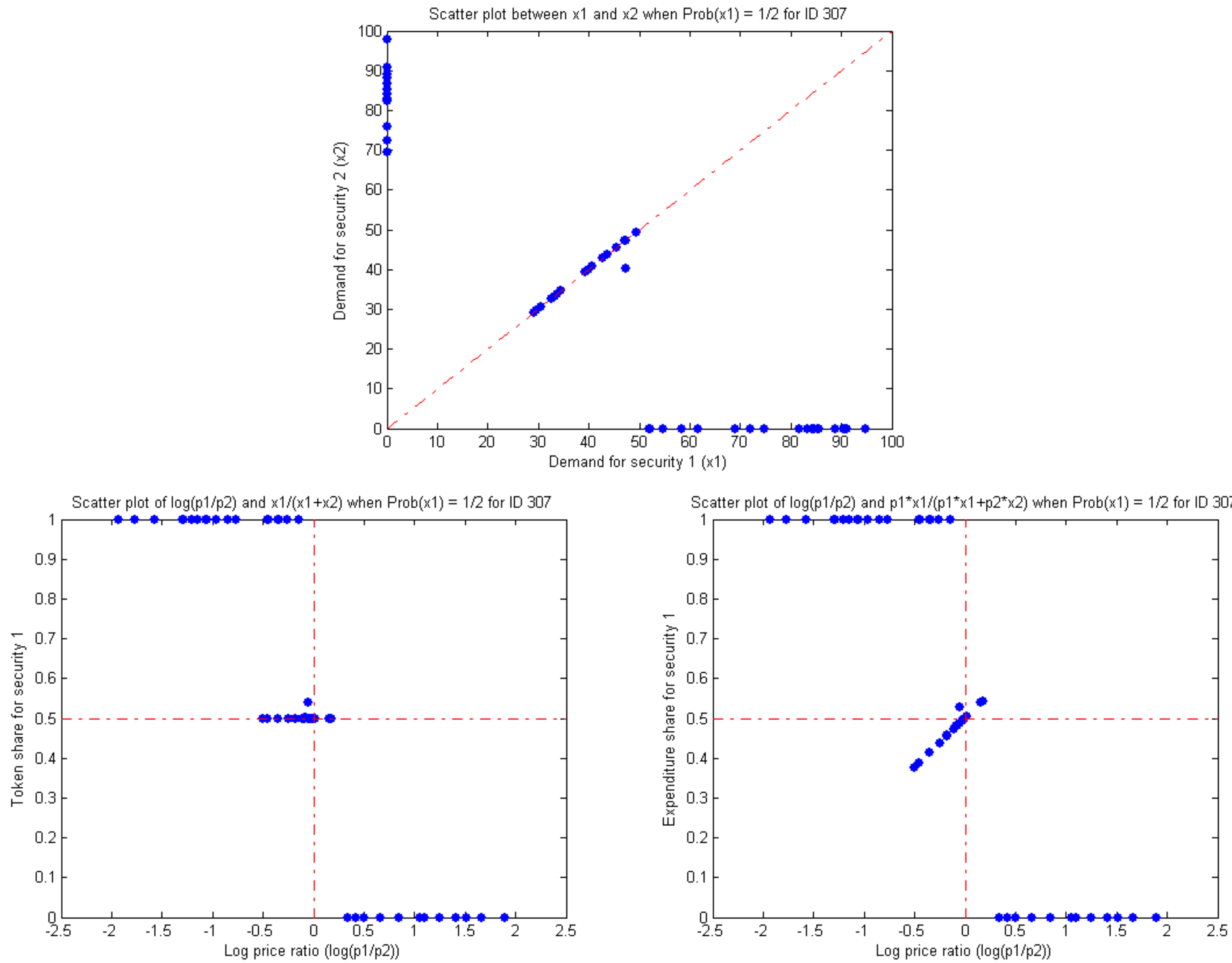

Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 205
Token share for security 1
Log price ratio (log(p1/p2))


Scatter plot of log(p1/p2) and p1*x1/(p1*x1+p2*x2) when Prob(x1) = 1/2 for ID 205
Expenditure share for security 1
Log price ratio (log(p1/p2))

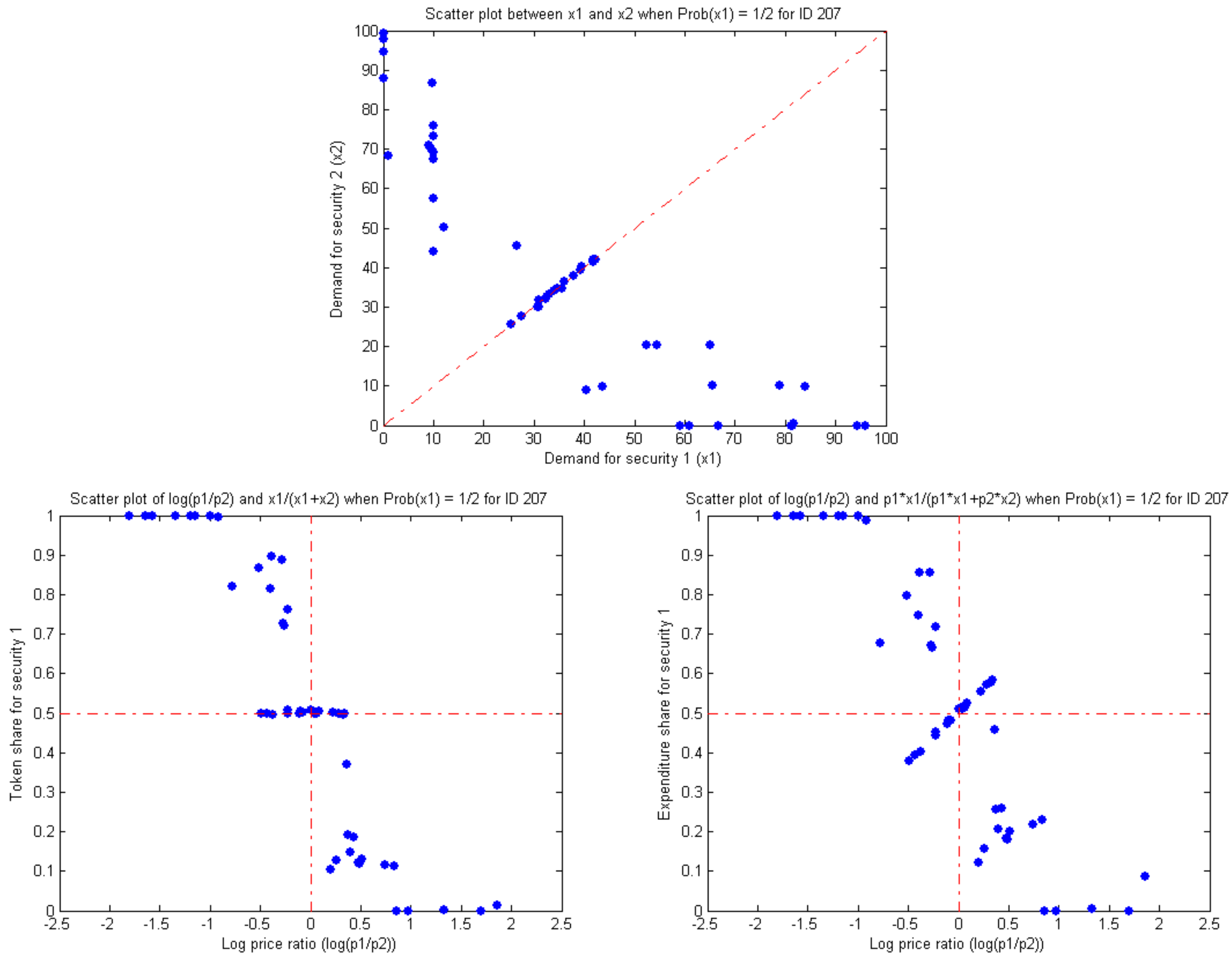

Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 303
Demand for security 2 (x2)
Demand for security 1 (x1)


Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 303
Token share for security 1
Log price ratio (log(p1/p2))

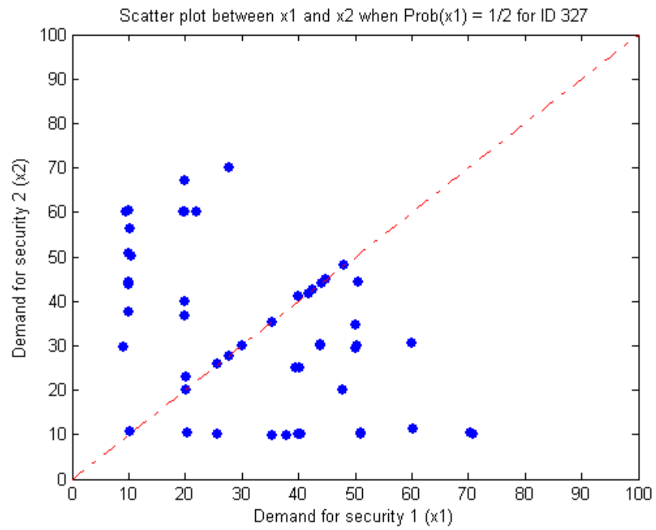

Scatter plot of log(p1/p2) and p1*x1/(p1*x1+p2*x2) when Prob(x1) = 1/2 for ID 303
Expenditure share for security 1
Log price ratio (log(p1/p2))



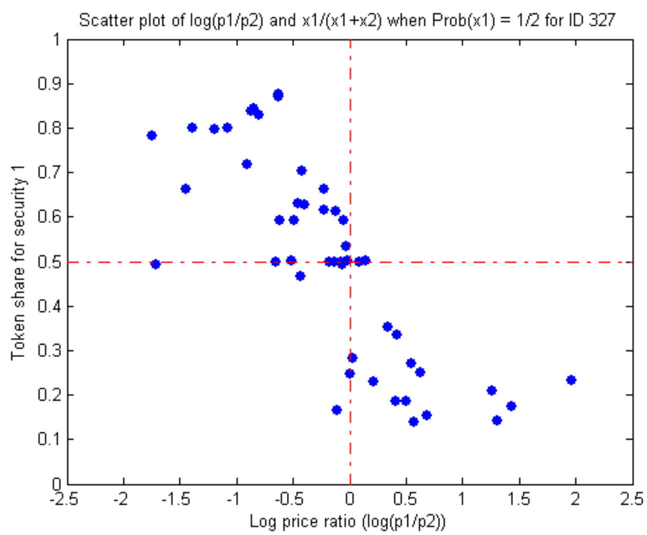
Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 307
Demand for security 2 (x2)
Demand for security 1 (x1)
Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 307
Token share for security 1
Log price ratio (log(p1/p2))
Scatter plot of log(p1/p2) and p1*x1/(p1*x1+p2*x2) when Prob(x1) = 1/2 for ID 307
Expenditure share for security 1
Log price ratio (log(p1/p2))



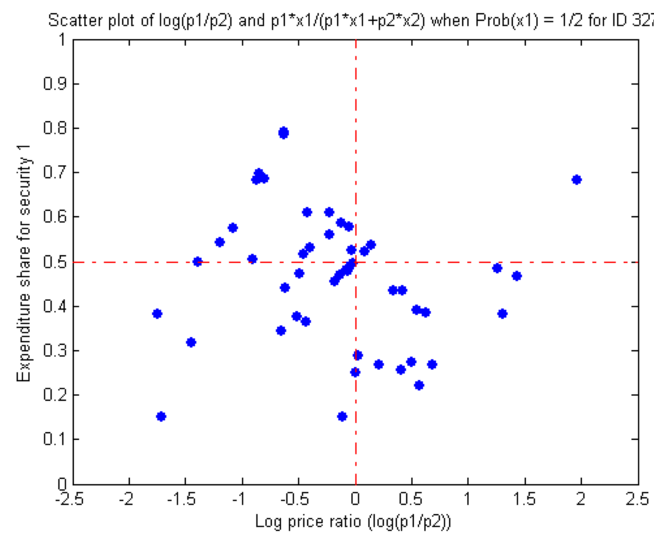
Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 207
Demand for security 2 (x2)
Demand for security 1 (x1)
Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 207
Token share for security 1
Log price ratio (log(p1/p2))
Scatter plot of log(p1/p2) and p1*x1/(p1*x1+p2*x2) when Prob(x1) = 1/2 for ID 207
Expenditure share for security 1
Log price ratio (log(p1/p2))



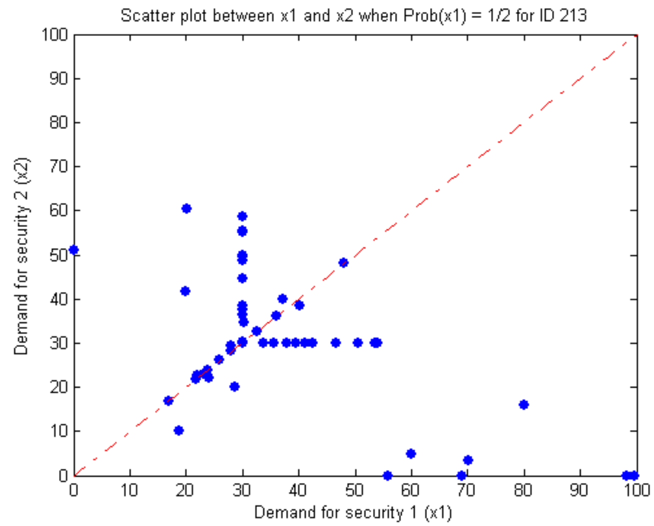
Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 327
Demand for security 2 (x2)
Demand for security 1 (x1)

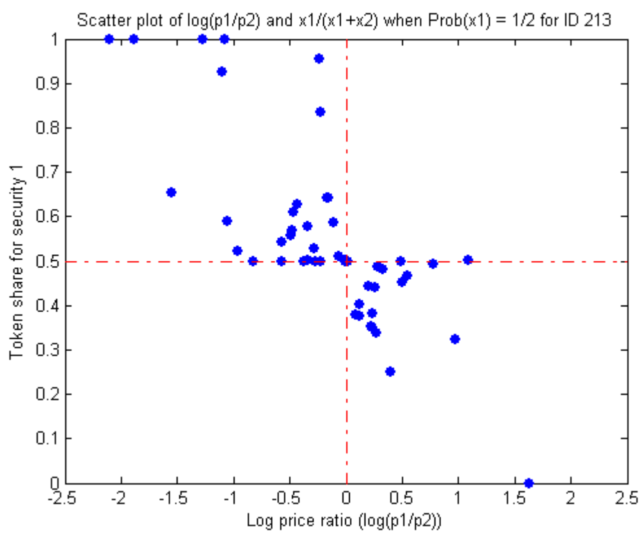


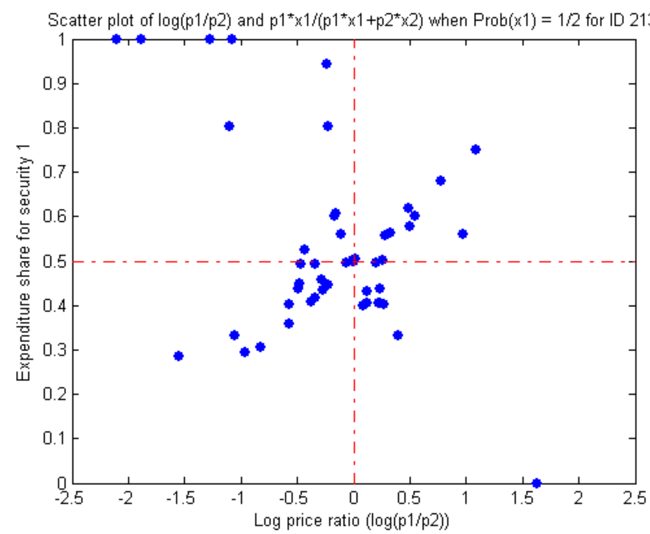
Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 327
Token share for security 1
Log price ratio (log(p1/p2))



Scatter plot of log(p1/p2) and p1*x1/(p1*x1+p2*x2) when Prob(x1) = 1/2 for ID 327
Expenditure share for security 1
Log price ratio (log(p1/p2))


Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 213
Demand for security 2 (x2)
Demand for security 1 (x1)


Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 213
Token share for security 1
Log price ratio (log(p1/p2))


Scatter plot of log(p1/p2) and p1*x1/(p1*x1+p2*x2) when Prob(x1) = 1/2 for ID 213
Expenditure share for security 1
Log price ratio (log(p1/p2))

Loss aversion/tolerance

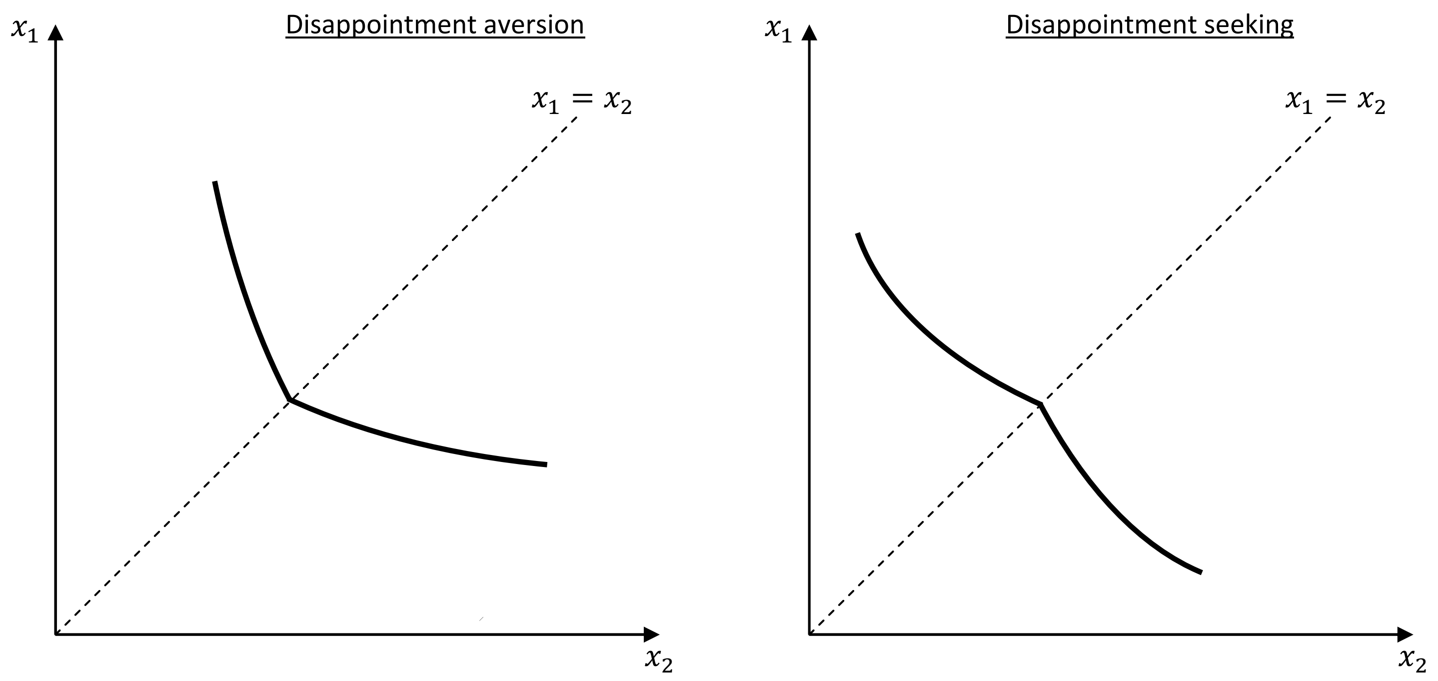
Suppose the underlying utility function over portfolios takes the form

$$\min\left\{\alpha u\left(x\right)+u(y), u(x)+\alpha u\left(y\right)\right\},$$

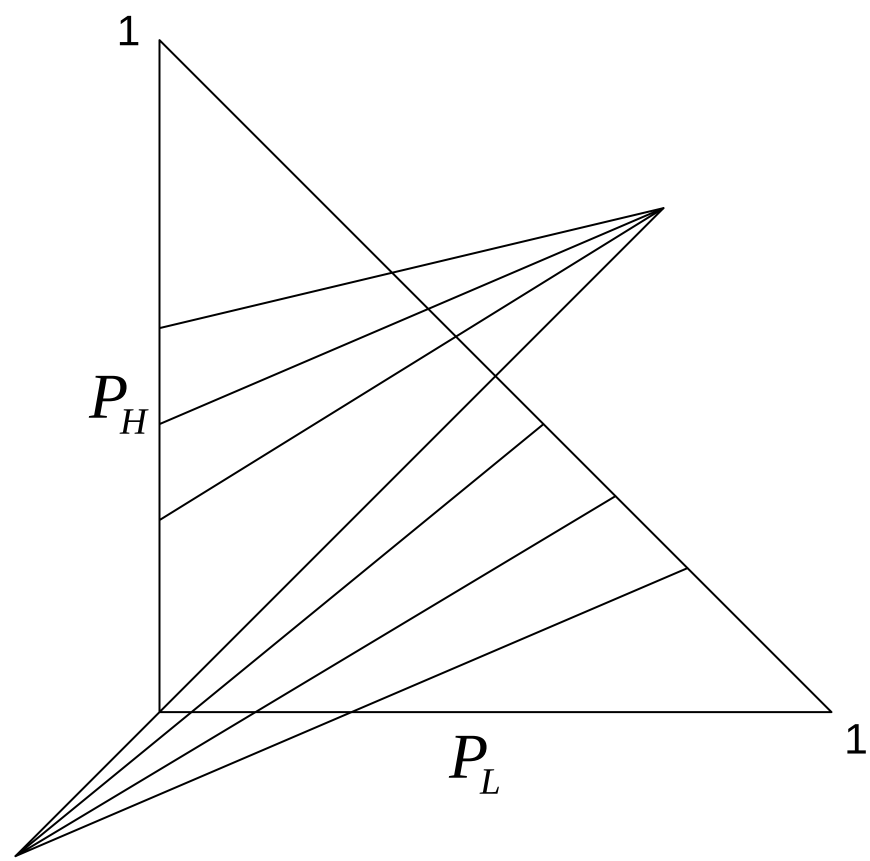
where $\alpha \geq 1$ measures loss aversion and $u(\cdot)$ measures risk aversion using CRRA or CARA.

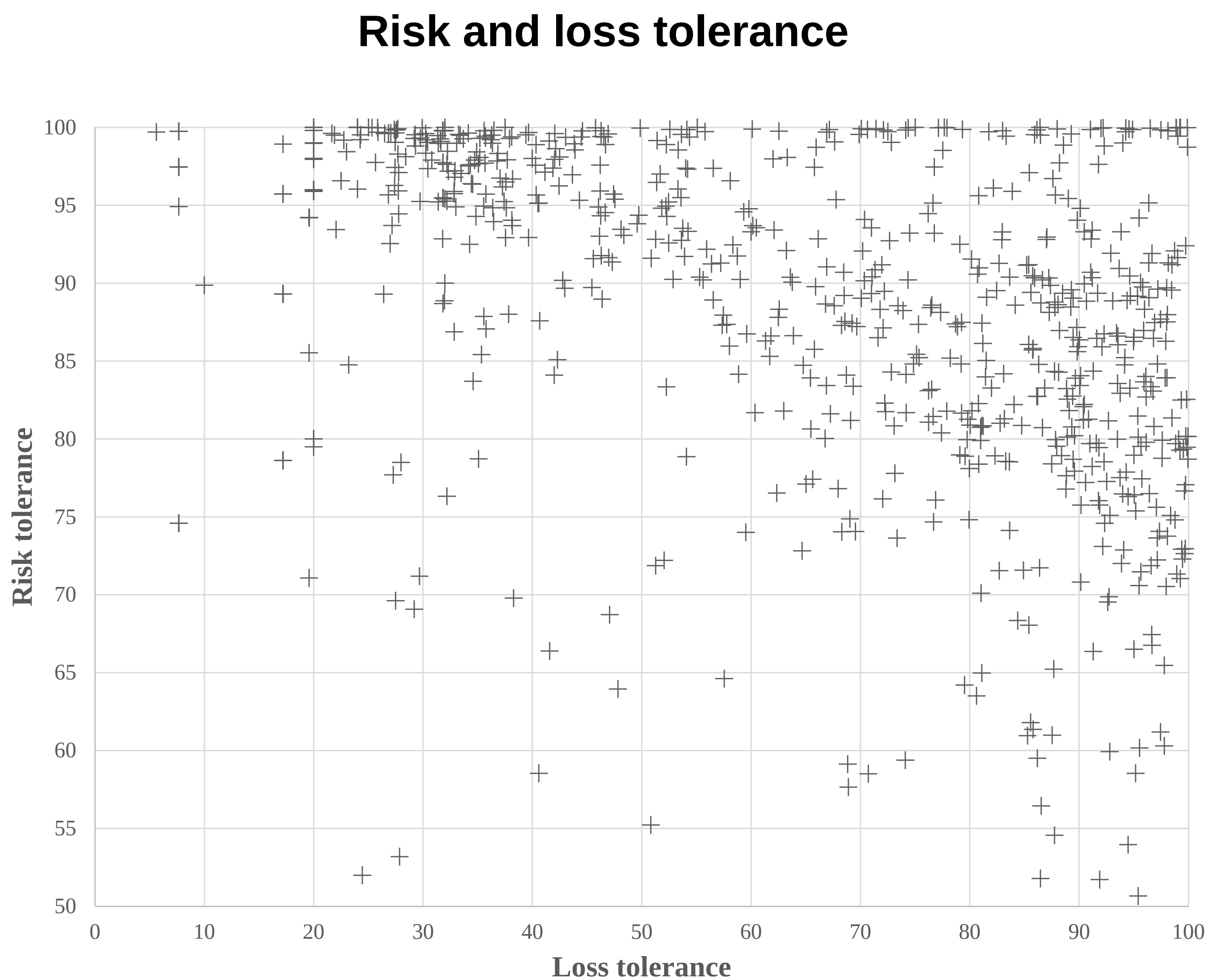
If $\alpha > 1$ there is a kink at the point where $x = y$ and if $\alpha = 1$ we have loss neutrality (standard EUT representation).

The indifference map of Gul (1991)



The indifference map of Gul (1991) in the Marschak-Machina triangle




Risk and loss tolerance
Risk tolerance
Loss tolerance
100
95
90
85
80
75
70
65
60
55
50
0
10
20
30
40
50
60
70
80
90
100

Ambiguity aversion

Ambiguity aversion

- The distinction between settings with risk and ambiguity dates back to at least the work of Knight (1921).

- Ellsberg (1961) countered the reduction of subjective uncertainty to risk with several thought experiments.

- A large theoretical literature (axioms over preferences) has developed models to accommodate this behavior.

Experiments à la Ellsberg

Consider the following four two-color Ellsberg-type urns (Halevy, 2007):

I. 5 red balls and 5 black balls

II. an unknown number of red and black balls

III. a bag containing 11 tickets with the numbers 0-10; the number written on the drawn ticket determines the number of red balls

IV. a bag containing 2 tickets with the numbers 0 and 10; the number written on the drawn ticket determines the number of red balls

A model of ambiguity aversion and loss/disappointment aversion

- If both loss aversion and ambiguity aversion are present in the data, we need a structural model in order to disentangle the two effects.

- In order to allow for kinks at portfolios where $x_s = x_{s'}$ for any $s \neq s'$, we make use of the rank-dependent utility (RDU) model of Quiggin (1982).

- This is a generalization of the SEU model that replaces probabilities with *decision weights* when calculating the value of expected utility.

- In Quiggin (1982), the decision weight of each payout depends only on its (known) probability and its ranking position.

Following α-MEU, we assume that the unknown probabilities π_1 and π_3 are skewed using the weights α and $1 - \alpha$:

$$x_{\text{min}} = \min\{x_1, x_3\}$$

is given a probability weight $\frac{2}{3}\alpha$ and

$$x_{\text{max}} = \max\{x_1, x_3\}$$

is given probability weight $\frac{2}{3}(1 - \alpha)$ where the parameter $\frac{1}{2} \leq \alpha \leq 1$ measures the degree of ambiguity aversion.

The utility of a portfolio $\mathbf{x} = (x_1, x_2, x_3)$ takes the form

I. $x_2 \leq x_{\text{min}}$

$$\beta_1 u(x_2) + \beta_2 u(x_{\text{min}}) + (1 - \beta_1 - \beta_2)\, u(x_{\text{max}})$$

II. $x_{\text{min}} \leq x_2 \leq x_{\text{max}}$

$$\beta_3 u(x_{\text{min}}) + (\beta_1 + \beta_2 - \beta_3)\, u(x_2) + (1 - \beta_1 - \beta_2)\, u(x_{\text{max}})$$

III. $x_{\text{max}} \leq x_2$

$$\beta_3 u(x_{\text{min}}) + \beta_4 u(x_{\text{max}}) + (1 - \beta_3 - \beta_4)\, u(x_2)$$

where

$$\begin{aligned}
\beta_1 &= w\left(\tfrac{1}{3}\right), \\
\beta_2 &= w\left(\tfrac{2}{3}\alpha + \tfrac{1}{3}\right) - w\left(\tfrac{1}{3}\right), \\
\beta_3 &= w\left(\tfrac{2}{3}\alpha\right), \\
\beta_4 &= w\left(\tfrac{2}{3}\right) - w\left(\tfrac{2}{3}\alpha\right),
\end{aligned}$$

and the mapping from the four parameters $\beta_1, \ldots, \beta_4$ to two parameters δ and γ is as follows:

$$\begin{aligned}
\beta_1 &= \tfrac{1}{3} + \gamma, \\
\beta_2 &= \tfrac{1}{3} + \delta, \\
\beta_3 &= \tfrac{1}{3} + \gamma + \delta, \\
\beta_4 &= \tfrac{1}{3} - \delta.
\end{aligned}$$

The parameter δ measures the degree of ambiguity aversion and the parameter γ measures the degree of loss aversion:

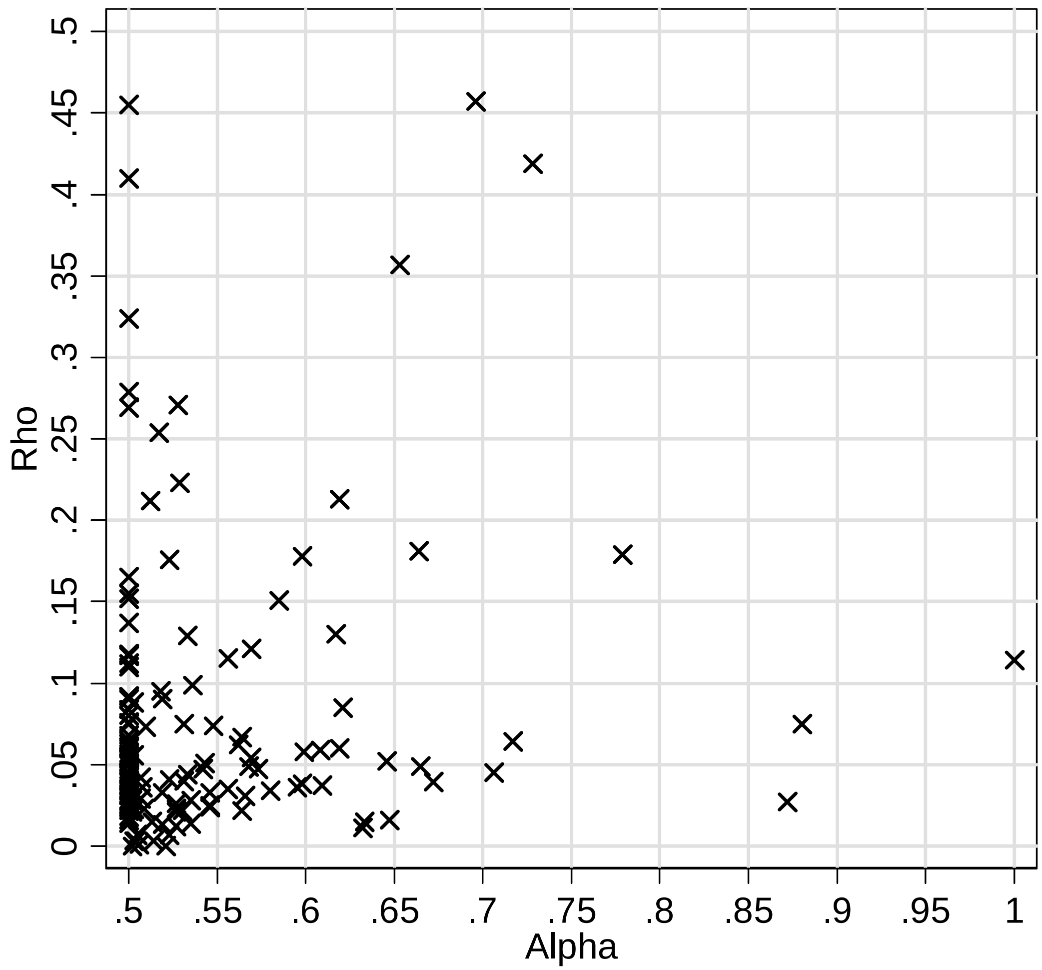
- $\delta \geq 0$ and $\gamma = 0$ – kinked specification

- $\delta = 0$ and $\gamma \geq 0$ – loss/disappointment aversion (Gul, 1991)

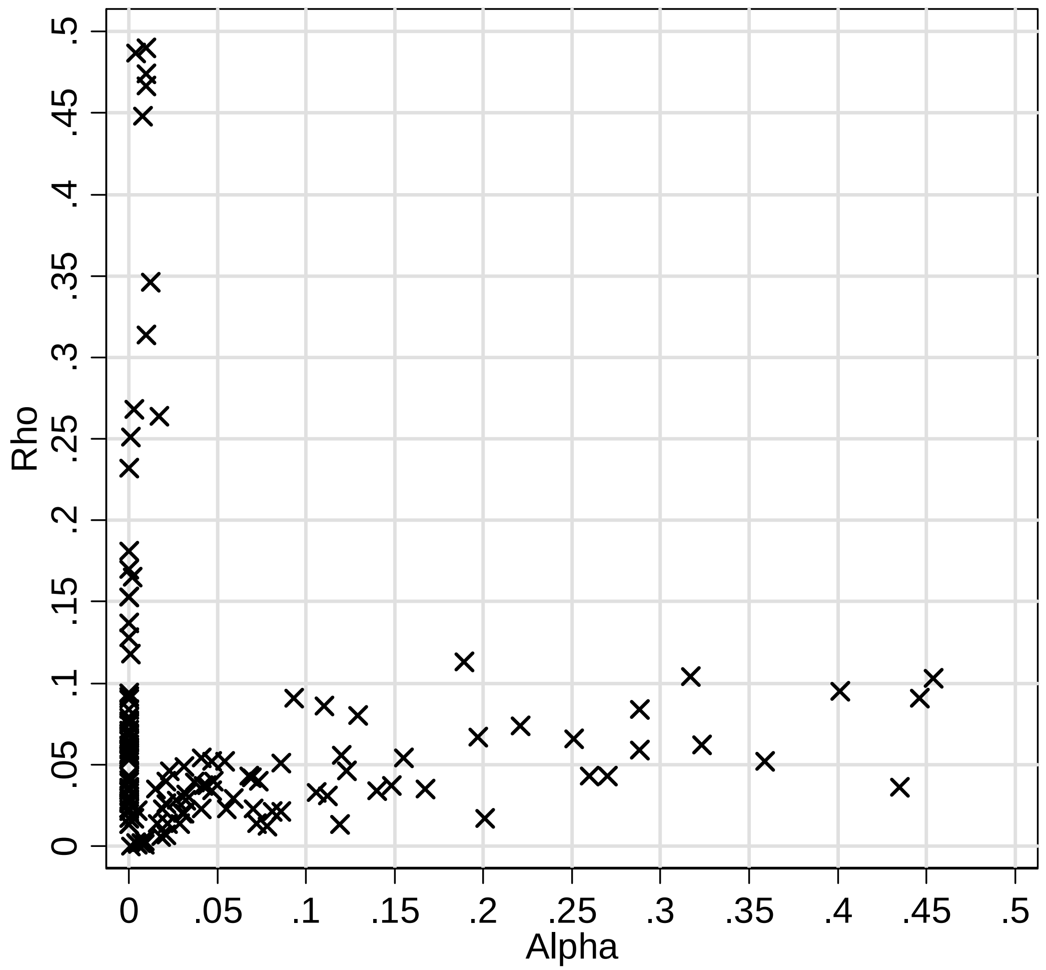
- $\delta = 0$ and $\gamma = 0$ – standard SEU representation.

The indifference curves will have kinks where $x_s = x_{s'}$ and agents will choose portfolios that satisfy $x_s = x_{s'}$ for a non-negligible set of prices.

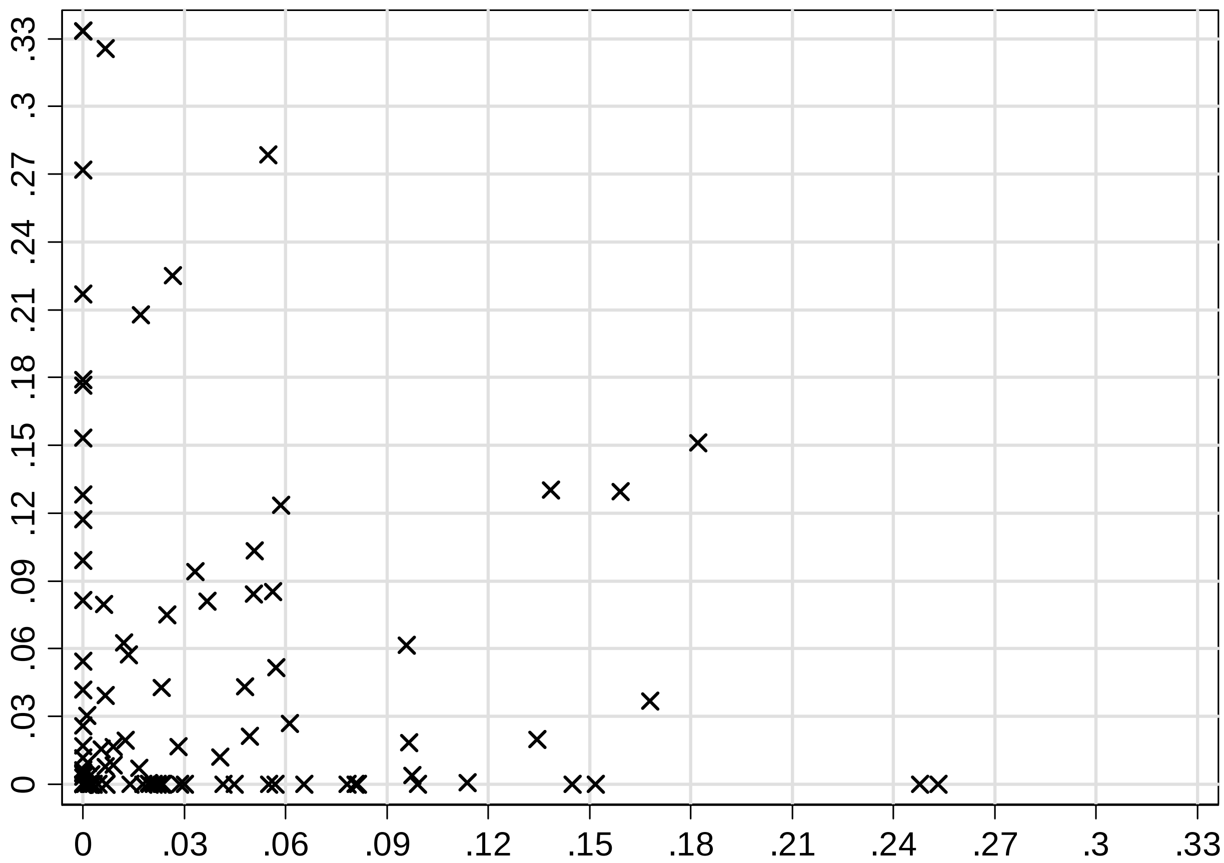
Scatterplot of the estimated parameters – kinked specification



Scatterplot of the estimated parameters – smooth specification



Scatterplot of the estimated parameters – generalized kinked specification (ambiguity - horizontal axis / loss - vertical axis)



Econometric results

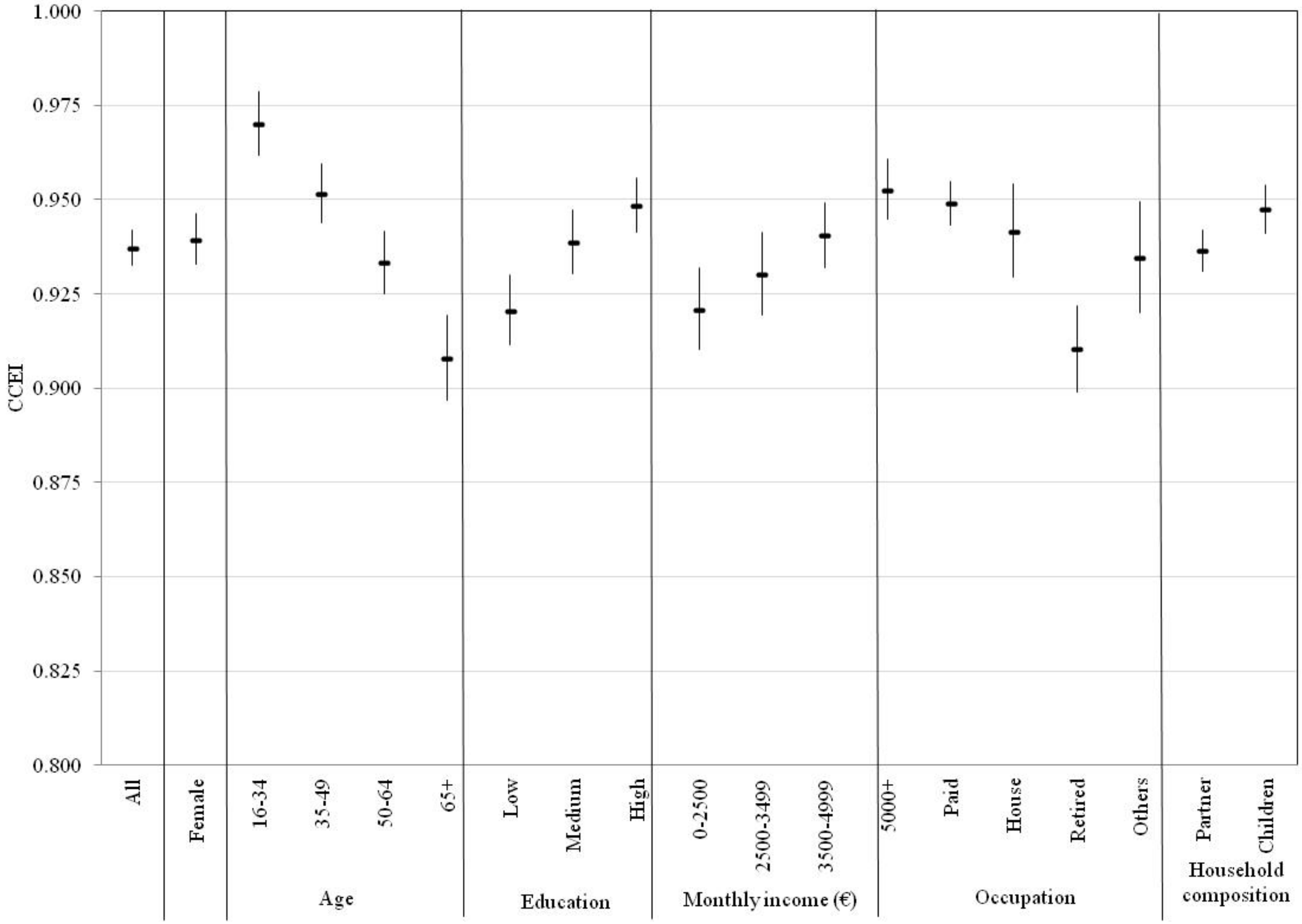
The vast majority of the subjects are well described by the loss- and ambiguity-neutral SEU model. The remainder appear to have a significant degree of loss and/or ambiguity aversion

		Ambiguity Neutral	Ambiguity Averse	Total
Loss	Neutral	60.4	16.7	77.1
	Averse	18.1	4.9	22.9
	Total	78.5	21.5	

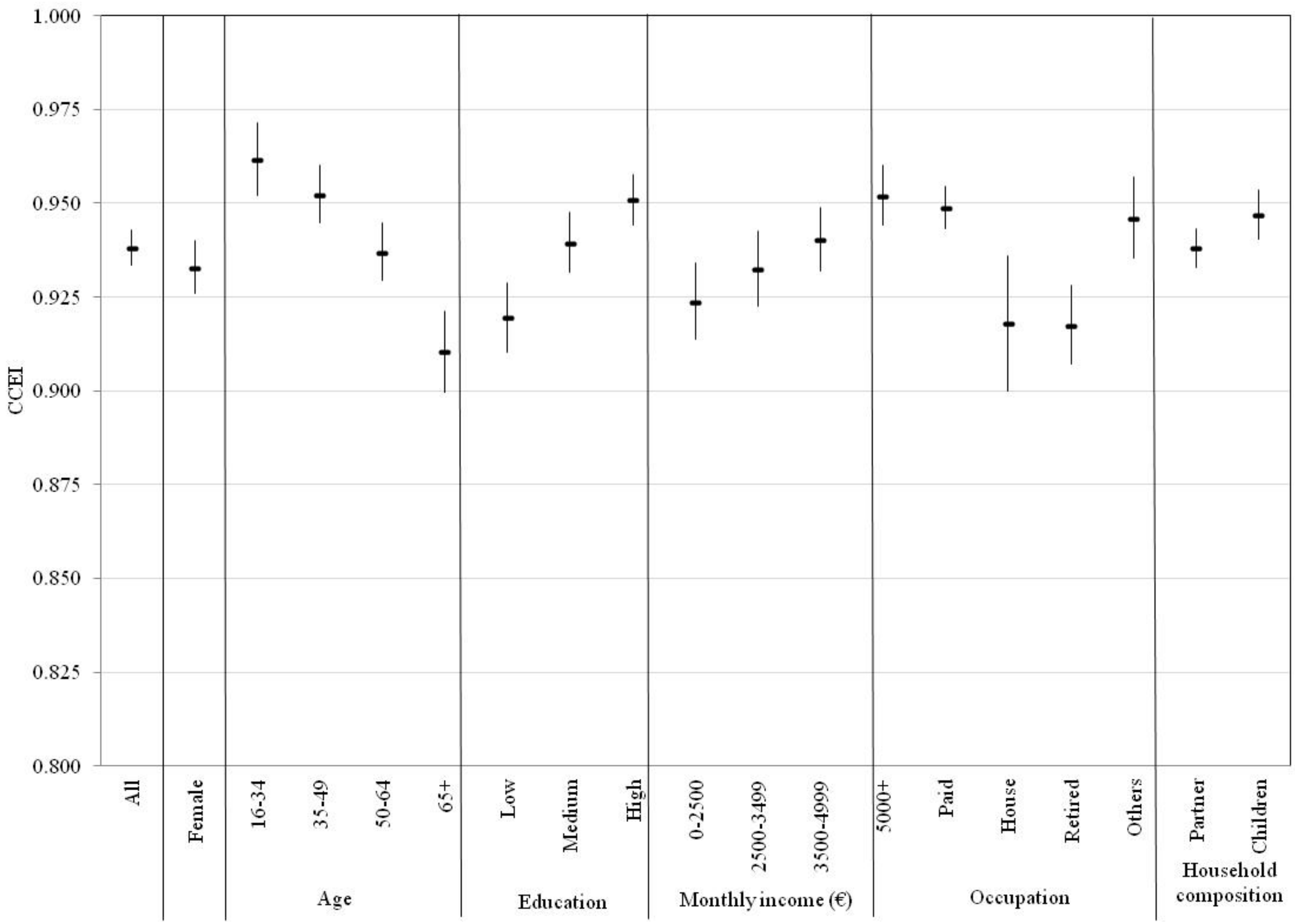
There is considerable heterogeneity in both $\hat{\delta}_n$ or $\hat{\gamma}_n$ and that their values are not correlated ($r^2 = 0.029$).

Time preferences

Mean CCEI scores: income in a few days and income 60 days after that



Mean CCEI scores: income in 60 days and income another 60 days after that



Stationarity, time invariance, and time consistency

- Time discount rates decline as tradeoffs are pushed into the temporal distance.
  - Subjects often choose the larger and later of two rewards when both are distant in time, but prefer the smaller and earlier one as both rewards draw nearer to the present.

- Interpreted as non-constant time discounting, these preference reversals have important implications.
  - Under standard assumptions, non-constant time discounting implies time-inconsistency – self-control problems and a demand for commitment thus emerge.

Stationarity

$\succsim_t$ is stationary if for every $t, t' \geq 0$ and $\Delta_1, \Delta_2 \geq 0$

$$(x, t+\Delta_1) \sim_t (x', t+\Delta_2) \Longleftrightarrow (x, t'+\Delta_1) \sim_t (x', t'+\Delta_2).$$

Ranking does not depend on the distance from t. Tested in the standard static experiment.

Time invariance

$\{\succsim_t\}_{t=1}^T$ is time-invariant if for every $t, t' \geq 0$ and $\Delta_1, \Delta_2 \geq 0$

$$(x, t+\Delta_1) \sim_t (x', t+\Delta_2) \Longleftrightarrow (x, t'+\Delta_1) \sim_{t'} (x', t'+\Delta_2).$$

Ranking does not depend on a calendar time (payments are evaluated relative to a "stopwatch time").

Time consistency

$\{\succsim_t\}_{t=1}^{T}$ is time-consistent if for every $t, t' \geq 0$ and $\Delta_1, \Delta_2 \geq 0$

$$(x, t+\Delta_1) \sim_t (x', t+\Delta_2) \Longleftrightarrow (x, t+\Delta_1) \sim_{t'} (x', t+\Delta_2).$$

Ranking does not change as the evaluation perspective changes from t to t'. Time consistency precludes dynamic preference reversals.

$\Longrightarrow$ These properties are pair-wise independent, but any two properties imply the third (Halevy, 2014)

A GARP test of stationarity

A non-parametric econometric approach for testing whether there is a single preference ordering that can rationalize all intertemporal choices (for a given subject):

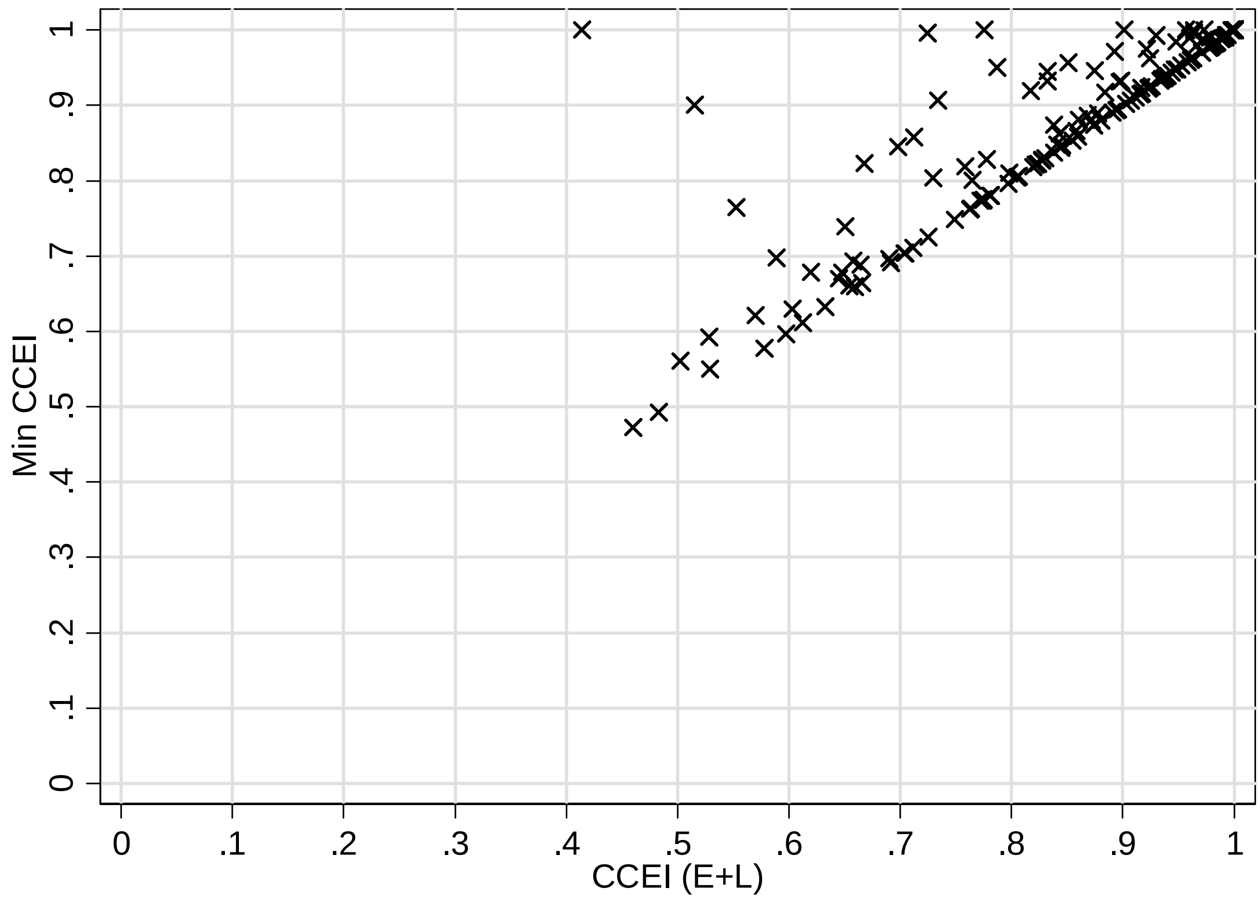
– Combine dataset E with dataset L.

– Compute the consistency score for this combined dataset.

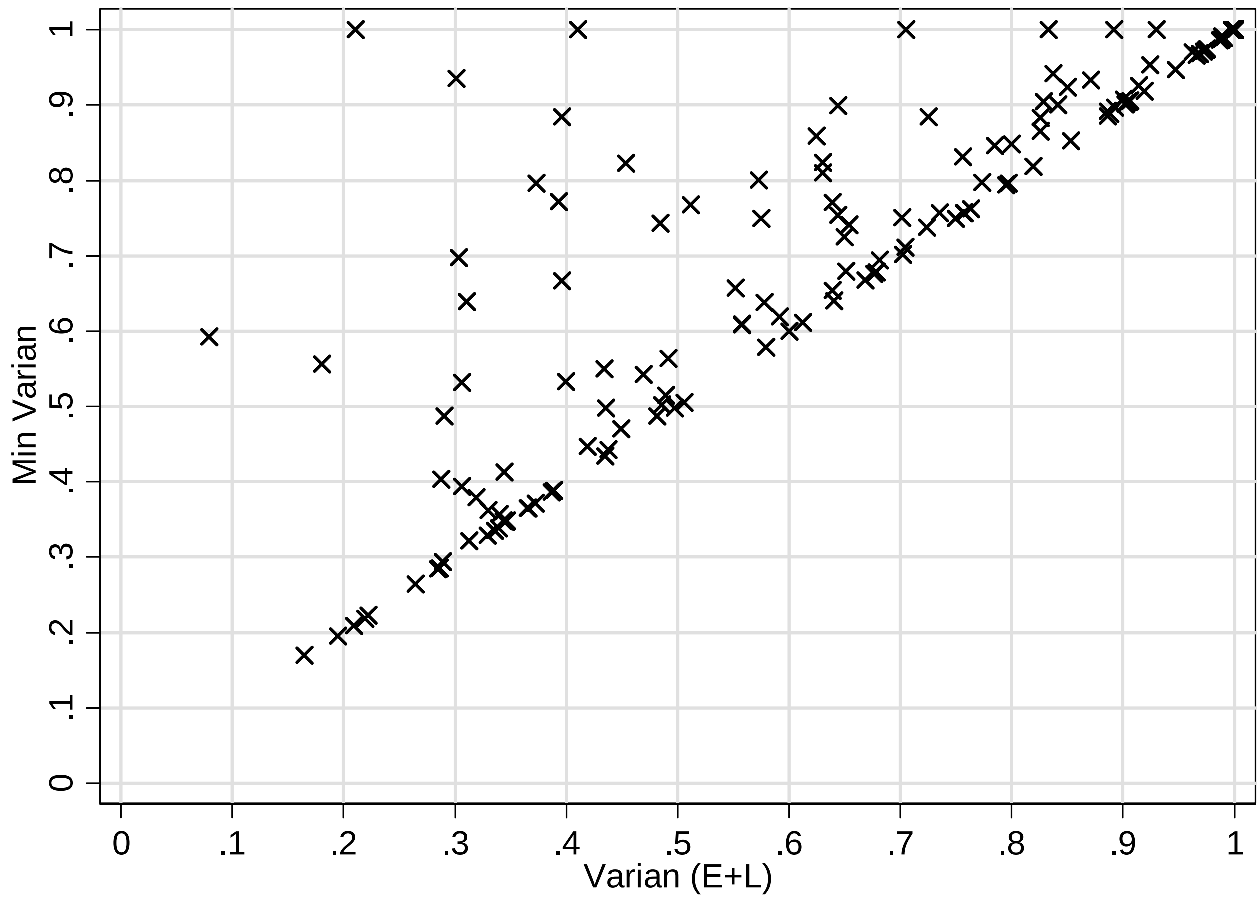
– Compare that number to the minimum score in each of the separate treatments.

The score for the combined dataset can be no bigger than the minimum of the scores for the separate datasets E and L.

The minimum (vertical axis) and combined (horizontal axis) CCEI scores



The minimum (vertical axis) and combined (horizontal axis) Varian (1990, 1991) scores



A statistical (permutation) test

To obtain a distribution for the test statistic under the null hypothesis (stationarity):

[1] Rearrange (permute) the choice, randomly reassigning the choice from dataset E or L to each budget line.

[2] Compute the consistency score for each permutation and construct the joint probability distribution of the min and max scores.

[3] Compare the actual min and max scores with their permutation distribution.

The relationship between permutation test and β

$\hat{\beta}$	Stationary	Nonstationary
$\hat{\beta} < 1$	27 54.0%	23 46.0%
$\hat{\beta} = 1$	110 84.0%	21 16.0%
$\hat{\beta} > 1$	20 66.7%	10 33.3%
Total	157 74.4%	54 25.6%