Character and Candidates: A View from Decision Theory*

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Abstract

Campaigns for political office often center on the "character" of the candidates. Moral issues aside, there is one aspect of a candidate's character that voters clearly should care about: attitude toward risk. If there is a linkage between the candidate's attitudes toward risk in the *private* domain and in the *public* domain then (this aspect of) the candidate's character provides important information about (future) policy-making decisions. This paper formalizes this issue and identifies such linkage. The strength of the link depends on the amount the voter observes/infers and on the degree of rationality the voter ascribes to the candidate. (*JEL* Classification Numbers: D72, D81.)

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1 Introduction

Campaigns for political office, and especially for the U.S. Presidency, often place a great deal of emphasis on the personal character of the candidates – but why? Do voters care – and should they care – whether a candidate has an illicit affair or smokes in secret or invests speculatively or exaggerates athletic accomplishments? It seems to us that there are two obvious reasons why voters do or should care about the personal character of candidates. The first reason is *moral*: (some) voters believe the personal character of the candidate – as evidenced in choices and behavior in the personal domain – is, in and of itself, a qualification for office. The second reason is a *empirical*: (some) voters believe that the character of the candidate provides useful predictions of performance in office. (Whether or not character *does* provide such predictions of performance in office has been the subject of considerable debate in the political science literature, which we discuss below.)

But there is a third reason as well, which is *theoretical*, and is the focus of this paper: voters observe choices that candidates make in their personal lives, especially choices that involve risk; revealed preference means that voters can use these observations to infer the candidate's attitude toward risk in the personal domain; if the candidate's preferences are consistent (an assumption that is universal in economic theory) it might be possible for the voter to *deduce* the candidate's attitude toward risk in the social domain from the candidate's attitude toward risk in the personal domain. This paper provides necessary and sufficient conditions that such deductions be possible. As we show, the required conditions depend on the degree of rationality that voters ascribe to the candidate: the greater the degree of rationality ascribed to the candidate, the easier it will be to deduce the candidate's attitude toward risk in the social domain from the candidate's attitude toward risk in the personal domain.

That voters *should* care about the candidate's attitude toward risk in the social domain seems completely obvious: almost every President makes (foreign) policy choices that involve some risk of war. More prosaically, the President may propose but it is Congress that disposes, so voters should like to know (to use a recent example) not only the candidate's preferences over stimulus proposals whose outcome is certain but also over stimulus proposals whose outcomes are uncertain: would the candidate favor a proposal that would result in an $800 Billion stimulus for sure or a proposal that would result in a $1.6 Trillion stimulus with probability 2/3 and no stimulus at all with probability 1/3?

That voters do learn *something* about candidates' attitudes toward risk in the personal domain – and that they make use of what they learn – is perhaps most clearly illustrated by the case of Gary Hart. Hart was a leading candidate for the 1988 Democratic presidential nomination but withdrew from the race when he was found on his boat with a woman not his wife. Nelson (2009), among others, argued that the issue was less what Hart's behavior revealed about his morality and more what it revealed about his attitude toward risk: "Hart's extramarital escapades ... were politically harmful less because of his moral weakness than because of the recklessness the incidents illuminated in his character."

In our formal model, we consider a candidate for office, characterized by a fixed preference relation $\succeq$ over lotteries on *social states*. A social state has both a *private component* (which matters only to the candidate) and a *public component* (which matters to the candidate and to other members of society). The voter does not observe the entire preference relation $\succeq$ but only its restriction $\succeq_1$ to the set consisting of social states and lotteries on *private states* (social states in which the public component is fixed at some status quo). Given the partial preference relation $\succeq_1$, we ask whether it is possible for the voter to deduce the entire preference relation $\succeq$; that is, we ask whether $\succeq_1$ has a *unique* extension to the full domain of lotteries on social states. Put differently, we ask whether it is possible for the voter to deduce the candidate's attitude toward social risk from knowledge of the candidate's attitude toward personal risk.

We consider two settings: in the first, the sets of social states, public components and private components are all finite; in the second the sets of social states and private components are continuous (and preferences are assumed continuous as well) – as would seem natural if "wealth" were one dimension of the private component. The comparison between the two settings is a little more subtle than it might appear. The assumptions we make on what the voter observes are weaker in the finite setting – when there is less to observe – but these weaker assumptions provide more bite in the continuous setting. Which setting is more appropriate seems a matter of taste that we leave to the reader.

The possibility of deducing the entire preference relation $\succeq$ from the partial preference relation $\succeq_1$ – of deducing the candidate's attitude toward social risk from his/her attitude toward personal risk – depends on the amount the voter observes/infers about the candidate's preferences – revealed directly through statements made by the candidate and revealed indirectly through choices.[1] In the finite setting (but not in the continuous setting), it also depends on the degree of rationality the voter ascribes to the candidate, and in particular on whether the voter believes/assumes that the candidate's preferences obey the axioms of Expected Utility theory or believes/assumes only that the candidate's preferences obey some weaker criteria (in particular, a very weak version of the Independence Axiom that does not have the usual consequences):

- In the finite setting, (the assumption the voter makes about) the extent of candidate rationality matters: assuming the usual axioms of individual choice under uncertainty together with the Independence Axiom, a necessary and sufficient condition that the voter be able to deduce the candidate's preferences toward social risk is that (the voter

[1] Presumably the voter learns some aspects of the candidate's preferences from statements the candidate makes and infers other aspects from the candidate's observed choices. In keeping with the spirit of revealed preference, we ignore this distinction.

infers that) the candidate finds every social state to be indifferent to some personal lottery; without the Independence Axiom, a necessary and sufficient condition that the voter be able to deduce the candidate's preferences toward social risk is that (the voter infers that) the candidate finds every social state to be indifferent to some personal state.

- In the continuous setting, the extent of candidate rationality no longer matters: with either assumption, a necessary and sufficient condition that the voter be able to deduce the candidate's preferences toward social risk is that (the voter infers that) the candidate finds every social state to be indifferent to some personal state. Once the results are stated, the proofs in the finite setting are quite straightforward; the proof in the continuous setting is not straightforward. In each case, the inability of the voter to deduce the candidate's full preference ordering over social lotteries means that there is more than one preference relation over social lotteries that is consistent with the voter's observations; from the point of view of the voter, the candidate's preferences are *indeterminate* – and the extent of this indeterminacy can be (and typically is) quite large.

In both cases we provide necessary and sufficient conditions on how much the voter must observe in order that it be possible to deduce $\succeq$ from $\succeq_1$; if the voter believes/assumes that the candidate's preferences obey the axioms of Expected Utility theory then fewer observations are necessary than if the voter believes/assumes only that the candidate's preferences obey some weaker criteria. In either case, however, it will be necessary for the voter to observe/infer a great deal about the candidate's attitude toward personal risk in order to deduce the candidate's attitude toward social risk.

In our formal analysis, we assume a great deal about what the voter observes. We assume that the voter assigns probabilities to all uncertain events, that the voter assigns the same probabilities as the candidate, and that the voter observes many preference comparisons involving (random) outcomes that have purely personal consequences and (deterministic) outcomes that have both personal and social consequences. We make these strong assumptions in the spirit of giving the voter the *best possible chance* to deduce the candidate's attitude toward social risk. As we shall see, even with these assumptions, the voter is faced with a very difficult task.

We emphasize again that we address a *theoretical* question: can a voter *deduce* a candidate's attitude toward social risk from the candidate's attitude toward personal risk (and some other information)? We have nothing to say about the *empirical* question of whether there is a correlation between a candidate's attitude toward personal risk and his/her attitude toward social risk. We also emphasize that we are agnostic about the voter's *desiderata* regarding the candidate's attitude toward risk: the voter might prefer the candidate to be risk averse or to be risk loving. Rather we are concerned solely with whether the voter can deduce the candidate's attitude toward risk in the public domain from observations of the candidate's attitude toward risk in the personal domain.

The remainder of the paper is organized as follows. Section 2 provides a discussion of some related literature. Section 3 provides the template for analysis. Section 4 contains our main results and proofs and Section 5 concludes.

2 Related Literature

From a purely technical point of view, our work poses a problem in decision theory: under what circumstances is a preference relation over some set of lotteries completely determined by its restriction to a subset of lotteries? Grant et al. (1992), which is closest to the present work, pose the problem in the context of lotteries whose outcomes are commodity bundles and lotteries whose outcomes are monetary payoffs. Given fixed prices for commodities, they ask for conditions that guarantee that preferences over lotteries whose outcomes are commodity bundles are completely determined by the restriction of those preferences to lotteries whose outcomes are monetary payoffs; the sufficient condition they identify is one we call *Degenerate Independence*. However, because our intent is different from Grant et al. (1992), we ask different questions and face quite different issues.

In particular, although prices play a crucial role for Grant et al. (1992) (prices mediate between monetary outcomes and consumption bundles), prices play no role at all in our setting. More subtly, the central issue in our setting is whether all choices in a larger set (social choices) have equivalents (are viewed as indifferent to) choices in a smaller set (personal choices) or to lotteries on the smaller set (personal lotteries). In Grant et al. (1992) it is assumed that all choices in the larger set have equivalents in the smaller set; the central issue is whether this condition is strong enough to determine preferences over lotteries. (We return to this point later when we introduce Degenerate Independence.)

In the realm of political economy, Kartik and McAfee (2007) study the influence of character in a model of electoral competition in which voters have preferences over both policy platforms and "character" – which is identified with "always keeping campaign promises." In their model, character is an exogenous characteristic: some candidates have character, some do not. Candidates without character are strategic (they make campaign promises in order to maximize the probability of being elected); candidates with character are not (they make only campaign promises they will keep). Because only the candidates without character are strategic, the model induces a signalling game between these candidates and the voters. Kartik and McAfee (2007) show that incorporating character in this way has far-reaching implications, including violation of the median voter theorem.

As we have noted, a substantial literature in political science argues that personal character is an important predictor of Presidential conduct. The argument is made most famously and forcefully in a classic book *The Presidential Character* (1972), by James Barber, who writes: "Character is the force, the motive power, around which the person gathers his view of the world, and from which his style receives its impetus. The issues will change; the character of the president will not." Barber argues in particular that candidate's character provides "a realistic estimate of what will endure into a man's White House years." A number of Presidents – real and fictional – and Presidential aides agree:

> With all the power that a President has, the most important thing to bear in mind is this: You must not give power to a man unless, above everything else, he has character. Character is the most important qualification the President of the United States can have. – Richard Nixon

> For the past several months ... [my opponent] ... has suggested that being President of this country was, to some extent, about character ... I have been President for three years and two days and I can tell you without hesitation that being President of this country is entirely about character. – Andrew Shepard[2]

> In a president, character is everything. A president does not have to be brilliant ... He does not have to be clever; you can hire clever ... You can hire pragmatic, and you can buy and bring in policy wonks. But you cannot buy courage and decency, you cannot rent a strong moral sense. A president must bring those things with him... He needs to have, in that much maligned word, but a good one nonetheless, a "vision" of the future he wishes to create. But a vision is worth little if a president does not have the character – the courage and heart – to see it through. – Peggy Noonan[3]

Neither Barber nor Noonan nor the quoted Presidents define character but Barber and others argue (explicitly or implicitly) that character is revealed by personal choices – and early in life. As Barber puts it "the personal past foreshadows the presidential future." Such an argument would seem coherent – and of use to voters – only if the candidate's attitude with respect to social policy – and in particular toward social risk – after achieving office can be deduced from the candidate's attitude with respect to personal choices – and in particular toward personal risk – before achieving office. Such a deduction would seem

[2] Fictional President, portrayed by Michael Douglas in the film *The American President* (1995), responding to a political opponent's attack on his character in a surprise appearance in the White House press room.

[3] Political writer and columnist for *The Wall Street Journal* and former speechwriter and Special Assistant to Ronald Reagan (who believed that "you can tell a lot about a fella's character" by his way of eating jelly beans).

require the existence of a strong *linkage* between the candidate's attitudes toward personal risk and social risk. Barber argues that such a strong link exists. At the other extreme, it is sometimes argued that the constraints imposed by the institutional aspects of the Presidency completely outweigh any possible influence of personal character. Lyons (1997) and Nelson (2009) provide excellent discussions of the arguments.[4,5]

3 Framework

We consider a candidate for public office. From the point of view of the candidate, each *social state* has both a *private component* and a *public component*. To formalize this idea, we take as given three sets X, Z, Ω, where Ω is the set of social states, X is the set of private components, Z is the set of public components, and $\Omega \subset X \times Z$. We allow for the possibility that some private components and public components are incompatible so that the inclusion $\Omega \subset X \times Z$ might be proper. We view $x \in X$ as a proxy for things that affect the candidate but not society as a whole, and $z \in Z$ as a proxy for things that affect society (of which the candidate is a member) as a whole.

We are somewhat agnostic about the specific natures of X, Z in part because voters may differ in their knowledge of social states, in which states they find relevant, and in what they observe about the candidate. Looking ahead this means that the linkage between the candidate's attitudes toward personal risk and social risk may be stronger for some voters than for others. To avoid triviality, we assume X and Z are not singletons and that Ω contains at least three states; otherwise X, Z, Ω are arbitrary.

For any subset $\Theta \subset \Omega$, we write $L(\Theta)$ for the set of (finite) lotteries over states in Θ. We frequently write $\sum_i p_i \omega_i$ for the lottery that yields the state ω_i with probability p_i. We identify the state $\omega \in \Omega$ with the degenerate lottery that yields the state ω for sure. We take as given a *reference state* $\omega_0 = (x_0, z_0) \in \Omega$, which we identify as the current social state. $P = X \times \{z_0\}$ is the set of social states in which the public component is

[4] Again, some voters might care about a candidate's personal choices on purely moral grounds, independent of the implications for choices the candidate might make or policies the candidate might follow when he or she actually assumes office – but that is *not* the argument being made by Barber and others. As Jonathan Yardley concluded "in Washington, and wherever else two or more politicians may gather, he who does not get caught has 'character' and he who gets caught has none."

[5] Other aspects of behavior might also matter, especially if they are viewed as signaling "strength" or "weakness". The reader may recall that Edmund Muskie was widely regarded as the leading candidate for the Democratic Presidential nomination in 1972 until, at a press conference during the New Hampshire primary, Muskie gave an emotional response to attacks on his wife. (Many press accounts of this incident even reported that Muskie cried.) This emotional incident was widely viewed as revealing "weakness" and appears to have fatally damaged Muskie's candidacy. It would be interesting to carry out an analysis linking "weakness in the private domain" with "weakness in the public domain" – but this does not seem easy; certainly we do not know how to do it.

fixed to be z_0 and only the private component x varies; we assume $P \subset \Omega$. Because z_0 is fixed, choices between states in P or between lotteries on P have implications only for the candidate alone; choices between states in Ω or between lotteries on Ω have both personal and social implications. We refer to elements of P as *personal states* and to lotteries in $L(P)$ as *personal lotteries*; we refer to elements of Ω as *social states* and lotteries in $L(\Omega)$ are *social lotteries*.

The candidate's true relation $\succeq$ over $L(\Omega)$ – that is, the candidate's comparisons of all social lotteries – is fixed, but not known to the voter. The voter seeks to *deduce* the candidate's comparisons between *all* lotteries but must base this deduction on observation/inference of only a *subset* of all comparisons and on the degree of rationality the voter ascribes to the candidate. We posit that the voter observes/infers comparisons between social states (on the basis of policy statements, for instance) and personal lotteries. This assumption is formalized by assuming the voter knows, not the entire complete relation $\succeq$ on $L(\Omega)$, but rather only the restriction $\succeq_1$ to $L(P) \cup \Omega$.

We posit that the voter assumes the candidate's preferences satisfy familiar requirements: Completeness, Transitivity, Archimedean (Continuity), Reduction of Compound Lotteries and the Sure Thing Principle. (The last implies that we may and do identify the lottery $\sum_i p_i \omega$ with the certain state ω.) The voter also assumes one of two alternative additional rationality requirements. The strong rationality requirement is the familiar Independence Axiom.

Independence If $\alpha_i, \alpha_i' \in L(\Omega)$ for $i = 1, \ldots k$, $\alpha_i \succeq \alpha_i'$ for each i and $p = (p_1, \ldots, p_k)$ is a probability vector, then

$$\sum_{i=1}^{k} p_i \alpha_i \succeq \sum_{i=1}^{k} p_i \alpha_i'$$

In conjunction with our other requirements, Independence implies that the candidate's preference relation admits an Expected Utility representation. A much weaker rationality requirement, adapted from Grant et al. (1992), is Degenerate Independence.

Degenerate Independence If $\omega_i, \omega_i' \in \Omega$ for $i = 1, \ldots k$, $\omega_i \succeq \omega_i'$ for each i and $p = (p_1, \ldots, p_k)$ is a probability vector, then

$$\sum_{i=1}^{k} p_i \omega_i \succeq \sum_{i=1}^{k} p_i \omega_i'$$

Degenerate Independence is a *much* weaker assumption than Independence because it compares only lotteries whose outcomes are primitives – social states – rather than lotteries whose outcomes are themselves lotteries. Almost all decision-theoretic models that have been proposed as alternatives to Expected Utility obey Degenerate Independence. In the setting of Grant et al. (1992), the primitives are consumption bundles; the implication of Degenerate Independence is that only indifference sets matter, and not specific bundles in those indifference sets. In our setting the primitives are social states rather than consumption bundles; the implication is that only indifference sets matter, and not particular states in those indifference sets.[6]

Example A simple example will illustrate the impact of the different rationality assumptions in our setting. Suppose that there is a worst personal state $W = (x^w, z_0)$ and a best personal state $B = (x^b, z_0)$, that $\omega \in \Omega$ is some other social state, and that the voter observes that $\omega \sim \frac{1}{2}W + \frac{1}{2}B$; Figure 1A (top left panel) illustrates all of this in the familiar Marschak-Machina probability triangle. If the voter assumes that the candidate's preferences conform to Expected Utility, then the candidate's indifference curve through the points $\omega, \frac{1}{2}W + \frac{1}{2}B$ is a straight line, and all other indifference curves are straight lines parallel to this one; in particular, the voter can deduce that $\frac{1}{2}\omega + \frac{1}{2}B \sim \frac{1}{4}W + \frac{3}{4}B$, as illustrated in Figure 1B (top right panel).

However, if the voter does not assume that the candidate's preferences conform to Expected Utility, but only to some weaker axiom such as Betweenness (Chew, 1989, and Dekel, 1986), then all indifference curves are again straight lines but they need not be parallel; in particular, it may be that $\frac{1}{2}\omega + \frac{1}{2}B \sim \frac{1}{8}W + \frac{7}{8}B$ as illustrated in Figure 1C (bottom left panel). And if the voter assumes that the candidate's preferences satisfy only Degenerate Independence, then indifference curves need not be straight lines; all that can be said is that indifference curves are "upward sloping" – see Figure 1D (bottom right panel). In these cases, based on information available, the voter cannot infer the shapes of the candidate's indifference curves.

[Figure 1 here]

To formulate our results precisely, we isolate two definitions:

[6] To clarify, both Grant et al. (1992) and we pose (versions of) the following question: if we know preferences of an individual over a set of outcomes Ω and over the set of lotteries with outcomes in some subset $P \subset \Omega$, can we deduce preferences over lotteries with outcomes in all of Ω? For us, Ω is a set of social choices and P is a set of personal choices; for Grant et al. (1992), Ω is a set of consumption bundles and P is a set of monetary outcomes (identified with specific consumption bundles). Prices play a crucial role for Grant et al. (1992) (prices mediate between monetary outcomes and consumption bundles); prices play no role at all in our setting. More subtly, the central issue in our setting is whether all choices in Ω have equivalents in P or in lotteries with outcomes in P; in the setting of Grant et al. (1992) it is always the case (virtually by definition) that all choices in Ω have equivalents in P. A related issue about substitutions arises in Anscombe and Aumann (1963): is a horse race viewed as equivalent to the spin of a roulette wheel, independently of the prizes that are awarded?

Personal state equivalent The social state $\omega \in \Omega$ admits a *personal state equivalent* if there is a personal state $(x, z_0) \in P$ such that $\omega \sim (x, z_0)$.

Personal lottery equivalent The social state $\omega \in \Omega$ admits a *personal lottery equivalent* if there is a personal lottery $\alpha = \sum p_i \omega_i \in L(P)$ such that $\omega \sim \alpha$.

Because we identify states with degenerate lotteries it is clear that if state ω admits a personal state equivalent then it also admits a personal lottery equivalent. Note that personal state or lottery equivalents, if they exist, need not be unique. For interpretation, suppose that X is private wealth. If (x_0, z_0) is the current social state, and $\omega \in \Omega$ admits a personal state equivalent (x, z_0), then we can interpret the wealth difference $x - x_0$ as the amount of money that would be required to bribe the candidate to veto a change from the current state (x_0, z_0) to the alternative ω.[7] If $\omega \in \Omega$ admits a personal lottery equivalent $\sum p_i(x_i, z_0)$ we can interpret $\sum p_i(x_i - x_0, z_0)$ as a gamble that the candidate would be willing to accept to veto a change from the current state (x_0, z_0) to the alternative ω.

4 The Linkage

The questions we have in mind can now be formulated in the following way: If the voter observes $\succeq_1$ and assumes that the candidate's preferences satisfy Degenerate Independence or Independence, can the voter deduce $\succeq$? In different words: is $\succeq$ uniquely determined by $\succeq_1$? As we show below, stringent assumptions on rationality and observability are necessary to guarantee that the voter can deduce $\succeq$, and when these assumptions are not satisfied, the voter will find that there are many lotteries in $L(\Omega)$ over which the preference ordering of the candidate is indeterminate. For simplicity, we begin in Theorems 1, 2 and 3 by assuming that X, Z (and hence Ω) are finite; in Theorem 4, we assume X is continuous.

Theorem 1 *Assume that the voter observes $\succeq_1$ and that the voter assumes that the candidate's preferences satisfy Independence. Then the following are equivalent:*

(a) every social state $\omega \in \Omega$ is ranked between the worst and best personal states,
(b) every social state $\omega \in \Omega$ admits a personal lottery equivalent,
(c) $\succeq$ can be deduced from $\succeq_1$.

Proof. That (a) implies (b) follows immediately from the Archimedean property of preferences.

[7] If X is not private wealth, we might interpret the change from x_0 to x as the change in personal circumstance that would make the candidate willing to veto a change from the current state (x_0, z_0) to the alternative ω.

To see that (b) implies (c), consider lotteries $\sum p_i\omega_i$ and $\sum q_j\omega_j$. In view of (b), we can find personal lotteries α_i and β_j such that $\omega_i \sim \alpha_i$ and $\omega_j \sim \beta_j$ for each i, j. Independence guarantees that $\sum p_i\omega_i \sim \sum p_i\alpha_i$ and $\sum q_j\omega_j \sim \sum q_j\beta_j$ so

$$\sum p_i\omega_i \succeq \sum q_j\omega_j \iff \sum p_i\alpha_i \succeq \sum q_j\beta_j \iff \sum p_i\alpha_i \succeq_1 \sum q_j\beta_j$$

Thus $\succeq$ can be deduced from $\succeq_1$.

Finally, to see that (c) implies (a), suppose not. Because X and hence P is finite, there are worst and best personal states; say $(x^w, z_0), (x^b, z_0)$ respectively. Define lower sets and upper sets:

$$W = \{\omega \in \Omega : \omega \prec (x^w, z_0)\} \text{ and } B = \{\omega \in \Omega : \omega \succ (x^b, z_0)\}$$

The supposition that (a) is false means that at least one of W, B is not empty. Choose a utility function $u : \Omega \to \mathbb{R}$ that yields an Expected Utility representation of $\succeq$. Let f be any strictly increasing function such that $f(t) = t$ for every $t \in [u(x^w, z_0), u(x^b, z_0)]$. Define a preference relation $\succeq_f$ on $L(\Omega)$ by

$$\sum p_i\omega_i \succeq \sum q_j\omega_j \iff \sum p_i [f \circ u(\omega_i)] \succeq \sum q_j [f \circ u(\omega_j)]$$

Evidently, $\succeq_f$ extends $\succeq_1$ but there are infinitely many different choices of f that yield different preference relations $\succeq_f$; i.e., $\succeq$ cannot be deduced from $\succeq_1$. ■

Condition (a) in Theorem 1 – that every social state is ranked between the worst and best personal states – has an interpretation that may provide some insight. To say that every social state is (weakly) worse than the best personal state $B = (x^b, z_0)$ is to say that, no matter how good an outcome is achievable for society, there is always a sufficiently large "bribe" (literally, if X is personal wealth; figuratively otherwise) that the candidate would accept to veto that outcome. To say that every social state is (weakly) better than the worst personal state $W = (x^w, z_0)$ is to say that, no matter how bad an outcome is for society, the candidate does not find it to be worse than leaving society in its current reference outcome and giving the candidate his/her worst private outcome. So the assumption that a candidate ranks every state of the world between the best and worst private states is an assumption about the extent to which the candidate is selfish – rather than altruistic. Theorem 1 tells us that it is easier for the voter to deduce the preferences of a selfish candidate than of an altruistic candidate. If that seems strange, remember that we are only asking about the voter's ability to deduce preferences of the candidate – not about the voter's preferences over candidates.

If the voter assumes less rationality on the part of the candidate, deduction requires stronger assumptions.

Theorem 2 *Assume that the voter observes $\succeq_1$ and that the voter assumes that the candidate's preferences satisfy only Degenerate Independence. Then the following are equivalent:*

(a) every social state $\omega \in \Omega$ admits a personal state equivalent
(b) $\succeq$ can be deduced from $\succeq_1$.

Proof. To see that (a) implies (b), assume that every social state ω admits a personal state equivalent $(x(\omega), z_0)$. Degenerate Independence implies that $\sum p_i\omega_i \sim \sum p_i(x(\omega_i), z_0)$ for every lottery $\sum p_i\omega_i \in (\Omega)$. Hence given two lotteries $\sum p_i\omega_i, \sum q_j\omega_j$ it follows from transitivity that

$$\begin{aligned} \sum p_i\omega_i, \succeq \sum q_j\omega_j \quad &\iff \quad \sum p_i(x(\omega_i), z_0) \succeq \sum q_j(x(\omega_j, z_0)) \\ &\iff \quad \sum p_i(x(\omega_i), z_0) \succeq_1 \sum q_j(x(\omega_j, z_0)) \end{aligned}$$

That is, $\succeq$ can be deduced from $\succeq_1$.

To see that (b) implies (a), suppose that some social state does *not* admit a personal state equivalent. Use Debreu's (1954) representation theorem to choose a utility function $U : L(\Omega) \to \mathbb{R}$ that represents $\succeq$. Choose any strictly increasing continuous function $F : [0, 1] \to [0, 1]$ be such that $F(0) = 0, F(1) = 1$. Write $S \subset \Omega$ for the set of social states that do admit a personal state equivalent and $\Omega \setminus S$ for the complementary set of states that do not admit a personal state equivalent. Note that $P \subset S$ so neither of the sets $S, \Omega \setminus S$ is empty. Every lottery $\alpha \in L(\Omega)$ can be decomposed uniquely as a compound lottery over lotteries in $S, \Omega \setminus S$

$$\sum p_i\omega_i = p\left[\sum_{\omega_i \in S} \left(\frac{p_i}{p}\right)\omega_i\right] + (1-p)\left[\sum_{\omega_i \in \Omega\setminus S} \left(\frac{p_i}{1-p}\right)\omega_i\right]$$

where

$$p = \sum_{\omega_i \in S} p_i$$

We can define a preference relation $\succeq_F$ on $L(\Omega)$ by

$$p\sigma + (1-p)\tau \succeq p'\sigma' + (1-p')\tau' \iff F(p)U(\sigma) + F(1-p)U(\tau) \geq F(p')U(\sigma') + F(1-p')U(\tau')$$

Because $F(0) = 0$ and $F(1) = 1$, $\succeq_F$ extends $\succeq_1$. However, there are infinitely many different choices of F that yield different preference relations $\succeq_F$, so $\succeq$ cannot be deduced from $\succeq_1$. ■

We have made the strong assumption that the voter observes many comparisons between social states and personal lotteries. It might be more natural to make the weaker assumption that the voter observes only comparisons between social states and comparisons between personal lotteries but does not observe comparisons between social states and personal lotteries. Formally, this means that the voter observes only the partial relation $\succeq_0$ whose graph is

$$\text{graph}(\succeq_0) = \text{graph}(\succeq) \cap \left(\left[L(P) \times L(P) \right] \cup \left[\Omega \times \Omega \right] \right)$$

For comparison, note that the graph of $\succeq_1$ is

$$\text{graph}(\succeq_1) = \text{graph}(\succeq) \cap \left(\left[L(P) \cup \Omega \right] \times \left[L(P) \cup \Omega \right] \right)$$

Theorem 3 *Assume that the voter observes $\succeq_0$ and that the voter assumes that the candidate's preferences satisfy Degenerate Independence. Then, whether or not the voter assumes the candidate's preferences satisfy Independence, the following are equivalent:*
(a) every social state $\omega \in \Omega$ admits a personal state equivalent
(b) $\succeq$ can be deduced from $\succeq_0$.

Proof. The proof that (a) implies (b) is exactly the same as in the proof of Theorem 2 above. To see that (b) implies (a), we proceed almost as in the proof of Theorem 2 above, except that the distortion is at the level of the utility of states rather than at the level of probabilities. Suppose therefore that some social state does *not* admit a personal state equivalent. Let $U : L(\Omega) \to \mathbb{R}$ be any utility function that represents $\succeq$. Write $S \subset \Omega$ for the set of social states that do admit a personal state equivalent and $\Omega \setminus S$ for the complementary set of states that do not admit a personal state equivalent; neither of the sets $S, \Omega \setminus S$ is empty. Let $h : \mathbb{R} \to \mathbb{R}$ be any strictly increasing function with the property that $h(U(\omega)) = U(\omega)$ for every $\omega \in S$. Every lottery $\alpha \in L(\Omega)$ can be decomposed uniquely as a compound lottery over lotteries in $L(S), L(\Omega \setminus S)$:

$$\sum p_i \omega_i = p \left[\sum_{\omega_i \in S} \left(\frac{p_i}{p} \right) \omega_i \right] + (1-p) \left[\sum_{\omega_i \in \Omega \setminus S} \left(\frac{p_i}{1-p} \right) \omega_i \right]$$

where

$$p = \sum_{\omega_i \in S} p_i$$

Hence we may define a utility function $U_h : L(\Omega) \to \mathbb{R}$ by

$$U_h \left(\sum p_i \omega_i \right) = p\, U \left(\sum_{\omega_i \in S} \left(\frac{p_i}{p} \right) \omega_i \right) + (1-p) \left(\sum_{\omega_i \in \Omega \setminus S} \left[\frac{p_i}{1-p} \right] \left[h \circ U(\omega_i) \right] \right)$$

and a preference relation $\succeq_h$ on $L(\Omega)$ by

$$\alpha \succeq_h \alpha' \Longleftrightarrow U_h(\alpha) \geq U_h(\alpha')$$

It is easily checked that every choice of h leads to a preference relation that extends $\succeq_0$ but infinitely many different choices of h lead to different extensions. Moreover, if we begin with an utility function U that satisfies the Expected Utility property, every choice of h leads to a utility function U_h that satisfies the Expected Utility property. ■

Because we assume that the voter can observe preferences on $L(P)$, the richer is X is larger will the set of private choices that the voter might observe and (conceivably) the easier it might be for the voter to infer $\succeq$ by observing $\succeq_0$ or $\succeq_1$. If X is private wealth, it might be natural to assume that $X = [0, \infty)$ and conceivably the voter might observe/infer many personal states – but only in very restricted circumstances will it be possible for the voter to deduce $\succeq$.

To formalize this idea, we assume in what follows that X and Z are separable metric spaces and that X is connected. (This seems especially natural if we identify X with personal wealth.) In this context it seems natural to insist that $\succeq$ be continuous (in social states and probabilities) and locally non-satiated (that is, for every social state $\omega \in \Omega$ and every open neighborhood W of ω there is some $\omega' \in \Omega$ such that $\omega' \succ \omega$). It is probably not reasonable to assume that a voter "observes" preferences over an infinite set – but a voter who observes preferences over a finite set might draw inferences about a much larger set of preferences. For instance: the voter might assume the candidate has constant relative risk aversion with regard to personal wealth, and use a small number of observations to estimate the coefficient of relative risk aversion.

Theorem 4 *In order that $\succeq_1$ admit a unique extension to $L(\Omega)$ it is necessary and sufficient that that every social state be the limit of states that admit a personal state equivalent.*

Proof. First use the utility representation theorem of Debreu (1954) to choose a continuous utility function $U : L(\Omega) \to \mathbb{R}$ that represents $\succeq$; that is,

$$A \succeq B \Leftrightarrow U(A) \geq U(B).$$

Set

$$M = \sup_{x \in X} U(x, z_0) \ , \ m = \inf_{x \in X} U(x, z_0),$$

and define

$$\begin{array}{lcl} Q_+ & = & \{\alpha \in \Omega : U(\alpha) > M\} \ , \\ Q_- & = & \{\alpha \in \Omega : U(\alpha) < m\}, \\ E & = & \alpha \in \Omega : m \leq U(\alpha) \leq M\}. \end{array}$$

Continuity of U implies that Q_+, Q_- are open and E is closed; evidently we can decompose

$$\Omega = Q_+ \cup E \cup Q_-$$

Let PE be the set of social states that admit a personal state equivalent. A preliminary result will be useful: For each $\alpha \in \Omega \setminus PE$ either

$$U(\alpha) \geq U(\gamma) \text{ for all } \gamma \in P \quad \textit{or} \quad U(\alpha) \leq U(\gamma) \text{ for all } \gamma \in P.$$

To see this, suppose not. Then there are $\alpha \in \Omega \setminus P$ and $(x, z_0), (x', z_0) \in P$ such that

$$U(x, z_0) > U(\gamma) > U(x', z_0).$$

Because X is connected we can find an $x'' \in X$ such that $U(x'', z_0) = U(\alpha)$; that is, α admits a private equivalent. Since this is a contradiction, we conclude that no such γ, γ' exist, which establishes the assertion.

We can now show that at least one of Q_+, Q_- is not empty. By assumption, $\overline{PE} \neq \Omega$; let $\alpha \in \Omega \setminus \overline{PE}$. In view of the above, either $U(\alpha) \geq U(\gamma)$ for all $\gamma \in P$ or $U(\alpha) \leq U(\gamma)$ for all $\gamma \in P$. In the former case, $U(\alpha) \geq M$. Local non-satiation guarantees the existence of some $\alpha' \in \Omega$ such that $U(\alpha') > U(\alpha) \geq M$ so $\alpha' \in Q_+$ and $Q_+ \neq \emptyset$. In the latter case, $U(\alpha) \leq m$. If $U(\alpha) < m$ then $\alpha \in Q_-$ and we are done. If not then $U(\alpha) = m$ and local non-satiation guarantees the existence of a sequence $\{\alpha_n\} \subset \Omega$ such that $U(\alpha_n) > U(\alpha)$ and $\alpha_n \to \alpha$. Continuity of U guarantees that $U(\alpha_n) \to U(\alpha)$. However, $U(\alpha) = m = \inf_{x \in X} U(x, z_0)$ so for each sufficiently large n we can find $x_n, x'_n \in X$ such that $U(x_n, z_0) \geq U(\alpha_n) \geq U(x'_n, z_0)$. Connectedness of X and continuity of U implies that there is some $y_n \in X$ such that $U(y_n, z_0) = U(\alpha_n)$; that is, $\alpha_n \in PE$. Since $\alpha_n \to \alpha$, this means $\alpha \in \overline{PE}$, contrary to our assumption. We conclude that at least one of Q_+, Q_- is not empty. In what follows, we assume $Q_+ \neq \emptyset$; the argument if $Q_+ = \emptyset$ and $Q_- \neq \emptyset$ is similar.

Let $f : \mathbb{R} \to \mathbb{R}$ be any continuous, strictly increasing function such that $f(t) = t$ for $t \in [m, M]$ but f is non-linear everywhere else; e.g.,

$$f(t) = \begin{cases} (t-m)^k - m & \text{if} \quad t < m \\ t & \text{if} \quad m \leq t \leq M \\ (t-M)^k - M & \text{if} \quad t > M \end{cases}$$

where $k \geq 3$ is an odd integer. Set $U_f = f \circ U$. In view of the decomposition $\Omega = Q_+ \cup E \cup Q_-$, we can uniquely decompose every lottery $A \in L(\Omega)$ as

$$A = \sum_{i \in I} p_i \alpha_i + \sum_{j \in J} q_j \beta_j \tag{1}$$

where each $\alpha_i \in Q_+$, each $\beta_j \in E \cup Q_-$ and $\sum p_i + \sum q_j = 1$. Set

$$\overline{p} = \sum_{i \in I} p_i \; , \overline{q} = \sum_{j \in J} q_j.$$

We consider two cases: either Q_+ is closed or Q_+ is not closed.

Case 1 If Q_+ is closed define a continuous utility function $V_f : L(\Omega) \to \mathbb{R}$ by

$$V_f(A) = \sum_{i \in I} p_i U_f(\alpha_i) + \overline{q}\, U_f \left(\sum_{j \in J} (q_j / \overline{q}) \beta_j \right)$$

and let $\succeq_f$ be the preference relation induced by V_f. If $A \in L(P) \subset L(E \cup Q_-)$ then $V_f(A) = U_f(A) = U(A)$ (because $f(t) = t$ on $[m, M]$). If $A = (x, z) \in \Omega$ then $V_f(A) = U_f(x, z)$. In particular, $\succeq_f$ coincides with $\succeq_0$ on $\Omega \cup L(P)$. On the other hand, the non-linearity of f guarantees that $\succeq_f \neq \succeq$ on $L(\Omega)$, which is the desired result.

Case 2 If Q_+ is not closed the construction above need not yield a continuous utility function V_f or a continuous preference relation $\succeq_f$, so we proceed differently. Because Q_+ is not closed, the set $\overline{Q_+} \cap E$ is not empty; fix any $\zeta \in \overline{Q_+} \cap E$. Using the representation (1), define

$$V_f(A) = \sum_{i \in I} p_i U_f(\alpha_i) + U_f \left(\sum_{j \in J} q_j \beta_j + p\, \zeta \right) - p \;\; U_f(\zeta) \tag{2}$$

and let $\succeq_f$ be the preference relation induced by V_f. If $A \in L(P) \subset L(E \cup Q_-)$ then each $p_i = 0$ so $V_f(A) = U_f(A)$. If $A = (x, z) \in Q_+$ then $\overline{p} = 1$, all $\alpha_i = (x, z)$ and q so

$$V_f(A) = \sum_{i \in I} p_i U_f(x, z) + U_f(\zeta) - U_f(\zeta) = U_f(x, z)$$

Thus, $\succeq_f$ coincides with $\succeq_0$ on $\Omega \cup L(P)$.

It remains only to show that V_f is continuous, so that $\succeq_f$ is continuous. To see that V_f is continuous, consider a lottery A and a sequence $\{A^n\}$ of lotteries with $A^n \to A$; we must show $V_f(A^n) \to V_f(A)$. Write

$$A^n = \sum_{i \in I} p_i^n \alpha_i^n + \sum_{j \in J} q_j^n \beta_j^n$$

By definition, the probabilities p_i^n, q_j^n and the points α_i^n, β_i^n converge; convergence of $V_f(A^n)$ to $V_f(A)$ is in doubt only when $\alpha_i^n \to \alpha_i \in \overline{Q^+} \cap E$ for some indices i. Let K be the set of such indices and let $K' = I \setminus K$ be the complementary set of indices. Then the decomposition of A has the form

$$A = \sum_{i \in K'} p_i \alpha_i + \left[\sum_{i \in K} p_i \alpha_i + \sum_{j \in J} q_j \beta_j \right]$$

where the first summation encompasses terms corresponding to Q_+, the term in brackets encompasses terms corresponding to $E \cup Q_-$, $p_i^n \to p_i$, $q_j^n \to q_i$, $\alpha_i^n \to \alpha_i$, $\beta_j^n \to \beta_j$ for all i, j.

Using (2) and the above decompositions and keeping in mind that $K \cup K' = I$ yields

$$\begin{aligned} V_f(A^n) &= \sum_{i \in K'} p_i^n U_f(\alpha_i^n) + \sum_{i \in K} p_i^n U_f(\alpha_i^n) + U_f\left(\sum_{j \in J} q_j^n \beta_j + \overline{p}_i^n \, \zeta \right) - \overline{p}_i^n \;\, U_f(\zeta) \\ V_f(A) &= \sum_{i \in K'} p_i U_f(\alpha_i) + U_f\left(\sum_{j \in J} q_j \beta_j + \sum_{i \in K} p_i \alpha_i + \overline{p} \, \zeta \right) - \overline{p} \;\, U_f(\zeta) \end{aligned}$$

where $\overline{p} = \sum_{K'} p_i$. Continuity of U_f entails that $U(\alpha_i^n) \to U(\alpha^n)$. By assumption $\alpha_i \in \overline{Q_+} \cap E$ so $U(\alpha_i) = U(\zeta)$. Hence

$$\sum_{i \in K'} p_i^n U_f(\alpha_i^n) + \sum_{i \in K} p_i^n U_f(\alpha_i^n) \to \sum_{i \in K'} p^n U_f(\alpha_i^n) + \sum_{i \in K} p^n U_f(\zeta).$$

Degenerate Independence entails that

$$U_f\left(\sum_{j \in J} q_j^n \beta_j + \overline{p}_i^n \, \zeta \right) \to U_f\left(\sum_{j \in J} q_j \beta_j + \sum_{i \in K} p_i \alpha_i + \overline{p} \, \zeta \right).$$

After grouping the probabilities p_i^n and p_i in the obvious way, it now follows as asserted that $V_f(A^n) \to V_f(A)$ as asserted. Hence V_f is continuous. This completes the proof. ■

5 Concluding Remarks

It is often said that private choices should remain private. As Paul Krugman has written "… I'm talking about professional mistakes. The other kinds of mistakes … are none of your business." This point of view seems reasonable when applied to Krugman, who is not a candidate for any public office, much less the Presidency. But, to the extent that mistakes are the consequences of attitudes toward risk and attitudes toward personal risk are indicative of attitudes toward social risk, then this point of view would seem mistaken when applied to candidates for public office and in particular the Presidency. Hence we *should* care about the private choices of candidates – at least to the extent that those choices involve risk.

However, it can be dangerously easy to err and infer too much from a linkage that is too weak or observation that is too imperfect. During (and after) the 1992 presidential campaign, stories were widely told about Bill Clinton's choices in the private domain, which – entirely aside from its moral content – were surely quite risky, and many pundits – and no doubt many voters – used these stories as the basis for predictions about his choices in the public domain. Such predictions did not stop with his election; a 1994 article in Newsweek, for instance, concluded that "... it may well be that this is one case where private behavior does give an indication of how a politician will perform in the arena." History is yet to write its judgement of that prediction, but voters have already done so: Clinton left office with the highest approval rating of any President in recent history.

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Figure 1. An illustration of the linkage between attitude toward personal risk and attitude toward social risk