UC Berkeley
Haas School of Business
Economic Analysis for Business Decisions
(EWMBA 201A)

Game Theory II

Applications (part 1)

Lectures 6-7
Sep. 12, 2009

Outline

This week

[1] The main ideas – review

[2] Strictly competitive games

[3] Oligopolistic competition

Next week

[4] Auctions

[5] Bargaining / negotiations

[6] Observational learning

A review of the main ideas

We study two (out of four) groups of game theoretic models:

[1] Strategic games – all players simultaneously choose their plan of action once and for all.

[2] Extensive games (with perfect information) – players choose sequentially (and fully informed about all previous actions).

A solution (equilibrium) is a systematic description of the outcomes that may emerge in a family of games. We study two solution concepts:

[1] Nash equilibrium – a steady state of the play of a strategic game (no player has a profitable deviation given the actions of the other players).

[1] Subgame equilibrium – a steady state of the play of an extensive game (a Nash equilibrium in every subgame of the extensive game).

$\Longrightarrow$ Every subgame perfect equilibrium is also a Nash equilibrium.

Example I (a 2×2 strategic game)

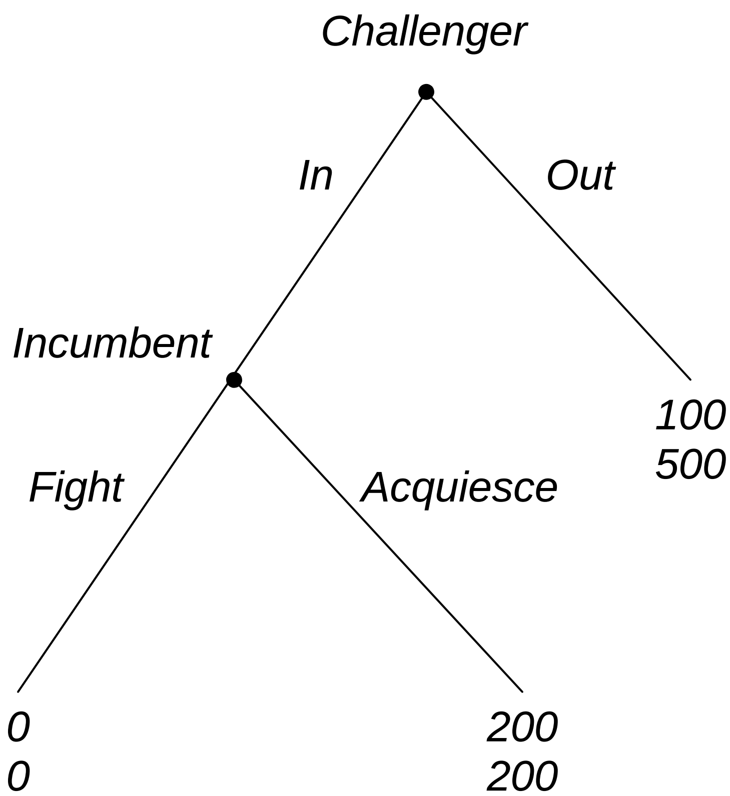
	B	S
B	$3, 1$	$0, 0$
S	$0, 0$	$1, 3$

This Battle of the Sexes (BoS) game has three Nash equilibria

$$(B, B),\ (S, S),\ \text{and}\ ((3/4, 1/4), (1/4, 3/4)).$$

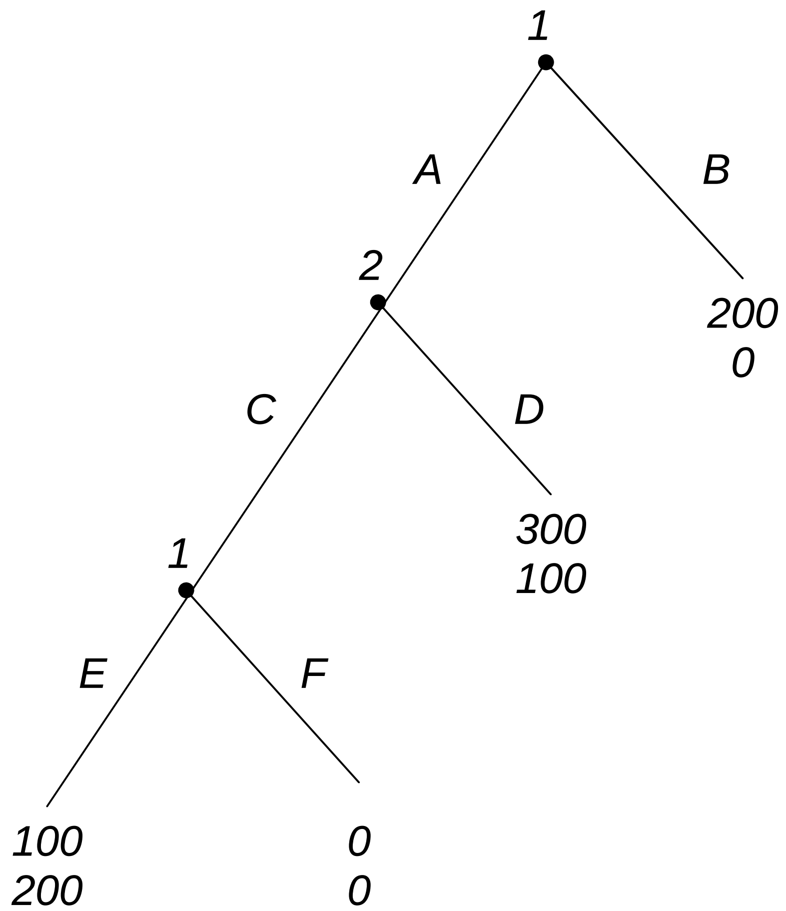
The last equilibrium is a mixed strategy equilibrium in which each player chooses B and S with positive probability (so each of the four outcome occurs with positive probability).

Example II (an entry game)

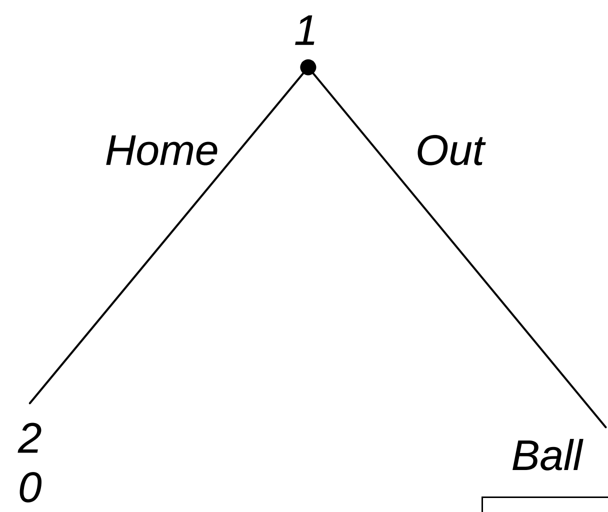


The game has two Nash equilibria (*In, Acquiesce*) and (*Out, Fight*) but only (*In, Acquiesce*) is a subgame perfect equilibrium.

Example III



Example IV (a game with simultaneous and sequential moves)



	Ball	*Show*
Ball	3,1	0,0
Show	0,0	1,3

Strictly competitive games

In strictly competitive games, the players' interests are diametrically opposed.

More precisely, a strategic two-player game is strictly competitive if for any two outcomes a and b we have

$$a \succsim_1 b \text{ if and only if } b \succsim_2 a.$$

A strictly competitive game can be represented as a zero-sum game

	L	R
T	$A, -A$	$B, -B$
B	$C, -C$	$D, -D$

This class of games is important for a number of reasons:

- A simple decision making procedure leads each player to choose a Nash equilibrium action.

- There are innumerable social and economic situations which are strictly competitive.

- In the game of business, a successful strategy is avoiding the zero-sum trap by reshaping the game.

$\Longrightarrow$ See Brandenburger & Nalebuff and Hermalin (Chapter 6).

Maxminimization

A maxminimizing strategy is a (mixed) strategy that maximizes the player's minimal payoff.

> A strategy that maximizes the player's expected payoff under the (very pessimistic) assumption that whatever she does the other player will act in a way that minimizes her expected payoff.

A pair strategies in a strictly competitive game is a Nash equilibrium if and only if each player's strategy is a maxminimizer (or a minimaximzer).

An example

	L	R
T	$2, -2$	$-1, 1$
B	$-1, 1$	$1, -1$

The maxminimizing strategy of player 1 is $(2/5, 3/5)$, which yields her a payoff of $1/5$.

Some history: the theory was developed by von Neumann in the late 1920s but the idea appeared two centuries earlier (Montmort, 1713-4).

Changing the game of business

(Brandenburger & Nalebuff)

To change a (strictly competitive) game one has to change on or more of its elements:

- Players (including yourself)
- Added values
- Rules
- Strategies
- Scope

Oligopolistic competition (PR 12.2-12.5)

Cournot's oligopoly model (1838)

- A single good is produced by two firms (the industry is a "duopoly").

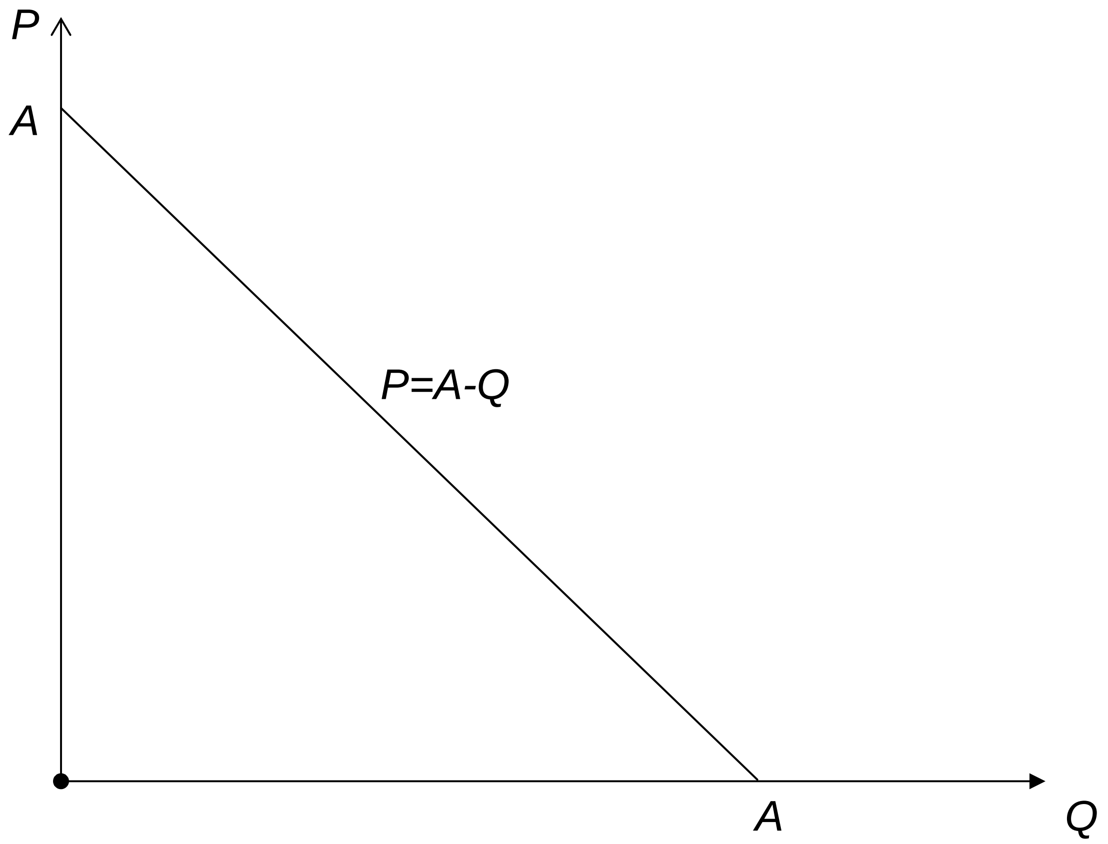
- The cost for firm $i = 1, 2$ for producing q_i units of the good is given by $c_i q_i$ ("unit cost" is constant equal to $c_i > 0$).

- If the firms' total output is $Q = q_1 + q_2$ then the market price is

$$P = A - Q$$

  if $A \geq Q$ and zero otherwise (linear inverse demand function). We also assume that $A > c$.

The inverse demand function



To find the Nash equilibria of the Cournot's game, we can use the procedures based on the firms' best response functions.

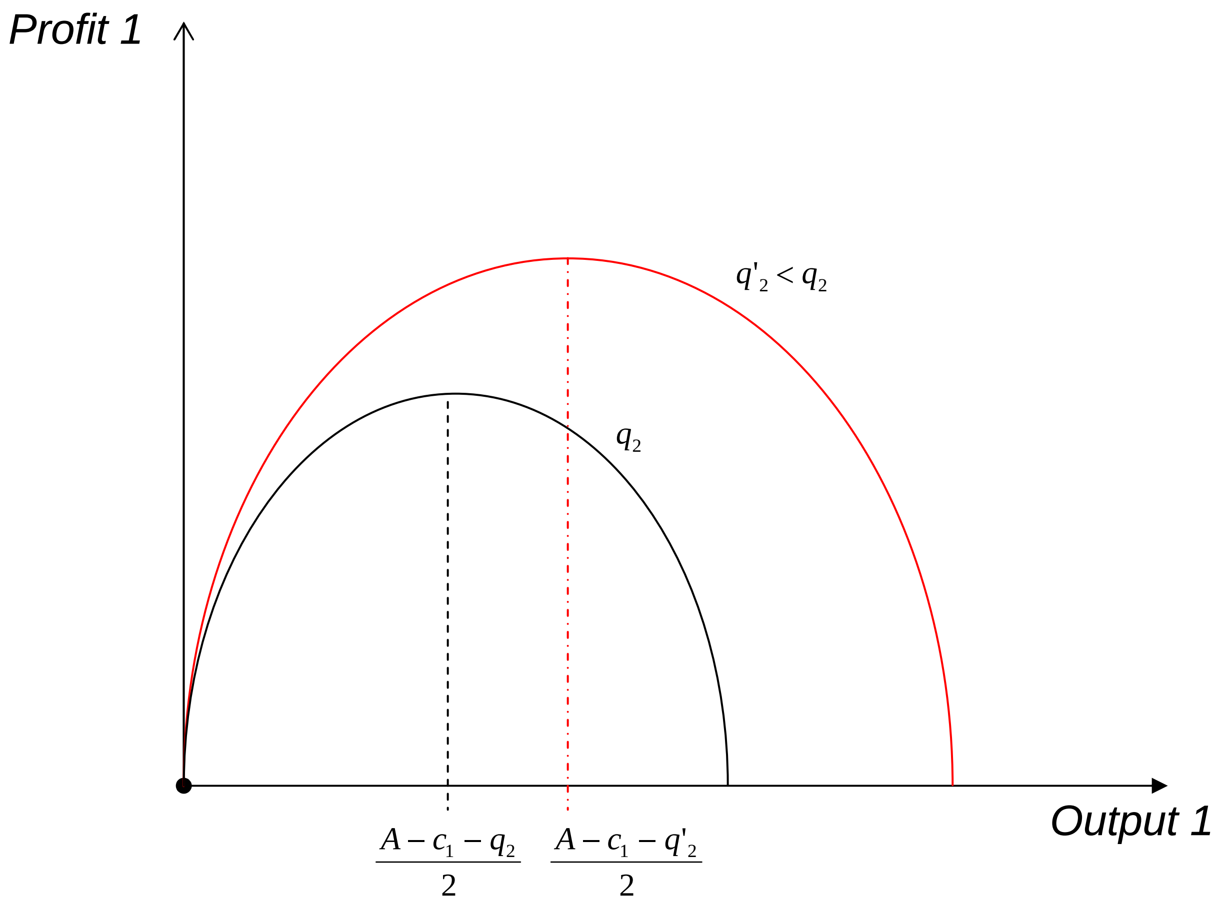
But first we need the firms payoffs (profits):

$$
\begin{aligned}
\pi_1 &= Pq_1 - c_1q_1 \\
&= (A - Q)q_1 - c_1q_1 \\
&= (A - q_1 - q_2)q_1 - c_1q_1 \\
&= (A - q_1 - q_2 - c_1)q_1
\end{aligned}
$$

and similarly,

$$\pi_2 = (A - q_1 - q_2 - c_2)q_2$$

Firm 1's profit as a function of its output (given firm 2's output)



To find firm 1's best response to any given output q_2 of firm 2, we need to study firm 1's profit as a function of its output q_1 for given values of q_2.

If you know calculus, you can set the derivative of firm 1's profit with respect to q_1 equal to zero and solve for q_1:

$$q_1 = \frac{1}{2}(A - q_2 - c_1).$$

We conclude that the best response of firm 1 to the output q_2 of firm 2 depends on the values of q_2 and c_1.

Because firm 2's cost function is $c_2 \neq c_1$, its best response function is given by

$$q_2 = \frac{1}{2}(A - q_1 - c_2).$$

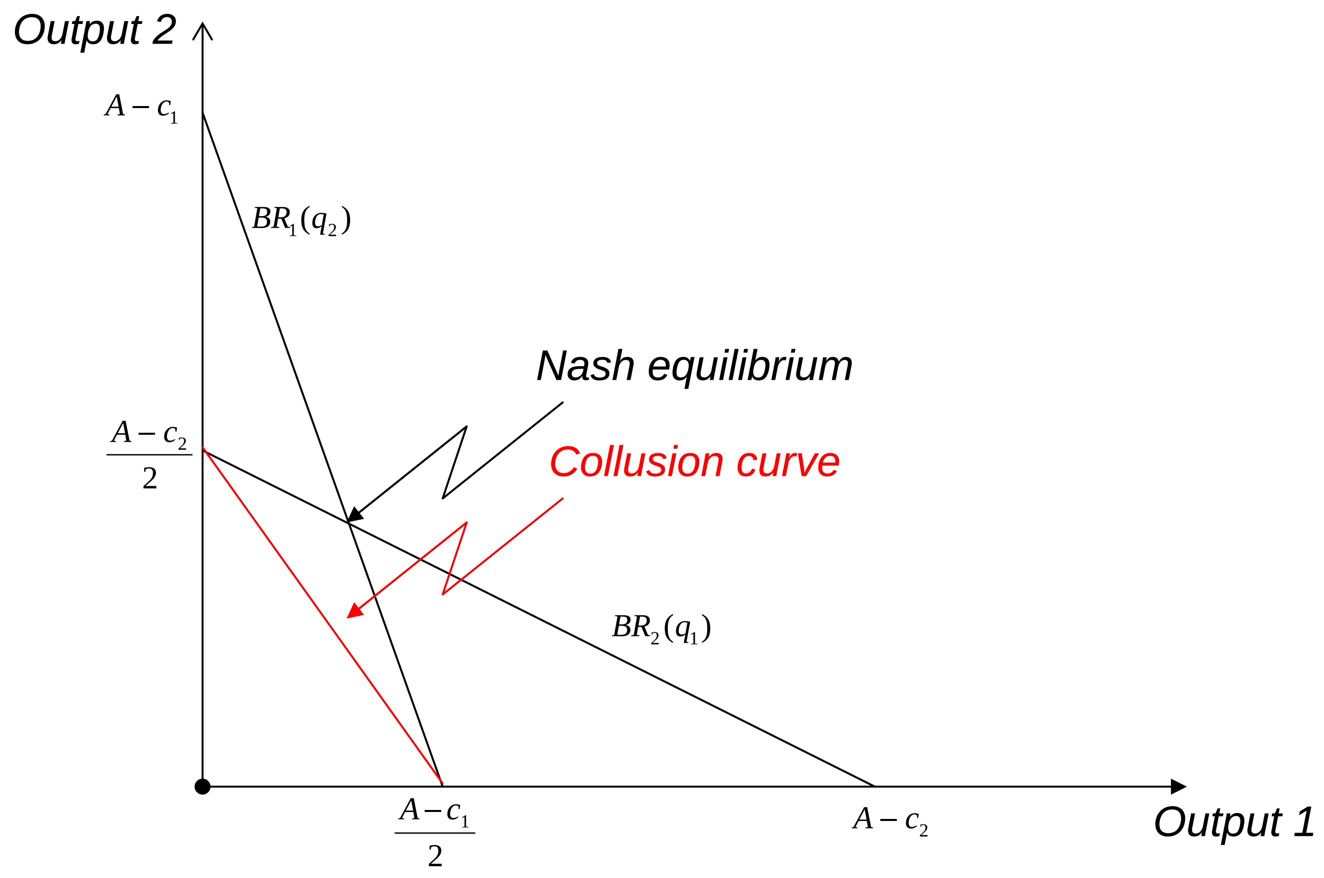
A Nash equilibrium of the Cournot's game is a pair (q_1^*, q_2^*) of outputs such that q_1^* is a best response to q_2^* and q_2^* is a best response to q_1^*.

From the figure below, we see that there is exactly one such pair of outputs

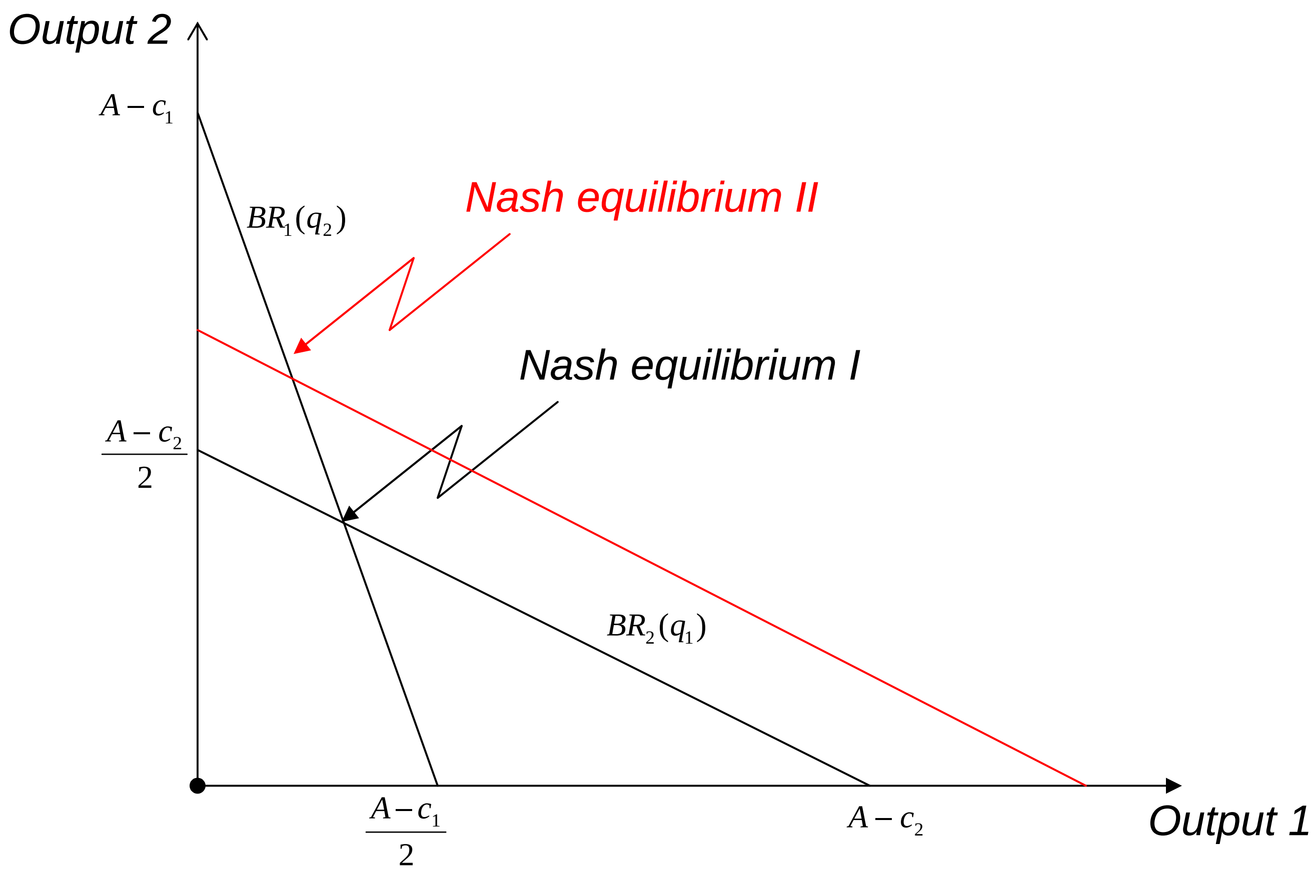
$$q_1^* = \frac{A+c_2-2c_1}{3} \quad \text{and} \quad q_2^* = \frac{A+c_1-2c_2}{3}$$

which is the solution to the two equations above.

The best response functions in the Cournot's duopoly game



Nash equilibrium comparative statics
(a decrease in the cost of firm 2)



A question: what happens when consumers are willing to pay more (*A* increases)?

In summary, this simple Cournot's duopoly game has a unique Nash equilibrium.

Two economically important properties of the Nash equilibrium are (to economic regulatory agencies):

[1] The relation between the firms' equilibrium profits and the profit they could make if they act collusively.

[2] The relation between the equilibrium profits and the number of firms.

[1] Collusive outcomes: in the Cournot's duopoly game, there is a pair of outputs at which *both* firms' profits exceed their levels in a Nash equilibrium.

[2] Competition: The price at the Nash equilibrium if the two firms have the *same* unit cost $c_1 = c_2 = c$ is given by

$$\begin{aligned} P^* &= A - q_1^* - q_2^* \\ &= \frac{1}{3}(A + 2c) \end{aligned}$$

which is above the unit cost c. But as the number of firm increases, the equilibrium price deceases, approaching c (zero profits!).

Stackelberg's duopoly model (1934)

How do the conclusions of the Cournot's duopoly game change when the firms move sequentially? Is a firm better off moving before or after the other firm?

Suppose that $c_1 = c_2 = c$ and that firm 1 moves at the start of the game. We may use backward induction to find the subgame perfect equilibrium.

– First, for *any* output q_1 of firm 1, we find the output q_2 of firm 2 that maximizes its profit. Nest, we find the output q_1 of firm 1 that maximizes its profit, *given the strategy* of firm 2.

Firm 2

Since firm 2 moves after firm 1, a strategy of firm 2 is a *function* that associate an output q_2 for firm 2 for each possible output q_1 of firm 1.

We found that under the assumptions of the Cournot's duopoly game Firm 2 has a unique best response to each output q_1 of firm 1, given by

$$q_2 = \frac{1}{2}(A - q_1 - c)$$

(Recall that $c_1 = c_2 = c$).

Firm 1

Firm 1's strategy is the output q_1 the maximizes

$$\pi_1 = (A - q_1 - q_2 - c)q_1 \quad \text{subject to} \quad q_2 = \tfrac{1}{2}(A - q_1 - c)$$

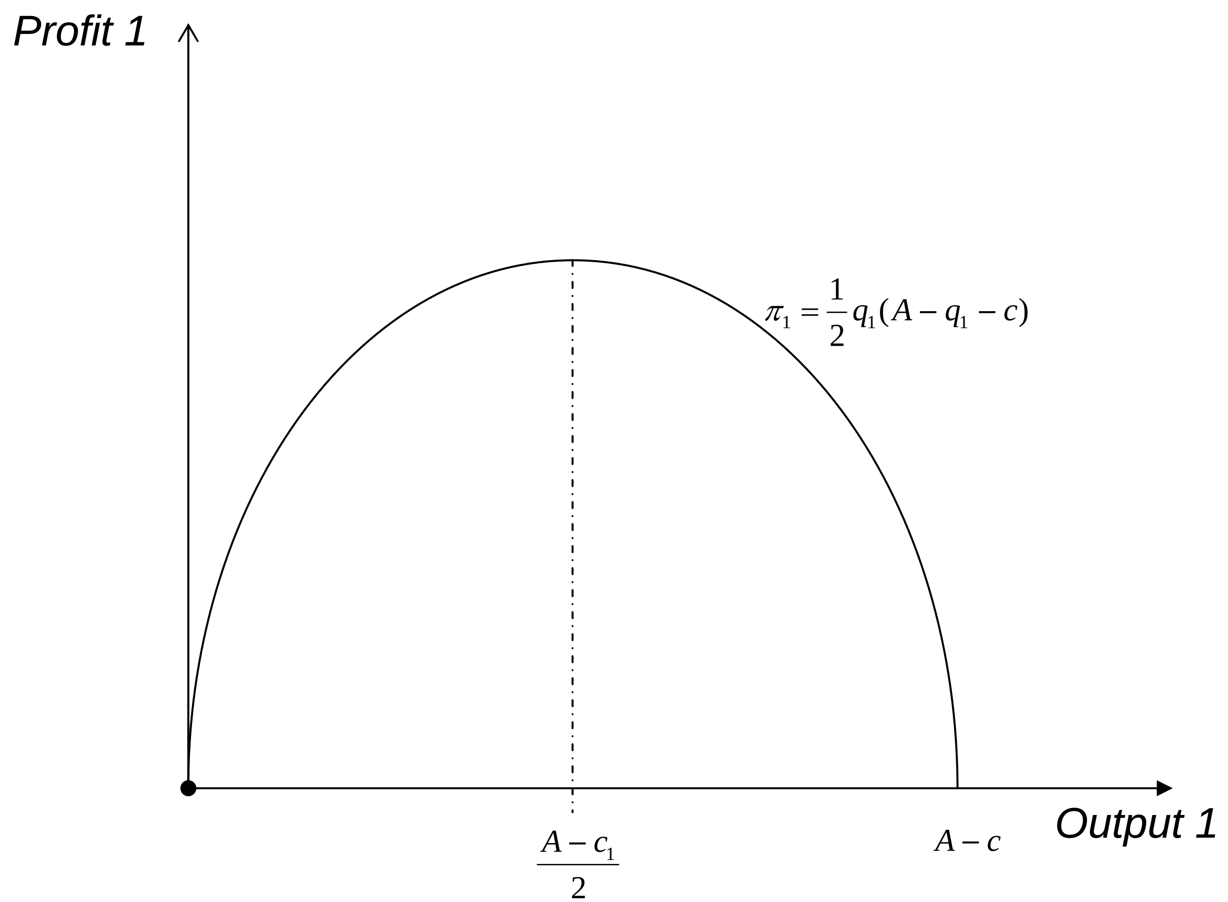
Thus, firm 1 maximizes

$$\pi_1 = (A - q_1 - (\frac{1}{2}(A - q_1 - c)) - c)q_1 = \frac{1}{2}q_1(A - q_1 - c).$$

This function is quadratic in q_1 that is zero when $q_1 = 0$ and when $q_1 = A - c$. Thus its maximizer is

$$q_1^* = \frac{1}{2}(A - c).$$

Firm 1's (first-mover) profit in Stackelberg's duopoly game



We conclude that Stackelberg's duopoly game has a unique subgame perfect equilibrium, in which firm 1's strategy is the output

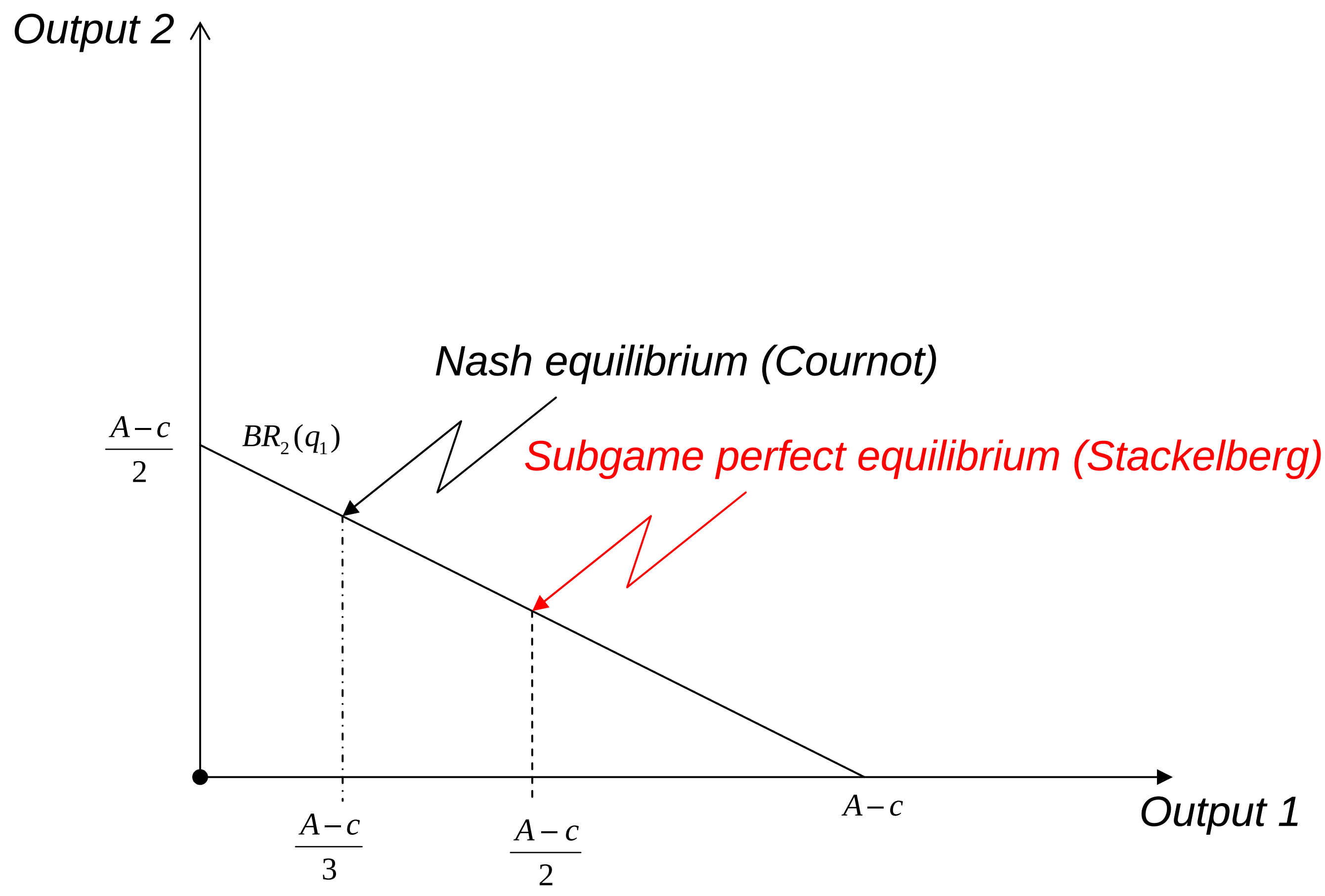
$$q_1^* = \frac{1}{2}(A - c)$$

and firm 2's output is

$$\begin{aligned} q_2^* &= \frac{1}{2}(A - q_1^* - c) \\ &= \frac{1}{2}(A - \frac{1}{2}(A - c) - c) \\ &= \frac{1}{4}(A - c). \end{aligned}$$

By contrast, in the unique Nash equilibrium of the Cournot's duopoly game under the same assumptions ($c_1 = c_2 = c$), each firm produces $\frac{1}{3}(A - c)$.

The subgame perfect equilibrium of Stackelberg's duopoly game



Bertrand's oligopoly model (1883)

In Cournot's game, each firm chooses an output, and the price is determined by the market demand in relation to the total output produced.

An alternative model, suggested by Bertrand, assumes that each firm chooses a price, and produces enough output to meet the demand it faces, given the prices chosen by *all* the firms.

$\Longrightarrow$ As we shell see, some of the answers it gives are different from the answers of Cournot.

Suppose again that there are two firms (the industry is a "duopoly") and that the cost for firm $i = 1, 2$ for producing q_i units of the good is given by cq_i (equal constant "unit cost").

Assume that the demand function (rather than the inverse demand function as we did for the Cournot's game) is

$$D(p) = A - p$$

for $A \geq p$ and zero otherwise, and that $A > c$ (the demand function in PR 12.3 is different).

Because the cost of producing each until is the same, equal to c, firm i makes the profit of $p_i - c$ on every unit it sells. Thus its profit is

$$\pi_i = \begin{cases} (p_i - c)(A - p_i) & \text{if } \; p_i < p_j \\ \frac{1}{2}(p_i - c)(A - p_i) & \text{if } \; p_i = p_j \\ 0 & \text{if } \; p_i > p_j \end{cases}$$

where j is the other firm.

In Bertrand's game we can easily argue as follows: $(p_1, p_2) = (c, c)$ is the unique Nash equilibrium.

Using intuition,

- If one firm charges the price c, then the other firm can do no better than charge the price c.

- If $p_1 > c$ and $p_2 > c$, then each firm i can increase its profit by lowering its price p_i slightly below p_j.

$\Longrightarrow$ In Cournot's game, the market price decreases toward c as the number of firms increases, whereas in Bertrand's game it is c (so profits are zero) even if there are only two firms (but the price remains c when the number of firm increases).

Avoiding the Bertrand trap

If you are in a situation satisfying the following assumptions, then you will end up in a Bertrand trap (zero profits):

[1] Homogenous products

[2] Consumers know all firm prices

[3] No switching costs

[4] No cost advantages

[5] No capacity constraints

[6] No future considerations