UC Berkeley
Haas School of Business
Berkeley MBA for Executives Program

Game Theory
(XMBA 296)

Block 5
Social learning
Aug 7-8, 2014

“Men nearly always follow the tracks made by others and proceed in their affairs by imitation.” Machiavelli (Renaissance philosopher)

Examples

Business strategy

- TV networks make introductions in the same categories as their rivals.

Finance

- The withdrawal behavior of small number of depositors starts a bank run.

Politics

– The solid New Hampshirites (probably) can not be too far wrong.

Crime

– In NYC, individuals are more likely to commit crimes when those around them do.

Why should individuals behave in this way?

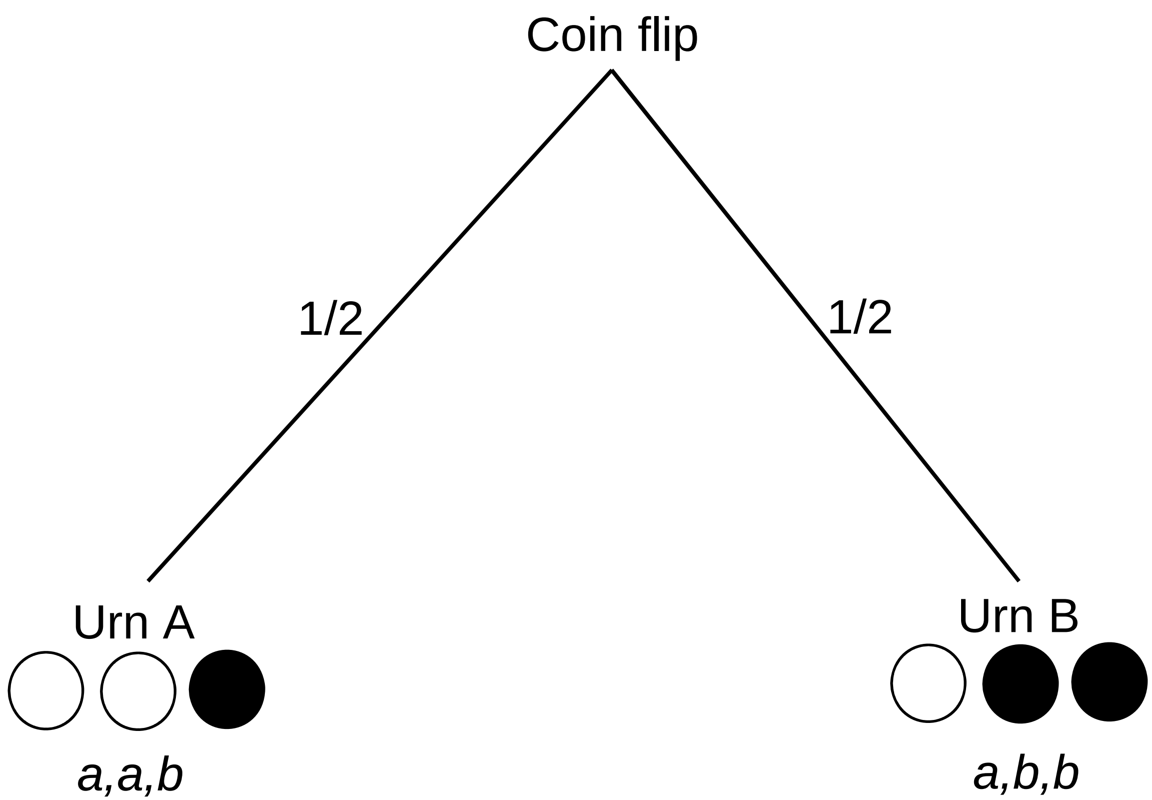
Several “theories” explain the existence of uniform social behavior:

- benefits from conformity
- sanctions imposed on deviants
- network / payoff externalities
- social learning

Broad definition: any situation in which individuals learn by observing the behavior of others.

The canonical model of social learning

- Rational (Bayesian) behavior
- Incomplete and asymmetric information
- Pure information externality
- Once-in-a-lifetime decisions
- Exogenous sequencing
- Perfect information / complete history


Coin flip
1/2
1/2
Urn A
a,a,b
Urn B
a,b,b

Bayes' rule

Let n be the number of a signals and m be the number of b signals. Then Bayes' rule can be used to calculate the posterior probability of urn A:

$$\begin{aligned}
\Pr(A\,|n,m) &= \frac{\Pr(A)\Pr(n,m\,|A)}{\Pr(A)\Pr(n,m\,|A)+\Pr(B)\Pr(n,m\,|B)} \\
&= \frac{(\frac{1}{2})(\frac{2}{3})^n(\frac{1}{3})^m}{(\frac{1}{2})(\frac{2}{3})^n(\frac{1}{3})^m+(\frac{1}{2})(\frac{1}{3})^m(\frac{2}{3})^n} \\
&= \frac{2^n}{2^n+2^m}.
\end{aligned}$$

An example

- There are two decision-relevant events, say A and B, equally likely to occur *ex ante* and two corresponding signals a and b.

- Signals are informative in the sense that there is a probability higher than $1/2$ that a signal matches the label of the realized event.

- The decision to be made is a prediction of which of the events takes place, basing the forecast on a private signal and the history of past decisions.

- Whenever two consecutive decisions coincide, say both predict A, the subsequent player should also choose A even if his signal is different b.

- Despite the asymmetry of private information, eventually every player imitates her predecessor.

- Since actions aggregate information poorly, despite the available information, such herds / cascades often adopt a suboptimal action.

What have we learned from Social Learning?

- Finding 1
  - Individuals 'ignore' their own information and follow a herd.
- Finding 2
  - Herds often adopt a wrong action.
- Finding 3
  - Mass behavior may be idiosyncratic and fragile.

Informational cascades and herd behavior

Two phenomena that have elicited particular interest are *informational cascades* and *herd behavior*.

- Cascade: agents 'ignore' their private information when choosing an action.

- Herd: agents choose the same action, not necessarily ignoring their private information.

- While the terms informational cascade and herd behavior are used interchangeably there is a significant difference between them.

- In an informational cascade, an agent considers it optimal to follow the behavior of her predecessors without regard to her private signal.

- When acting in a herd, agents choose the same action, not necessarily ignoring their private information.

- Thus, an informational cascade implies a herd but a herd is not necessarily the result of an informational cascade.

A model of social learning

Signals

- Each player $n \in \{1, ..., N\}$ receives a signal θ_n that is private information.

- For simplicity, $\{\theta_n\}$ are independent and uniformly distributed on $[-1, 1]$.

Actions

- Sequentially, each player n has to make a binary irreversible decision $x_n \in \{0, 1\}$.

Payoffs

– $x = 1$ is profitable if and only if $\sum_{n \le N} \theta_n \ge 0$, and $x = 0$ is profitable otherwise.

Information

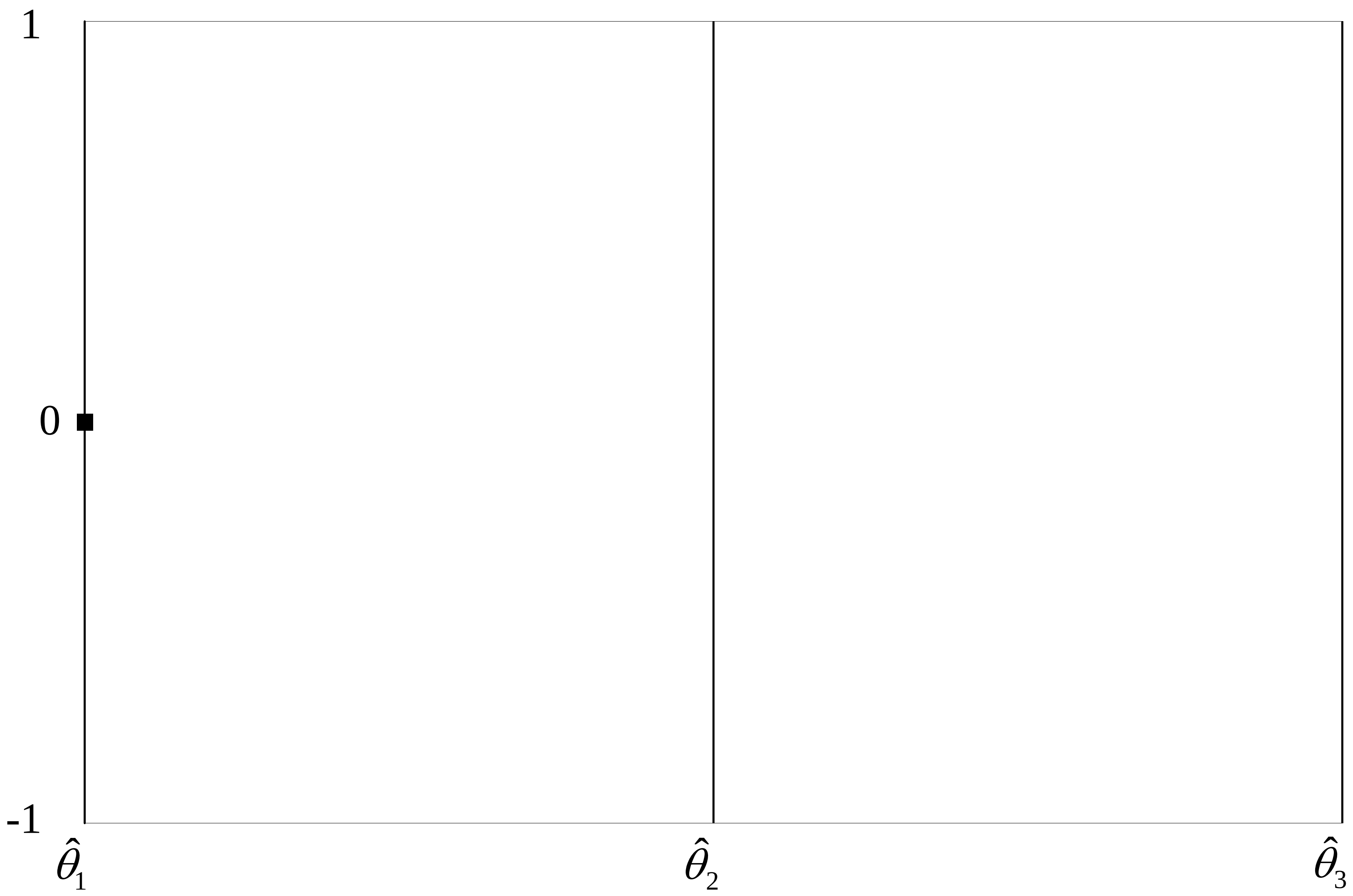
– Perfect information

$$\mathcal{I}_n = \{\theta_n, (x_1, x_2, ..., x_{n-1})\}$$

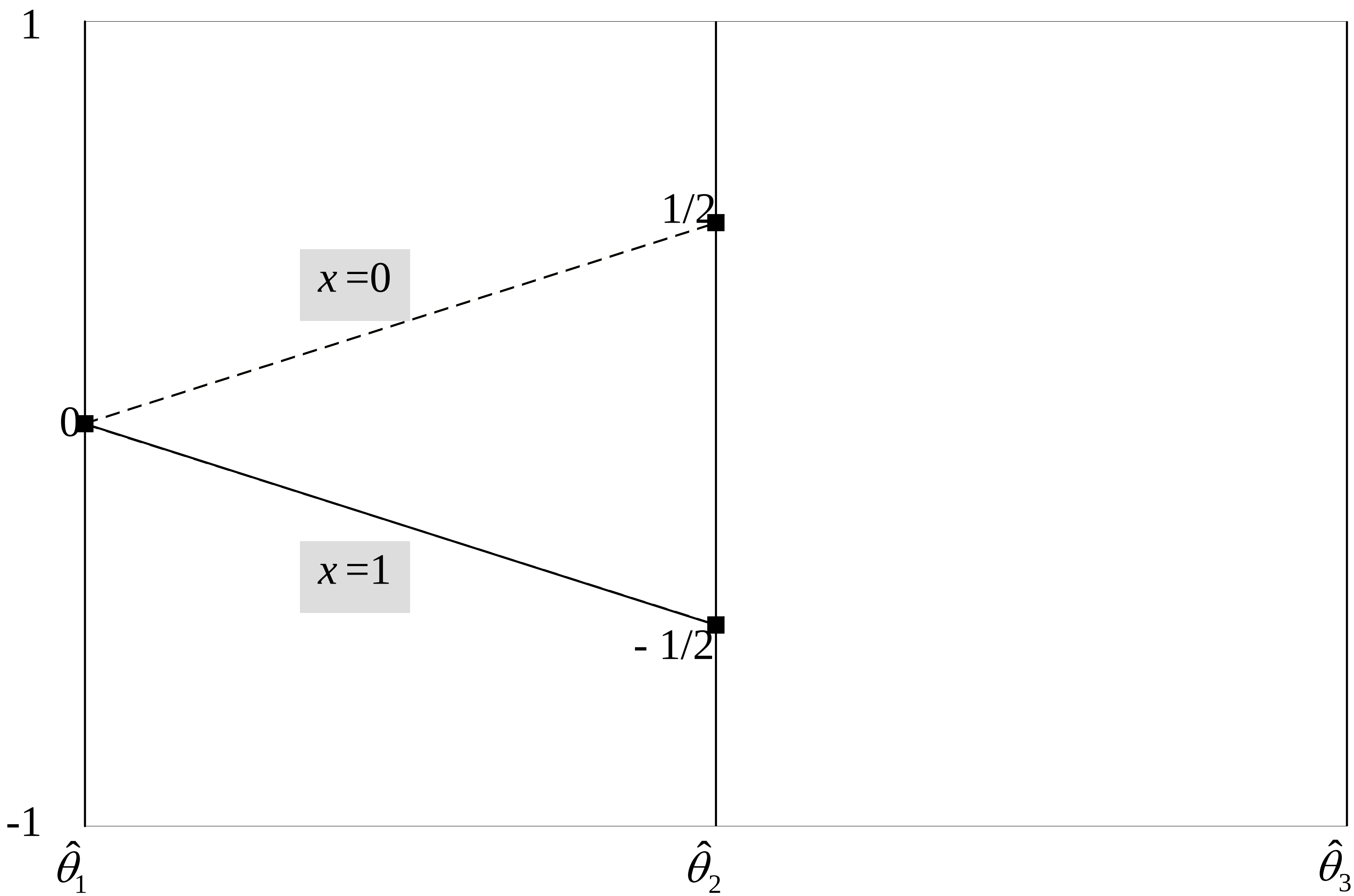
– Imperfect information

$$\mathcal{I}_n = \{\theta_n, x_{n-1}\}$$

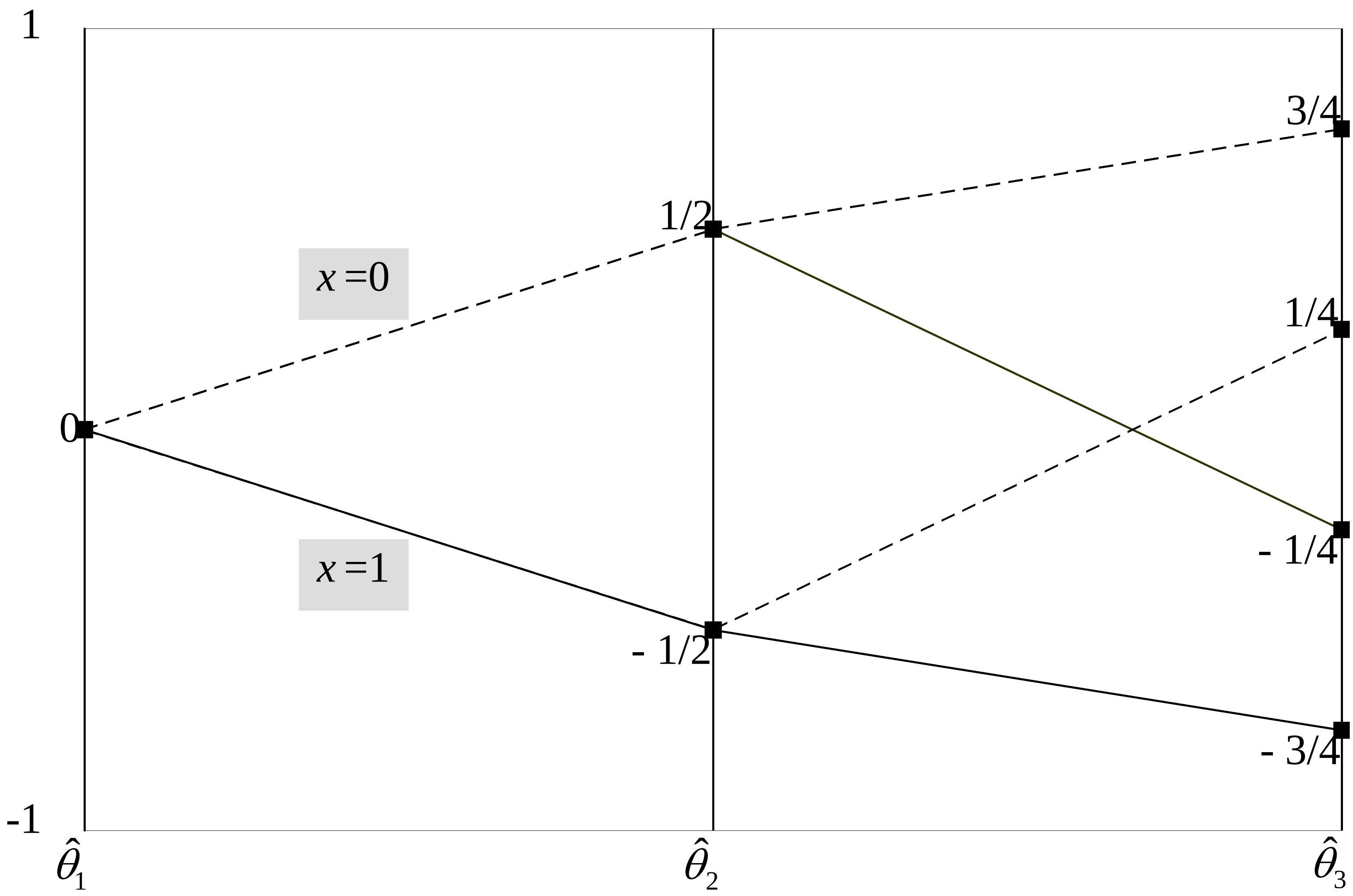
A three-agent example



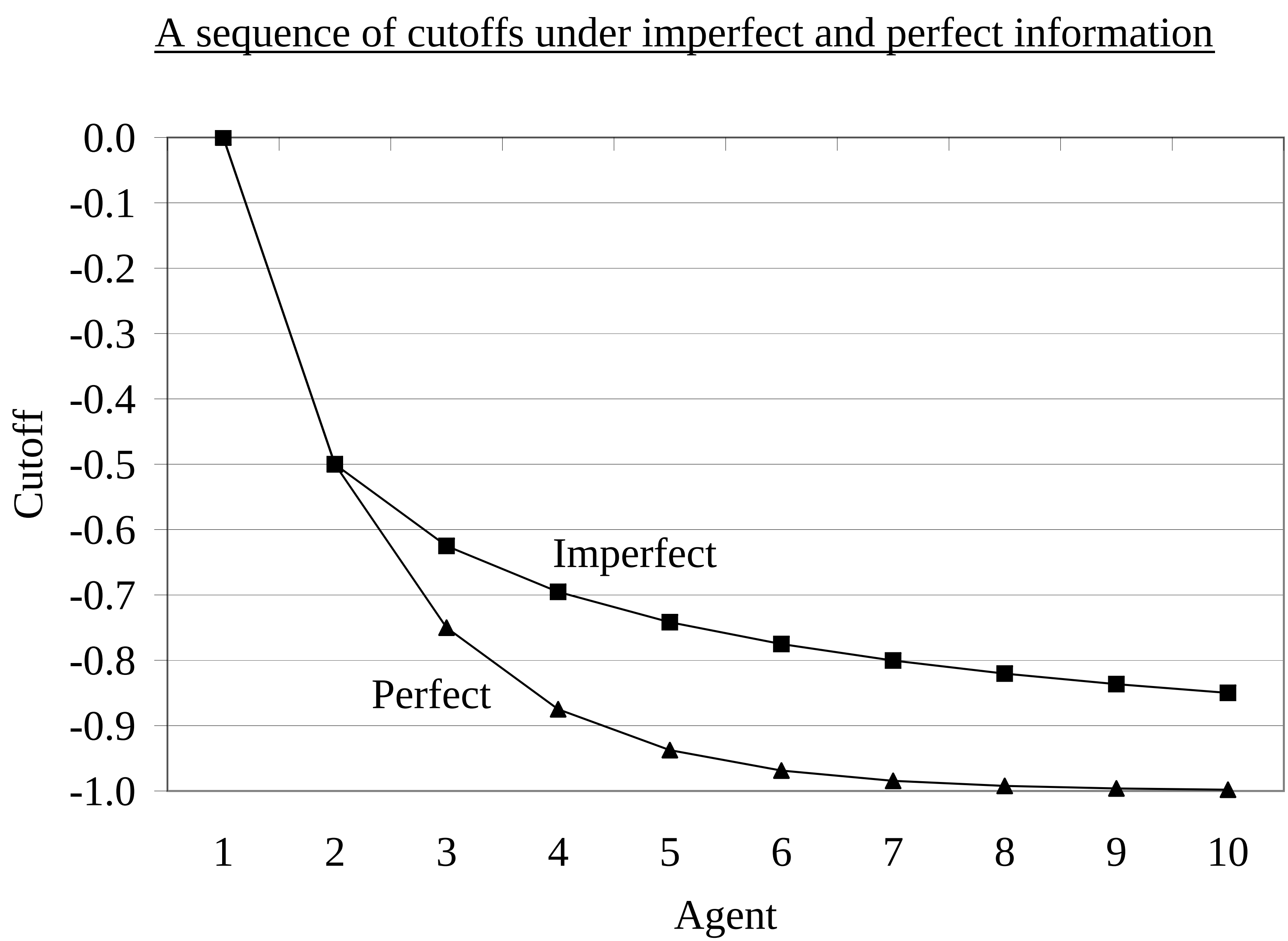
A three-agent example



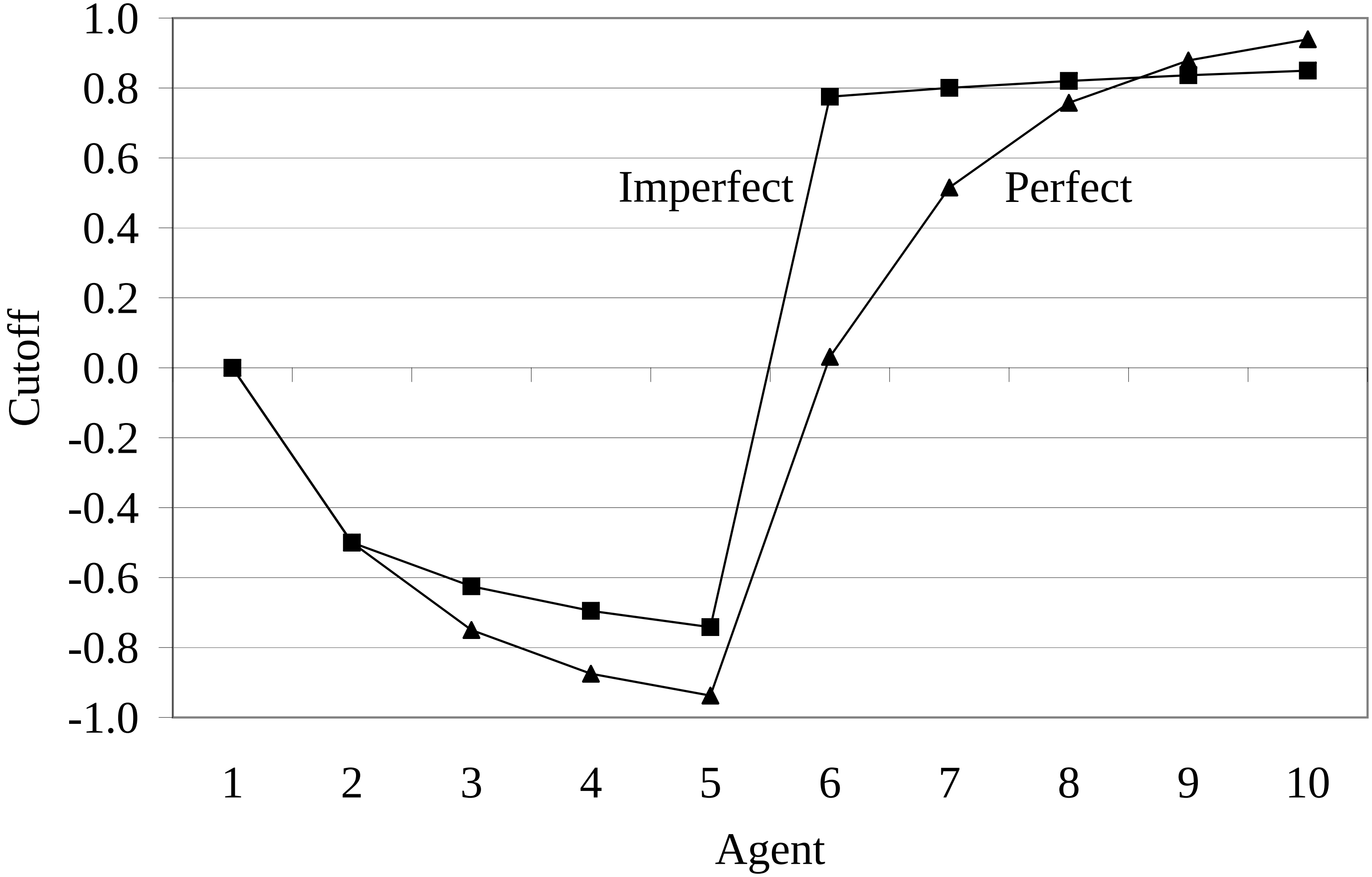
A three-agent example under perfect information



A three-agent example under imperfect information


A sequence of cutoffs under imperfect and perfect information
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
Cutoff
Imperfect
Perfect
1
2
3
4
5
6
7
8
9
10
Agent

A sequence of cutoffs under imperfect and perfect information



The decision problem

– The optimal decision rule is given by

$$x_n = 1 \text{ if and only if } \mathbb{E}\left[\sum_{i=1}^{N} \theta_i \mid \mathcal{I}_n\right] \geq 0.$$

Since $\mathcal{I}_n$ does not provide any information about the content of successors' signals, we obtain

$$x_n = 1 \text{ if and only if } \mathbb{E}\left[\sum_{i=1}^{n} \theta_i \mid \mathcal{I}_n\right] \geq 0$$

Hence,

$$x_n = 1 \text{ if and only if } \theta_n \geq -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n\right].$$

The cutoff process

– For any n, the optimal strategy is the *cutoff strategy*

$$x_n = \begin{cases} 1 & if \quad \theta_n \geq \hat{\theta}_n \\ 0 & if \quad \theta_n < \hat{\theta}_n \end{cases}$$

where

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n\right]$$

is the optimal history-contingent cutoff.

– $\hat{\theta}_n$ is sufficient to characterize the individual behavior, and $\{\hat{\theta}_n\}$ characterizes the social behavior of the economy.

Overview of results

Perfect information

– A cascade need not arise, but herd behavior must arise.

Imperfect information

– Herd behavior is impossible. There are periods of uniform behavior, punctuated by increasingly rare switches.

- The similarity:
  - Agents can, for a long time, make the same (incorrect) choice.
- The difference:
  - Under perfect information, a herd is an absorbing state. Under imperfect information, continued, occasional and sharp shifts in behavior.

- The dynamics of social learning depend crucially on the extensive form of the game.

- The key economic phenomenon that imperfect information captures is a succession of fads starting suddenly, expiring rather easily, each replaced by another fad.

- The kind of episodic instability that is characteristic of socioeconomic behavior in the real world makes more sense in the imperfect-information model.

As such, the imperfect-information model gives insight into phenomena such as manias, fashions, crashes and booms, and better answers such questions as:

– Why do markets move from boom to crash without settling down?

– Why is a technology adopted by a wide range of users more rapidly than expected and then, suddenly, replaced by an alternative?

– What makes a restaurant fashionable over night and equally unexpectedly unfashionable, while another becomes the 'in place', and so on?

The case of perfect information

The optimal history-contingent cutoff rule is

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_1, ..., x_{n-1}\right],$$

and $\hat{\theta}_n$ is different from $\hat{\theta}_{n-1}$ only by the information reveals by the action of agent $(n-1)$

$$\hat{\theta}_n = \hat{\theta}_{n-1} - \mathbb{E}\left[\theta_{n-1} \mid \hat{\theta}_{n-1}, x_{n-1}\right],$$

The cutoff dynamics thus follow the cutoff process

$$\hat{\theta}_n = \begin{cases} \frac{-1+\hat{\theta}_{n-1}}{2} & \text{if} \quad x_{n-1} = 1 \\ \frac{1+\hat{\theta}_{n-1}}{2} & \text{if} \quad x_{n-1} = 0 \end{cases}$$

where $\hat{\theta}_1 = 0$.

Informational cascades

– $-1 < \hat{\theta}_n < 1$ for any n so any player takes his private signal into account in a non-trivial way.

Herd behavior

– $\{\hat{\theta}_n\}$ has the martingale property by the Martingale Convergence Theorem a limit-cascade implies a herd.

The case of imperfect information

The optimal history-contingent cutoff rule is

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1}\right],$$

which can take two values conditional on $x_{n-1} = 1$ or $x_{n-1} = 0$

$$\begin{aligned}\overline{\theta}_n &= -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1} = 1\right],\\ \underline{\theta}_n &= -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1} = 1\right].\end{aligned}$$

where $\overline{\theta}_n = -\underline{\theta}_n$.

The law of motion for $\overline{\theta}_n$ is given by

$$\begin{aligned}\overline{\theta}_n = P(x_{n-2} = 1 | x_{n-1} = 1) \left\{ \overline{\theta}_{n-1} - \mathbb{E}\left[\theta_{n-1} \mid x_{n-2} = 1\right] \right\} \\ + P(x_{n-2} = 0 | x_{n-1} = 1) \left\{ \underline{\theta}_{n-1} - \mathbb{E}\left[\theta_{n-1} \mid x_{n-2} = 0\right] \right\},\end{aligned}$$

which simplifies to

$$\begin{aligned}\overline{\theta}_n &= \frac{1 - \overline{\theta}_{n-1}}{2} \left[\overline{\theta}_{n-1} - \frac{1 + \overline{\theta}_{n-1}}{2} \right] \\ &\quad + \frac{1 - \underline{\theta}_{n-1}}{2} \left[\underline{\theta}_{n-1} - \frac{1 + \underline{\theta}_{n-1}}{2} \right].\end{aligned}$$

Given that $\overline{\theta}_n = -\overline{\theta}_n$, the cutoff dynamics under imperfect information follow the cutoff process

$$\hat{\theta}_n = \begin{cases} -\frac{1+\hat{\theta}^2_{n-1}}{2} & \text{if} \quad x_{n-1} = 1 \\ \frac{1+\hat{\theta}^2_{n-1}}{2} & \text{if} \quad x_{n-1} = 0 \end{cases}$$

where $\hat{\theta}_1 = 0$.

Informational cascades

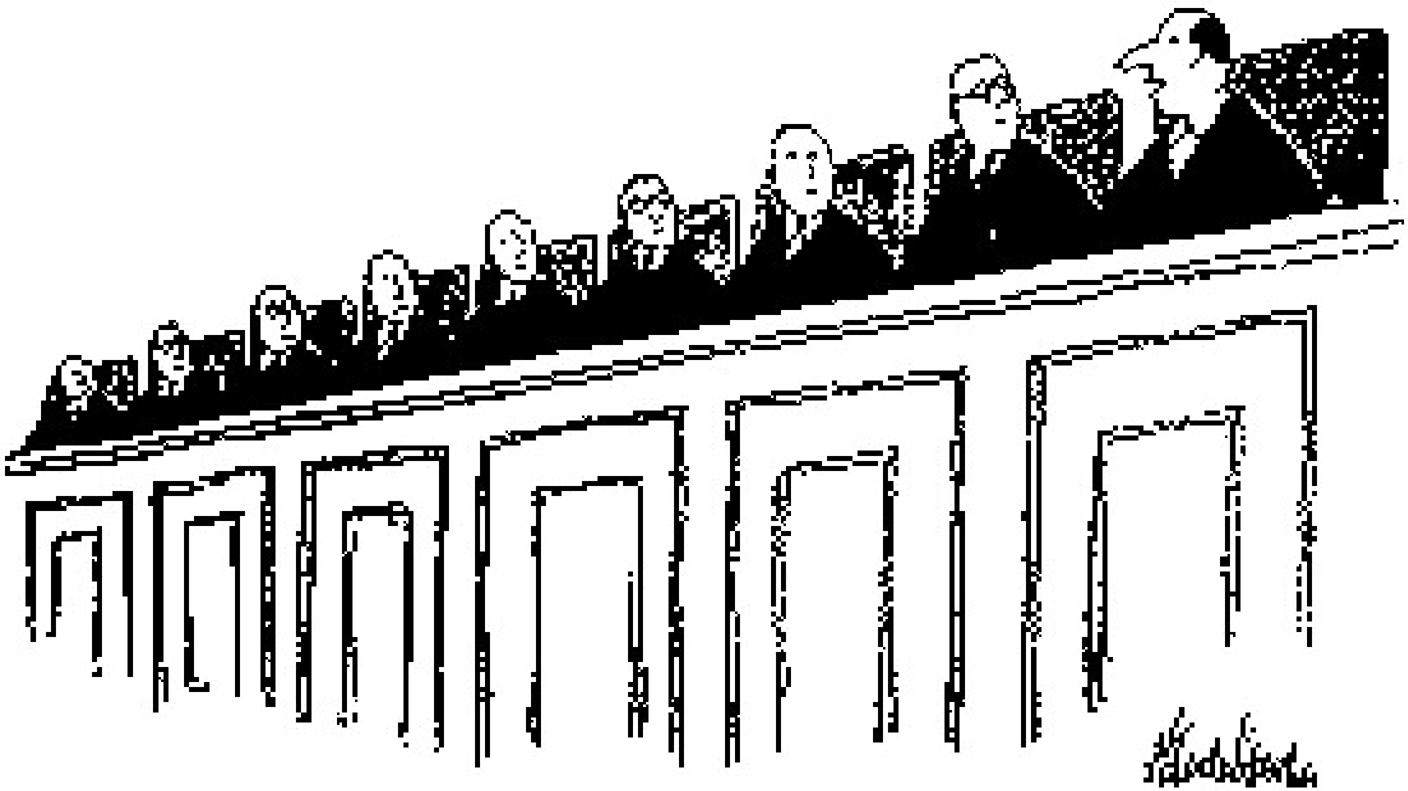
- $-1 < \hat{\theta}_n < 1$ for any n so any player takes his private signal into account in a non-trivial way.

Herd behavior

- $\{\hat{\theta}_n\}$ is not convergent (proof is hard!) and the divergence of cutoffs implies divergence of actions.

- Behavior exhibits periods of uniform behavior, punctuated by increasingly rare switches.

Sequential social-learning model:
Well heck, if all you smart cookies agree, who am I to dissent?



Imperfect information:
Which way is the wind blowing?!