8 Application of a Continuous Spatial Choice Logit Model

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8.1 Introduction

Travel demand predictions require aggregation over numerous spatial alternatives and spatially distributed individuals.[1] It is customarily done by dividing the area into zones that are taken as the relevant spatial alternatives and as homogeneous market segments of individuals. However, in general unbiased aggregate predictions cannot be obtained by using only average values of the independent variables in an individual choice model. Therefore an aggregation procedure that employs information about the distributions of the variables is required.[2] An efficient aggregation procedure is particularly important in sketch-planning models that are designed for large spatial analysis units and limited input data requirements.

The methodology developed in this chapter employs continuous mathematical functions, expressed in terms of spatial coordinates, for the spatial choice models and for the distribution of individuals, spatial alternatives, and spatial attributes. The prediction of aggregate travel demand is achieved by integrating a continuous spatial choice model over the areas of the relevant zones.

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1. Travel and spatial choice models are described in many references including Domencich and McFadden (1975), Richards and Ben-Akiva (1975), Ben-Akiva et al. (1976), Ben-Akiva and Atherton (1977), and Spear (1977).

2. Procedures for aggregating choice models over individuals were investigated by Talvitie (1973), Westin (1974), McFadden and Reid (1975), Koppelman (1975) and Landau (1976). The alternative forms of representing the distributions are reviewed in Koppelman (1975) for aggregation over individuals and in Watanatada and Ben-Akiva (1977) for aggregation over alternatives and individuals.

The logit model has been extensively applied to spatial and nonspatial choice problems.[3] It is described here in its conventional discrete form and is then applied in a continuous form as a spatial choice model expressed in terms of two-dimensional coordinates to represent the location of the spatial alternatives.

The chapter concludes with a brief description of an application of the methodology to an aggregate prediction model of urbanized area travel demand using Monte Carlo simulation techniques. This model can be used in the framework of the multimodal national urban transportation policy-planning model described in Weiner (1976) and known as TRANS (transportation resource allocation study); it is referred to as MIT-TRANS (Watanatada and Ben-Akiva 1977).

8.2 Basic Definitions

Denote the probability of an individual t, choosing an elemental alternative s as $P_t(s)$.[4] The expected number of individuals choosing alternative s, T_s, is the sum of individual's choice probabilities for alternative s:

$$T_s = \sum_{t=1}^{T} P_t(s),$$

where T is the number of individuals in the aggregate group.

The foregoing relationship represents an aggregation of individuals for an elemental alternative. In the case where a group of elemental alternatives, rather than the elemental alternatives themselves, is of interest, aggregation of alternatives is performed for each individual t by summation of elemental alternatives' choice probabilities:

$$P_t(j) = \sum_{s \in j} P_t(s),$$

where $P_t(j)$ denotes the probability that individual t will choose an elemental alternative in group j.

3. The logit model and its derivation from a theory of utility maximization, properties, and econometric analysis techniques are given in McFadden (1974).

4. Elemental alternatives in a choice process are defined such that the individual chooses one and only one of them. That is, elemental alternatives are mutually exclusive in the same sense as elemental or atomic events in probability.

The aggregation of individuals for a group of alternatives j expresses the expected number of individuals choosing an elemental alternative in group j:

$$T_j = \sum_{t=1}^{T} P_t(j) = \sum_{t=1}^{T} \sum_{s \in j} P_t(s).$$

8.3 Spatial Aggregation

Spatial aggregation involves elemental alternatives and individuals that are distributed over space. Choices of residential location, workplace, shopping destination, and so on are characterized by spatially defined alternatives. The geographic distribution of individuals and spatial alternatives and their interrelationships in space in terms of transportation level-of-service attributes are essential inputs for predicting aggregate travel demand.

Consider an origin zone as a group of individuals and a destination zone as a group of elemental spatial alternatives. The elemental spatial alternatives are housing units (in the choice of residential locations), jobs (in the choice of workplace), and so on. The expected number of individuals at an origin i selecting an alternative in destination j is

$$T_{ij} = \sum_{t \in i} \sum_{s \in j} P_t(s).$$

To illustrate the relationship between spatial and nonspatial alternatives, let $P_t(m \mid s)$ be the probability of individual t choosing mode m given that he travels to spatial alternative s. The expected number of trips from origin i to destination j by mode m is given by

$$T_{ijm} = \sum_{t \in i} \sum_{s \in j} P_t(m \mid s) P_t(s).$$

8.4 The Discrete Logit Model

Assume the logit model for the spatial choice probability

$$P_t(s) = \frac{e^{V_{st}}}{\sum_{s=1}^{M} e^{V_{st}}}, \tag{8.1}$$

where

V_{st} = the average utility of elemental spatial alternative s to individual t,[5]

M = the number of available elemental spatial alternatives.

The independent variables of a spatial choice model that enter the utility functions are transportation level-of-service attributes by different modes, times of day, and facilities to the elemental spatial alternatives, L; locational attributes, or attraction variables, of the elemental alternatives, A; and socioeconomic characteristics of the individual, S.

To obtain choice probabilities for aggregate alternatives, partition the space of available alternatives into nonoverlapping subsets of elemental alternatives, or destinations, and sum the logit choice probabilities as follows:

$$P_t(j) = \frac{\sum_{s=1}^{M_j} e^{V_{st}}}{\sum_{j=1}^{J} \sum_{s=1}^{M_j} e^{V_{st}}}, \tag{8.2}$$

where

$\sum_{j=1}^{J} M_j = M$,

J = the number of destination zones,

M_j = the number of elemental spatial alternatives in zone j.

Define

$$\bar{K}_{jt} = \frac{1}{M_j} \sum_{s=1}^{M_j} e^{V_{st}} \tag{8.3}$$

and

$$\tilde{V}_{jt} = \ln \bar{K}_{jt}. \tag{8.4}$$

Substitute (8.3) and (8.4) in (8.2):

5. In the derivation of logit as a random utility model the utility of an alternative is written as $U_{st} = V_{st} + \varepsilon_{st}$, where ε_{st} is the random unobserved utility.

$$P_t(j) = \frac{e^{\tilde{V}_{jt} + \ln M_j}}{\sum_{j=1}^{J} e^{\tilde{V}_{jt} + \ln M_j}}. \tag{8.5}$$

This model for aggregate spatial alternatives was investigated by Lerman (1975) and McFadden (1977) who consider the problem of estimating $\tilde{V}_{jt}$.[6] Lerman (1975) used a Taylor series expansion around the mean

$$\bar{V}_{jt} = \frac{1}{M_j} \sum_{s=1}^{M_j} V_{st} \tag{8.6}$$

to investigate the sensitivity of the model to higher moments of the distribution of the attributes of elemental alternatives within zones. McFadden (1977) employed the transformation

$$\tilde{V}_{jt} = \bar{V}_{jt} + \ln \frac{1}{M_j} \sum_{s=1}^{M_j} e^{V_{st} - \bar{V}_{jt}} \tag{8.7}$$

to show that under the assumption that for large zones V_{st} are normally distributed the difference $\tilde{V}_{jt} - \bar{V}_{jt}$ approaches $1/2\ \sigma_{jt}{}^2$, where $\sigma_{jt}{}^2$ is the within zone variance of the utility V_{st}. Thus in all cases

$$\tilde{V}_{jt} \geq \bar{V}_{jt},$$

and the equality holds for perfectly homogeneous zones. This implies that, in order to use average zonal values for spatial aggregation, the zones should be defined to be homogeneous or to have equal within zone variances.

This also holds for the nested, or sequential, logit model described in McFadden (1977) and Ben-Akiva and Lerman (1977). The choice probability for $s \in j$ is

$$P_t(s) = \frac{e^{V_{st}}}{\sum_{s=1}^{M_j} e^{V_{st}}} \cdot \frac{e^{\mu \tilde{V}_{jt} + \mu \ln M_j}}{\sum_{j=1}^{J} e^{\mu \tilde{V}_{jt} + \mu \ln M_j}}, \tag{8.8}$$

6. An alternative approach described in Lerman (1975) is to define the utility of a zone as the maximum of the elemental alternatives' utilities. Under the logit assumption the expected value of the maximum is

$$\bar{V}_{jt} + \ln M_j,$$

and the model in (8.5) is obtained.

where $0 \le \mu \le 1$ is an additional parameter, indicating the degree of similarity among unobserved attributes of elemental alternatives in the zones, or communities. The model in (8.1) is a special case of (8.8) when $\mu = 1$. The major disadvantage of this generalization for applications to spatial aggregation is the need to retain the definition of the zones used to define μ.

8.5 Spatial Aggregation Using Continuous Functions

The discrete summation form cannot be used in actual applications when the numbers of spatial alternatives and individuals are large, because complete enumeration would require astronomical amounts of data and computation. There are many possible ways to represent spatial distributions, depending on the level of detail desired. One way to generalize the definition of spatial aggregation is to employ mathematical functions expressed in terms of two-dimensional coordinates to represent the geographic distributions of the spatial alternatives, individuals, and the attributes.

Define a spatial choice function, denoted by $G_k(p, q \mid x, y)$, as the probability of an individual of type k located at point (x, y) choosing one spatial alternative located at point (p, q). This is a unique surface for individual type k located at (x, y) which is a function of[7]

$L(p, q; x, y)$ = transportation level-of-service attributes between origin point (x, y) and destination point (p, q),

$A(p, q)$ = locational attributes of the elemental alternatives at point (p, q),

S_k = socioeconomic characteristics of individual type k.

Define spatial density functions for spatial alternatives and individuals as follows:

$M(p, q)$ = density of elemental spatial alternatives at point (p, q),

$H_k(x, y)$ = density of individuals of type k at point (x, y).

The integral of the spatial choice function over all available alternatives M, must equal one. Thus the spatial choice function is defined such that

7. The derivation in McFadden (1976) implies that a unique spatial choice function exists if the choice probabilities are absolutely continuous with respect to the number of elemental spatial alternatives.

$$\iint_M G_k(p, q \mid x, y) M(p,q) dpdq = 1.$$

If it is only a function of the attributes of alternatives at (p, q), it is the continuous logit model. In the more general case it can be a function of the entire distribution of attributes of alternatives with respect to (x, y).

The expected number of individuals from zone i selecting alternatives in zone j can now be derived as follows:

1. by aggregation over spatial alternatives to obtain the probability that individual of type k located at (x, y) will choose an alternative in zone j:

$$P_k(j \mid x, y) = \iint_{\substack{\text{zone} \\ j}} G_k(p, q \mid x, y) M(p, q) dpdq, \tag{8.9}$$

2. by aggregation over individuals to obtain the expected number of individuals of type k located in zone i who will select an alternative in zone j:

$$T_{kij} = \iint_{\substack{\text{zone} \\ i}} P_k(j \mid x, y) H_k(x, y) dxdy. \tag{8.10}$$

The total number of trips from zone i to zone j is

$$\begin{aligned} T_{ij} &= \sum_k T_{kij} \\ &= \sum_k \iint_{\substack{\text{zone} \\ i}} \iint_{\substack{\text{zone} \\ j}} G_k(p, q \mid x, y) M(p, q) H_k(x, y) dpdqdxdy. \end{aligned} \tag{8.11}$$

It is also possible to repeat these steps to derive other quantities. For example, let $D(p, q; x, y)$ be the distance between points (p, q) and (x, y). Then the expected miles of travel for the origin/destination pair (i, j) is given by

$$MT_{ij} = \sum_k \iint_{\substack{\text{zone}\\ i}} \iint_{\substack{\text{zone}\\ j}} D(p,q;x,y)G_k(p,q \mid x,y)M(p,q)H_k(x,y)\,dpdqdxdy. \tag{8.12}$$

8.6 The Continuous Logit Model

The definition of an aggregate spatial choice probability can be rewritten as

$$P_t(j) = \iint_{\substack{\text{zone}\\ j}} G_t(p,q)M(p,q)\,dpdq, \tag{8.13}$$

where the subscript t is used to denote an individual type k at location (x, y). The continuous logit model is obtained by assuming the independence from irrelevant alternatives, IIA, property (McFadden 1976).[8] It implies that the spatial choice probabilities for any feasible subset of spatial alternatives, M^1, can be written as

$$P_t(j \mid M^1) = \frac{\iint_j K_t(p,q)M(p,q)\,dpdq}{\iint_{M^1} K_t(p,q)M(p,q)\,dpdq}, \tag{8.14}$$

where $K_t(p, q)$ is a spatial choice function defined in terms of attributes at (p, q). The spatial choice probability of an infinitesimal area $(dpdq)$ is given by

$$P_t(dpdq \mid M^1) = \frac{K_t(p,q)M(p,q)\,dpdq}{\iint_{M^1} K_t(p,q)M(p,q)\,dpdq}. \tag{8.15}$$

8. Alternatively the spatial choice function could be defined as the product $G_t(p, q)M(p, q)$ and the elemental alternative as the unit area. The continuous logit model can then be viewed as the infinitesimal limit of the discrete logit model (Ben-Akiva et al. 1976).

As in equation (8.4) define

$V_t(p,q) = \ln K_t(p,q)$,

and substitute it in (8.15) to obtain

$$P_t(dpdq \mid M^1) = \frac{e^{V_t(p,q)} M(p,q)dpdq}{\iint\limits_{M^1} e^{V_t(p,q)} M(p,q)dpdq}$$

$$= \frac{e^{V_t(p,q)+\ln M(p,q)dpdq}}{\iint\limits_{M^1} e^{V_t(p,q)+\ln M(p,q)dpdq}}, \tag{8.16}$$

where $V_t(p,q)$ can be interpreted as the average utility to individual t of a spatial alternative at (p, q) and $M(p, q)dpdq$ the number of elemental spatial alternatives in the infinitesimal area $dpdq$.

To derive the discrete logic model, the feasible choice set is partitioned into groups of elemental alternatives as in (8.2), and the model (8.14) is rewritten as

$$P_t(j \mid M) = \frac{\iint\limits_{j} K_t(p,q)M(p,q)dpdq}{\sum_{j=1}^{J} \iint\limits_{j} K_t(p,q)M(p,q)dpdq}. \tag{8.17}$$

Define

$$\bar{K}_{jt} = \frac{1}{M_j} \iint\limits_{j} K_t(p,q)M(p,q)dpdq, \tag{8.18}$$

where

$$M_j = \iint\limits_{j} M(p,q)dpdq.$$

Substitution of (8.18) and (8.4) in (8.17) yields the discrete logit model for aggregate spatial alternatives in (8.5).

8.7 A Parametric Example of Spatial Aggregation

The concept of spatial aggregation and the continuous logit model can be demonstrated by means of a specific travel demand model. The purpose of the example is to illustrate (1) the conversion of spatial choice models from their original discrete form to the continuous form, (2) the linkage between spatial choices and nonspatial choices in travel demand forecasting, and (3) the use of parametric distributions over space.

Consider a joint shopping destination-mode logit choice model of the form

$$P_t(md) = \frac{e^{V_{tmd}}}{\sum_{m'} \sum_{d'} e^{V_{tm'd'}}},$$

where

$P_t(md)$ = the probability that an individual trip maker t will travel to shop at destination d by mode m,

V_{tmd} = the average utility of mode m and destination d for individual t.

V_{tmd} is assumed to have a simplified specification, as follows:

$$V_{tmd} = a_1 S_{tm} + a_2 TC_{tmd} + a_3 IT_{tmd} + a_4 OT_{tmd} + a_5 \ln(Q_d) + \ln(A_d)$$

where

S_{tm} = a measure of socioeconomic characteristics of individual t for mode m,

TC_{tmd} = round trip out-of-pocket travel cost,

IT_{tmd} = round trip in-vehicle travel time,

OT_{tmd} = round trip out-of-vehicle travel time,

A_d = area of destination d,

Q_d = measure of attraction density at destination d—for example, retail employment per acre (the logarithmic form for Q in the utility function is used for convenience, although it is not necessary),

$a_1, \ldots, a_5$ = unknown parameters.

In this model the destinations are treated as groups of relatively homogeneous elemental spatial alternatives. Grouping of spatial alternatives is necessary for model estimation because data on spatial alternatives are only available for discrete area units (e.g., retail employment by zone). Because the notion of elemental alternatives for the shopping destination choice is not well defined, the unit area is taken as the measure of an elemental alternative. Thus the number of elemental alternatives contained in destination d is proportional to A_d, with the attraction variable Q_d representing the locational attributes of these spatial alternatives. The coefficient for $\ln(A_d)$ is constrained to unity to ensure that the model is linearly homogeneous with respect to the size of the aggregate spatial alternatives. That is, doubling the area of a destination will double the odds for choosing that destination. The property of linear homogeneity of a spatial choice model is needed to guarantee that the model will be applicable to any level of geographic aggregation.[9]

For simplicity rewrite the utility function into three groups of variables:

$$V_{tmd} = \alpha_{tm} + \beta_{tmd} + \ln(A_d \gamma_d),$$

where

$$\alpha_{tm} = a_1 S_{tm};$$
$$\beta_{tmd} = a_2 TC_{tmd} + a_3 IT_{tmd} + a_4 OT_{tmd},$$
$$\gamma_d = Q_d^{a_5}.$$

Substitute this definition of the utility function in the model to get

$$P_t(md) = \frac{e^{\alpha_{tm}} A_d \gamma_d e^{\beta_{tmd}}}{\sum\limits_{m'} e^{\alpha_{tm'}} \sum\limits_{d'} A_{d'} \gamma_{d'} e^{\beta_{tm'd'}}}.$$

To convert the discrete form model to the continuous form, we use a system of polar coordinates (L, θ) for the location of the spatial alternatives (the unit areas) with respect to individual t, a trip maker of type k residing at (x, y). We express the generalized travel cost (β_{tmd}) and the attraction measure (γ_d) in terms of (L, θ): β_{tmd} is replaced by $\beta_{tm}(L, \theta)$, and γ_d is replaced by $\gamma(L, \theta)$. Furthermore, because the unit area is the measure of an

9. To maintain this property when there are two or more size variables, it is necessary to replace the variable A_d with a linear function of size variables with unknown parameters. This results in a logit model with utility functions that are not linear in the parameters. An estimator for this case was developed by Daly (1978).

elemental alternative, we take the infinitesimal area ($LdLd\theta$) as the measure of the number of elemental alternatives. Then the probability of the individual traveling to the infinitesimal area ($LdLd\theta$) by mode m is given by

$$P_t(m, LdLd\theta) = G_t[m, (L, \theta)]LdLd\theta$$
$$= \frac{e^{\alpha_{tm}}\gamma(L,\theta)e^{\beta_{tm}(L,\theta)}LdLd\theta}{\sum_{m'} e^{\alpha_{tm'}} \int_0^B \int_0^{2\pi} \gamma(L,\theta)e^{\beta_{tm'}(L,\theta)}LdLd\theta},$$

where the summation in the denominator is replaced by an area integral over θ and L with an upper limit on travel distance B which could be set to infinity if the space of alternatives is not constrained.

The function $G_t[m, (L, \theta)]$ is the joint choice function for mode m and destination at point (L, θ). The spatial choice function for the model, expressed as the probability of the trip maker t choosing one spatial alternative at (L, θ), is derived as the sum of $G_t[m, (L, \theta)]$ over all modes:

$$G_t(L, \theta) = \sum_m G_t[m, (L, \theta)]$$
$$= \frac{\sum_m e^{\alpha_{tm}}\gamma(L,\theta)e^{\beta_{tm}(L,\theta)}}{\sum_m e^{\alpha_{tm}} \int_0^B \int_0^{2\pi} \gamma(L,\theta)e^{\beta_{tm}(L,\theta)}LdLd\theta}.$$

These functions can be used to derive required predictions. For example, to predict the probability of choosing mode m, we integrate $G_t[m, (L, \theta)]$ over the spatial alternatives space:

$$P_t(m) = \int_0^B \int_0^{2\pi} G_t[m, (L, \theta)]LdLd\theta.$$

To predict the trip length distribution by specific mode m, we first obtain the spatial choice function for (L, θ) conditional on mode m:

$$G_t[(L, \theta) \mid m] = \frac{G_t[m, (L, \theta)]}{P_t(m)}.$$

Then the trip length distribution by mode m is given by

$$f_t(L \mid m) = \int_0^{2\pi} G_t[(L,\theta) \mid m] L d\theta, \quad 0 \leq L \leq B,$$

and the expected trip length by mode m is

$$\bar{L}_{tm} = \int_0^B L f_t(L \mid m) dL.$$

Note that the expressions for mode choice probability and trip length distribution entail integrals in a generally intractable form which requires numerical integration

8.8 Continuous Logit with Featureless Plane

Under simplifying assumptions on the functional forms of the independent variables, a solution exists in closed form. Consider the following continuous spatial choice logit model without mode choice:

$$P(LdLd\theta) = \frac{e^{\beta(L,\theta) + \ln \gamma(L,\theta)} L dL d\theta}{\int_0^B \int_0^{2\pi} e^{\beta(L,\theta) + \ln \gamma(L,\theta)} L dL d\theta},$$

$0 \leq L \leq B$ and $0 \leq \theta \leq 2\pi$, and assume that the attraction measure is constant across all destinations,

$$\gamma(L,\theta) = \gamma,$$

and the transportation level of service is a linear function of distance,

$$\beta(L,\theta) = -bL.$$

Under these assumptions the following results are derived:

1. Consumer surplus, or the expected utility from the choice, equals the natural logarithm of the denominator of the model (Ben-Akiva and Lerman 1977):

$$CS = \ln \int_0^{2\pi} \int_0^B e^{-bL+\ln \gamma} L dL d\theta = \ln \frac{2\pi\gamma}{b^2}[1 - e^{-bB}(bB + 1)],$$

$$\lim_{B \to \infty} CS = \ln \frac{2\pi\gamma}{b^2}.$$

If the term CS is divided by the cost coefficient in the utility function (e.g., coefficient a_2 in the parametric example), the consumer surplus will become expressable in monetary units.

2. Spatial choice function

$$G(L, \theta) = \frac{e^{-bL+\ln \gamma}}{\int_0^{2\pi} \int_0^B e^{-bL+\ln \gamma} L dL d\theta} = \frac{b^2 e^{-bL}}{2\pi[1 - e^{-bB}(bB + 1)]},$$

$$\lim_{B \to \infty} G(L, \theta) = \frac{b^2}{2\pi} e^{-bL}.$$

3. Trip length probability density function[10]

$$f(L) = \int_0^{2\pi} G(L, \theta) L d\theta = \frac{b^2 L e^{-bL}}{1 - e^{-bB}(bB + 1)},$$

$$\lim_{B \to \infty} f(L) = b^2 L e^{-bL}$$

4. Trip length cumulative distribution function

$$F(L) = \frac{1 - e^{-bL}(bL + 1)}{1 - e^{-bB}(bB + 1)},$$

$$\lim_{B \to \infty} F(L) = 1 - e^{-bL}(bL + 1).$$

5. Average trip length

$$\bar{L} = \frac{1}{b}[2 - Bf(B)],$$

10. This result for $B \to \infty$ was also obtained by Goodwin (1975) in a similar derivation.

$$\lim_{B \to \infty} \bar{L} = \frac{2}{b}.$$

6. The effect of B on CS and $\bar{L}$, or the effect of time/distance budget B on consumer surplus, is given by

$$\frac{\partial CS}{\partial B} = f(B) \geq 0,$$

and

$$\frac{\partial^2 CS}{\partial B^2} = \frac{f(B)}{B}[1 - bB - f(B)] \leq 0,$$

where the second inequality is derived from the condition that $\bar{L} \leq B$. Thus as B increases, the CS increases monotonically at a diminishing rate. The effect on average trip length is given by

$$\frac{\partial \bar{L}}{\partial B} = -\frac{f(B)}{B}[2 - bB - f(B)] \geq 0,$$

where the term in brackets can be shown to be nonpositive from the condition that $\bar{L} \leq B$.

For the parametric example with mode choice consider the following transportation level of service

$$\beta_{tm}(L, \theta) = -a_{tm} - b_{tm}L,$$

where a_{tm} is the generalized fixed travel cost and b_{tm} is the generalized marginal travel cost per unit distance for mode m. These generalized travel costs include both out-of-pocket costs and travel times weighted by model parameters.

The (marginal) probability for mode m becomes

$$P_t(m) = \frac{b_{tm}^{-2} e^{a_{tm} - a_{tm}} [1 - e^{-b_{tm}B}(b_{tm}B + 1)]}{\sum_{m'} b_{tm'}^{-2} e^{a_{tm'} - a_{tm'}} [1 - e^{-b_{tm'}B}(b_{tm'}B + 1)]}.$$

The trip length distribution by mode m is

$$f_t(L \mid m) = L b_{tm}^2 e^{-b_{tm}L} [1 - e^{-b_{tm}B}(b_{tm}B + 1)]^{-1},$$

and the expected trip length is

$$\bar{L}_{tm} = 2b_{tm}^{-1}\left[1 - \frac{B}{2} f_t(B \mid m)\right].$$

In other words, the average trip length by mode is inversely proportional to the mode's generalized marginal travel cost per unit trip length. (The generalized travel costs not only vary with mode but also with trip purpose and socioeconomic characteristics.) For B approaching infinity, the elasticity of average modal trip length with respect to the unit generalized travel cost for the mode is equal to minus one. It should be noted, however, that in this rudimentary analysis we have ignored the elasticities of trip frequency and mode shares.

The resulting trip length distribution by mode for B equal to infinity is a gamma two distribution of familiar shape, as shown in figure 8.1 (for $B < \infty$ the gamma distribution becomes truncated), that was observed for a number of urban areas (A. M. Voorhees and Associates 1968, 1970). This indicates that even highly simplifying assumptions for the distributions may still result in qualitatively meaningful predictions. The behavioral implications of the conditional trip length distribution can be seen by plotting it for modes with significantly different level-of-service characteristics, for example, walk and auto, as shown in figure 8.2.

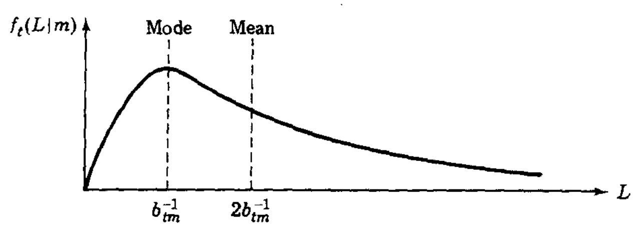

Figure 8.1
Trip length distribution

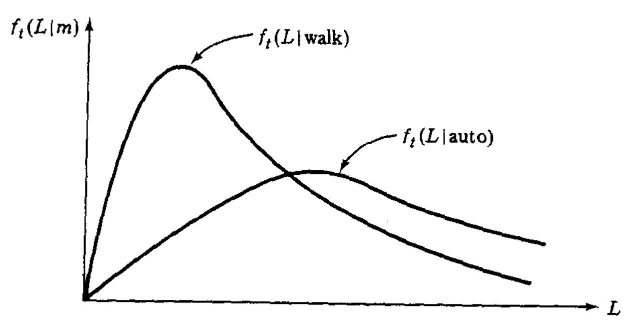

Figure 8.2
Trip length distribution for different modes

This example is intended for illustrative purposes only, since the assumed attractiveness and level-of-service distributions for the spatial alternatives are clearly unlikely to be even approximate in real urban situations. To achieve more accurate predictions, more realistic mathematical functions should be used to describe these distributions. However, because solutions to the integrals rarely exist in closed form, the general approach to perform spatial aggregation must be based on numerical integration techniques.

8.9 Basic Operations of the MIT-TRANS Model[11]

The MIT-TRANS model represents an extreme form of test for the feasibility and validity of the spatial aggregation methodology, because it treats an entire urban area as a single zone. The model is based on the application of Monte Carlo simulation, as the main numerical integration technique, to forecast trip generation, trip distribution, and modal split, with a system of disaggregate travel demand models. There are seven disaggregate models for both work and nonwork trips linked together (outputs from one model become inputs to lower-hierarchy models). Examples of predictions are the number of trips made, mode shares, person-miles of travel, vehicle-miles, and average vehicle occupancy rates for both work and nonwork trips, number of automobiles per family, and so on. These predictions are policy-sensitive, as reflected by the fact that they embody elastic travel demand models for the choices of workplace, auto ownership, mode to work, nonwork travel frequency, destination, and mode.

It should be noted that the MIT-TRANS model in its present form represents only the demand component of an overall policy evaluation package which must also include a supply component and evaluation procedures. Future extensions of the MIT-TRANS model include the development of network abstract transportation supply and traffic assignment models and the integration of these models and the aggregation procedure into an equilibrium framework. (In lieu of a complete supply-demand equilibrium framework, a set of level-of-service relationships describing a spatial distribution of the equilibrium conditions of an existing transportation system with externally specified parameters is being used in the current MIT-TRANS model.)

11. A detailed description of this model is given in Watanatada and Ben-Akiva (1977).

The operations of the MIT-TRANS model are summarized schematically in figure 8.3. It accepts three sets of inputs: (1) the aggregate city geometry and land use distribution parameters, (2) the urbanized area's socioeconomic characteristics, and (3) the specifications of a transportation policy alternative. These policy specifications are used to modify the level-of-service relationships which have been calibrated for the base conditions. The Monte Carlo aggregation procedure—a numerical integration procedure—operates on these inputs, the disaggregate choice models, and the (modified) level-of-service relationships to produce aggregate travel demand forecasts for the urbanized area. The forecasts can be disaggregated by market segments such as by income group.

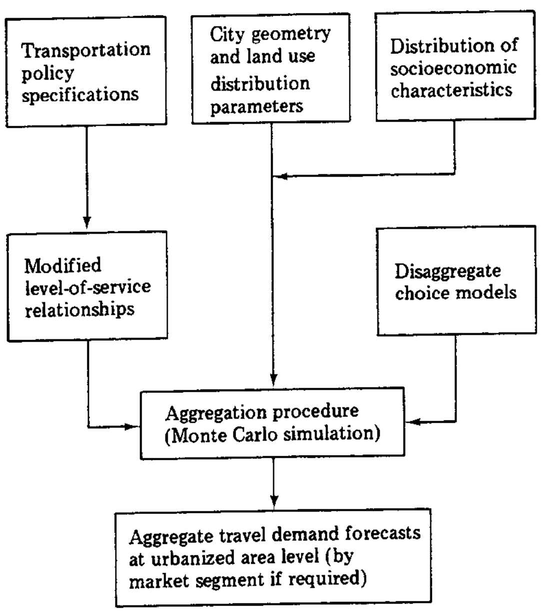

Figure 8.3
Basic operations of MIT-TRANS model

The urbanized area is modeled as a quasi-circular shape with the origins (home ends of trips) and destinations (nonhome ends) defined by sets of coordinates (R, λ) and (r, ϕ), or $(L, \theta \mid R, \lambda)$, respectively, as depicted in figure 8.4. For each of three income classes the household density function such as negative exponential is assumed (figure 8.5). The spatial alternatives—jobs, shopping destinations, and social recreational facilities—are also represented by employment density functions (negative exponential) and other functions describing locational attributes. The parameters of these density functions can be easily estimated from total counts of population and employment for a central city and its entire metropolitan area.

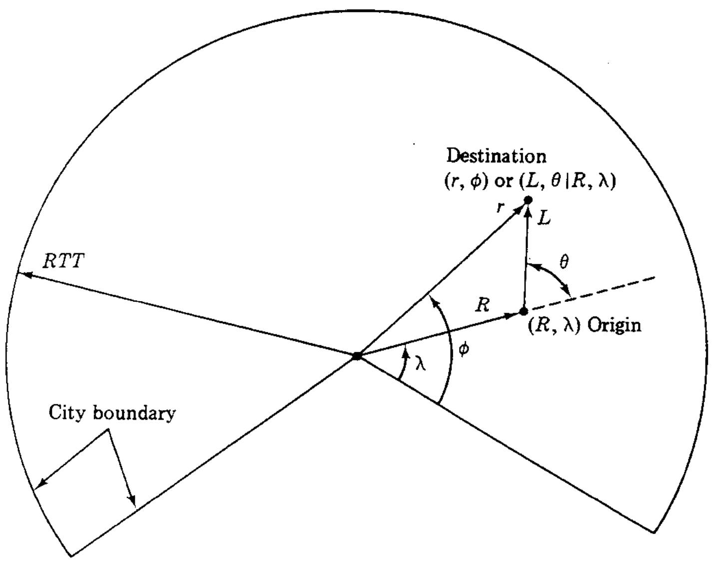

Figure 8.4
City geometry and system of coordinates

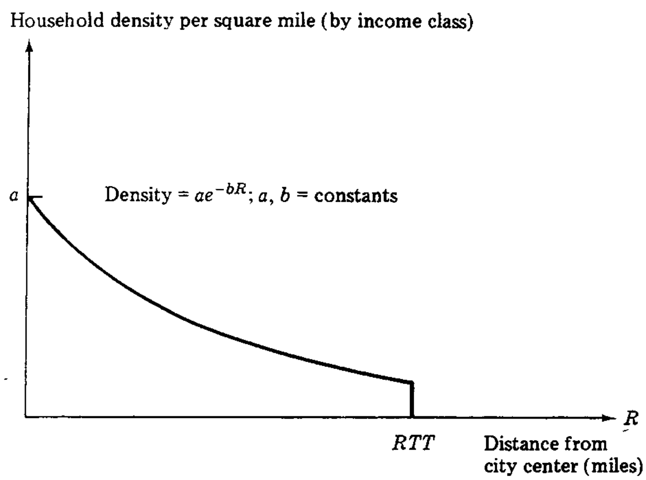

Figure 8.5
Negative exponential distribution of households

The transportation level-of-service functions by mode and time of day are expressed in terms of trip geometry variables, which are in turn functions of the coordinates of the trip ends.

MIT-TRANS also includes a procedure, similar to the one used by Duguay et al. (1976), to obtain the distribution of socioeconomic characteristics of the urban area population by generating a sample of households from available data. The procedure can operate on samples of disaggregate observations from the census public use sample, or any other household survey, and available aggregate data from surveys or published sources for the past years, from forecasts, or from explicit future scenarios.

The operations of the Monte Carlo aggregation procedure, summarized in figure 8.6, include the following basic steps:

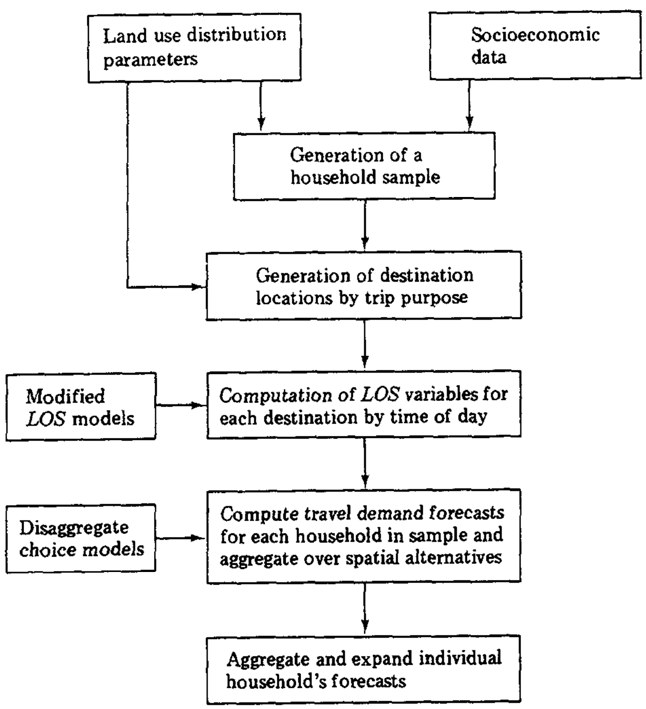

Figure 8.6
Monte Carlo aggregation procedure

1. Determine household sample size.
2. Generate sample of households for forecast year, characterizing each household by (R, λ) location and a set of socioeconomic attributes.
3. Determine sample size of spatial alternatives by purpose.
4. Generate sample of spatial alternatives by purpose for each household in the sample, defining each destination by (L, θ) coordinates.
5. Modify appropriate attributes of the alternatives for policy analysis.
6. Apply linked demand models for each household in the sample.
7. Expand sample forecasts to population market segments.
8. Compare forecasts against base case for policy analysis.

The Monte Carlo approach was selected to circumvent the extreme complexity of setting the bounds of the integrals for L and θ and to allow the use of alternative parametric distributions. The flexibility of the technique is demonstrated for an integral taken from a continuous logit model for an individual at (R, λ), which for simplicity is written as

$$I_j = \iint_j K(L,\theta) M(L,\theta) L dL d\theta,$$

where the origin of the coordinates (L, θ) is at (R, λ).

The most simple technique is to draw points $(L, \theta)_n$ uniformly over the area j and obtain the following unbiased estimator:

$$\hat{I}_j = \frac{A_j}{N_j} \sum_{n=1}^{N_j} K(L,\theta)_n M(L,\theta)_n,$$

where A_j is the area of zone j and N_j the number of points drawn in zone j.

Alternatively, since the input distribution of spatial alternatives is given in terms of (r, ϕ), it is possible, for example, to draw directly from the negative exponential distributions shown in fig. 8.5. (It involves drawing from a gamma distribution.) In this case

$$\hat{I}_j = \frac{M_j}{N_j} \cdot \sum_{n=1}^{N_j} K(L,\theta)_n.$$

However, with regard to shopping trips most destinations are expected to be 4 to 5 miles from the trip maker's home. Uniform sampling or sampling from an urbanized area employment distribution generate many locations outside the potential destination area with a very low value of $K(L, \theta)$. Therefore a more efficient sampling technique based on the knowledge of the entire integrand is also employed. The basic principle of importance sampling is to find a probability density function $f_j(L, \theta)$, such that

$$\frac{K(L,\theta)M(L,\theta)L}{f_j(L,\theta)}$$

varys as little as possible (Hammersley and Handscomb 1965). Drawing from $f_j(L, \theta)$ results in

$$\hat{I}_j = \frac{1}{N_j} \sum_{n=1}^{N_j} \frac{K(L,\theta)_n M(L,\theta)_n L_n}{f_j(L,\theta)_n}.$$

It is necessary, however, to find $f_j(L, \theta)$ that is simple enough to allow locations $(L, \theta)_n$ to be drawn conveniently. The featureless plane example presented earlier suggests for the entire urbanized area the gamma distribution with a parameter

$$b = \frac{2}{\bar{L}}$$

which can be calculated from generally available information on average trip lengths. In the implementation of this procedure some efficiency was lost because locations were sampled from a gamma distribution over an infinite space and therefore some fell outside the urbanized area.

The MIT-TRANS model was programmed in Fortran for an IBM 370/168 computer. It required about 0.6 CPU minutes per policy run with a standard error of about 1 percent of predicted average passenger miles of travel.

The model was calibrated for the 1968 metropolitan Washington, D.C., area and then used to forecast 1975 conditions as a validation test. Between 1968 and 1975 there were substantial changes in the metropolitan Washington area. The major changes in travel behavior include increased auto ownership, increased vehicle miles of travel per household, and decreased transit patronage. All the forecasted changes agree with these trends.

Apart from Washington, D.C., the model was also calibrated for the Minneapolis-St. Paul area, which has two central business districts. The elasticities produced by the model for both the Washington, D.C., and the Twin Cities are comparable to before-and-after empirical evidence and forecasts obtained from other more detailed studies.

Several Monte Carlo sampling experiments were conducted to investigate the statistical properties of the model. It was found empirically that a small number of sampled destinations would result in minimal bias and optimal efficiency.

The empirical results have led to the basic conclusion with respect to the applicability of disaggregate travel demand models and Monte Carlo techniques for aggregate sketch-planning predictions. The travel demand-forecasting methodology proposed operates with readily available aggregate input data, while still maintaining the full degree of policy sensitivity available in recently developed systems of disaggregate models. The most important future extensions of the methodology are the incorporation of supply and traffic assignment models and the development of a version of MIT-TRANS for multiple zones of varying sizes.

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