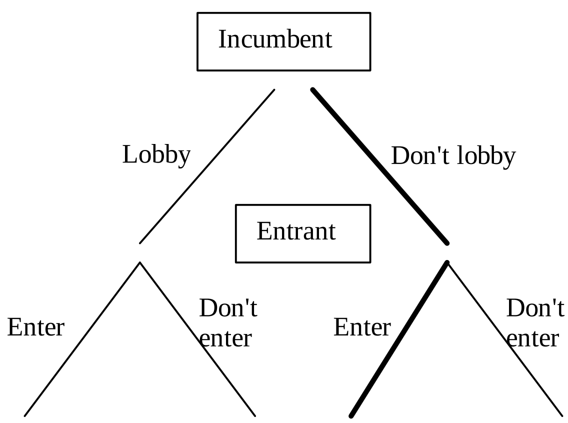
**Economics 100A** **Prof. McFadden**
**Fall 2001** **Due the week of 11/26/01**

# Answer Key to Problem Set #11

1. Currently, there is an incumbent monopoly in the market. Next year, a potential entrant may enter. The incumbent needs to make a decision on whether or not it should spend $50 to lobby the government into passing legislation which places a lump-sum tax of $100 on the potential entrant if it enters. If the potential entrant stays out of the market it makes zero profit and the incumbent firm makes a monopoly profit, $\pi_m>0$, minus expenditures on lobbying if any ($50 or $0). If the potential entrant enters the market, it gets duopoly profit, $\pi_d$=$200, minus the tax if any, and the incumbent gets duopoly profit minus the lobbying costs if any.

   A. Show the game in extensive form. What strategies will be played?

   *The following game tree describes the sequential move game. Generally, let T equal the lump-sum tax and b equal the lobbying expense):*

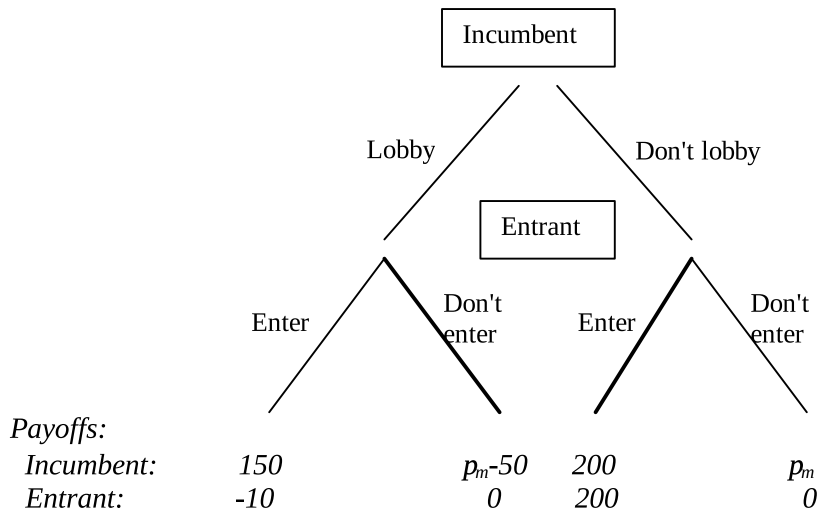



| *Payoffs:* | | | | |
|---|---|---|---|---|
| *Incumbent:* | $p_d-b=150$ | $p_m-b=p_m-50$ | $p_d=200$ | $p_m$ |
| *Entrant:* | $p_d-T=100$ | *0* | $p_d=200$ | *0* |

*If the incumbent lobbies, the entrant still finds it profitable to enter (100>0). And if the incumbent knows this, it will not lobby since 200>150. Therefore, the strategies played are "don't lobby" by the incumbent and "enter" by the entrant, yielding payoffs of $200 for each participant.*

   B. Suppose the tax on the potential entrant is $210 instead of $100. Repeat part A. How large must $\pi_m$ be for the monopolist to want to lobby?

*Now the entrant will stay out of the market if the incumbent lobbies (-10<0). So the incumbent can make "$p_m$-50" if it lobbies, and "200" if it does not (when it doesn't, the entrant enters and the incumbent therefore makes duopoly profit). For the incumbent to want to lobby, $p_m$-50>200, or $p_m$>250. The game tree is shown as follows:*

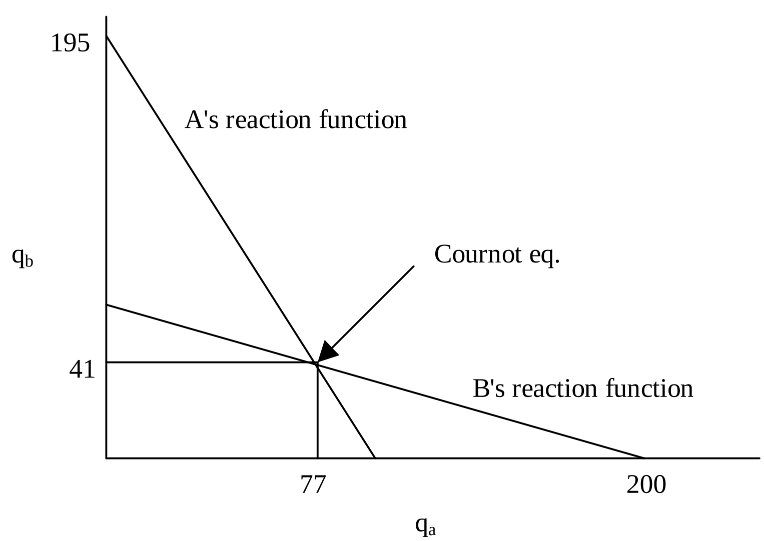



2. Consider a market in which two firms produce a homogeneous product. Market demand is given by $Q_d(P)$=200-$P$. The cost functions for firm A and firm B are $TC_a(q_a)=5q_a$ and $TC_b(q_b)=0.5q_b^2$, respectively.

   A. Find the Cournot equilibrium quantities supplied by each firm. Graph your result using reaction functions. Find the market price, and calculate profits for each firm.

*To determine the Cournot equilibrium, we first calculate the reaction function for each firm. Firm A's residual demand is $P=200-q_a-q_b$ and $MR_a=(200-q_b)-2q_a$. By equating $MR_a$ and $MC_a$ and rearranging, we get $q_a=97.5-0.5q_b$, A's reaction function. Similarly for B, $MR_b=(200-q_a)-2q_b=MC_b=q_b$. Rearranging, we get $q_b=66.67-0.33q_a$, B's reaction function. With two equations and two unknowns, we substitute for $q_b$ in A's reaction function to solve for $q_a$.*
*$q_a=97.5-0.5(66.67-0.33q_a)$.* ➔ $\boxed{q_a=77}$
*Substituting $q_a$ into B's reaction function gives us* $\boxed{q_b=41}$.

*The following graph of reaction functions shows exactly where the Cournot equilibrium is located:*



*Substituting $q_a$ and $q_b$ into the demand equation gives us the market price:*
$P=200-77-41$
$\boxed{P=\$82}$

*Finally, find firm profits:*
$p_a=(82)(77)-[5(77)]=\boxed{\$5929}$
$p_b=(82)(41)-[0.5(41)^2]=\boxed{\$2521.5}$

B. Now suppose that firm A chooses how much to produce before firm B does (i.e. firm A is a Stackelberg leader, B a follower). Calculate quantities, market price and profit for each firm[1].

*Firm A will choose its output $q_a$ to maximize its profits, subject to the reaction function of Firm B. That is, A faces demand $P=200-q_a-q_b$, but it also knows B's reaction function. We can plug that into the demand faced by A.*
$P=200-q_a-[66.67-(1/3)q_a]=133.33-(2/3)q_a$
*A's MR is therefore:* $MR_a=133.33-(4/3)q_a$.
*Setting $MR_a=MC_a$ gives* $133.33-(4/3)q_a=5$, *or* $\boxed{q_a=96.25}$.
*Substituting $q_a$ into B's reaction function gives* $\boxed{q_b=34.58}$.
$P=200-96.25-34.85=\boxed{69.2}$
$p_a=(69.2)(96.25)-[5(96.25)]=\boxed{\$6179.25}$
$p_b=(69.2)(34.58)-[0.5(34.58)^2]=\boxed{\$1795}$.

C. Now consider the case where total social welfare is maximized. Find market quantity, quantities supplied by each of the two firms, and market price.

*Total welfare is maximized when the price is equal to the marginal cost (the competitive equilibrium where market demand equals market supply). Market supply is a horizontal sum of each firm's supply curve. Firm A's supply curve is $P=5$ since $MC_a=AC_a=5$ and Firm B's supply curve is $P=q_b$ since $MC_b=q_b$. Thus market supply is $P=Q_s$ for $Q\leq 5$ and $P=5$ for $Q>5$. Under perfect competition, $Q_s=Q_d$. Using the inverse demand equation we find $200-Q=5$, yielding a total quantity $\boxed{Q=195}$ and $\boxed{P=\$5}$. Since B's costs are lower for quantity $\leq 5$, B produces the first 5 units of output and A produces the rest. If Firm A produces all of the output alone there will be no producer surplus and we lose the surplus that can be obtained by allowing Firm B to produce 5 units of output.*
$\boxed{q_a=190}$
$\boxed{q_b=5}$
$\boxed{p_a=0}$
$\boxed{p_b=(5)(5)-[0.5(5)^2]=\$12.5}$

D. Compare firm output, total output and price for parts A through C. Do your values make sense?

*As expected, we observe more production and a lower price when social welfare is maximized. When Firm A can move first it produces more than it does in the Cournot case, implying a first-mover advantage. We find a similar effect if B moved first compared to the Cournot case.*

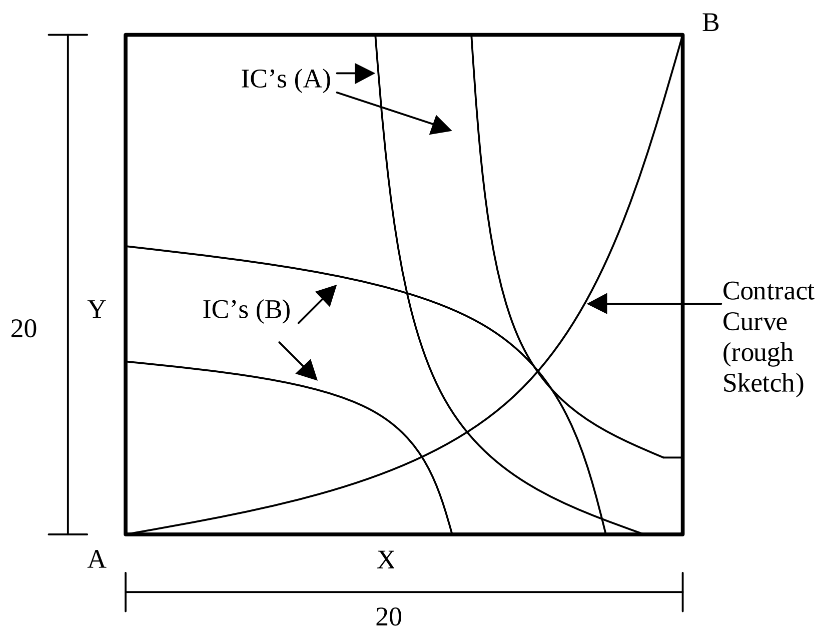
[1] Apologies for forcing you to get out your calculator.

3. The only two consumers in an exchange economy, consumer A and consumer B, consume the only two goods, X and Y, in the economy. There are 20 units of X available and 20 units of Y.

A. If A and B have identical preferences, mutually beneficial trade cannot occur. True or false? Explain.

*False. Mutually beneficial trade will not occur only when the allocation of resources among A and B is already efficient. In the case of our two-consumer economy,* $MRS_A = MRS_B$ *indicates an efficient allocation of goods. If* $MRS_A$ *is not equal to* $MRS_B$*, the two consumers will be able to arrange a mutually beneficial trade (I assume identical* <u>*convex*</u> *preferences).*

B. Assume A's preferences are described by $U_A = X_A^{0.5} Y_A^{0.5}$ and B's preferences are described by $U_B = 2X_B Y_B$, where $X_A$, $Y_A$, $X_B$, $Y_B$ are the consumption of $X$ and $Y$ by consumers A and B, respectively. Draw an Edgeworth box describing this scenario. Label the lengths of the sides, draw a few indifference curves for each consumer and (roughly) sketch the contract curve.




C. Now derive an <u>equation</u> for the contract curve. (Hint: Compute the MRS for consumer A as a function of $X_A$ and $Y_A$, and for consumer B as a function of $X_B$ and $Y_B$. Impose the restriction that total consumption of $X$ should equal the total units of $X$ available, and do the same for $Y$. Rearrange to get an equation of $Y_A$ as a function of $X_A$ (your result should look pretty simple)).

*When on the contract curve,* $MRS_A = MRS_B$*. Therefore,*

$$\frac{MU_X^A}{MU_Y^A} = \frac{MU_X^B}{MU_Y^B} \quad \Rightarrow \quad \frac{0.5X_A^{-0.5}Y_A^{0.5}}{0.5X_A^{0.5}Y_A^{-0.5}} = \frac{2Y_B}{2X_B}$$

$$\Rightarrow \quad \frac{Y_A}{X_A} = \frac{Y_B}{X_B}$$

*Since $X_A + X_B = 20$ and $Y_A + Y_B = 20$, we substitute $X_B = 20 - X_A$ and $Y_B = 20 - Y_A$ into the above equation and then solve for $Y_A$ as a function of $X_A$.*

$$\Rightarrow \quad \frac{Y_A}{X_A} = \frac{20 - Y_A}{20 - X_A}$$

$$\Rightarrow \quad 20Y_A - X_A Y_A = 20X_A - X_A Y_A$$

$$\Rightarrow \quad 20Y_A = 20X_A$$

$$\Rightarrow \quad Y_A = X_A$$

*The contract curve is a straight diagonal line connecting A's origin to B's origin (note that this is unlike the curved "sketch" drawn in part B. We have a linear contract curve, but this is not always the case).*

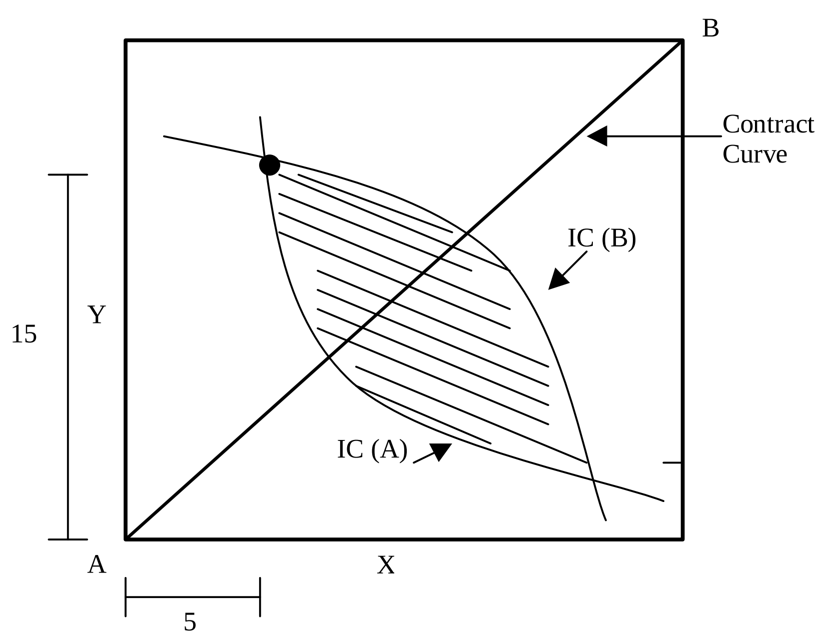
D. Suppose that consumer A has an initial endowment of 5 units of *X* and 15 units of *Y*, and consumer B has the remainder of what's available. Show, using the concept of MRS and the Edgeworth Box, that a trade could benefit both consumers.

$$MRS_A = \frac{-Y_A}{X_A} = \frac{-15}{5} = -3$$

$$MRS_B = \frac{-Y_B}{X_B} = \frac{-5}{15} = -\frac{1}{3}$$

$$MRS_A \neq MRS_B$$

*In the following graph, any point in the shaded area represents a new allocation of X and Y that benefits both consumers relative to the original endowment. So any trade which puts them into this area is beneficial (Pareto improving).*



E. Assume the consumers can trade as much as they like at the prices of $P_X$=1 and $P_Y$=1. If starting out with the same endowments as in part (D), how much will each consumer want to buy/sell of each good? Is the result a competitive equilibrium?

*Each consumer's demand will be at a point such that (1) $P_X/P_Y$ = MRS, and (2) the amount spent on the desired bundle of goods is equal to the value of the initial endowment.*
*For consumer A:*

$$\frac{P_X}{P_Y} = \frac{Y_A}{X_A} \quad \Rightarrow \quad Y_A = X_A$$

*And the cost of the initial endowment = cost of the new bundle* ➔ $1(10) + 1(10) = 1X_A + 1Y_A$
*(1)* $Y_A = X_A$
*(2)* $X_A + Y_A = 20$
➔ $\boxed{X_A = 10}$ *and* $\boxed{Y_A = 10}$

*For consumer B:*

$$\frac{1}{1} = \frac{Y_B}{X_B} \Rightarrow Y_B = X_B$$

*And* $1(10) + 1(10) = X_B + Y_B$
➔ $\boxed{X_B = 10}$ *and* $\boxed{Y_B = 10}$

*A wishes to sell 5 units of Y and buy 5 units of X.*
*B wishes to sell 5 units of X and buy 5 units of Y.*
*This is a competitive equilibrium.*

F. Assume the consumers can trade as much as they like at the prices of $P_X$=2 and $P_Y$=1. If starting with equal endowments (each individual has 10 units of both good *X* and good *Y*), how much will each consumer want to buy/sell of each good? Is this result a competitive equilibrium?

*For consumer A:*
*(1)* $P_X/P_Y = Y_A/X_A$ ➔ $2X_A=Y_A$
*(2)* *value of initial endowment = cost of new bundle* ➔ $2(10)+1(10)=2X_A+Y_A$
➔ $30=2X_A+Y_A$
*Substitute and solve:*
$X_A=7.5$
$Y_A=15$

*For consumer B:*
*(1)* $P_X/P_Y = Y_B/X_B$ ➔ $2X_B=Y_B$
*(2)* $30=2X_B+Y_B$
*Substitute and solve:*
$X_B=7.5$
$Y_B=15$

*A wishes to sell 2.5 units of X and buy 5 units of Y.*
*B also wants to sell 2.5 units of X and buy 5 units of Y.*
*Since supply is not equal to demand in these markets for X and Y, this is NOT a competitive equilibrium.*

G. Suppose consumer C considers 2 units of *X* a perfect substitute for 1 unit of *Y* and consumer D considers 3 units of *X* a perfect substitute for 1 unit of *Y* (these ratios hold at <u>all</u> consumption levels). Now, consumers C and D are the only two consumers in an exchange economy. Repeat part (C).

*Firstly, I should have mentioned that you can't solve for the contract curve in the same way as part (C). It's easier to see what the contract curve will look like on a graph.*

*To sketch, realize that consumer C is just as happy with "2 units of X and no units of Y" or "no units of X and 1 unit of Y". And because X and Y are perfect substitutes for this consumer, indifference curves are linear.*

$U_C(X,Y) = X + 2Y$ *(any utility function with constant MRS=1/2 is correct)*

*Consumer D is just as happy with "3 units of X and no units of Y" or "no units of X and 1 unit of Y". Again, X and Y are perfect substitutes for this consumer, and therefore indifference curves are linear.*

$U_D(X,Y) = X + 3Y$ *(any utility function with constant MRS = 1/3 is correct)*

*$MRS_C$ never equals that of $MRS_D$ (both are constants in this example). Therefore, we get corner solutions and the contract curve follows the border of the Edgeworth Box.*