

Problem Set #3 -- Answer Key¹
Problem Set #3

- Venus Williams likes both tennis rackets and tennis shoes. She has many of both. Her marginal rate of substitution (MRS) of rackets for shoes is 3, meaning that given the opportunity, she is willing to trade 3 tennis rackets for 1 pair of shoes, or vice versa. Unused rackets and shoes may be returned to the local sporting goods store for a refund. The current price for a racket is \$200 and the price for a pair of shoes is \$100. Suggest a way for Venus to make herself better off.

*Answer: Since Venus has many tennis rackets (Y) and tennis shoes (X) she hasn't worn, she can return or exchange them. Since her $MRS_{YforX}=3$, she would be **willing** to give up 3 rackets for one pair of shoes. However, since the price ratio is $P_X/P_Y=\$100/\$200=1/2$, she's only **required** to give up 1/2 racket for each extra pair of shoes (if you don't like the way that sounds, an equivalent statement is that she's required to give up 1 racket for every 2 extra pairs of shoes). This is a good deal for Venus, who would make herself happier by trading in some tennis rackets in return for some shoes at the market rate.*

- A government considers two policy options, (i) a cash grant of \$100 and (ii) a certificate good for \$100 worth of food. Is a wealthy person more likely than a poor person to prefer the cash grant? Explain using graphs.

Recall that a budget constraint line (BC) will shift out by \$100 for an individual with \$100 additional income. For one receiving \$100 in food stamps, the BC shifts out by \$100 along one segment only; this new BC also has a kink because one cannot spend more on other goods than the maximum affordable pre-policy. Now consider two cases, shown in Figures 2A and 2B:

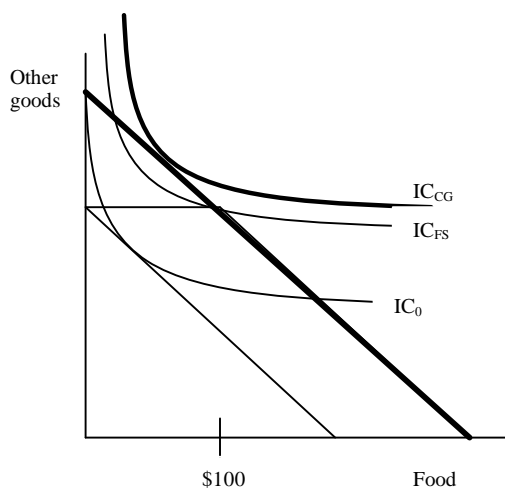


Figure 2A

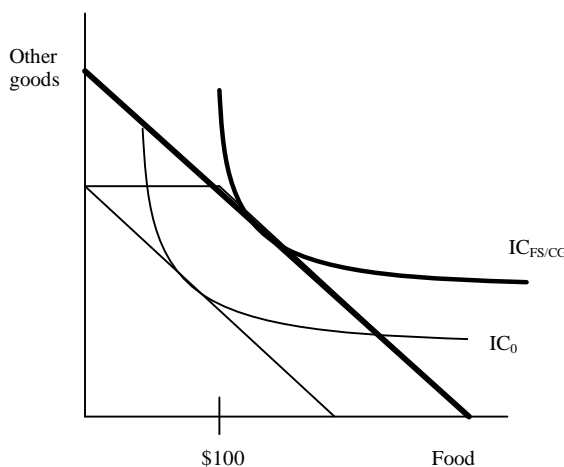


Figure 2B

¹ Prepared by Steve Warmerdam

An individual with indifference curves (IC) as shown in Fig. 2A prefers \$100 in cash to \$100 in food stamps. The dotted BCs and ICs are those under the cash grant program, and the solid-line BCs and ICs are those under the food stamp program. Note that under the food stamp program the IC will shift out from IC_0 to IC_{FS} ; under the cash grant policy, the IC would shift out from IC_0 to IC_{CG} , a higher (and thus more desirable) IC. Someone with ICs as shown in Fig. 2B will be indifferent between cash grant and food stamp policies--both policies shift IC_0 by the same amount (the new IC being $IC_{FS/CG}$).

So which graph is representative of a poor individual, and which represents a wealthy one? Fig. 2A represents someone whom, when given \$100 cash, still chooses to spend less than \$100 total on food. Such an individual will prefer the cash to food stamps because it allows the individual to take full advantage of the subsidy while continuing to spend less than \$100 on food. Fig. 2B represents someone spending more than \$100 on food when given \$100 cash. This type of person is indifferent between the two policies. If one believes that wealthy individuals are likely to spend more than \$100 on food, and poor individuals are more likely to spend less than \$100 on food (since their limited income must cover other necessities (e.g. housing)), then the poor person will prefer the cash over the food stamps. The wealthy person is indifferent.

Note that as the budget line shifts farther and farther out into the northeast direction (indicating greater wealth), the percentage of the budget line to the left of 100 units of food falls. Thus the likelihood of a consumer being concerned with a preference of cash over food stamps becomes smaller and income rises.

3. Stuart's utility function for goods X and Y is represented as $U(X,Y)=X^{0.2}Y^{0.8}$. Assume his income is \$100 and the prices of X and Y are \$10 and \$20, respectively.
- Express his marginal rate of substitution (MRS) between goods X and Y. As the amount of X increases relative to the amount of Y along the same indifference curve, does the MRS increase or decrease? Explain.

$$MU_X = 0.2(Y/X)^{0.8} \text{ and } MU_Y = 0.8(X/Y)^{0.2}$$

$$MRS_{YforX} = -MU_X/MU_Y = -Y/4X$$

The MRS (in absolute value) gets smaller as the amount of X increases relative to Y. In other words, the more X (and less Y) one has, the less of Y one is willing to give up in order to obtain an additional unit of X.

- What is his optimal consumption bundle (X^* , Y^*), given income and prices of the two goods?

At the optimal consumption bundle, the MRS is equal to the ratio of prices. That is,

$MRS_{YforX} = -P_X/P_Y$. Plugging in our prices and the results from (a),

$$-Y/4X = -10/20 \rightarrow -Y/4X = -1/2 \rightarrow 2Y=4X \rightarrow Y=2X$$

The budget constraint must hold as well: $I=P_XX+P_Y Y \rightarrow 100=10X+20Y$

We now have two equations and two unknowns. Solve for (X^* , Y^*):

$$100=10X+20(2X) \rightarrow 100=50X \rightarrow \boxed{X^*=2}$$

$$Y=2X=2(2) \rightarrow \boxed{Y^*=4}$$

- How will this bundle change when all prices double and income is held constant? When all prices double AND income doubles?

The utility function does not change and therefore the formula for MRS does not change. Prices change, but the ratio $-P_X/P_Y$ does not. Therefore, from $MRS_{YforX} = -P_X/P_Y$ we still get $Y=2X$.

The budget constraint changes to $100=20X+40Y$.

Solving as before (with two equations, two unknowns), we get $(X^*, Y^*) = (1, 2)$

If all prices and income change proportionally, the optimal bundle does not change.

- d. Derive the demand curve for good X and demand curve for good Y.

Solve for X^* and Y^* as before, except this time do not plug in explicit values for P_X , P_Y and I .

$$MRS_{YforX} = -P_X/P_Y \rightarrow -Y/4X = -P_X/P_Y \rightarrow P_Y Y = 4P_X X$$

$$\text{Budget constraint: } I = P_X X + P_Y Y$$

$$\text{Now let's solve for } X^* \text{ by substituting out } Y. I = P_X X + 4P_X X = 5P_X X \rightarrow$$

$$X^*(P_X; I) = I/(5P_X)$$

Holding income constant at \$100 gives us a demand curve of

$$\boxed{X^*(P_X; I=100) = 20/P_X}$$

We can solve for Y^* by substituting out X :

$$P_Y Y = 4P_X X \rightarrow Y = (4P_X X)/P_Y \rightarrow Y = (4P_X [I/(5P_X)])/P_Y$$

$$Y^*(P_Y; I) = 4I/(5P_Y)$$

Holding income constant at \$100 gives us a demand curve of

$$\boxed{Y^*(P_Y; I=100) = 80/P_Y}$$

Now a government subsidy program lowers the price of Y from \$20 per unit to \$10 per unit.

- e. Calculate and graphically show the change in good Y consumption resulting from the program.

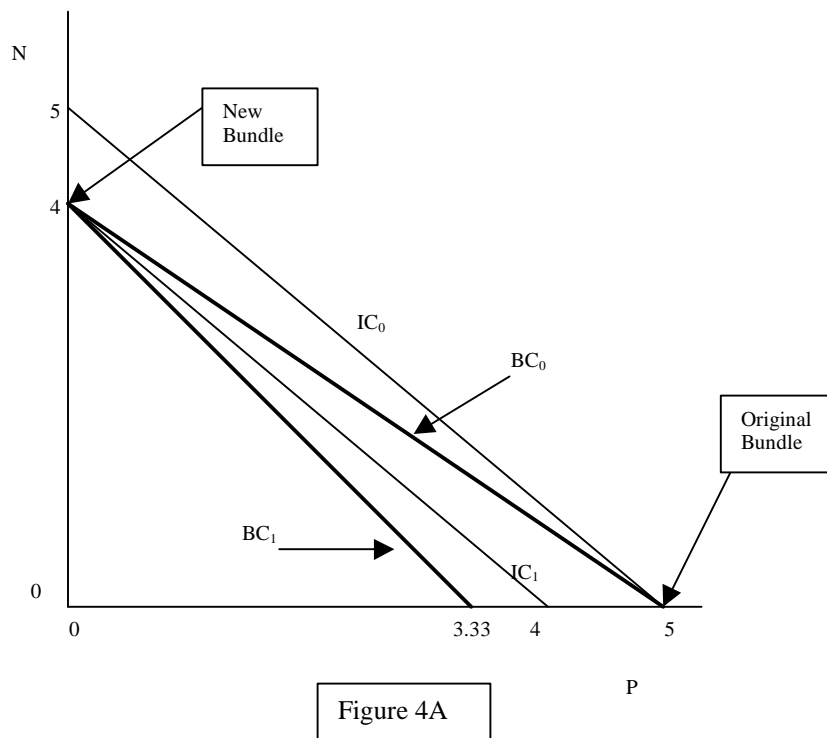
To calculate the new bundle we could go through the same procedure as in part (b) or simply use the demand equations derived in part (d). X^* does not depend on P_Y so $X^* = 2$. Y^* is a function of its own price and, using demand from (d), $\boxed{Y^* = 80/P_Y = 80/10 = 8}$. This change is depicted in Figure 3.

- f. Graphically show the change in consumption attributable to the separate income and substitution effects.

No calculations were required for this part. Refer to Figure 3 for the separate income and substitution effects. Note that the substitution effect for good Y may be found by sketching a BC_{sub} with the new price ratio (parallel to BC_1) but tangent to the original indifference curve IC_0 .² The corresponding point tells us what a person facing the new price ratio would purchase if utility were somehow held constant at its original level. The income effect brings us from this substitution point to Y^* .

² Chapter 8 of Varian discusses two different types of substitution effects--Slutsky (Section 8.1) and Hicks (Section 8.8). The Slutsky substitution effect measures the change in demand when prices change but consumer's **purchasing power** is held constant. The Hicks substitution effect measures the change in demand when prices change but consumer's **utility** is held constant. Both methods are correct; simply keep in mind that in this key, the Hicks method of Section 8.8 is used.

Figure 4A sketches the changes. Since peaches and nectarines are perfect substitutes, the utility function will have a linear form (such as $U(P,N)=P+N$). Indifference curves are therefore linear. As the price changes she shifts to a lower indifference curve and thus her utility falls.



- c. How much must her budget increase in order to return to the original utility level?

In order to achieve her old level of utility (not necessarily her old bundle), she needs enough income to increase total consumption of fruit to five units. Nectarines are the cheaper good, and by purchasing one additional nectarine she will return to her original indifference curve. Thus she needs an additional \$1.25 if she is to return to her original utility level.

- d. Derive and graph the demand curve for nectarines.

Isobel consumes I/P_N nectarines so long as $P_N < P_P$; she is indifferent as to how she divides income between peaches and nectarines if $P_N = P_P$; finally, she demands no nectarines if $P_N > P_P$. We can now graph the demand curve for nectarines (note: Income and price of peaches are held constant when sketching the nectarine demand curve. In Figure 4B I assume income is \$5 and peach price is \$1. You may have used a constant peach price of \$1.50, which is also acceptable).

- e. Derive and graph the Engel curve for nectarines (under the assumption that the price of peaches is \$1.50 a pound).

When nectarines are the cheaper good, Isobel will spend all her income on them. The number of nectarines she will buy is determined using $N=I/P_N$. Rearranging gives $I=P_N N$, so the Engel curve is a straight line with a slope of P_N . This is shown in Figure 4C.

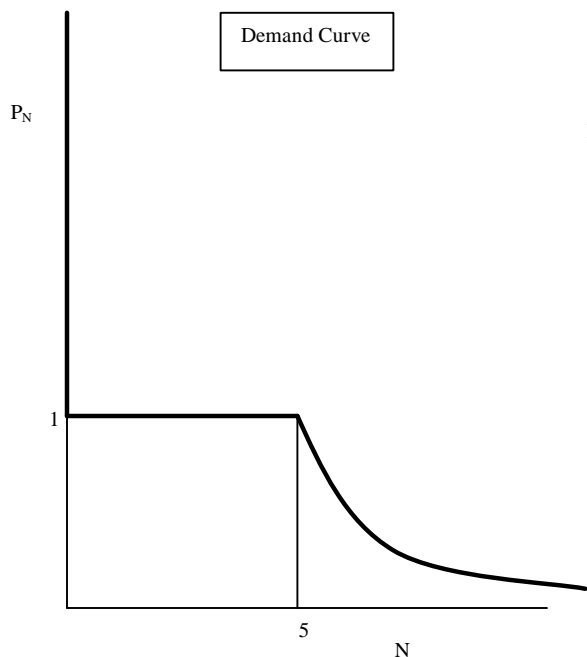


Figure 4B

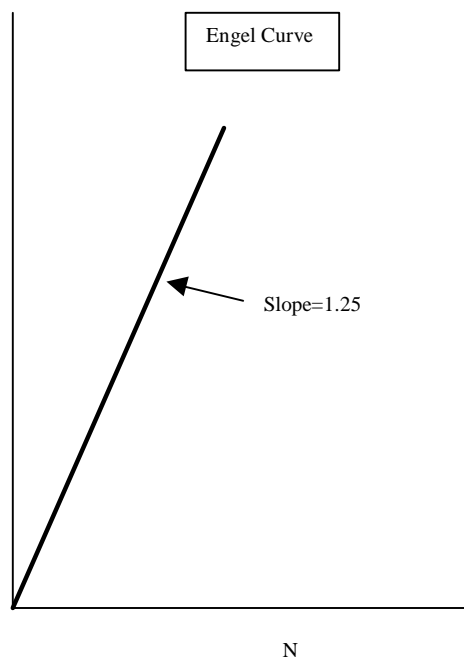


Figure 4C

5. Twenty years ago, Dmitri consumed bread which cost him 10 kopeks a loaf and potatoes which cost him 13 kopeks a sack. With his income of 266, he bought 11 loaves of bread and 12 sacks of potatoes. Today he has an income of 510. Bread now costs him 20 kopeks a loaf and potatoes cost him 20 kopeks a sack. Assuming his preferences haven't changed, when was he better off? Explain.

Previously, Dmitri spent all his income on the purchase of 11 loaves of bread and 12 sacks of potatoes. We can see this by plugging values into his original budget constraint (let B represent bread and S represent potatoes ("S" is for "spuds")).

$$I^0 = P_B^0 B^0 + P_S^0 S^0 \rightarrow 266 = (10)(11) + (13)(12)$$

It's now 20 years later. Given new prices and income, will Dmitri be able to afford his original bundle? If so, then he won't be worse off. Using the new prices, let's find how much income he needs in order to purchase his original bundle:

$$P_B^1 B^0 + P_S^1 S^0 \rightarrow (20)(11) + (20)(12) = 460$$

He can afford his original bundle. In fact, this bundle only costs 460, leaving him with additional income ($510 - 460 = 50$) to spend. Therefore, he will be able to move to a higher indifference curve and thus become better off than he was 20 years ago.

6. Prudence maximized her utility subject to her budget constraint. Then prices changed. After the change, she became better off. Therefore we can conclude that the new bundle costs more at the old prices than the old bundle did. True or false? Explain.

Figure 6 shows the situation described above. At original prices and income, Prudence maximizes utility by consuming at point A, where MRS (from indifference curve IC_0) equals the price ratio (from budget constraint BC_0). We know that she becomes better off after the price change; therefore, her new consumption choice, point B, must be located on a higher indifference curve, IC_1 . If she is maximizing utility subject to her budget constraint, the new bundle is located where MRS (from indifference curve IC_1) equals the price ratio (of a new budget constraint, sketched as BC_1).

The question's statement is true--the new bundle costs more at the old prices than the old bundle did. How do we know this? At old prices, the slope of any budget constraint must equal that of BC_0 . A inward parallel shift of BC_0 indicates the taking away of income, and an outward parallel shift of BC_0 indicates the addition of income. A parallel shift of BC_0 that hits point B will require an outward shift, indicating that the new bundle costs more at old prices than the old bundle did. Note that this holds regardless of where point B is drawn on the new indifference curve IC_1 .

