Economic 100A Prof. D. McFadden
Fall 2001

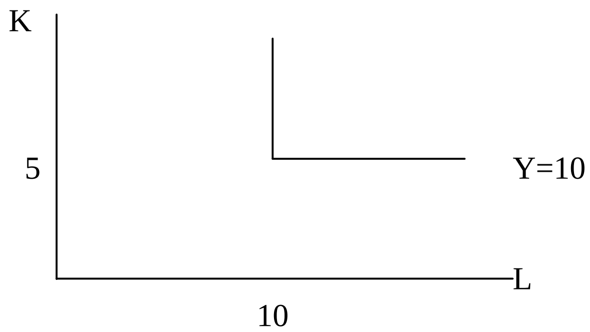
Microeconomic Analysis
Problem set 7
Suggested Answers

1.

| Prod. Fn. | $MP_1$ | $MP_2$ | TRS | Scale |
|---|---|---|---|---|
| $x_1^{1/4}x_2^{3/4}$ | $\frac{1}{4}(x_1^{-3/4}x_2^{3/4})$ | $\frac{3}{4}(x_1^{1/4}x_2^{-1/4})$ | $-1/3(x_2/x_1)$ | Constant |
| $x_1 + (x_2)^{1/2}$ | $1$ | $\frac{1}{2}(x_2^{-1/2})$ | $-2/(x_2^{-1/2})$ | Decreasing |
| $(x_1^{1/3}+x_2^{1/3})^3$ | $(x_1^{1/3}+x_2^{1/3})^2 x_1^{-2/3}$ | $(x_1^{1/3}+x_2^{1/3})^2 x_2^{-2/3}$ | $-x_1^{-2/3}/x_2^{-2/3}$ | Constant |

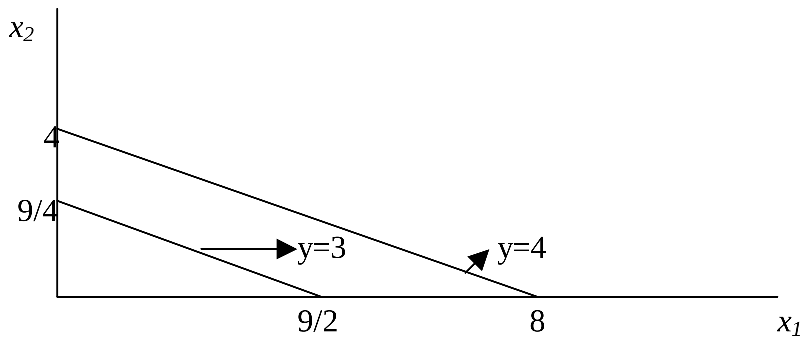
2. a) This production function has constant returns to scale. To see this just scale both inputs by a factor $t > 0$: $\min\{tL,2(tK)\} = \min\{tL,t2K\} = t*\min\{L,2K\}$.

b) Because the factors are complements, the isoquants are L-shaped:




c) The most cost-effective way to produce 10 units of output is to use 10 units of labor and 5 unit of capital. This is true regardless of the relative prices of capital and labor.

3. a)




b) Note that the factors are perfect substitutes so only one will be used. The marginal product of factor 1 is smaller than the marginal product of factor 2. Further, we know that $P*MP_i$=factor price $i$, at the optimum, hence:

$4*(1/2)*((2x_1 + 4x_2)^{-1/2})*4=3$. Solving for $x_2$, noticing that $x_1=0$, gives us the solution:
$x_1^*=0$ ; $x_2^*=16/9$ and profit-maximizing output is $8/3$.

4. a) $y=\min\{S/6,B/12\}$, i.e. perfect complements production function.
   b) $y(tS,tB)=\min\{(tS)/6,(tB)/12\}=t*\min\{S/6,B/12\}=t*y$, hence CRS.
   c) $\min_{S,B} w_S S+w_B B$ s.t. $y=\min\{S/6,B/12\}$,

$$S\geq 0$$
$$B\geq 0$$

Calculus is not appropriate here because the kink of the perfect complement function.

For any given level of output $y$, the firm requires $y=S/6$ units of $S$, so: $S=6y$ is the conditional demand function for $S$.

For any given level of output $y$, the firm requires $y=B/12$ units of $B$, so: $B=12y$ is the conditional demand function for $B$.

   d) The cost function is therefore $c(w_S,w_B,y)= w_S S^*+w_B B^*= w_S 6y+w_B 12y= 6y(w_S+2w_B)$
   This is a long run cost function because none of the inputs (i.e. neither S nor B) are fixed.