Econ 100A
Fall 2001 Prof. Daniel McFadden

Problem Set 8

1. True/False: state your reasons clearly and succinctly.

a) A profit-maximizing firm will always minimize costs.

True.

b) In the long run a firm always operates at the minimum level of average costs for the optimally sized plant to produce a given amount of output.

False.

2. Assume downward sloping or flat demand and a U-shaped LRAC curve. In each of the following situations, determine graphically and/or verbally:
a) Does the firm have the cost-minimizing amount of capital given its output level? If not, should the firm increase or decrease its amount of capital given its output?
b) Does the firm have the profit maximizing level of output given its amount of capital? If not, should the firm increase or decrease its level or output, given its capital?
If the situation is impossible, state why.

Answers: first, it is to be understood that capital input is here regarded as the fixed costs---fixed in the short run, that is. So the two questions for each case come down to your judgment in the short term and long term. Secondly, refer to Fig.21.6 and 21.10 for a graphical understanding of the explanation.

i) SRAC > LRAC, SRMC > LRMC, MR = SRMC

MR=SRMC shows that the firm is at an output level that maximizes the profit, given its capital in the short term. However, because SRMC>LRMC and SRAC>LRAC (See Fig. 21.10), the amount of capital is not optimized to minimize costs at the present output level. The firm should increase its capital input in the long run so as to move cost curves to the right.

ii) SRAC > LRAC, SRMC < LRMC, MR = SRMC

Similar to above in the short run. But now SRMC<LRMC, which means that the cost curves are to the left of optimum. Capital input must be decreased in the long run to shift costs to the left.

iii) SRAC < LRAC, SRMC > LRMC, MR = SRMC

Impossible: SRAC is always greater than LRAC.

iv) SRAC > LRAC, SRMC > LRMC, MR > SRMC

The output is not optimized even in the short run, because MR>SRMC. The output should be increased to maximize profit in the short run. Long run decision regarding capital is the same as i).

v) SRAC > LRAC, SRMC < LRMC, MR > SRMC

The output is not optimized even in the short run, because MR>SRMC. The output should be increased to maximize profit in the short run. Long run decision regarding capital is the same as ii). Graphically, this means that SRAC curve should move to the left along LRAC, to which it runs tangent at optimum output level.

vi) SRAC = LRAC, SRMC = LRMC, MR > SRMC

SRAC = LRAC means that the capital input is most appropriate for the given output level in the long run. However, since MR > SRMC, the given output

level is lower than it should be to maximize profit, so y needs to be increased in the short run to y^*, causing SRAC* > LRAC*, and SRMC* > LRMC*. This means that SRAC curve must shift to the right to re-establish the equilibrium between SRAC and LRAC. So an injection of capital is required in the long run, and output will be increased.

vii) SRAC = LRAC, SRMC > LRMC, MR = SRMC

Impossible. Where SRAC= LRAC and MR = SRMC, SRMC must equal LRMC.

viii) SRAC = LRAC, SRMC =LRMC, MR =SRMC

The company reaches the optimum level in both long run and short run.

3. A firm has a production technology given by: $Q = L^{0.25} * K^{0.25}$. Initial input prices are given by w =1 and r = 1. Suppose for the moment that the amount of capital is fixed at K= 4.

a) Is the marginal product of labor diminishing? Why?

$MP_L(\overline{K} = 4) = \frac{\partial}{\partial L}(L^{0.25} * 4^{0.25}) = \frac{\sqrt{2}}{4} L^{-\frac{3}{4}}$.

The marginal product of labor decreases as L increases due to a fixed amount of capital level in the short-run: diminishing marginal returns.

b) Find the short run total cost function, and then the short-run marginal and average cost curves.

Recall the definition of short run cost function:

$C^s(Q) = w * L(Q) + r * \overline{K}$.

By plugging a fixed amount of capital K = 4, into the production function, we easily derive the conditional demand for labor for a given level of output Q:

$L^s(Q) = \frac{1}{4} Q^4$.

The short run cost function is therefore

$C^s(Q) = 1 * \frac{1}{4} Q^4 + 1 * 4 = \frac{1}{4} Q^4 + 4$.

Differentiating cost by q gives

$SMC(Q) = Q^3$:

and dividing by q gives

$SAC(Q) = \frac{1}{4} Q^3 + \frac{4}{Q}$.

Suppose now that the period is long enough that both inputs can be adjusted.

c) Find the capital-labor ratio.

Note that the expansion path is the curve through tangency points between Isoquants and input price ratio. From the equation

$MRTS_{LK}(L, K) = \frac{w}{r}$

or equivalently

$wL = rK$,

we derive K = L asthe long run expansion path.

d) Find the returns to scale of this production technology.

$(aL)^{0.25} * (aK)^{0.25} = a^{0.5} L^{0.25} K^{0.25} < aL^{0.25} K^{0.25}$

for a given a > 1. Therefore, this production technology exhibits 'decreasing returns to scale'.

e) If the rental price of capital is doubled, what happens to the profit maximizing output level?

Assume there is an entry barrier for this market even in the long run. Otherwise, price

competition between firms drives market equilibrium price down to '0'. New expansion path comes from:

$MRTS_{LK}(L,K) = \frac{1}{2},\ L = 2K$.

With production function together, we derive the conditional demand for labor and capital,

$K(Q) = \frac{1}{\sqrt{2}}Q^2,\ L(Q) = \sqrt{2}Q^2$

for a given output level Q. Hence the long run cost function reduces to

$C(Q) = 1*\sqrt{2}Q^2 + 2*\frac{1}{\sqrt{2}}Q^2 = 2\sqrt{2}Q^2$.

Accordingly,

$LAC(Q) = 2\sqrt{2}Q$ and $LMC(Q) = 4\sqrt{2}Q$.

By applying profit maximization rule of

$P = LMC(Q^*)$,

we obtain

$Q^* = \frac{4\sqrt{2}}{P}$.

Profit by producing Q* is equal to

$\pi = (p - 2\sqrt{2}) * \frac{4\sqrt{2}}{P}$

when

$P \geq 4\sqrt{2}$.

'Shut down' and 'zero' profit if

$P < 4\sqrt{2}$.

4. Suppose a firm faces a cost function of $C = 8 + 4q + q^2$, so that its marginal cost is $MC = 4 + 2q$.

a) What is the firm's fixed cost, F?

F=8.

b) What is the formula for the firm's variable cost, VC?

$VC=4q + q^2$

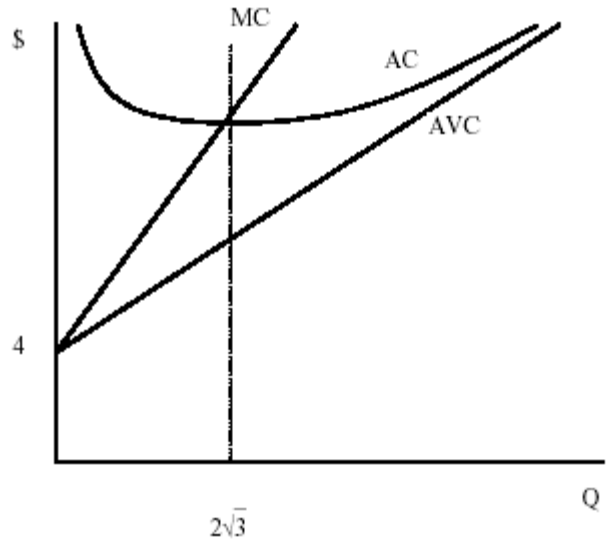
c) What is the formula for the average cost, AC?

$AC = 8/q + 4 + q$

d) What is the formula for average variable cost, AVC?

AVC= 4 + q.

e) On a diagram, draw the AC, AVC, and MC curves.



5. Suppose a firm's short run cost curves were found to be:
Total Cost = SRTC = $1 + 2Q + Q^2$
Marginal Cost = SRMC = 2 + 2Q ,where Q is output.
Assume the firm behaves as a price-taker and sells its output at P = $8 per unit (that is, its demand curve is flat at P = $8).
a) If the firm maximizes profits, how much will it produce?
Marginal Cost = SRMC = 2 + 2Q^* = P = 8.
So Q* = 3.
b) What are the marginal, average, and total cost at that point?
At Q*=3, SRMC* = 8.
SRTC* = 16.
AC = SRTC*/Q* = 5.33.
c) What is the firm's profit?
P = 3*8 – 16 = 8.