Solutions

Economics 100A
Fall 2001
Prof. Daniel McFadden

Quiz #1

September 26, 2001
(by: Peter Adams)

Name:

SID #:

Section: **115** **116**

Instructions

You have 50 minutes to complete the exam.
Mark only on the exam.
Do not separate the pages.
Show all work.
Partial credit will be awarded where applicable.

1 Multiple Choice (10 Points)

CIRCLE ONLY the statements that are TRUE. If a statement is FALSE, DO NOT circle the response. A question may have ALL, MULTIPLE, or NO true statements. You will be rewarded for the TRUE statements you select and penalized for marking FALSE statements as TRUE. It is not in your best interest to simply guess, as it is possible to achieve a NEGATIVE score. If you circle ALL of the statements, you will end up with ZERO POINTS. Unless otherwise stated, always assume the standard two good scenario, (x_1, x_2).

$$(p_1 + t_1)x_1 + (p_2 + t_2)x_2 = m$$

Problem 1 *Budget Constraints* $\Rightarrow (p_1 + p_1/2)x_1 + (p_2 + \frac{p_2}{2})x_2 = m$

Consider the standard two good budget constraint, $p_1x_1 + p_2x_2 = m$. If a quantity tax, $t_1 = \frac{p_1}{2}$, is applied to good 1 and a quantity tax, $t_2 = \frac{p_2}{2}$, is applied to good 2, then...

$\Rightarrow \frac{3}{2}p_1x_1 + \frac{3}{2}p_2x_2 = m$

- This is equivalent to $p_1x_1 + p_2x_2 = \frac{2}{3}m$.
- The budget constraint will not change, because both prices are affected by a tax of the same proportion.

$\Rightarrow p_1x_1 + p_2x_2 = \frac{2}{3}m$

- The budget constraint will make a parallel shift OUT.
- The budget constraint will make a parallel shift IN.

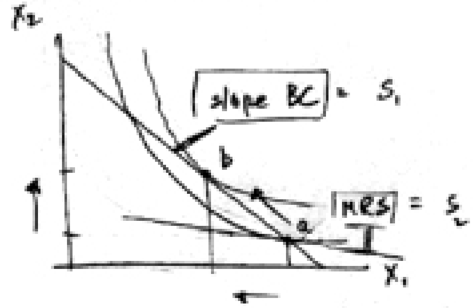
Problem 2 *Preferences*

Consider a single consumer and two bundles, A and B. If $A \succsim B$ and $B \succsim A$, where $\succsim$ means "at least as good as," then we can conclude that,

- This consumer has intransitive preferences.
- This consumer is indifferent between bundle A and B (i.e. $A \sim B$).
- If another bundle, C, is strictly preferred by this consumer to bundle B (i.e. $C \succ B$), then bundle C is strictly preferred to bundle A (i.e. $C \succ A$).
- Bundles A and B lie on DIFFERENT indifference curves.

$|s_2| < |s_1|$

* move from (a) towards (b)



Problem 3 *Indifference Curves*

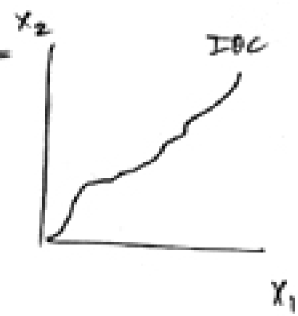
$MRS_{12} = \dfrac{dx_2}{dx_1}$. This is the marginal rate of substitution of good 1 for good 2. It is the amount of good 2 with which you must be compensated for a unit loss of good 1. (Do not be confused. Think of x_2 on the $y - axis$ and x_1 on the $x - axis$.)

- Along the budget line, if $|MRS_{12}| < \dfrac{p_1}{p_2}$, then you would like MORE of good 2 and LESS of good 1.
- Along the budget line, if $|MRS_{12}| < \dfrac{p_1}{p_2}$, then you would like LESS of good 2 and MORE of good 1.
- Indifference curves can cross if preferences are NOT convex. ⟸ NEVER
- The absolute value of the slope of an indifference curve, or $|MRS_{12}|$, is ALWAYS equal to the price ratio, $\dfrac{p_1}{p_2}$.

NOT TRUE for corner solutions ← only at optimal bundle (x^*) when x_1^* and x_2^* > 0

Problem 4 *Engel Curves*

- Normal goods have upward sloping Engel curves. $\dfrac{dm}{dx} > 0$
- Inferior goods have downward sloping Engel curves. $\dfrac{dm}{dx} < 0$
- If the income offer curve (IOC) or income expansion path is strictly increasing, then the Engel curve for each good is upward sloping. =
- The Engel curves for perfect substitutes are always linear, but the Engel curves for perfect complements can be either linear or nonlinear.



Always linear for BOTH perfect substitutes and complements

Problem 5 *Demand*

- For an ordinary good, the income effect (IE) always goes in the SAME direction as the substitution effect (SE).
- A good is EITHER always a normal good OR always an inferior good, but cannot be BOTH (i.e. sometimes normal and sometimes inferior).
- If a good is normal, then the good is also an ordinary good.
- Giffen goods have upward sloping demand functions.

Slutsky Equation

$\dfrac{dx}{dp} > 0 \Rightarrow$

ordinary good $\dfrac{dx}{dp} = \dfrac{dx^s}{dp} - x \cdot \dfrac{dx}{dm}$ if normal (+)

$(-) = (-) \underbrace{- (+)}_{(-)}$

5 pts

2 Short Answer (20 Points)

Answer all parts of the following question in the space provided. If you need additional space, use the back of each page. Scratch paper is available, if necessary.

Problem 6 *Lloyd Christmas only consumes Atomic peppers (A) and 12-packs of Beer (B). Suppose Lloyd can purchase the following bundles of Atomic Peppers and Beer (A, B). He can afford bundles of (40, 10) and (12, 24).*

1. Solve for the relative price of Atomic peppers to Beer $\left(\frac{p_A}{p_B} =\right)$?

$40 P_A + 10 P_B = m$

$12 P_A + 24 P_B = m$

$\Rightarrow P_A (40 - 12) = P_B (24 - 10)$

$28 P_A = 14 P_B$

$\frac{P_A}{P_B} = \frac{14}{28} = \frac{1}{2}$

2. Let $p_A = 5$. Solve for the price of Beer ($p_B =$) and the level of income ($m =$).

$5(40) + 10 P_B = m$ $\qquad$ If $P_A = 5 \rightarrow P_B = 10$

$\Rightarrow 5(40) + 10(10) = 300$

$m = 300$

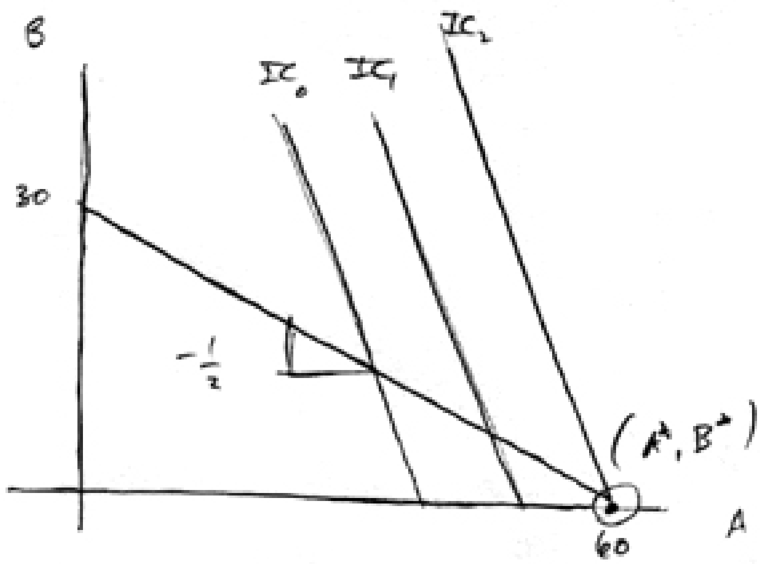
$P_B = 10$

5 pts

3. If Lloyd's $MRS_{AB} = \frac{dB}{dA}$ of Atomic peppers for Beer is *CONSTANT* and equal to -2, what TYPE of goods are Atomic peppers and Beer?

perfect substitutes

4. Draw Lloyd's budget set with Atomic peppers (A) on the $x-axis$ and Beer (B) on the $y-axis$. Label the two intercepts, the slope, and draw some indifference curves implied by a CONSTANT $MRS_{AB} = -2$.



$(A^* B^*) = (60, 0)$

5. What is the optimal consumption bundle (A^*, B^*) for Lloyd? Show this on your graph above.

5pts

6. Suppose Lloyd's preferences change from those in (3). His new preferences imply the following demand function for Beer, $B^* = \frac{p_A(m)}{p_B(p_B+p_A)}$. Using $m = 360$, $p_A = 4$, and $p_B = 8$, determine Lloyd's optimal bundle (A^*, B^*).

$$B^* = \frac{4}{8}\frac{(360)}{12} = 15$$

$$\Rightarrow 4(A^*) + 8(15) = 360$$

$$4A = 240$$

$$A^* = 60$$

$$(A^*, B^*) = (60, 15)$$

7. Suppose a quantity subsidy (s) is imposed on the price of Beer (p_B) such that the price of Beer is cut in half $\left(\text{i.e. } s = \frac{p_B}{2}\right)$. Using $m = 360$, $p_A = 4$, and $p_B = 8$, what is the *NEW* optimal bundle (A^{**}, B^{**}) after you impose the subsidy?

$$\Rightarrow P_B' = P_B - S = P_B - \frac{P_B}{2} = \frac{1}{2}P_B = 4$$

$$\Rightarrow B^{**} = \frac{4(360)}{4(8)} = 45$$

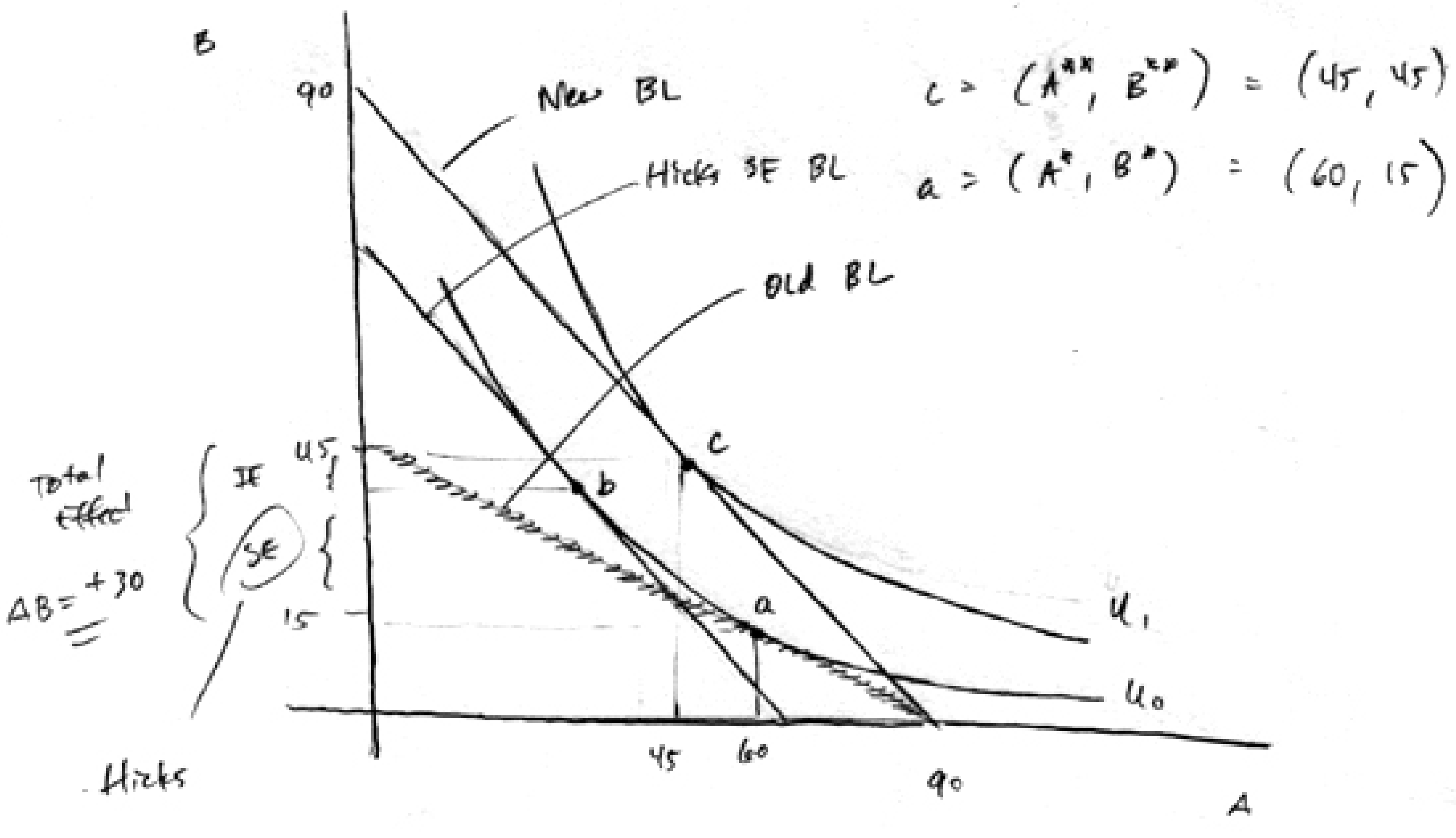
$$(A^{**}\ B^{**}) = (45, 45)$$

$$\Rightarrow 4A^{**} + 4(45) = 360$$

$$4A^{**} = 180 \qquad A^{**} = 45$$

5pts

8. .Graph Lloyd's ORIGINAL budget set from part (6) with Atomic peppers (A) on the $x-axis$ and Beer (B) on the $y-axis$. Label the intercepts, optimal bundle $a = (A^*, B^*)$, and tangent indifference curve (U_0). On the SAME graph, draw Lloyd's NEW budget set (i.e. after the price subsidy). Label the intercepts, the new optimal bundle $c = (A^{**}, B^{**})$, and tangent indifference curve (U_1). Assume "nice, smooth" convex indifference curves.



9. What is the TOTAL EFFECT of the price change on the demand for Beer (ΔB). Above, show graphically the income effect (IE) and substitution effect (SE) on demand for Beer. You are NOT required to give specific numerical values for each effect.

$\Delta B = +30$

Optional/Extra Credit (5 Points)

10. Suppose Lloyd's preferences can be described by this utility function, $u(A,B) = \sqrt{A} + \sqrt{B}$. DERIVE the demand curves for good A and good B as a function of prices (p_A, p_B) and income (m). In other words, solve Lloyd's standard utility maximization problem for $A^* = f(p_A, p_B, m)$ and $B^* = g(p_A, p_B, m)$. To receive full credit, you must show your work. DO NOT solve for a specific optimal consumption bundle, rather find the functional form that characterizes the optimal bundle for all prices (p_A, p_B) and income (m).

① MRS cond

② BC

① $\frac{MU_A}{MU_B} = \frac{P_A}{P_B}$ $\qquad \frac{\frac{1}{2}A^{-1/2}}{\frac{1}{2}B^{-1/2}} = \frac{P_A}{P_B}$

$\frac{\sqrt{B}}{\sqrt{A}} = \frac{P_A}{P_B} \Rightarrow B = \frac{P_A^2}{P_B^2} A$

② $P_A A + P_B \left[\frac{P_A^2}{P_B^2} A \right] = m$

$A \left(P_A + \frac{P_A^2}{P_B} \right) = m$ $\qquad A^* = \frac{m}{P_A \left(1 + \frac{P_A}{P_B} \right)}$

$B^* = \frac{P_A \, m}{P_B^2 \left(1 + \frac{P_A}{P_B} \right)}$