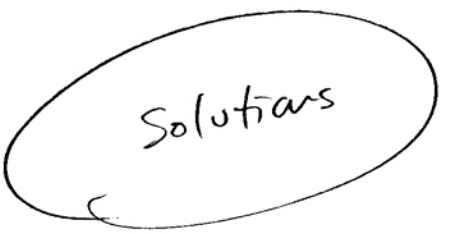
Economics 100A
Fall 2001
Prof. Daniel McFadden

Quiz #2

October 17, 2001
(by: Peter Adams)



Name:

SID #:

Section: **115** **116**

Instructions

You have 55 minutes to complete the exam.
Mark only on the exam.
Do not separate the pages.
Show all work.
Partial credit will be awarded where applicable.

4 pts

1 Endowments (10 minutes)

Problem 1 *Consider the standard labor supply problem, where C=weekly consumption, L=weekly hours of leisure, w=hourly wage, p=price of consumption, m=weekly non-wage income, and $\bar{L}$=weekly endowment of leisure/work hours.*

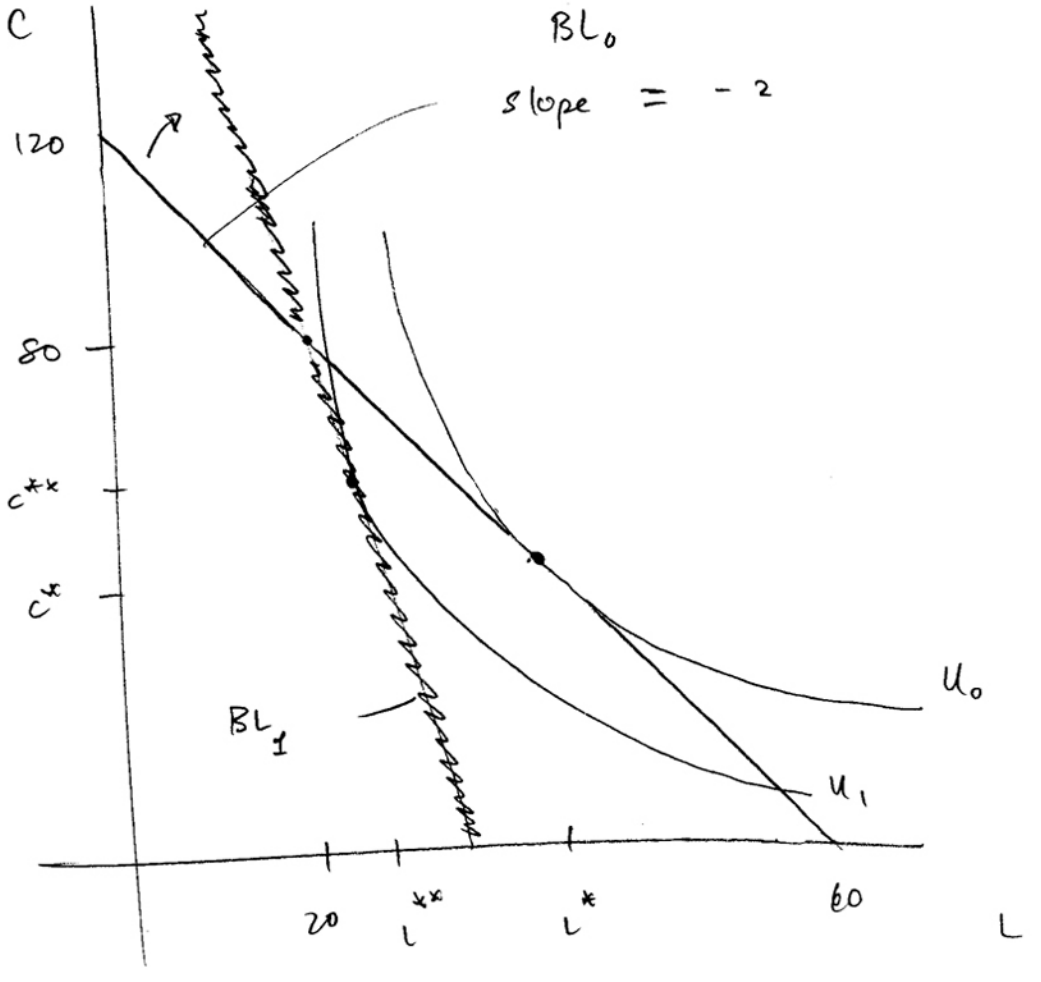
1. If $m = 80$, $p = 1$, $\bar{L} = 20$, $w = 2$, write the budget constraint?
2. Suppose $\bar{L}$ is NOT the maximum amount of leisure (L) you can consume, rather you can "purchase" additional hours of leisure with non-wage income. Graph the budget constraint from (a) with leisure (L) on the $x-axis$ and consumption (C) on the $y-axis$ under the assumption that you are NOT limited to a maximum of $L = \bar{L}$.
   (a) Suppose you are a NET BUYER of leisure. Indicate on your graph some optimal bundle (L^*, C^*) and the associated indifference curve (U_0) consistent with you being a NET BUYER of leisure. Also indicate on your graph the endowment $(\bar{L}, m)$, the slope of the budget line, and the intercepts. Note: (L^*, C^*) will NOT be numerical values.
   (b) If the wage increases ($w \uparrow$) and you REMAIN a NET BUYER of leisure, what can you say, if anything, about your new level of utility (U_1) relative to (U_0). Show this on your graph by labelling a new optimal bundle (L^{**}, C^{**}) and the associated indifference curve (U_1).
   (c) Can you say, DEFINITIVELY, whether remaining a NET BUYER after the wage increase results in MORE or LESS consumption of leisure (L)?

(1) $pC + wL = m + w\bar{L}$

$\Rightarrow C + 2L = 80 + 2(20)$

$* \quad C + 2L = 120$

(2) (a), (b)



(c). No, you cannot. Its ambiguous

could draw new Budget Line (BL_1) and

U_1 such that new optival bundle (l^{**}, c^{**})

has $l^{**} > l^*$.

4 pts

2 Intertemporal Choice (5 minutes)

Problem 2 *Suppose you live for 3 periods. Your consumption in each period is* (c_1, c_2, c_3) *and your income in each period is* (m_1, m_2, m_3).

1. Write an expression for the (present value or future value) intertemporal budget constraint. Assume the interest rate is CONSTANT between periods at rate (r).
2. Write an expression for the (present value or future value) intertemporal budget constraint when the interest rate is NOT constant between periods. Assume the interest rate between period 1 and period 2 is (r) and the interest rate between period 2 and period 3 is (s).

(1)

$$c_1 + \frac{c_2}{(1+r)} + \frac{c_3}{(1+r)^2} = m_1 + \frac{m_2}{(1+r)} + \frac{m_3}{(1+r)^2} \qquad PV =$$

OR

$$(1+r)^2 c_1 + (1+r) c_2 + c_3 = \qquad FV =$$

$$(1+r)^2 m_1 + (1+r) m_2 + m_3$$

(2)

$$c_1 + \frac{c_2}{(1+r)} + \frac{c_3}{(1+r)(1+s)} = m_1 + \frac{m_2}{(1+r)} + \frac{m_3}{(1+r)(1+s)} \qquad PV =$$

OR

$$(1+r)(1+s) c_1 + (1+s) c_2 + c_3 = \qquad FV =$$

$$(1+r)(1+s) m_1 + (1+s) m_2 + m_3$$

4 pts

3 Uncertainty (5 minutes)

Problem 3 *Suppose you face the following gamble or lottery. You can earn 1 of 3 possible grades in this class: an "A", a "C", or an "F", with the following probabilities:* $\pi_A = \frac{2}{10}$; $\pi_C = \frac{6}{10}$, *and* $\pi_F = \frac{2}{10}$. *Your current wealth* (w) *is* \$400. *If you receive an "A", you GAIN (e.g. I pay you)* \$500. *However, if you get an "F", you LOSE (e.g. you pay me)* \$300. *If you receive a "C", you DO NOT GAIN OR LOSE anything. Assume your utility function, defined over wealth, is* $U(w) = \sqrt{w}$.

1. What is your expected utility (EU)? [Hint: be sure to calculate your total wealth in each "state"]
2. What is the certainty equivalent level of wealth (w^*)?

(1) $EU = \pi_A u(w_A) + \pi_C u(w_C) + \pi_F u(w_F)$

$w_A = 400 + 500 = 900$

$w_C = 400 + 0 = 400$

$w_F = 400 - 300 = 100$

$EU = \frac{2}{10}\sqrt{900} + \frac{6}{10}\sqrt{400} + \frac{2}{10}\sqrt{100}$

$= 6 + 12 + 2 = 20$

(2) $u(w^*) = EU \Rightarrow \sqrt{w^*} = 20$

$\Rightarrow w^* = 20^2 = 400$

6 pts

4 Elasticity of Demand (10 minutes)

Problem 4 *Consider the following market demand function:* $Q(p) = D(p) = 108 - p^3$.

1. Find an expression for the price elasticity of demand $\epsilon(p)$ as a function of price.
2. At what price level (p) is market demand UNIT ELASTIC?
3. If $p = 2$, what is the price elasticity of demand, (ϵ) or $|\epsilon|$? Is it ELASTIC, UNIT ELASTIC, or INELASTIC?

(1) $\epsilon = \frac{p}{q} \cdot \frac{dq}{dp}$, $\frac{dq}{dp} = -3p^2$

$\Rightarrow \epsilon = \frac{p}{q} \cdot (-3p^2) = \frac{-3p^3}{108 - p^3}$ *

(2) set $|\epsilon| = 1 \Rightarrow \frac{3p^3}{108 - p^3} = 1$

$\Rightarrow 3p^3 = 108 - p^3 \Rightarrow 4p^3 = 108$

$\Rightarrow p^3 = 27 \Rightarrow p = 3$ * *

(3) $p = 2 \Rightarrow \epsilon = \frac{-3(2)^3}{108 - (2)^3} = \frac{24}{100} = .24$

INELASTIC

S P*

5 Equilibrium Analysis (25 minutes)

Problem 5 *Consider the following market demand function,* $D(p) = 20 - \frac{1}{4}p$, *and market supply function,* $S(p) = p - 10$.

1. Solve for the NO TAX equilibrium price and quantity (p^*, q^*)?

Set $D(p) = S(p)$

$\Rightarrow \quad 20 - \frac{1}{4}p = p - 10$

$\Rightarrow \quad 30 = \frac{5}{4}p$

$\Rightarrow \quad p^* = 24$

$\Rightarrow \quad q^* = 14 \quad \left(\begin{matrix} D(24) \\ S(24) \end{matrix} = 14 \right)$

2. Suppose the government imposes an AD VALOREM TAX (τ) of 100%. In other words, the ad valorem tax rate is $\tau = 1$. The government is effectively doubling the price. Find the new equilibrium under the ad valorem tax? [Hint: when solving for the price under the ad valorem tax, recall that an ad valorem tax affects price by $(1+\tau)p = (p + \tau p)$]

   (a) What is the new equilibrium quantity ($\hat{q}$)?

   (b) What is the effective price faced by suppliers ($\hat{p}^S$)?

   (c) What is the price faced by consumers ($\hat{p}^D$)?

   (d) What is the size of the tax ($\tau\hat{p}$)?

   (e) How much tax revenue (R) is raised?

where $p_D = p_S + \tau p_S$

under Ad Valorem Tax, solve

$D(p_S + \tau p_S) = S(p_S)$ for $\tau = 1$

$\Rightarrow \quad D(2p_S) = S(p_S)$

$$20 - \frac{1}{4}(2P_s) = P_s - 10$$

$$\Rightarrow \quad 20 - \frac{1}{2}P_s = P_s - 10$$

$$\Rightarrow \quad 30 = \frac{3}{2}P_s$$

$$\Rightarrow \quad \hat{P}_s = 20$$

(a) $S(P_s) = S(20) = 20 - 10 = \boxed{10}$ $\hat{q}$ *

(b) $\hat{P}_s = \boxed{20}$ *

(c) $\hat{P}_D = \hat{P}_s + \tau \hat{P}_s = 2\hat{P}_s = \boxed{40}$ *

(d) $\tau_P = \hat{P}_D - \hat{P}_s = \boxed{20}$ *

or $\tau \hat{P}_s = 20$

(e) $R = tax \cdot q = \tau \hat{P}_s \cdot \hat{q}$

$= 20(10) = \boxed{200}$ *

6 pts

* see page 8 for Graph!

3. What is the CHANGE in Consumer Surplus (ΔCS)?
4. What is the CHANGE in Producer Surplus (ΔPS)?
5. What is the Deadweight Loss (DWL) from the tax?
6. Who bears the greater burden of the tax revenue (R), consumers or producers? What is the value of each?

(3) $\Delta CS = CS_2 - CS_1$

$CS_1 = \frac{1}{2}(80-24)\cdot 14 = 392$

$CS_2 = \frac{1}{2}(80-40)\cdot 10 = 200$

$\Delta CS = 200 - 392 = \boxed{-192}$ *

(4) $\Delta PS = PS_2 - PS_1$

$PS_1 = \frac{1}{2}(24-10)\,14 = 98$

$PS_2 = \frac{1}{2}(20-10)\,10 = 50$

$\Delta PS = 50 - 98 = \boxed{-48}$ *

(5) DWL = $\frac{1}{2}$ tax · Δq

= $\frac{1}{2}(20)(14-10)$ = **40** *

(6) Share of Tax Burden (Revenue):

Consumers: $(40-24)\cdot 10$ = **160** *

Producers: $(24-20)\cdot 10$ = **40** *

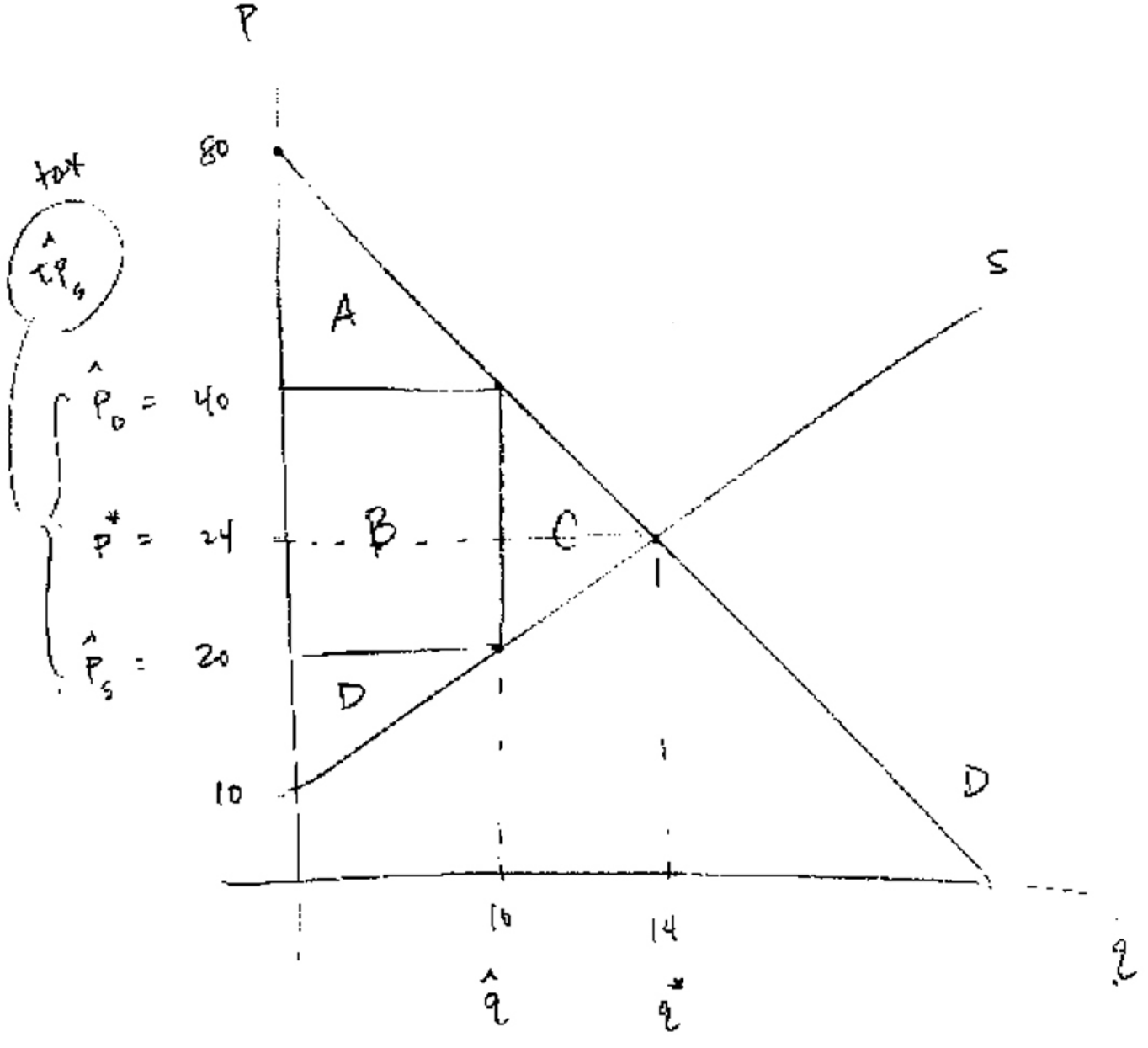
Total Tax Rev. = 200

* Consumers bear greater share

[160 > 40]

3 pts

7. Graph the two equilibria discussed above, labelling the intercepts and relevant prices and quantities. **LABEL or IDENTIFY** all of the areas under the demand curve and above the supply curve according to following criteria: *A=CS after tax, B=government revenue, C=DWL, D=PS after tax?*



3 $p^{1/3}$

5.1 Extra Credit

8. Suppose, instead of the ad valorem tax, the government imposes a QUANTITY TAX (t) such that the equilibrium level of output under the quantity tax is $q = 12$. Solve for the new equilibrium and give values for the following.

   (a) What is the effective price faced by suppliers (p^S)?

   (b) What is the price faced by consumers (p^D)?

   (c) What is the size of the quantity tax ($t = p^D - p^S$)?

   (d) How much Government Revenue (R) is raised?

   (e) What is the CHANGE in DWL (ΔDWL) in switching from the AD VALOREM tax to the QUANTITY tax?

If $\bar{q} = 12$ (After Tax Eq. quantity)

(a) $S(P_S) = 12 \Rightarrow 12 = P_S - 10$

$\boxed{P_S = 22}$ *

(b) $D(P_D) = 12 \Rightarrow 12 = 20 - \frac{P_D}{4}$

$\Rightarrow \frac{1}{4} P_D = 8 \Rightarrow \boxed{P_D = 32.}$ *

(c) $t = P_D - P_S = 32. - 22 = \boxed{10}$ *

(d) $R = t \cdot \bar{q}$

$= 10 \cdot 12 = \boxed{120}$

(e) Δ DWL from Ad valorem Tax to Quantity Tax

$DWL_{Av} = 40$ [from Part 5]

$DWL_{Q} = \frac{1}{2} tax \cdot \Delta q$

$= \frac{1}{2} 10 \cdot (14 - 12) = \boxed{10}$

$\Delta DWL = \boxed{-30}$ *