Economics 240A, Section 2: Multiple Regression

Michael R. Roberts
Department of Economics and
Department of Statistics
University of California at Berkeley

March 26, 2000

1 Problems

Here are some new problems. Some are easy, some are hard but if you want to learn this stuff you've got to do problems. Unless otherwise stated, assume we are working in the classical regression framework which assumes:

- $E(y) = X\beta$ (or $E(\varepsilon) = 0$)
- $\text{cov}(y) = \sigma^2 I$ (or $\text{cov}(\varepsilon) = \sigma^2 I$)
- X is full rank and nonstochastic.

1. A couple of people wanted a little refresher on matrix algebra so...

   Expand the matrix product

   $$X = \left(\left[AB + (CD)'\right]\left[(EF)^{-1} + GH\right]\right)'$$

   Assume all matrices are square and E and F are invertible (i.e. nonsingular).

2. A matrix, A, is nilpotent if $\lim_{k\to\infty} A^k = 0$. Prove that a necessary and sufficient condition for a symmetric matrix to be nilpotent is that all of its eigenvalues be less than 1 in absolute value.

3. Consider the least squares regression of y on X where $X \in R^{n\times k}$ and includes a constant. Consider an alternative set of regressors, $Z = XP$, where $P \in R^{k\times k}$ is a nonsingular matrix. Thus, each column of Z is a mixture of some of the columns of X.

(a) Prove that the residual vectors in the regressions of y on X and y on Z are identical. What relevance does this have to the question of changing the fit of a regression by changing the units of measurement of the independent variables?

(b) Prove that the cosine of the angle between y and it's projection into X and the cosine of the angle between y and it's projection into Z is the same.

4. Powell said and I quote, "You must know how to find the inverse of a 2x2 matrix in order to hope to pass this class". That said...Find the inverse of

$$P = \begin{pmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{pmatrix}$$

What are the eigenvalues of P?

5. Is var(y) a parameter or a random variable? Scalar or vector?

6. Is cov(y) is a parameter or a random variable? Scalar or vector?

7. Is var($\hat{y}$) a parameter or a random variable? Scalar or vector?

8. Is cov($\hat{y}$) is a parameter or a random variable? Scalar or vector?

9. Is e (the OLS residuals) observable or unobservable? What about ε? What is the difference between e and ε?

10. Show that $E(e) = 0$

11. Find cov(e) and cov($\hat{y}$)

12. (FWL Theorem) Consider partitioning the design matrix of the classical regression framework.

$$y = X\beta + \varepsilon = X_1\beta_1 + X_2\beta_2 + \varepsilon$$

Show that b_2, the OLS estimator of the vector β_2, may be obtained by the following process

(a) Regress each column of X_2 on X_1. Take the residual vector from each of these regressions and form a matrix, X_2^*.

(b) Regress y on X_1 and save the residuals in a vector, y^*

(c) Regress y^* on X_2^* to get the coefficients b_2^*. These coefficients are equal to the b_2 you would have gotten if you'd run the original full regression.

(Hint: Partition the normal equations and solve for b_2)

13. Consider the *least absolute deviation (LAD) line* given by the values of c and d that minimize

$$\sum_{t=1}^{T} |y_t - (c + dx_t)|$$

Show that, for a dataset with three observations, $(x_1, y_1), (x_1, y_2)$, and (x_3, y_3) (note the first two $x's$ are the same), any line that goes through (x_3, y_3) and lies between (x_1, y_1) and (x_1, y_2) is a least absolute deviation line.