Econ. 240A, Spring 2000 D. McFadden

PROBLEM SET 5 (Hypothesis Testing + Midterm Review)

Due Monday, March 6, with Section Discussion on March 7,9

(Questions 2-12 are adapted from old midterm exams)

1. Use the data set nyse.txt in the class data area, and the variable RNYSE giving the daily rate of return on the New York Stock Exchange. For the purpose of this exercise, make the maintained hypothesis that the observations are independent and identically normally distributed. Let μ denote the population mean and σ^2 denote the population variance of RNYSE.

a. Test H_o: $\mu = 0.0003$ versus H_1: $\mu \neq 0.0003$ at significance level $\alpha = 0.04$. At $\alpha = 0.01$. What is the power of each of these tests against the alternative $\mu = 0.0005$?

b. Test H_o: $\mu \geq 0.0003$ versus H_1: $\mu < 0.0003$ at significance level $\alpha = 0.05$. What is the power of this test against the alternative $\mu = 0.0005$?

c. Test H_o: $\sigma^2 = 0.0001$ versus H_1: $\sigma^2 \neq 0.0001$ at significance level $\alpha = 0.01$. What is the power of this test against the alternative $\sigma^2 = 0.000095$?

d. Some analysts claim that opening of international capital markets in the 1980's improved the productivity of capital in large multinational corporations, and this has in turn led to higher mean returns to equity. Make the maintained hypothesis that the variance of returns is constant over the full observation period in nyse.txt. Test the hypothesis that mean return after January 1, 1985 was the same as the mean return prior to that date, versus the alternative that it was not. Use $\alpha = 0.01$.

e. Some analysts claim that the introduction of dynamic hedging strategies and electronic trading, beginning around January 1, 1985, has made the stock market more volatile than it was previously. Test the hypothesis that the variance in RNYSE after that date was higher than the variance before that date, versus the alternative that it was smaller. Use $\alpha = 0.02$. Do not maintain the hypothesis of a common mean return in the two periods.

2. The table gives the investment rate X_i in 8 developed countries. Assume that the X_i are i.i.d. draws from a normally distributed population.

Country	Ratio of Gross Fixed Capital Formation to GDP (Pct.,1993)
Japan	30.1
Germany	22.7
Netherlands	19.7
France	18.9
Canada	18.2
Italy	17.1
U.S.A.	16.1
U.K.	14.9

(a) Test the hypothesis that the population mean investment rate is no greater than 17.0 using a significance level of 95 percent. Be specific about the test statistic, its distribution under the null, and the critical level you would use.

(b) Compute the power of this test against the alternative that the mean is 20.0. Be specific about the distribution that would be used for the power calculation. Give numerical values for the parameters of this distribution, substituting s for σ if necessary. Give a numerical value for the power.

3. Suppose a random sample of size 4 is drawn from a uniform distribution on $[0,\theta]$. You want to test H_o: $\theta \le 2$ versus H_1: $\theta > 2$ by rejecting the null if $Max(X_n) > K$. Find the value of K that gives significance level $\alpha = 0.05$. Construct the power curve for this test.

4. A random sample $X_1,...,X_N$ is drawn from a normal density. The variance is known to be 25. You want to test the hypothesis H_o: $\mu \ge 2$ versus the alternative H_1: $\mu < 2$ at significance level $\alpha = 0.01$, and you would like to have power $\boldsymbol{\pi} = 0.99$ against the alternative $\mu = 1$. What sample size do you need?

5. Let $X_1,...,X_N$ be a random sample from a density whose mean is μ and variance is σ^2. Consider estimators of μ of the form $m = \sum_{n=1}^{N} a_{Nn}X_n$, where the a_{Nn} are non-random weights. Under what conditions on the weights is m unbiased? Among unbiased estimators of this form, what weights give minimum variance?

6. A husband and wife are both laid off when the local auto assembly plant closes, and begin searching for new jobs on the same day. The number of weeks Y the wife takes to find a job has a geometric density, $f_Y(y) = p^{y-1}(1-p)$, for y = 1,2,..., where p is a parameter. The number of weeks X it takes the husband to find a job is independent of Y, and also has a geometric density, $f_X(x) = q^{x-1}(1-q)$, for x = 1,2,..., where q is a parameter. The parameters have the values p = 0.5 and q = 0.75. Useful facts about the geometric density $f(z) = r^{z-1}(1-r)$ for z = 1,2,...are (I) $\mathbf{E}_Z(Z-1)\cdots(Z-n) \equiv \sum_{z=1}^{\infty} (z-1)\cdots(z-n)r^{z-1}(1-r) \equiv (r/(1-r))^n\, n!$ for n = 1,2,.. and (ii) $\Pr(Z > t) \equiv \sum_{z=1}^{\infty} r^{z-1}(1-r) \equiv r^t$.

a. What is the expected value of the difference between the lengths of the unemployment spells of the husband and the wife?

b. If the wife is unemployed for at least 6 weeks, what is the expectation of the total number of person-weeks of unemployment insurance the couple will receive, assuming benefits continue for a person as long as he or she is unemployed?

c. What is the probability that the unemployment spell for the husband is greater than that for the wife?

d. What is the expected time until at least one member of the couple is employed?

e. What is the expected time until <u>both</u> husband and wife are employed?

7. Let $X_1,...,X_N$ be a random sample from a uniform distribution on $[0,\theta]$, where θ is an unknown parameter. Show that $T = [(N+1)/N]\cdot Max(X_n)$ is an unbiased estimator for θ. What is its variance? What is the asymptotic distribution of $N(T-\theta)$?

8. You decide to become a bookmaker for the next Nobel prize in Economics. Three events will determine the outcomes of wagers that are placed:

U. The prize will go to a permanent resident of the U.S.

R. The prize will go to an economist politically more conservative than Milton Friedman

A. The prize will go to someone over 70 years of age

(a) You are given the following probabilities: $P(A|U) = 5/6$, $P(A|R) = 4/5$, $P(A|R\&U) = 6/7$, $P(U|A\&R) = 3/4$, $P(R|A) = 4/7$, $P(R|A\&U) = 3/5$. Find $P(R|U)$, $P(A\&R|U)$, $P(A\cup R|U)$.

(b) If in addition, you are given $P(U) = 3/5$, find $P(A)$, $P(R)$, $P(A\&R)$, $P(A\&R\&U)$.

© You want to sell a ticket that will pay \$2 if one of the events U,R,A occurs, \$4 if two occur, and \$8 if all three occur. What is the minimum price for the ticket such that you will not have an expected loss?

9. If X is standard normal, what is the density of $|X|$? Of $\exp(X)$? Of $1/X$?

10. You wish to enter a sealed bid auction for a computer that has a value to you of \$3K if you win the auction. You believe that each competitor will make a bid that is uniformly distributed on the interval [\$2K,\$3K].

(a) If you know that the number N of other bidders is 3, what is the probability that all competitor's bids are less than \$2.9K? What should you bid to maximize your expected profit?

(b) Suppose the number N of other bidders is unknown, but you believe N is Poisson distributed with an expected value $\mathbf{E}N = 5$. What is the probability that the maximum bid from your competitors is less than x? What should you bid to maximize expected profit?

11. Random Variables X and Y are bivariate normal, with $\mathbf{E}X = 1$, $\mathbf{E}Y = 3$, and $Var(X) = 4$, $Var(Y) = 9$, $Covariance(X,Y) = 5$.

(a) What is the mean of $Z = 2X - Y$?

(b) What is the variance of $Z = 2X - Y$?

© What is the conditional mean of Z given $X = 5$?

(d) What is the conditional variance of Z given $X = 5$?

12. What is the probability that the larger of two random observations from any continuous distribution will exceed the population median?