Econ. 240A, Spring 1998 D. McFadden

PROBLEM SET 1 (Review of Elementary Probability theory)

(Due Monday, Feb. 2, with discussion in section on Feb. 4)

1. Prove that a σ-field of events contains *countable* intersections of its members.

2. It is known that 0.2 percent of the population is HIV-positive. It is known that a screening test for HIV has a 10 percent chance of incorrectly showing positive if you are not, and a 2 percent chance of incorrectly showing negative if you are in truth positive. What proportion of the population that tests positive is in truth positive?

3. Ramanathan, problems 2.4, 2.5, 2.7.

4. An airplane has 40 seats. The probability that a ticketed passenger shows up for the flight is 0.95, and the events that any two different passengers show up is statistically independent. If the airline sells 45 seats, what is the probability that the plane will be overbooked? How many seats can the airline sell, and keep the probability of overbooking to 5 percent or less?

5. Prove that the expectation $\mathbf{E}(X - c)^2$ is minimized when $c = \mathbf{E}X$.

6. Prove that the expectation $\mathbf{E}|X - c|$ is minimized when c = median(X).

7. A sealed bid auction has an economist for sale to the highest of n bidders. You are bidder 1. Your experience is that the bids of each other bidder is distributed with a Power distribution $F(X) = X^{\alpha}$ for $0 \le X \le 1$. Your profit if you are successful in buying the economist at price y is 1 - y. What should you bid to maximize your expected profit?

8. A random variable X has a normal distribution if its density is $f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-(x-\mu)^2/2\sigma^2}$, where μ and σ^2 are parameters. Prove that X has mean μ and variance σ^2. Prove that $\mathbf{E}(X-\mu)^3 = 0$ and $\mathbf{E}(X-\mu)^4 = 3\sigma^4$.