

PROBLEM SET 3 (Stochastic Limits)

(Due Monday, Feb. 23, with discussion in section on Feb. 18)

1. Ignoring adjustments for family composition and location, an American family is said to be below the poverty line if its annual income is less than \$14,800 per year. Let Y_i be the income level of family i , drawn randomly and independently from this population, and let Q_i be one if Y_i is less than \$14,800, zero otherwise. Family income can obviously never be larger than GDP, so that it is bounded above by a (very big) constant G , and cannot be negative.

2. Hoeffding's inequality says that if independent identically distributed random variables Y_i satisfy $EY_i = 0$ and $|Y_i| \leq B$, then

$$\text{Prob}\left(\frac{1}{n} \sum_{i=1}^n Y_i > \varepsilon\right) < \exp(-\varepsilon^2 n / 2B^2) .$$

Use this inequality to show that $\frac{1}{n} \sum_{i=1}^n Y_i \xrightarrow{\text{as}} 0$.

3.