Econ. 240A, Spring 1998 D. McFadden

PROBLEM SET 4 (Estimation)

(Due Monday, March 2, with discussion in section on Feb. 25)

1. If h(t) is a convex function and t = T(**x**) is a statistic, then **E**h(T) $\geq$ h(**E**T), with the inequality strict when h is not linear over the support of T. When h is a concave function, **E**h(T) $\leq$ h(**E**T). If T is an unbiased estimator of a parameter σ^2, what can you say about $\sqrt{h(T)}$ as an estimator of σ and exp(h(T)) as an estimator of exp(σ^2)?

2. A simple random sample i = 1,...,n is drawn from a binomial distribution b(K,1,p); i.e., K_i = 1 with probability p and K_i = 0 with probability 1-p. Which of the following statistics are sufficient: a. $(K_1,...,K_n)$ b. $(K_1^2,[K_2+...+K_n]^2)$ c. $\bar{K}$ d. $(\bar{K},[K_1^2+...+K_n^2])$ e. $[K_1^2+...+K_n^2]$?

3. You want to estimate mean consumption from a random sample of households i = 1,...,n. You have two alternative income measures, C_{1i} which includes the value of in-kind transfers and C_{2i} which excludes these transfers. You believe that $\bar{C}_1$ will overstate consumption because in-kind transfers are not fully fungible, but $\bar{C}_2$ will understate consumption because these transfers do have value. After some investigation, you conclude that $0.7\cdot\bar{C}_1 + 0.3\cdot\bar{C}_2$ is an unbiased estimator of mean consumption. Your friend Dufus proposes instead the following estimator: Draw a random number between 0 and 1, report the estimate $\bar{C}_2$ if this random number is less than 0.3, and report the estimate $\bar{C}_1$ otherwise. Is the Dufus estimator unbiased? Does it pass the test of ancillarity?

4. Suppose T(**x**) is an unbiased estimator of a parameter θ, and that T has a finite variance. Show that T is *inadmissible* by demonstrating that $(1-\lambda)\cdot$T(**x**) + $\lambda\cdot 17$ for λ some small positive constant has a smaller mean square error. (This is called a Stein shrinkage estimator. The constant 17 is obviously immaterial, zero is often used.)