

**PROBLEM SET 5 (hypothesis testing)**

(Due Monday, March 9, with discussion in section on March 4)

1. The data set on the reverse gives the proportion of female-headed households (i.e., no husband present) in each State that are below the poverty line. These data are also available in machine-readable form at <http://econ.berkeley.edu/240a> as an ASCII file (ps5.txt) and as an Excel spreadsheet (ps5.xls). Obtain class accounts from Sam Thompson. Use TSP, SST, Excel, QuatroPro, or the statistical package of your choice to do these calculations. The file ps5.out is SST output containing solutions, to be looked at after you have done the exercises yourself.

a. Assume that these proportions are independently and identically normally distributed across States. Compute minimum variance unbiased estimators of the population mean and variance. Also estimate population means and variances separately for Coastal States (e.g., New England, Mid-Atlantic, South-Atlantic, Pacific) and Inland States. (i) Test the hypothesis that the population mean is 0.30, using a 10 percent significance level. What is the power of this test against the alternative that the population mean is 0.32? (ii) Test the hypothesis that the variance is the same in Coastal and Inland States. What is the power of this test against the alternative that the variance in Coastal States is 50% higher than the variance in Inland States. (iii) Test the hypothesis that the means in Coastal and Inland States are the same, conditioned on the maintained hypothesis that the variance is the same in both groups of States. What is the power of this test against the alternative that the Inland mean is 0.03 higher than the Coastal mean?

b. Assume that the State proportions are drawn from a CDF  $F(q) = q^\alpha$  for  $0 \leq q \leq 1$ , where  $\alpha$  is a positive parameter. (i) What additional assumptions are needed to guarantee that a Maximum Likelihood estimator of  $\alpha$  is consistent and asymptotically normal? (ii) Compute the MLE for  $\alpha$ , and estimate its asymptotic standard error. (iii) Do a large sample Wald test of the hypothesis that  $\alpha = 0.5$ . Do a large sample LR test of this hypothesis. (iv) Do a large sample LR test of the hypothesis that  $\alpha$  is the same in Coastal and Inland States. What is the power of your test against the alternative that  $\alpha_{\text{Coastal}} - \alpha_{\text{Inland}} = 0.5$ ?