Econ. C103, 2003 Daniel McFadden

Problem Set 10. Example Final Exam Questions
(For practice, not to be handed in)

1. There are J firms in an industry. Each can try to convince Congress to give the industry a subsidy. Let H_j denote the hours of effort put in by firm j, and let $w_jH_j^2$, where w_j is a positive constant, be the cost of this effort to firm j. When the effort levels of the firms are $(H_1,...,H_J)$, the value to each firm of the subsidy that gets approved is $\alpha\sum_{j=1}^{J} H_j + \beta\prod_{j=1}^{J} H_j$, where α and β are constants. Show that each firm has a strictly dominant strategy if and only if β = 0. What is this dominant strategy when β = 0?

2. [Hint: difficult, use a separating hyperplane argument] Prove that in a two-player game with finite sets of actions, if action a_1 for player 1 is never a best response for any mixed strategy of player 2, then a_1 is strictly dominated by some mixed strategy for player 1.

3. Consider a bargaining game in which two firms are considering a joint venture that will earn a profit of one million dollars, but they must agree on how to split the profit. Bargaining works as follows: Each makes a bid for a share of the one million. If the sum of the two bids exceeds one million, then the bargaining fails and both get nothing. If the sum of the two bids is less than one million, each gets his bid and the remainder goes to charity What are each player's strictly dominated strategies? What are their weakly dominated strategies? What are the pure strategy Nash equilibria for this game?

4. The centipede game for two players works as follows: Each player starts with one dollar. Starting with player 1, they alternately say "Stop" or "Continue". When a player says "Continue", one dollar is taken from her pile, and two dollars are added to her opponent's pile. As soon as either player says "Stop", the game terminates, and each receives the dollars in her own pile. Alternately, the game stops if each player's pile reaches five dollars. (A) Draw the extensive form of this game. (B) Find the unique subgame perfect Nash equilibrium.

5. Two firms, 1 and 2, have competitive products in the same market, with prices p_1 and p_2, and marginal costs $m_1 < m_2$. The demands for goods 1and 2 are

$$x_1 = \min\{2, 1 - \beta(p_1 - p_2)\},$$
$$x_2 = \max\{0, 1 - \beta(p_2 - p_1)\}.$$

Find Nash equilibrium price strategies for the two firms. Be sure to include the possibilities that both firms produce, or only firm 1 produces, with firm 2 poised to enter under profitable conditions.

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6. Two identical objects are to be auctioned in sequence to three symmetric bidders whose independent private values for an object are drawn from a known common distribution $G(v)$, using English auctions. (The value of a second object to a single bidder is zero.) Show that a subgame perfect Nash equilibrium assigns the two objects to the bidders with the two highest values, and the price paid by each will be the third highest value.

7. Each respondent in a poll of $N+1$ consumers is asked to submit a bid giving the maximum cost to them at which they would vote “Yes” for provision of a particular public good, and told that the good will be provided if a majority of those polled are in favor at the actual cost per capita C that provision of the good turns out to require. It is emphasized that the actual cost per capita will be charged to everyone if and only if the good is provided, so that in particular there is no linkage from their bid to the amount they will have to pay if the good is provided. Is this mechanism incentive-compatible, so that each consumer bids her true value?