

PROBLEM SET 6 (Sample Midterm Questions)

1. State and prove Walras' Law. Does Walras' Law hold when an economy is not in Walrasian equilibrium?
2. Is it true that if the assumptions of the second welfare theorem hold, then it is always possible to find an incomes policy that results in a Walrasian equilibrium?
3. If the initial endowment is on the contract curve, then there must always be a competitive equilibrium in which no trade takes place. True or false? Explain.
4. Jack Spratt's utility function is $U^J(F,L)=L$. His wife's utility function is $U^W(F,L)=F$. If Jack's initial endowment is 70 units of F and 35 units of L and if Jack's wife's initial endowment is 42 units of F and 70 units of L, then in an Edgeworth box for Jack and his wife, an allocation of F and L will be Pareto optimal only if it is at a corner of the box. True or false? Explain.
5. An economy has 2000 people. 1000 of them have utility functions $U(x_1,x_2)=x_1+x_2$ and 1000 of them have utility functions $U(x_1,x_2)=\min(2x_1,x_2)$. Everybody has an initial allocation of 1 unit of x_1 and 1 unit of x_2 . Find the competitive equilibrium prices and consumptions for each type of person.
6. A firm has two variable factors and a production function $f(x_1,x_2)=(2x_1+4x_2)^{1/2}$. What is the firm's profit function?
7. Darlene's utility function is $U(x_1,x_2,x_3)=x_1^4x_2^7x_3$. If her income doubles and prices remain unchanged, her demand for good x_2 will more than double. True or false? Explain.
8. A consumer has the utility function $U(x_1,x_2)=\min(x_1^2,2x_2)$. If the price of good x_1 is zero and the price of good x_2 is $p_2 > 0$, then the consumer's demand function for good x_2 is $y/2p_2$. True or false?
9. Angela's utility function is $x_1+x_2^{1/2}$. It is possible that if her income is very high, an increase in income will not make her spend more on x_2 . True or false? Explain.
10. Prove that it is impossible for a person to have a demand curve that slopes upward at all prices.
11. Prudence was maximizing her utility subject to her budget constraint. Then prices changed. After the price change she is better off. Therefore the new bundle costs more at the old prices than the old bundle did. True or false? Explain.
12. A consumer has the indirect utility function $V(p,y) = \log y - \log(p_1+p_2)$. If this person had an initial endowment (1,0), what is her price offer curve? What is the income elasticity of demand for good 1? The own price elasticity of demand for good 1?

13. With some services, e.g. checking accounts, phone service, or pay TV, a consumer is offered a choice of two or more payment plans. One can either pay a high "entry fee" and get a low price per unit of service or pay a low entry fee and a high price per unit of service. Suppose you have an income of \$100. There are two plans. Plan A has an entry fee of \$20 with a price of \$2 per unit. Plan B has an entry fee of \$40 with a price of \$1 per unit for using the service. Let x_1 be expenditure on other goods and x_2 be consumption of the service. a) Write down the budget equation that you would have after you paid the entry fee for each of the two plans. b) If your utility function is x_1x_2 , how much x_2 would you choose in each case? c) Which plan would you prefer? Explain.

14. A firm has a profit function $p_1^2/(p_2+p_3)$. What is the own-price elasticity of output?

15. An indirect utility function is said to have *Gorman form* if $V(p,y) = (y - b(p))/a(p)$, where $a(p)$ and $b(p)$ are concave, linear homogeneous non-decreasing functions of the price vector p . Show that the Engle curves for this indirect utility function are straight lines.

16. An economist named Apple decides that expenditure functions, giving the minimum income necessary to achieve utility level u at prices p , are too anonymous, so he defines a function giving the minimum endowment in apples necessary to achieve utility level u at prices p . Of course, he calls this the apple function. Prove that the apple function is concave and homogeneous of degree zero in prices, non-decreasing in the price of every good except apples, and decreasing in the price of apples.

17. Production is sometimes usefully described by what is termed an activity analysis model. For example, output might be given by $b_1x_1 + b_2x_2$, the requirement for input 1 by $a_{11}x_1 + a_{12}x_2$, and the requirement for input 2 by $a_{21}x_1 + a_{22}x_2$, where x_1 and x_2 are non-negative *activity levels* and production is limited by supplies c_1 and c_2 of the respective inputs. Write down a Lagrangian for the problem of maximizing output. Write down the dual linear program.

18. A two-person, two-good exchange economy has utility functions $U^1(x_{11},x_{21})$ and $U^2(x_{12},x_{22})$, and endowments $(\omega_{11},\omega_{21})$ for Ms. 1 and $(\omega_{12},\omega_{22})$ for Ms.2. Set up a constrained optimization problem for maximizing Ms. 1's utility subject to the constraint that Ms. 2 achieve at least the utility level provided by her initial endowment. Find the Lagrangian for this problem, and give the Lagrangian multipliers an economic interpretation.

19. An economy has two firms. The first firm uses labor input to produce apples, with constant returns to scale. The second firm uses inputs of labor and apples to produce breadfruit, again with constant returns. If leisure, apples, and breadfruit are all consumed in positive quantities by the consumers in this economy, how will equilibrium prices be influenced by a shift in consumer tastes toward breadfruit and away from leisure?

20. Is $1/[p_1/y + 2(p_1p_2)^{1/2}/y + p_2/y]$ an indirect utility function?