PRODUCTION ECONOMICS:
A DUAL APPROACH TO THEORY AND APPLICATIONS

VOLUME 2

CONTRIBUTIONS TO ECONOMIC ANALYSIS

111

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J. TINBERGEN

Editors

D.W. JORGENSON

J. WAELBROECK

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

PRODUCTION ECONOMICS: A DUAL APPROACH TO THEORY AND APPLICATIONS

Volume 2
Applications of the Theory of Production

Editors:
MELVYN FUSS and DANIEL McFADDEN

1978

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

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ISBN North-Holland for this series: 0 7204 3100 X
ISBN North-Holland for this volume: 0 444 85013 9

Publishers:

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

Sole distributors for the U.S.A. and Canada:

ELSEVIER NORTH-HOLLAND INC.
52 VANDERBILT AVENUE
NEW YORK, N.Y. 10017

Library of Congress Cataloging in Publication Data
Main entry under title:

Production economics.

(Contributions to economic analysis ; 110-111)
Includes bibliographies and indexes.
CONTENTS: v. 1. The theory of production.--v. 2. Applications of the theory of production.
1. Production (Economic theory) I. Fuss, Melvyn A. II. McFadden, Daniel. III. Series.
HB241.P74 338'.001 78-19128
ISBN 0-444-85014-7 (set)
Vol 1; 0 444 85012 0
Vol 2; 0 444 85013 9

PRINTED IN THE NETHERLANDS

INTRODUCTION TO THE SERIES

This series consists of a number of hitherto unpublished studies, which are introduced by the editors in the belief that they represent fresh contributions to economic science.

The term 'economic analysis' as used in the title of the series has been adopted because it covers both the activities of the theoretical economist and the research worker.

Although the analytical methods used by the various contributors are not the same, they are nevertheless conditioned by the common origin of their studies, namely theoretical problems encountered in practical research. Since for this reason, business cycle research and national accounting, research work on behalf of economic policy, and problems of planning are the main sources of the subjects dealt with, they necessarily determine the manner of approach adopted by the authors. Their methods tend to be 'practical' in the sense of not being too far remote from application to actual economic conditions. In addition they are quantitative rather than qualitative.

It is the hope of the editors that the publication of these studies will help to stimulate the exchange of scientific information and to reinforce international cooperation in the field of economics.

The Editors

PREFACE

The traditional starting point of production theory is a set of physical technological possibilities, often described by a production or transformation function. The development of the theory then parallels the process of firm operation, with the firm seeking to achieve its goals subject to the limitation of its technology and of the economic environment. The results are *constructed* input demands and output supplies, expressed as functions of the technology and the economic environment.

An alternative approach to production theory is to start directly from observed economic data – supplies, demands, prices, costs, and profits. The advantage of such an attack is that the theory can be formulated directly in terms of the causal *economic* relationships that are presumed to hold, without the intervening constructive steps required in the traditional theory. Because this approach is not bound by computational tractability in the step from production technology to economic observations, the prospect is opened for more satisfactory models of complex production problems.

It would at first appear that a theory of production couched in terms of economic observables would be less fundamental than one based on the physical technology, and that one could never be sure in an economic theory of consistency with a physical model. However, the theory of *production duality* establishes that the two approaches are equivalent and equally fundamental. Using duality, the technology underlying an economic model can be reconstructed and tested for compatibility with physical laws, as necessary. Then, the main thrust of analysis can be devoted to developing the structure and relationships of observed economic variables.

The purpose of these volumes is to develop the theory of production from the standpoint of the "dual" – the relationships between economic observables which are dual to the physical technology. The spirit of our treatment is the view that the end purpose of production theory is econometric study of economic problems involving technological limitations. The volumes emphasized the empirical implications of the theory, and therefore the development of the theoretical concepts proceeds with

an eye towards the econometric framework inherent in empirical applications. We hold the view that there is an intimate, symbiotic relationship between theory and econometrics, and that development of a fully successful economic analysis of production requires an integration of theoretical and econometric ideas in a unified approach. The papers in the two volumes of *Production Economics* represent an attempt to achieve this ideal.

The theory of production duality had its beginnings in the work of Hotelling (1932), Hicks (1946), Roy (1942), and Samuelson (1947). A pioneering book by Shephard (1953) provided the first comprehensive treatment of the subject and proof of the basic duality of cost and production. Extensions of the formal theory of duality were later made by McFadden (1962), Uzawa (1964), Shephard (1970), and Diewert (1971). Many of the basic duality results were also obtained by Gorman (1970), working independently. In a paper on the estimation of returns to scale, Nerlove (1963) utilized a cost function to derive econometric estimating equations. Subsequent work by McFadden (1964), Diewert (1969a,b), Christensen, Jorgenson and Lau (1971), and others have established the use of dual cost and profit functions as a basic tool in econometric production analysis.

It is possible to trace the origins of the present volumes back to 1961 when D. McFadden worked as a research assistant to M. Nerlove and H. Uzawa at Stanford University. The contributions of Uzawa (1962, 1964), McFadden (1962, 1963), and Nerlove (1963) date from that period. The empirical implications of duality theory were developed in McFadden (1964 and 1966). The first explicit empirical application of dual flexible functional forms appeared in Diewert's (1969a) study of labor demand functions for the Canadian Department of Manpower and Immigration. The generalized Leontief function [Diewert (1971)] was introduced in that study. The subsequent generation and empirical application of flexible functional forms received their major impetus from McFadden (1966) and Diewert (1969a,b).

Applications of the basic duality concepts continued to evolve at the University of California, Berkeley, during the years 1968–1970 under the auspices of the Project for the Optimization and Evaluation of Economic Growth. The introduction of the translog function by Christensen, Jorgenson, and Lau (1971, 1973), the nested generalized Leontief form by Fuss (1970, 1977b), the hybrid generalized Leontief form by Hall (1973), and the generalized CES form by Denny (1974a) all result from research begun at that time. A. Belinfante, T. Cowing, and P. Frenger also

were associated with the Economic Growth Project at various times. M. Bruno was a visiting scholar at M.I.T., together with D. McFadden, in 1971 when his chapter was written.

The idea of collecting a group of studies in duality under a common cover grew out of a seminar series held at the Economic Growth Project during the summer of 1969. A tentative title, *An Econometric Approach to Production Theory*, was chosen at that time. A number of the papers which appear in this volume have been referenced under that title. Since that time, the contents of the volumes evolved through several additions and deletions and M. Fuss joined D. McFadden as a co-editor. We feel that the current title more accurately reflects the spirit and content of the books.

Production Economics is divided into two main parts. Volume 1 contains the basic theoretical analysis of the duality of cost, profit, and production and a number of investigations of specific functional forms. Volume 2 contains the empirical applications. In keeping with the spirit of this work, these applications draw heavily on the analysis of Volume 1. Details of the contents of both volumes can be found in the two introductions.

The editors have been unable to standardize notation throughout the volumes; however, the notation in each chapter is self-contained. In almost all cases, upper case boldface letters denote sets, lower case boldface letters denote vectors. Upper and lower case Roman and Greek letters are used variously to denote scalars and functions. Derivatives are denoted variously by subscripts (the symbol for the variable with respect to which derivatives are being taken, or the ordinal position of this variable among the arguments), primes, the ∇ operator, or the usual notation $\partial f/\partial x$.

The editors wish to acknowledge the contributions that many individuals have made to the preparation of *Production Economics*. Dale Jorgenson and Zvi Griliches have provided encouragement and ideas. A large intellectual debt is owed to K.J. Arrow, W.M. Gorman, L. Hurwicz, M. Nerlove, and H. Uzawa, whose work provided the background for most of the developments in these books. We thank the contributors, who have displayed stoic patience and goodwill in the lengthy process of refereeing and publication. We also wish to acknowledge the help of several scholars who participated in the early planning, and who have published related work elsewhere: T. Cowing (1974), W. E. Diewert (1971, 1974a), R.E. Hall (1973), C.K. Liew (1976), and M. Ohta (1975).

To G. Katagiri and N. Katagiri goes the credit for careful typing and editing of the manuscript.

The editors accept responsibility for all errors not allocatable to individual contributors. Finally, we thank our wives, Beverlee and Susan, for tolerance and encouragement through the lengthy process of bringing these volumes to completion.

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INTRODUCTION

This volume contains applications of the theoretical concepts developed in Volume 1. Several of the applications are themselves theoretical in nature, e.g., the chapters by Bruno, Diewert, and Diamond–McFadden–Rodriguez. However, the majority of the chapters constitute empirical studies designed to illustrate the advantages in estimation of the dual approach to production economics.

One of the most direct applications of duality theory is to the concept of value-added. This application is developed in Part III. The fact that value-added is a function of primary inputs and the prices of outputs and intermediate inputs implies that the value-added function is a member of the class of restricted profit functions analyzed extensively by McFadden in Chapter I.1. Bruno and Diewert in Chapters III.1 and III.2 utilize the properties of restricted profit functions to develop corresponding ones for value-added functions. They also analyze the biases inherent in using single- and double-deflated value-added in place of the correct gross output in production function and factor productivity studies. Functional separability, fixed intermediate input proportions, and constant relative intermediate goods prices emerge as three possible hypotheses, which if satisfied, may lead to a justification of the use of value-added. In Chapter III.3, Denny and May test these hypotheses for Canadian manufacturing using a translog cost function specification for the underlying technology. They reject each of the three hypotheses, thus casting doubt on the validity of the extensive use of the value-added specification.

The majority of papers in Part IV (those by McFadden, Belinfante, Fuss, and Cowing) represent empirical investigations using data drawn from the same source: the fossil-fuel electricity generation industry. This very detailed data set provides pooled cross-section time-series observations on individual plants and firms permitting a correspondingly detailed investigation of the characteristics of technology. In Chapter IV.1, McFadden applies variants of the CES cost function in order to analyze and estimate substitution elasticities. Belinfante, in Chapter

IV.3, uses an approach closely related to the Divisia indexing procedure to measure the extent and bias of disembodied and embodied technical change. In Chapter IV.4, Fuss implements empirically a nested generalized Leontief cost structure developed in detail elsewhere [Fuss (1970 and 1977b)], which is a member of the class of nested models explored in Chapter II.4. This model, which provides a detailed specification of the structure of technology, is used to test the "putty–clay" hypothesis for electricity generation. This hypothesis is not rejected on the basis of the evidence presented, but a more restrictive structure which assumes fixed proportions both *ex ante* and *ex post* is rejected. Cowing applies the duality approach to an investigation of the Averch–Johnson (1962) model of regulatory behavior in Chapter IV.5. In this chapter a modified Hotelling's lemma is developed and used to generate a system of net supply functions from the quadratic approximation to an arbitrary profit function (see Chapter II.1). This derivation explicitly takes into account the rate of return constraint through an endogenously determined Lagrangian multiplier. Cowing finds that his results are in general agreement with other recent empirical investigations which indicate that the Averch–Johnson thesis has empirical relevance. However, in addition Cowing's model allows inter-firm and inter-temporal comparisons of the effects of regulation.

The chapters by McFadden, Belinfante, Fuss, and Cowing can be seen as an historical evolution in the use of a data set with common antecedents. Electricity generation data were continuously collected and revised during the period 1963–71 at the University of California, Berkeley, and later at the State University of New York at Binghamton. Original versions of these chapters appeared in 1964 (McFadden), 1969 (Belinfante), 1971 (Fuss), and 1975 (Cowing). Also part of the historical evolution are studies by Cowing (1970 and 1974) and Ohta (1975).

Chapter IV.2 by Diamond, McFadden, and Rodriguez is the one non-empirical chapter in Part IV. However, the concepts developed in this chapter have empirical relevance (and are utilized extensively in Belinfante's study). Among these concepts is the Diamond–McFadden Impossibility Theorem, now well-known from Nerlove's (1967) survey of the estimation of CES production functions. This theorem states that there is a non-identifiability of the elasticity of substitution and the bias of technical change in the absence of *a priori* hypotheses on the structure of technical change. Belinfante's study can be viewed as an example of how such a structure might be imposed.

Production characteristics have most often been estimated using

much more aggregate data than that used in Part IV. Part V presents two studies which apply dual forms to aggregate data. Denny and Pinto's study in Chapter V.1 applies the translog cost function to economy-wide Canadian data using aggregates of consumption, investment, imports, capital, and labor. They estimate a production structure which has consumption and investment as outputs, and capital, labor, and imports as inputs. Tests for separability and non-jointness indicate that while the separability hypothesis should be rejected, the non-joint hypothesis cannot be. The marginal rates of transformation between outputs are shown to be particularly sensitive to the imposition of separability or non-jointness as maintained hypotheses. In Chapter V.2, Frenger applies the generalized Leontief cost function to an analysis of the production structure implied by the Norwegian input–output tables. He tests the Leontief fixed coefficient model (which underlies most uses of input–output tables) for three industrial sectors of the Norwegian economy (textiles, construction, and metals). The two models' predictive abilities are also compared. In general, the fixed coefficient model is rejected both with respect to the hypothesis tests and the prediction comparisons. This empirical investigation is followed by an analysis of possible biases inherent in the use of inconsistent price aggregates in the estimation of separable cost functions, with specific emphasis on the effects this might have on the estimated substitution parameters.

Most of the empirical chapters in Volume 2 present estimates of production structures obtained by estimating a complete system of net supply functions with theoretical constraints such as linear homogeneity and symmetry imposed. This procedure provides many more degrees of freedom in estimation than the more common ones of estimating the production function directly or utilizing the first-order condition for a single input. As long as the implied behavioral and exogeneity of prices assumptions are reasonable, we can expect to obtain more accurate estimates of the parameters of the production structure that are of interest. The force of the dual approach to empirical production economics is the ease with which the required net supply systems are generated and interpreted. The chapters of this volume provide documentation for this assertion.