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INTRODUCTION TO THE SERIES

This series consists of a number of hitherto unpublished studies, which are introduced by the editors in the belief that they represent fresh contributions to economic science.

The term 'economic analysis' as used in the title of the series has been adopted because it covers both the activities of the theoretical economist and the research worker.

Although the analytical methods used by the various contributors are not the same, they are nevertheless conditioned by the common origin of their studies, namely theoretical problems encountered in practical research. Since for this reason, business cycle research and national accounting, research work on behalf of economic policy, and problems of planning are the main source of the subjects dealt with, they necessarily determine the manner of approach adopted by the authors. Their methods tend to be 'practical' in the sense of not being too far remote from application to actual economic conditions. In addition they are quantitative rather than qualitative.

It is the hope of the editors that the publication of these studies will help to stimulate the exchange of scientific information and to reinforce international cooperation in the field of economics.

The Editors

PREFACE

The traditional starting point of production theory is a set of physical technological possibilities, often described by a production or transformation function. The development of the theory then parallels the process of firm operation, with the firm seeking to achieve its goals subject to the limitation of its technology and of the economic environment. The results are *constructed* input demands and output supplies, expressed as functions of the technology and the economic environment.

An alternative approach to production theory is to start directly from observed economic data – supplies, demands, prices, costs, and profits. The advantage of such an attack is that the theory can be formulated directly in terms of the causal *economic* relationships that are presumed to hold, without the intervening constructive steps required in the traditional theory. Because this approach is not bound by computational tractability in the step from production technology to economic observations, the prospect is opened for more satisfactory models of complex production problems.

It would at first appear that a theory of production couched in terms of economic observables would be less fundamental than one based on the physical technology, and that one could never be sure in an economic theory of consistency with a physical model. However, the theory of *production duality* establishes that the two approaches are equivalent and equally fundamental. Using duality, the technology underlying an economic model can be reconstructed and tested for compatibility with physical laws, as necessary. Then, the main thrust of analysis can be devoted to developing the structure and relationships of observed economic variables.

The purpose of these volumes is to develop the theory of production from the standpoint of the "dual" – the relationships between economic observables which are dual to the physical technology. The spirit of our treatment is the view that the end purpose of production theory is econometric study of economic problems involving technological limitations. The volumes emphasized the empirical implications of the theory, and therefore the development of the theoretical concepts proceeds with

an eye towards the econometric framework inherent in empirical applications. We hold the view that there is an intimate, symbiotic relationship between theory and econometrics, and that development of a fully successful economic analysis of production requires an integration of theoretical and econometric ideas in a unified approach. The papers in the two volumes of *Production Economics* represent an attempt to achieve this ideal.

The theory of production duality had its beginnings in the work of Hotelling (1932), Hicks (1946), Roy (1942), and Samuelson (1947). A pioneering book by Shephard (1953) provided the first comprehensive treatment of the subject and proof of the basic duality of cost and production. Extensions of the formal theory of duality were later made by McFadden (1962), Uzawa (1964), Shephard (1970), and Diewert (1971). Many of the basic duality results were also obtained by Gorman (1970), working independently. In a paper on the estimation of returns to scale, Nerlove (1963) utilized a cost function to derive econometric estimating equations. Subsequent work by McFadden (1964), Diewert (1969a,b), Christensen, Jorgenson and Lau (1971), and others have established the use of dual cost and profit functions as a basic tool in econometric production analysis.

It is possible to trace the origins of the present volumes back to 1961 when D. McFadden worked as a research assistant to M. Nerlove and H. Uzawa at Stanford University. The contributions of Uzawa (1962, 1964), McFadden (1962, 1963), and Nerlove (1963) date from that period. The empirical implications of duality theory were developed in McFadden (1964 and 1966). The first explicit empirical application of dual flexible functional forms appeared in Diewert's (1969a) study of labor demand functions for the Canadian Department of Manpower and Immigration. The generalized Leontief function [Diewert (1971)] was introduced in that study. The subsequent generation and empirical application of flexible functional forms received their major impetus from McFadden (1966) and Diewert (1969a,b).

Applications of the basic duality concepts continued to evolve at the University of California, Berkeley, during the years 1968–1970 under the auspices of the Project for the Optimization and Evaluation of Economic Growth. The introduction of the translog function by Christensen, Jorgenson, and Lau (1971, 1973), the nested generalized Leontief form by Fuss (1970, 1977b), the hybrid generalized Leontief form by Hall (1973), and the generalized CES form by Denny (1974a) all result from research begun at that time. A. Belinfante, T. Cowing, and P. Frenger also

were associated with the Economic Growth Project at various times. M. Bruno was a visiting scholar at M.I.T., together with D. McFadden, in 1971 when his chapter was written.

The idea of collecting a group of studies in duality under a common cover grew out of a seminar series held at the Economic Growth Project during the summer of 1969. A tentative title, *An Econometric Approach to Production Theory*, was chosen at that time. A number of the papers which appear in this volume have been referenced under that title. Since that time, the contents of the volumes evolved through several additions and deletions and M. Fuss joined D. McFadden as a co-editor. We feel that the current title more accurately reflects the spirit and content of the books.

Production Economics is divided into two main parts. Volume 1 contains the basic theoretical analysis of the duality of cost, profit, and production and a number of investigations of specific functional forms. Volume 2 contains the empirical applications. In keeping with the spirit of this work, these applications draw heavily on the analysis of Volume 1. Details of the contents of both volumes can be found in the two introductions.

The editors have been unable to standardize notation throughout the volumes; however, the notation in each chapter is self-contained. In almost all cases, upper case boldface letters denote sets, lower case boldface letters denote vectors. Upper and lower case Roman and Greek letters are used variously to denote scalars and functions. Derivatives are denoted variously by subscripts (the symbol for the variable with respect to which derivatives are being taken, or the ordinal position of this variable among the arguments), primes, the ∇ operator, or the usual notation $\partial f/\partial x$.

The editors wish to acknowledge the contributions that many individuals have made to the preparation of *Production Economics*. Dale Jorgenson and Zvi Griliches have provided encouragement and ideas. A large intellectual debt is owed to K.J. Arrow, W.M. Gorman, L. Hurwicz, M. Nerlove, and H. Uzawa, whose work provided the background for most of the developments in these books. We thank the contributors, who have displayed stoic patience and goodwill in the lengthy process of refereeing and publication. We also wish to acknowledge the help of several scholars who participated in the early planning, and who have published related work elsewhere: T. Cowing (1974), W. E. Diewert (1971, 1974a), R.E. Hall (1973), C.K. Liew (1976), and M. Ohta (1975).

To G. Katagiri and N. Katagiri goes the credit for careful typing and editing of the manuscript.

The editors accept responsibility for all errors not allocatable to individual contributors. Finally, we thank our wives, Beverlee and Susan, for tolerance and encouragement through the lengthy process of bringing these volumes to completion.

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INTRODUCTION

Volume 1 develops the theory of production from the standpoint of economic observables – prices, demands, supplies, cost, and profit – utilizing duality to relate this approach to the underlying production technology. The papers of Part I set out models of production and basic duality theorems, and discuss theoretical applications of these models.

In Chapter I.1, McFadden provides an introduction to cost, revenue, and profit functions. The first twelve sections of his chapter provide a detailed description of properties of production and cost functions, duality, the geometry of cost functions, and the comparative statics of the firm using cost functions. The remainder of the chapter introduces the concept of the restricted profit function – of which cost, revenue, and total profit functions are special cases – and utilizes the mathematical theory of convex conjugate and polar reciprocal forms to deduce the properties implied for the restricted profit function by various properties on the technology, and vice versa. Of particular interest are Tables 1, 3, and 4 listing dual properties; Tables 2 and 5 listing composition rules for concave functions which can be used to construct functional forms or deduce theorems on production structure; and Tables 6 and 7 summarizing the duality mappings holding for restricted profit functions.

In Chapter I.2, Hanoch shows how formal duality theory can be used to generate new functional forms for cost and production functions. This chapter explores the use and implications of structural assumptions on technology, cost, and profit in the specification of functional forms.

In Chapter I.3, Lau applies the restricted profit function to a variety of theoretical production problems. Using the classical theory of Legendre transformations, he develops a convenient formal calculus for working with derivatives of dual production and profit functions. Lau establishes the implications for the profit function of various homotheticity and separability properties, and develops a number of specific functional forms. He considers the formulation in terms of the profit function of measures of the elasticity of technical substitution and rates of technical change. Finally, he explores the structure of production in multiple-output firms and its implications.

Part II concentrates on the development of functional forms for econometric analysis, and the interaction of functional and stochastic specification. In Chapter II.1, Fuss, McFadden, and Mundlak set out the criteria that might be used to choose among functional forms, and use these criteria to compare many of the econometric forms appearing in the literature. The issue of stochastic specification is surveyed in the context of an extended example.

In Chapter II.2, McFadden outlines a general procedure for generating linear-in-parameters functional forms, and establishes conditions under which an arbitrary restricted profit function can be approximated to the second order by a specified approximating form.

In Chapter II.3, Hanoch applies the concepts of symmetric duality and polar production functions to develop specific functional forms for the study of substitutability in multiple-factor production functions.

In Chapter II.4, Fuss and McFadden develop a nested generalized Leontief functional form for the econometric representation of an *ex ante–ex post* production structure, and suggest methods for the analysis of technological flexibility within this structure.

This volume has a series of mathematical appendices which develop some of the concepts and tools used. Appendix A.1 gives a self-contained treatment of the theory of definite quadratic forms subject to constraint. Appendix A.2 surveys necessary and sufficient conditions for the use of classical Lagrangian methods for constrained optimization. The third appendix is a survey of convex analysis – the mathematical theory of convex sets and functions. In addition to outlining the standard theory, this appendix develops new results on the behavior of polar reciprocal convex correspondences. Appendix A.4 develops methods for imposing or testing concavity on a fitted production or cost function.