Online Supplemental Appendix to "Generalized Jackknife Estimators of Weighted Average Derivatives"

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This supplemental appendix has the following four main sections:

- **Appendix A**: proofs for the Lemmas stated in the main text.
- **Appendix B**: proofs of the uniform convergence rates derived for kernel estimators.
- **Appendix C**: details on the ROT bandwidths choice derivations.
- **Appendix D**: further simulations results.

Further details on these proofs, the proofs of Theorems 1 – 3, other derivations and the simulations are available upon request from the authors.

1. Appendix A: Proofs of Lemmas

1.1. Proof of Lemma A-1. Expanding $\hat{\mathbf{s}}_n(\mathbf{x};\mathbf{H}_n)$ around $\mathbf{s}(\mathbf{x})$ and suppressing the dependence on $\mathbf{H}_n$, we have

$$\hat{\mathbf{s}}_n(\mathbf{x}) = \hat{\mathbf{s}}^{**}(\mathbf{x}) - \frac{w(\mathbf{x})}{f(\mathbf{x})^2\hat{f}_n(\mathbf{x})}\delta_n(\mathbf{x})^2\left[\dot{\boldsymbol{\delta}}_n(\mathbf{x}) + \ell(\mathbf{x})\delta_n(\mathbf{x})\right],$$

where $\delta_n(\mathbf{x}) = \hat{f}_n(\mathbf{x}) - f(\mathbf{x})$ and $\dot{\boldsymbol{\delta}}_n(\mathbf{x}) = \partial\hat{f}_n(\mathbf{x})/\partial\mathbf{x} - \partial f(\mathbf{x})/\partial\mathbf{x}$.

Because $\Delta_{0,n} = o_p(1)$ it follows from a simple bounding argument that for any $\varepsilon > 0$ there exists a constant C_ε such that, for n sufficiently large,

$$\sup_{\mathbf{x}\in\mathcal{W}} \|\hat{\mathbf{s}}_n(\mathbf{x}) - \hat{\mathbf{s}}^{**}(\mathbf{x})\| \leq C_\varepsilon\Delta_{0,n}^2\Delta_{1,n} \tag{A-1}$$

with probability no less than $1-\varepsilon$. If (A-1) holds and if $\Delta_{0,n}^2\Delta_{1,n} = o_p\left(n^{-1/2}\right)$, then

$$\left\|\hat{\boldsymbol{\theta}}_n - \hat{\boldsymbol{\theta}}_n^{**}\right\| \leq C_\varepsilon\left(n^{-1}\sum_{i=1}^n |y_i|\right)\Delta_{0,n}^2\Delta_{1,n} = o_p\left(n^{-1/2}\right),$$

where the equality uses $\mathbb{E}[|y|] < \infty$. This establishes (6) in case (i).

Next, suppose $\Delta_{0,n}\Delta_{1,n} = o_p\left(n^{-1/2}\right)$. Then, by the triangle inequality and the result for case (i),

$$\left\|\hat{\boldsymbol{\theta}}_n - \hat{\boldsymbol{\theta}}_n^{*}\right\| \leq \left\|\hat{\boldsymbol{\theta}}_n - \hat{\boldsymbol{\theta}}_n^{**}\right\| + \left\|\hat{\boldsymbol{\theta}}_n^{**} - \hat{\boldsymbol{\theta}}_n^{*}\right\| = \left\|\hat{\boldsymbol{\theta}}_n^{**} - \hat{\boldsymbol{\theta}}_n^{*}\right\| + o_p\left(n^{-1/2}\right),$$

so validity of (6) in case (ii) follows from the fact that

$$\left\|\hat{\boldsymbol{\theta}}_n^{**} - \hat{\boldsymbol{\theta}}_n^{*}\right\| \leq C\left(n^{-1}\sum_{i=1}^n |y_i|\right)\Delta_{0,n}\Delta_{1,n} = o_p\left(n^{-1/2}\right),$$

where the inequality uses the elementary bound $\sup_{\mathbf{x}\in\mathcal{W}}\|\hat{\mathbf{s}}_n^{**}(\mathbf{x}) - \hat{\mathbf{s}}^{*}(\mathbf{x})\| \leq C\Delta_{0,n}\Delta_{1,n}$, in which $C = \sup_{\mathbf{x}\in\mathcal{W}}\left[|w(\mathbf{x})|\left(1+|\ell(\mathbf{x})|\right)/f(\mathbf{x})^2\right] < \infty$. ■

1.2. Proof of Lemma A-4. Part (a) is a standard result on the bias of kernel estimators, while part (b) follows from change of variables and simple bounding arguments. For instance,

$$\begin{aligned}
&\mathbb{E}\left[F(\mathbf{z}_1)^2 K_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{x}_2)^2\|\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{x}_3)\|^2\right] \\
&= \mathbb{E}\left[\int_{\mathbb{R}^d}\int_{\mathbb{R}^d} F(\mathbf{z}_1)^2 K_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{r})^2\|\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{t})\|^2 f(\mathbf{r})\, f(\mathbf{t})\, \mathrm{d}\mathbf{t}\mathrm{d}\mathbf{r}\right] \\
&= |\mathbf{H}_n|^{-2}\operatorname{tr}\left(\mathbf{H}_n^{-2}\right)\mathbb{E}\left[\int_{\mathbb{R}^d}\int_{\mathbb{R}^d} F(\mathbf{z}_1)^2 K(\mathbf{u})^2\|\dot{\mathbf{K}}(\mathbf{v})\|^2 f(\mathbf{x}_1-\mathbf{H}_n\mathbf{u})\, f(\mathbf{x}_1-\mathbf{H}_n\mathbf{v})\, \mathrm{d}\mathbf{v}\mathrm{d}\mathbf{u}\right] \\
&\leq d\,|\mathbf{H}_n|^{-2}\lambda_{\max}\left(\mathbf{H}_n^{-2}\right) C_f^2\mathbb{E}[F(\mathbf{z})^2]\int_{\mathbb{R}^d} K(\mathbf{u})^2\mathrm{d}\mathbf{u}\int_{\mathbb{R}^d}\|\dot{\mathbf{K}}(\mathbf{v})\|^2\mathrm{d}\mathbf{v} = O(|\mathbf{H}_n|^{-2}\lambda_{\max}\left(\mathbf{H}_n^{-2}\right)),
\end{aligned}$$

where $C_f = \sup_{\mathbf{x}\in\mathbb{R}^d} f(\mathbf{x})$. ■

1.3. Proof of Lemma A-2. Defining

$$\mathbf{V}_i^{\mu} = \mathbf{V}_i - \mathbb{E}[\mathbf{V}_i] = y_i\mathbf{s}(\mathbf{x}_i) - \boldsymbol{\theta}, \qquad \mathbf{V}_i = y_i\mathbf{s}(\mathbf{x}_i),$$

$$\mathbf{V}_{ij}^{\mu}(\mathbf{H}) = \mathbf{V}_{ij}(\mathbf{H}) - \mathbb{E}[\mathbf{V}_{ij}(\mathbf{H})], \qquad \mathbf{V}_{ij}(\mathbf{H}) = -y_i\frac{w(\mathbf{x}_i)}{f(\mathbf{x}_i)}\left[\dot{\mathbf{K}}_{\mathbf{H}}(\mathbf{x}_i-\mathbf{x}_j) + \ell(\mathbf{x}_i)K_{\mathbf{H}}(\mathbf{x}_i-\mathbf{x}_j)\right],$$

we have the decomposition

$$\begin{aligned}
\hat{\boldsymbol{\theta}}_n^*(\mathbf{H}) &= n^{-1}\sum_{i=1}^n \mathbf{V}_i + n^{-2}\sum_{i=1}^n\sum_{j=1}^n \mathbf{V}_{ij}(\mathbf{H}) \\
&= \mathbb{E}\left[\hat{\boldsymbol{\theta}}_n^*(\mathbf{H})\right] + n^{-1}\sum_{i=1}^n \mathbf{V}_i^{\mu} + n^{-2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left[\mathbf{V}_{ij}^{\mu}(\mathbf{H}) + \mathbf{V}_{ji}^{\mu}(\mathbf{H})\right] + n^{-2}\sum_{i=1}^n \mathbf{V}_{ii}^{\mu}(\mathbf{H}),
\end{aligned}$$

where $n^{-2}\sum_{i=1}^n \mathbf{V}_{ii}^{\mu}(\mathbf{H}_n) = o_p\left(n^{-1/2}\right)$ because

$$\mathbb{V}\left[n^{-2}\sum_{i=1}^n \mathbf{V}_{ii}^{\mu}(\mathbf{H}_n)\right] = n^{-3}\mathbb{V}\left[\mathbf{V}_{11}(\mathbf{H}_n)\right] = n^{-1}\left(\frac{K(\mathbf{0}_d)}{n\,|\mathbf{H}_n|}\right)^2\mathbb{V}\left[y\frac{w(\mathbf{x})}{f(\mathbf{x})}\ell(\mathbf{x})\right] = o\left(n^{-1}\right).$$

The proof for $\hat{\boldsymbol{\theta}}_n^A = \hat{\boldsymbol{\theta}}_n^*(\mathbf{H}_n)$ will be completed by showing that

$$n^{-1}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left[\mathbf{V}_{ij}^{\mu}(\mathbf{H}_n)+\mathbf{V}_{ji}^{\mu}(\mathbf{H}_n)\right] = n^{-1}\sum_{i=1}^{n}\boldsymbol{\varphi}(\mathbf{z}_i) + o_p\left(n^{-1/2}\right),$$

where

$$\boldsymbol{\varphi}(z) = \boldsymbol{\psi}(z) - [y\mathbf{s}(\mathbf{x}) - \boldsymbol{\theta}] = \frac{\partial}{\partial \mathbf{x}}[w(\mathbf{x})g(\mathbf{x})] - w(\mathbf{x})g(\mathbf{x})\ell(\mathbf{x}).$$

To do so, let $\mathbb{E}_i$ denote conditional expectation given $\mathbf{z}_i$ and for any positive sequence $\{r_n\}$, let $\mathbf{X}_n = O_2(r_n)$ and $\mathbf{X}_n = o_2(r_n)$ be shorthand for $\overline{\lim}_{n\to\infty}\mathbb{E}[\|\mathbf{X}_n\|^2]/r_n^2 < \infty$ and $\lim_{n\to\infty}\mathbb{E}[\|\mathbf{X}_n\|^2]/r_n^2 = 0$, respectively.

Because $\lambda_{\max}(\mathbf{H}_n) \to 0$ and $n\,|\mathbf{H}_n|\,\lambda_{\min}(\mathbf{H}_n^2) \to \infty$,

$$\begin{aligned}\mathbf{V}_{ij}(\mathbf{H}_n) &= -y_i\frac{w(\mathbf{x}_i)}{f(\mathbf{x}_i)}\left[\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{x}_i-\mathbf{x}_j) + \ell(\mathbf{x}_i)K_{\mathbf{H}_n}(\mathbf{x}_i-\mathbf{x}_j)\right] \\ &= O_2\left(1/\sqrt{|\mathbf{H}_n|\,\lambda_{\min}(\mathbf{H}_n^2)}\right) = o_2\left(\sqrt{n}\right),\end{aligned}$$

where the second equality uses Lemma A-4 (b). Therefore, by the projection theorem for variable U-statistics (e.g., Powell, Stock, and Stoker (1989, Lemma 3.1)),

$$n^{-2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left[\mathbf{V}_{ij}^{\mu}(\mathbf{H}_n)+\mathbf{V}_{ji}^{\mu}(\mathbf{H}_n)\right] = n^{-1}\sum_{i=1}^{n}\mathbb{E}_i\left[\mathbf{V}_{ij}^{\mu}(\mathbf{H}_n)+\mathbf{V}_{ji}^{\mu}(\mathbf{H}_n)\right] + o_p\left(n^{-1/2}\right),$$

where, by Lemma A-4 (a),

$$\mathbb{E}_i[\mathbf{V}_{ij}(\mathbf{H}_n)] = -y_i\frac{w(\mathbf{x}_i)}{f(\mathbf{x}_i)}\left[\dot{\mathbf{b}}(\mathbf{x}_i;\mathbf{H}_n) + \ell(\mathbf{x}_i)b(\mathbf{x}_i;\mathbf{H}_n)\right] = O_2\left(\lambda_{\max}\left(\mathbf{H}_n^P\right)\right) = o_2(1)$$

and, using integration by parts and change of variables,

$$\begin{aligned}
\mathbb{E}_i[\mathbf{V}_{ji}(\mathbf{H}_n)] &= -\int_{\mathbb{R}^d} g(\mathbf{r})w(\mathbf{r})\left[\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{r}-\mathbf{x}_i)+\ell(\mathbf{r})K_{\mathbf{H}_n}(\mathbf{r}-\mathbf{x}_i)\right]\mathrm{d}\mathbf{r} \\
&= \int_{\mathbb{R}^d}\left(\frac{\partial}{\partial \mathbf{r}}\left[g(\mathbf{r})w(\mathbf{r})\right]\right)K_{\mathbf{H}_n}(\mathbf{r}-\mathbf{x}_i)\mathrm{d}\mathbf{r}-\int_{\mathbb{R}^d} g(\mathbf{r})w(\mathbf{r})\ell(\mathbf{r})K_{\mathbf{H}_n}(\mathbf{r}-\mathbf{x}_i)\mathrm{d}\mathbf{r} \\
&= \int_{\mathbb{R}^d}\frac{\partial}{\partial \mathbf{x}}\left[g(\mathbf{x}_i+\mathbf{H}_n\mathbf{t})w\left(\mathbf{x}_i+\mathbf{H}_n\mathbf{t}\right)\right]K(\mathbf{t})\mathrm{d}\mathbf{t} \\
&\qquad -\int_{\mathbb{R}^d} g(\mathbf{x}_i+\mathbf{H}_n\mathbf{t})w(\mathbf{x}_i+\mathbf{H}_n\mathbf{t})\ell(\mathbf{x}_i+\mathbf{H}_n\mathbf{t})K(\mathbf{t})\mathrm{d}\mathbf{t} \\
&= \boldsymbol{\varphi}(\mathbf{z}_i)+o_2(1).
\end{aligned}$$

Using these results and the fact that $\mathbb{E}[\boldsymbol{\varphi}(\mathbf{z})]=0$ it is easy to show that

$$n^{-1}\sum_{i=1}^{n}\mathbb{E}_i\left[\mathbf{V}_{ij}^{\mu}(\mathbf{H}_n)+\mathbf{V}_{ji}^{\mu}(\mathbf{H}_n)\right]=n^{-1}\sum_{i=1}^{n}\boldsymbol{\varphi}(\mathbf{z}_i)+o_p\left(n^{-1/2}\right),$$

completing the proof for $\hat{\boldsymbol{\theta}}_n^A=\hat{\boldsymbol{\theta}}_n^*(\mathbf{H}_n)$.

Having established the result for $\hat{\boldsymbol{\theta}}_n^A=\hat{\boldsymbol{\theta}}_n^*(\mathbf{H}_n)$ the result for $\hat{\boldsymbol{\theta}}_n^A=\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)$ will follow if it can be shown that $\mathbb{V}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)-\hat{\boldsymbol{\theta}}_n^{*}(\mathbf{H}_n)]=o\left(n^{-1}\right)$. To do so, we employ the decomposition

$$\begin{aligned}
\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H})-\hat{\boldsymbol{\theta}}_n^{*}(\mathbf{H}) &= n^{-3}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\mathbf{V}_{ijk}(\mathbf{H}) \\
&= \mathbb{E}\left[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H})-\hat{\boldsymbol{\theta}}_n^{*}(\mathbf{H})\right]+n^{-3}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\mathbf{V}_{ijk}^{\mu}(\mathbf{H}),
\end{aligned}$$

where

$$\mathbf{V}_{ijk}(\mathbf{H})=y_i\frac{w(\mathbf{x}_i)}{f(\mathbf{x}_i)^2}\left[K_{\mathbf{H}}(\mathbf{x}_i-\mathbf{x}_j)-f(\mathbf{x}_i)\right]\left[\dot{\mathbf{K}}_{\mathbf{H}}(\mathbf{x}_i-\mathbf{x}_k)+\ell(\mathbf{x}_i)K_{\mathbf{H}}(\mathbf{x}_i-\mathbf{x}_k)\right],$$

and $\mathbf{V}_{ijk}^{\mu}(\mathbf{H})=\mathbf{V}_{ijk}(\mathbf{H})-\mathbb{E}[\mathbf{V}_{ijk}(\mathbf{H})]$.

The Hoeffding decomposition yields

$$\mathbb{V}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\mathbf{V}_{ijk}^{\mu}(\mathbf{H})\right]=\sum_{p=1}^{3}\binom{n}{p}\mathbb{V}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\mathbf{W}_{ijk}(p;\mathbf{H})\right],$$

where

$$\mathbf{W}_{ijk}(1;\mathbf{H})=\mathbb{E}_1[\mathbf{V}_{ijk}(\mathbf{H})]-\mathbb{E}[\mathbf{V}_{ijk}(\mathbf{H})],$$

$$\mathbf{W}_{ijk}(2;\mathbf{H})=\mathbb{E}_{1,2}[\mathbf{V}_{ijk}(\mathbf{H})]-\mathbb{E}_1[\mathbf{V}_{ijk}(\mathbf{H})]-\mathbb{E}_2[\mathbf{V}_{ijk}(\mathbf{H})]+\mathbb{E}[\mathbf{V}_{ijk}(\mathbf{H})],$$

$$\begin{aligned}\mathbf{W}_{ij_1j_2}(3;\mathbf{H}) &= \mathbb{E}_{1,2,3}[\mathbf{V}_{ijk}(\mathbf{H})]-\mathbb{E}_{1,2}[\mathbf{V}_{ijk}(\mathbf{H})]-\mathbb{E}_{1,3}[\mathbf{V}_{ijk}(\mathbf{H})]-\mathbb{E}_{2,3}[\mathbf{V}_{ijk}(\mathbf{H})]\\ &\quad +\mathbb{E}_1[\mathbf{V}_{ijk}(\mathbf{H})]+\mathbb{E}_2[\mathbf{V}_{ijk}(\mathbf{H})]+\mathbb{E}_3[\mathbf{V}_{ijk}(\mathbf{H})]-\mathbb{E}[\mathbf{V}_{ijk}(\mathbf{H})],\end{aligned}$$

with $\mathbb{E}_{1,2,3}[\mathbf{V}_{ijk}(\mathbf{H})]=\mathbb{E}[\mathbf{V}_{ijk}(\mathbf{H})|z_1,z_2,z_3]$, $\mathbb{E}_{2,3}[\mathbf{V}_{ijk}(\mathbf{H})]=\mathbb{E}[\mathbf{V}_{ijk}(\mathbf{H})|z_2,z_3]$, and so on. It therefore suffices to show that

$$\mathbb{V}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\mathbf{W}_{ijk}(p;\mathbf{H}_n)\right]=o\left(n^{5-p}\right),\qquad p\in\{1,2,3\}. \tag{A-2}$$

The proof of (A-2) for $p=1$ will be based on the relation

$$\mathbb{V}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\mathbf{W}_{ijk}(1;\mathbf{H})\right]=\mathbb{V}[\mathcal{W}_n(1;\mathbf{H})],$$

where

$$\begin{aligned}\mathcal{W}_n(1;\mathbf{H}) &= \mathbf{W}_{111}(1;\mathbf{H})+(n-1)\left[\mathbf{W}_{112}(1;\mathbf{H})+\mathbf{W}_{121}(1;\mathbf{H})+\mathbf{W}_{211}(1;\mathbf{H})\right]\\ &\quad +(n-1)\left[\mathbf{W}_{122}(1;\mathbf{H})+\mathbf{W}_{212}(1;\mathbf{H})+\mathbf{W}_{221}(1;\mathbf{H})\right]\\ &\quad +(n-1)(n-2)\left[\mathbf{W}_{123}(1;\mathbf{H})+\mathbf{W}_{213}(1;\mathbf{H})+\mathbf{W}_{231}(1;\mathbf{H})\right].\end{aligned}$$

Because $\mathbb{V}[\mathbf{W}_{ijk}(1;\mathbf{H})] \leq \mathbb{V}[\mathbb{E}_1[\mathbf{V}_{ijk}(\mathbf{H})]]$ for each $\{i,j,k\}$, the result $\mathbb{V}[\mathcal{W}_n(1;\mathbf{H}_n)] = o\left(n^4\right)$ can be established by means of polynomial (in n) bound on the second moment of each $\mathbb{E}_1[\mathbf{V}_{ijk}(\mathbf{H}_n)]$.

First,

$$\begin{aligned}\mathbb{E}_1[\mathbf{V}_{111}(\mathbf{H}_n)] &= y_1\frac{w(\mathbf{x}_1)}{f(\mathbf{x}_1)^2}\left[K_{\mathbf{H}_n}(\mathbf{0}_d) - f(\mathbf{x}_1)\right]\ell(\mathbf{x}_1)K_{\mathbf{H}_n}(\mathbf{0}_d) \\ &= \left|\mathbf{H}_n\right|^{-2}K(\mathbf{0}_d)^2 y_1\frac{w(\mathbf{x}_1)}{f(\mathbf{x}_1)^2}\ell(\mathbf{x}_1) - \left|\mathbf{H}_n\right|^{-1}K(\mathbf{0}_d)y_1\frac{w(\mathbf{x}_1)}{f(\mathbf{x}_1)^2}f(\mathbf{x}_1)\ell(\mathbf{x}_1) \\ &= O_2\left(\left|\mathbf{H}_n\right|^{-2}\right) = o_2\left(n^2\right).\end{aligned}$$

Next, using Lemma A-4 (a), change of variables, and simple bounding arguments,

$$\begin{aligned}\mathbb{E}_1[\mathbf{V}_{112}(\mathbf{H}_n)] &= y_1\frac{w(\mathbf{x}_1)}{f(\mathbf{x}_1)^2}K_{\mathbf{H}_n}(\mathbf{0}_d)\int_{\mathbb{R}^d}\left[\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{r}) + \ell(\mathbf{x}_1)K_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{r})\right]f(\mathbf{r})\mathrm{d}\mathbf{r} \\ &\quad -y_1\frac{w(\mathbf{x}_1)}{f(\mathbf{x}_1)^2}f(\mathbf{x}_1)\int_{\mathbb{R}^d}\left[\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{r}) + \ell(\mathbf{x}_1)K_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{r})\right]f(\mathbf{r})\mathrm{d}\mathbf{r} \\ &= y_1\frac{w(\mathbf{x}_1)}{f(\mathbf{x}_1)^2}\left[\left|\mathbf{H}_n\right|^{-1}K(\mathbf{0}_d) - f(\mathbf{x}_1)\right]\left[\dot{\mathbf{b}}(\mathbf{x}_1;\mathbf{H}_n) + \ell(\mathbf{x}_1)b(\mathbf{x}_1;\mathbf{H}_n)\right] \\ &= O_2\left(\left|\mathbf{H}_n\right|^{-1}\lambda_{\max}\left(\mathbf{H}_n^P\right)\right) = o_2\left(n\right).\end{aligned}$$

Similarly, it can be shown that

$$\mathbb{E}_1[\mathbf{V}_{121}(\mathbf{H}_n)] = O_2\left(\left|\mathbf{H}_n\right|^{-1}\lambda_{\max}\left(\mathbf{H}_n^P\right)\right) = o_2\left(n\right),$$

$$\mathbb{E}_1[\mathbf{V}_{211}(\mathbf{H}_n)] = O_2\left(\left|\mathbf{H}_n\right|^{-1}\lambda_{\max}\left(\mathbf{H}_n^{-1}\right)\right) = o_2\left(n\right),$$

$$\mathbb{E}_1[\mathbf{V}_{122}(\mathbf{H}_n)] = O_2\left(\left|\mathbf{H}_n\right|^{-1}\lambda_{\max}\left(\mathbf{H}_n^{-1}\right)\right) = o_2\left(n\right),$$

$$\mathbb{E}_1[\mathbf{V}_{212}(\mathbf{H}_n)] = O_2\left(\left|\mathbf{H}_n\right|^{-1}\right) = o_2\left(n\right),$$

$$\mathbb{E}_1[\mathbf{V}_{221}(\mathbf{H}_n)] = O_2\left(\left|\mathbf{H}_n\right|^{-1}\lambda_{\max}\left(\mathbf{H}_n^{-1}\right)\right) = o_2\left(n\right),$$

$$\mathbb{E}_1[\mathbf{V}_{123}(\mathbf{H}_n)] = O_2\left(\lambda_{\max}\left(\mathbf{H}_n^{2P}\right)\right) = o_2\left(1\right),$$

$$\mathbb{E}_1[\mathbf{V}_{213}(\mathbf{H}_n)] = O_2\left(\lambda_{\max}\left(\mathbf{H}_n^{2P}\right)\right) = o_2\left(1\right),$$

$$\mathbb{E}_1[\mathbf{V}_{231}(\mathbf{H}_n)] = O_2\left(\lambda_{\max}\left(\mathbf{H}_n^{P}\right)\lambda_{\max}\left(\mathbf{H}_n^{-1}\right)\right) = o_2\left(1\right),$$

from which (A-2) follows for $p = 1$.

The proofs of (A-2) are very similar for $p = 2$ and $p = 3$, so we give only the proof for $p = 3$, which is based on the relation

$$\mathbb{V}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\mathbf{W}_{ijk}(3;\mathbf{H})\right] = \mathbb{V}\left[\mathcal{W}_n(3;\mathbf{H})\right],$$

where

$$\mathcal{W}_n(3;\mathbf{H}) = \mathbf{W}_{123}(3;\mathbf{H})+\mathbf{W}_{132}(3;\mathbf{H})+\mathbf{W}_{213}(3;\mathbf{H})+\mathbf{W}_{231}(3;\mathbf{H})+\mathbf{W}_{312}(3;\mathbf{H})+\mathbf{W}_{321}(3;\mathbf{H})$$

and $\mathbb{V}\left[\mathbf{W}_{ijk}(3;\mathbf{H})\right] \leq \mathbb{V}[\mathbb{E}_{1,2,3}[\mathbf{V}_{ijk}(\mathbf{H})]]$ for each $\{i,j,k\}$.

Using Lemma A-4 (c) and $\mathbb{E}_{1,2,3}[\mathbf{V}_{123}(\mathbf{H}_n)] = \mathbf{V}_{123}(\mathbf{H}_n)$, with

$$\begin{aligned}\mathbf{V}_{123}(\mathbf{H}_n) &= y_1\frac{w(\mathbf{x}_1)}{f(\mathbf{x}_1)^2}\left[K_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{x}_2)-f(\mathbf{x}_1)\right]\left[\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{x}_3)+\ell(\mathbf{x}_1)K_{\mathbf{H}_n}(\mathbf{x}_1-\mathbf{x}_3)\right]\\ &= O_2\left(\left|\mathbf{H}_n\right|^{-1}\lambda_{\max}\left(\mathbf{H}_n^{-1}\right)\right) = o_2\left(n\right).\end{aligned}$$

The result $\mathbb{V}[\mathcal{W}_n(3;\mathbf{H}_n)] = o\left(n^2\right)$ follows from this and the fact that $\mathbf{W}_{123}(3;\mathbf{H})$, $\mathbf{W}_{132}(3;\mathbf{H})$, $\mathbf{W}_{213}(3;\mathbf{H})$, $\mathbf{W}_{231}(3;\mathbf{H})$, $\mathbf{W}_{312}(3;\mathbf{H})$, and $\mathbf{W}_{321}(3;\mathbf{H})$ are identically distributed. ■

1.4. Proof of Lemma A-3. Using the same notation as in the proof of Lemma A-2, we have

$$\begin{aligned}\mathbb{E}\left[\hat{\boldsymbol{\theta}}_n^*(\mathbf{H})\right] &= n^{-1}\sum_{i=1}^{n}\mathbb{E}[\mathbf{V}_i] + n^{-2}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathbb{E}[\mathbf{V}_{ij}(\mathbf{H})] \\ &= \mathbb{E}[\mathbf{V}_1] + n^{-1}\mathbb{E}[\mathbf{V}_{11}(\mathbf{H})] + \left(1-n^{-1}\right)\mathbb{E}[\mathbf{V}_{12}(\mathbf{H})],\end{aligned}$$

where $\mathbb{E}[\mathbf{V}_1] = \boldsymbol{\theta}$, $\mathbb{E}[\mathbf{V}_{11}(\mathbf{H})] = |\mathbf{H}|^{-1}\mathcal{B}_0^*$, and, using Lemma A-4 (a),

$$\begin{aligned}\mathbb{E}[\mathbf{V}_{12}(\mathbf{H}_n)] &= -\int_{\mathbb{R}^d} g(\mathbf{r})w(\mathbf{r})\left[\dot{\mathbf{b}}(\mathbf{r};\mathbf{H}_n) + \ell(\mathbf{r})b(\mathbf{r};\mathbf{H}_n)\right]\mathrm{d}\mathbf{r} \\ &= \mathcal{S}\left(\mathbf{H}_n\right) + o\left(\lambda_{\max}\left(\mathbf{H}_n^P\right)\right).\end{aligned}$$

This gives the first result in the Lemma. The proof of the second result is based on the expansion

$$\begin{aligned}&\mathbb{E}\left[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n) - \hat{\boldsymbol{\theta}}_n^*(\mathbf{H}_n)\right] \\ &= n^{-3}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\mathbb{E}[\mathbf{V}_{ijk}(\mathbf{H}_n)] \\ &= n^{-2}\mathbb{E}[\mathbf{V}_{111}(\mathbf{H}_n)] + n^{-1}\left(1-n^{-1}\right)\left(\mathbb{E}[\mathbf{V}_{112}(\mathbf{H}_n)] + \mathbb{E}[\mathbf{V}_{121}(\mathbf{H}_n)]\right) \\ &\qquad + n^{-1}\left(1-n^{-1}\right)\mathbb{E}[\mathbf{V}_{122}(\mathbf{H}_n)] + \left(1-n^{-1}\right)\left(1-2n^{-1}\right)\mathbb{E}[\mathbf{V}_{123}(\mathbf{H}_n)] \\ &= n^{-1}\left(1-n^{-1}\right)\mathbb{E}[\mathbf{V}_{122}(\mathbf{H}_n)] + O\left(n^{-2}|\mathbf{H}_n|^{-2} + \lambda_{\max}\left(\mathbf{H}_n^{2P}\right)\right),\end{aligned}$$

where the last equality uses Lemma A-4 (a) and simple bounding arguments to show that

$$\mathbb{E}[\mathbf{V}_{111}(\mathbf{H}_n)] = O\left(|\mathbf{H}_n|^{-2}\right), \qquad \mathbb{E}[\mathbf{V}_{112}(\mathbf{H}_n)] = O\left(|\mathbf{H}_n|^{-1}\lambda_{\max}\left(\mathbf{H}_n^P\right)\right),$$

and

$$\mathbb{E}[\mathbf{V}_{121}(\mathbf{h}_n)] = O\left(|\mathbf{H}_n|^{-1}\lambda_{\max}\left(\mathbf{H}_n^P\right)\right), \qquad \mathbb{E}[\mathbf{V}_{123}(\mathbf{h}_n)] = O\left(\lambda_{\max}\left(\mathbf{H}_n^{2P}\right)\right).$$

Now,

$$\begin{aligned}
\mathbb{E}[\mathbf{V}_{122}(\mathbf{H}_n)] &= \int_{\mathbb{R}^d}\int_{\mathbb{R}^d} g(\mathbf{r})\frac{w(\mathbf{r})}{f(\mathbf{r})^2}K_{\mathbf{H}_n}(\mathbf{r}-\mathbf{s})\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{r}-\mathbf{s})f(\mathbf{r})f(\mathbf{s})\mathrm{d}\mathbf{s}\mathrm{d}\mathbf{r} \\
&\quad + \int_{\mathbb{R}^d}\int_{\mathbb{R}^d} g(\mathbf{r})\frac{w(\mathbf{r})}{f(\mathbf{r})^2}\ell(\mathbf{r})K_{\mathbf{H}_n}(\mathbf{r}-\mathbf{s})^2 f(\mathbf{r})f(\mathbf{s})\mathrm{d}\mathbf{s}\mathrm{d}\mathbf{r} \\
&\quad - \int_{\mathbb{R}^d}\int_{\mathbb{R}^d} g(\mathbf{r})\frac{w(\mathbf{r})}{f(\mathbf{r})}\left[\dot{\mathbf{K}}_{\mathbf{H}_n}(\mathbf{r}-\mathbf{s}) + \ell(\mathbf{r})K_{\mathbf{H}_n}(\mathbf{r}-\mathbf{s})\right] f(\mathbf{r})f(\mathbf{s})\mathrm{d}\mathbf{s}\mathrm{d}\mathbf{r} \\
&= |\mathbf{H}_n|^{-1}\mathbf{H}_n^{-1}\int_{\mathbb{R}^d} g(\mathbf{r})\frac{w(\mathbf{r})}{f(\mathbf{r})}\left[\int_{\mathbb{R}^d} K(\mathbf{t})\dot{\mathbf{K}}(\mathbf{t})f(\mathbf{r}-\mathbf{H}_n\mathbf{t})\mathrm{d}\mathbf{t}\right]\mathrm{d}\mathbf{r} \\
&\quad + |\mathbf{H}_n|^{-1}\int_{\mathbb{R}^d} g(\mathbf{r})\frac{w(\mathbf{r})}{f(\mathbf{r})}\ell(\mathbf{r})\left[\int_{\mathbb{R}^d} K(\mathbf{t})^2 f(\mathbf{r}-\mathbf{H}_n\mathbf{t})\mathrm{d}\mathbf{t}\right]\mathrm{d}\mathbf{r} \\
&\quad + O\left(|\mathbf{H}_n|^{-1}\lambda_{\max}\left(\mathbf{H}_n^P\right)\right),
\end{aligned}$$

where

$$\begin{aligned}
&\int_{\mathbb{R}^d} g(\mathbf{r})\frac{w(\mathbf{r})}{f(\mathbf{r})}\left[\int_{\mathbb{R}^d} K(\mathbf{t})\dot{\mathbf{K}}(\mathbf{t})f(\mathbf{r}-\mathbf{H}_n\mathbf{t})\mathrm{d}\mathbf{t}\right]\mathrm{d}\mathbf{r} \\
&= \sum_{j=0}^{P}(-1)^{j+1}\sum_{\mathbf{l}\in\mathbb{Z}_+^d(j)}\frac{\mathbf{h}_n^{\mathbf{l}}}{\mathbf{l}!}\dot{B}_z(\mathbf{l})\dot{\mathbf{B}}_K(\mathbf{l}) + o(\lambda_{\max}\left(\mathbf{H}_n^P\right)),
\end{aligned}$$

$$\begin{aligned}
&\int_{\mathbb{R}^d} g(\mathbf{r})\frac{w(\mathbf{r})}{f(\mathbf{r})}\ell(\mathbf{r})\left[\int_{\mathbb{R}^d} K(\mathbf{t})^2 f(\mathbf{r}-\mathbf{H}_n\mathbf{t})\mathrm{d}\mathbf{t}\right]\mathrm{d}\mathbf{r} \\
&= \sum_{j=0}^{P}(-1)^{j}\sum_{\mathbf{l}\in\mathbb{Z}_+^d(j)}\frac{\mathbf{h}_n^{\mathbf{l}}}{\mathbf{l}!}\mathbf{B}_z(\mathbf{l})B_K(\mathbf{l}) + o\left(\lambda_{\max}\left(\mathbf{H}_n^P\right)\right).
\end{aligned}$$

Finally, because K is even, $B_K(\mathbf{l}) = 0$ whenever $\mathbf{l} \in \mathbb{Z}_+^d(j)$ for j odd, and $\dot{\mathbf{B}}_K(\mathbf{l}) = 0$

whenever $\mathbf{l} \in \mathbb{Z}_+^d(j)$ for j even. As a consequence,

$$\begin{aligned}
|\mathbf{H}_n|\,\mathbb{E}[\mathbf{V}_{122}(\mathbf{H}_n)] &= \sum_{j=0}^{P}(-1)^{j+1}\sum_{\mathbf{l}\in\mathbb{Z}_+^d(j)}\frac{\mathbf{h}_n^{\mathbf{l}}}{\mathbf{l}!}\dot{B}_z(\mathbf{l})\dot{\mathbf{B}}_K(\mathbf{l})\mathbf{H}_n^{-1}+\sum_{j=0}^{P}(-1)^{j}\sum_{\mathbf{l}\in\mathbb{Z}_+^d(j)}\frac{\mathbf{h}_n^{\mathbf{l}}}{\mathbf{l}!}\mathbf{B}_z(\mathbf{l})B_K(\mathbf{l}) \\
&\quad +O(\lambda_{\max}\left(\mathbf{H}_n^P\right)) \\
&= \sum_{j=0}^{\lfloor (P-1)/2\rfloor}\left\{\sum_{\mathbf{l}\in\mathbb{Z}_+^d(2j+1)}\frac{\mathbf{h}_n^{\mathbf{l}}}{\mathbf{l}!}\dot{B}_z(\mathbf{l})\dot{\mathbf{B}}_K(\mathbf{l})\mathbf{H}_n^{-1}+\sum_{\mathbf{l}\in\mathbb{Z}_+^d(2j)}\frac{\mathbf{h}_n^{\mathbf{l}}}{\mathbf{l}!}\mathbf{B}_z(\mathbf{l})B_K(\mathbf{l})\right\} \\
&\quad +O(\lambda_{\max}\left(\mathbf{H}_n^P\right)),
\end{aligned}$$

which gives the result. ■

2. APPENDIX B: UNIFORM CONVERGENCE RATES FOR KERNEL ESTIMATORS

2.1. Proof of Lemma B-1. Similarly to the proof of Newey (1994b, Lemma B.1), the proof consists of three steps, of which the first step is a truncation step, the second step is a discretization step, and the final step uses Bernstein's inequality to bound certain tail probabilities. To accommodate kernels with unbounded support, the second step borrows ideas from Hansen (2008). In the third step, we use Bernstein's inequality in two distinct ways (and employ a subsequence argument) in order to accommodate bandwidths that do not satisfy $n^{1-2/s}\,|\mathbf{H}_n|\,/\log n \to \infty$.

Given a sequence τ_n, let

$$\tilde{\Psi}_n(\mathbf{x}) = \frac{1}{n}\sum_{j=1}^{n}\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_j)Y_{jn}, \qquad Y_{jn} = Y_j\mathbf{1}\left(|Y_j| \le \tau_n\right),$$

denote a version of $\hat{\Psi}_n$ obtained by replacing Y_j with the truncated variable Y_{jn}. The processes $\hat{\Psi}_n(\cdot)$ and $\tilde{\Psi}_n(\cdot)$ coincide with a probability that can be made arbitrarily close to one (uniformly in n) by setting $\tau_n = C_\tau n^{1/s}$ for some large C_τ because

$$\begin{aligned}\mathbb{P}\left[\hat{\Psi}_n(\cdot) \neq \tilde{\Psi}_n(\cdot)\right] &\leq \mathbb{P}[Y_j \neq Y_{jn} \text{ for some } j] = \mathbb{P}[|Y_j| > \tau_n \text{ for some } j] \\ &\leq n\mathbb{P}[|Y| > \tau_n] \leq n\tau_n^{-s} C_Y(s),\end{aligned}$$

where $C_Y(s) = \mathbb{E}[|Y|^r] + \sup_{\mathbf{x}\in\mathbb{R}^d} \mathbb{E}[|Y|^r|\mathbf{X} = \mathbf{x}] f_{\mathbf{X}}(\mathbf{x})$ and the last inequality uses Markov's inequality. Also,

$$\begin{aligned}&\left|\mathbb{E}\left[\hat{\Psi}_n(\mathbf{x}) - \tilde{\Psi}_n(\mathbf{x})\right]\right| \\ &= \left|\mathbb{E}\left[Y\mathbf{1}\left(|Y| > \tau_n\right)\kappa_{\mathbf{H}_n}(\mathbf{x} - \mathbf{X})\right]\right| \\ &= \left|\int_{\mathbb{R}^d} \mathbb{E}\left[Y\mathbf{1}\left(|Y| > \tau_n\right)|\mathbf{X} = \mathbf{r}\right]\kappa_{\mathbf{H}_n}(\mathbf{x} - \mathbf{r}) f_{\mathbf{X}}\left(\mathbf{r}\right) \mathrm{d}\mathbf{r}\right| \\ &\leq \tau_n^{-(s-1)} \int_{\mathbb{R}^d} \mathbb{E}\left[|Y|^s \mathbf{1}\left(|Y| > \tau_n\right)|\mathbf{X} = \mathbf{r}\right]\left|\kappa_{\mathbf{H}_n}(\mathbf{x} - \mathbf{r})\right| f_{\mathbf{X}}\left(\mathbf{r}\right) \mathrm{d}\mathbf{r} \\ &\leq \tau_n^{-(s-1)} C_Y(s) C_\kappa, \qquad\qquad C_\kappa = \sup_{\mathbf{u}\in\mathbb{R}^d} |\kappa(\mathbf{u})| + \int_{\mathbb{R}^d} |\kappa(\mathbf{u})| \,\mathrm{d}u,\end{aligned}$$

so if $\tau_n = C_\tau n^{1/s}$, then

$$\sup_{\mathbf{x}\in\mathbb{R}^d} \left|\mathbb{E}[\hat{\Psi}_n(\mathbf{x})] - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right| = O\left(n^{1/s-1}\right) = o\left(\rho_n\right).$$

To complete the proof, it therefore suffices to show that

$$\sup_{\mathbf{x}\in\mathcal{X}_n} \left|\tilde{\Psi}_n(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right| = O_p\left(\rho_n\right), \qquad \tau_n = C_\tau n^{1/s}.$$

Remark. Hansen (2008, p. 740) employs $\tau_n = \rho_n^{-1/(s-1)} = o\left(n^{1/s}\right)$ in his truncation argument and shows that with this choice of τ_n

$$\left|\left(\tilde{\Psi}_n(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right) - \left(\hat{\Psi}_n(\mathbf{x}) - \mathbb{E}[\hat{\Psi}_n(\mathbf{x})]\right)\right| = O_p\left(\rho_n\right)$$

for every $\mathbf{x}$. It is unclear whether this pointwise rate of convergence holds uniformly in $\mathbf{x} \in \mathcal{X}_n$, so we err on the side of caution and set $\tau_n = C_\tau n^{1/s}$.

Continuing with the proof of Lemma B-1, we discretize by employing a sequence G_n (depending on $C_{X,n}$ and $\mathbf{h}_n$) and associated points $\{\mathbf{x}^*_{g,n} : j = 1, \cdots, G_n\}$ such that

$$\overline{\lim}_{n\to\infty} \frac{\log\left(G_n\right)}{\log n} < \infty \tag{B-1}$$

and

$$\mathcal{X}_n \subseteq \cup_{g=1}^{G_n} \mathcal{X}_{g,n}, \qquad \mathcal{X}_{g,n} = \{\mathbf{x} : \|\mathbf{x} - \mathbf{x}^*_{g,n}\| \leq \rho_n \lambda_{\min}\left(\mathbf{H}_n\right)\}. \tag{B-2}$$

It follows from (B-1) that $G_n = o\left(n^R\right)$ for some $R < \infty$, while (B-2) implies that, for any M,

$$\Pr\left[\sup_{\mathbf{x}\in\mathcal{X}_n} \left|\tilde{\Psi}_n(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right| > M\rho_n\right] \leq G_n \max_{1\leq g\leq G_n} \mathbb{P}\left[\sup_{\mathbf{x}\in\mathcal{X}_{g,n}} \left|\tilde{\Psi}_n(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right| > M\rho_n\right].$$

To complete the proof it therefore suffices to show that for any $R < \infty$, there is an M such that

$$\max_{1\leq g\leq G_n} \Pr\left[\sup_{\mathbf{x}\in\mathcal{X}_{g,n}} \left|\tilde{\Psi}_n(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right| > M\rho_n\right] = O\left(n^{-R}\right). \tag{B-3}$$

If $\mathbf{x} \in \mathcal{X}_{g,n}$ and $\rho_n \leq \delta_\kappa$, then for $\kappa^*_{\mathbf{H}}(\mathbf{x}) = |\mathbf{H}|^{-1}\kappa^*(\mathbf{H}^{-1}\mathbf{x})$,

$$\left|\kappa_{\mathbf{H}_n}(\mathbf{x} - \mathbf{X}_j) - \kappa_{\mathbf{H}_n}(\mathbf{x}^*_{g,n} - \mathbf{X}_j)\right| \leq \rho_n \kappa^*_{\mathbf{H}_n}(\mathbf{x}^*_{g,n} - \mathbf{X}_j), \qquad j = 1, \ldots, n,$$

so

$$\left|\tilde{\Psi}_n(\mathbf{x}) - \tilde{\Psi}_n(\mathbf{x}^*_{g,n})\right| \leq \rho_n \tilde{\Psi}^*_n(\mathbf{x}^*_{g,n}), \qquad \tilde{\Psi}^*_n(\mathbf{x}) = \frac{1}{n}\sum_{j=1}^{n} Y_{jn}\kappa^*_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_j).$$

Therefore, if $\rho_n \leq \delta_\kappa$, then

$$\begin{aligned}\sup_{\mathbf{x}\in\mathcal{X}_{g,n}} \left|\tilde{\Psi}_n(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right| \leq & \left|\tilde{\Psi}_n(\mathbf{x}^*_{g,n}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x}^*_{g,n})]\right| \\ & +\rho_n \left|\tilde{\Psi}^*_n(\mathbf{x}^*_{g,n}) - \mathbb{E}[\tilde{\Psi}^*_n(\mathbf{x}^*_{g,n})]\right| \\ & +2\rho_n \mathbb{E}\left[|\tilde{\Psi}^*_n(\mathbf{x}^*_{g,n})|\right],\end{aligned}$$

where

$$\begin{aligned}\mathbb{E}\left[|\tilde{\Psi}^*_n(\mathbf{x}^*_{g,n})|\right] &\leq \int_{\mathbb{R}^d} \mathbb{E}[|Y||\mathbf{X}=\mathbf{x}]\kappa^*_{\mathbf{H}_n}(\mathbf{x}^*_{g,n}-\mathbf{x}) f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x} \\ &\leq C_Y(1)C_{\kappa^*}, \qquad C_{\kappa^*} = \sup_{\mathbf{u}\in\mathbb{R}^d}\kappa^*(\mathbf{u}) + \int_{\mathbb{R}^d}\kappa^*(\mathbf{u})\mathrm{d}\mathbf{u}.\end{aligned}$$

As a consequence, if $\rho_n \leq \min(1,\delta_\kappa)$ and $M \geq 4C_Y(1)C_{\kappa^*}$, then

$$\begin{aligned}\mathbb{P}\left[\sup_{\mathbf{x}\in\mathcal{X}_{g,n}}\left|\tilde{\Psi}_n(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right| > M\rho_n\right] \leq & \mathbb{P}\left[\left|\tilde{\Psi}_n(\mathbf{x}^*_{g,n}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x}^*_{g,n})]\right| > M\rho_n/4\right] \\ & +\mathbb{P}\left[\left|\tilde{\Psi}^*_n(\mathbf{x}^*_{g,n}) - \mathbb{E}[\tilde{\Psi}^*_n(\mathbf{x}^*_{g,n})]\right| > M\rho_n/4\right].\end{aligned}$$

Because

$$\begin{aligned}&\left|Y_{jn}\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_j) - \mathbb{E}\left[Y_{jn}\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_j)\right]\right| \\ &\leq 2\tau_n\left|\mathbf{H}_n\right|^{-1}C_\kappa = 2C_\tau n^{1/s}\left|\mathbf{H}_n\right|^{-1}C_\kappa,\end{aligned}$$

and

$$\begin{aligned}
&\mathbb{V}\left[Y_{jn}\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_j)\right] \\
&\leq \mathbb{E}\left[Y_{jn}^2\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_j)^2\right] \\
&\leq \int_{\mathbb{R}^d}\mathbb{E}\left[|Y|^2|\mathbf{X}=\mathbf{r}\right]\kappa_{\mathbf{H}_n}(\mathbf{r}-\mathbf{X}_j)^2 f_{\mathbf{X}}\left(\mathbf{r}\right)\mathrm{d}\mathbf{r} \\
&\leq \left|\mathbf{H}_n\right|^{-1}C_Y(2)\int_{\mathbb{R}^d}\kappa\left(\mathbf{u}\right)^2\mathrm{d}\mathbf{u} \leq \left|\mathbf{H}_n\right|^{-1}C_Y(2)C_\kappa^2,
\end{aligned}$$

it follows from Bernstein's inequality that

$$\mathbb{P}\left[\left|\tilde{\Psi}_n(\mathbf{x}_{g,n}^*)-\mathbb{E}[\tilde{\Psi}_n(\mathbf{x}_{g,n}^*)]\right|>M\rho_n/4\right]\leq 2\exp\left[-\frac{n\left|\mathbf{H}_n\right|\rho_n^2M^2/32}{C_Y(2)C_\kappa^2+\frac{1}{6}MC_\tau C_\kappa\rho_n n^{1/s}}\right].$$

Similarly,

$$\mathbb{P}\left[\left|\tilde{\Psi}_n^*(\mathbf{x}_{g,n}^*)-\mathbb{E}[\tilde{\Psi}_n^*(\mathbf{x}_{g,n}^*)]\right|>M\rho_n/4\right]\leq 2\exp\left[-\frac{n\left|\mathbf{H}_n\right|\rho_n^2M^2/32}{C_Y(2)C_{\kappa^*}^2+\frac{1}{6}MC_\tau C_{\kappa^*}\rho_n n^{1/s}}\right],$$

so if $\rho_n \leq \min\left(1,\delta_\kappa\right)$ and $M \geq 4C_Y\left(1\right)C_{\kappa^*}$, then

$$\begin{aligned}
&\max_{1\leq g\leq G_n}\mathbb{P}\left[\sup_{\mathbf{x}\in\mathcal{X}_{g,n}}\left|\tilde{\Psi}_n(\mathbf{x})-\mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right|>M\rho_n\right] \\
&\leq 4\exp\left[-\frac{n\left|\mathbf{H}_n\right|\rho_n^2M^2/32}{C_Y(2)\max\left(C_\kappa,C_{\kappa^*}\right)^2+\frac{1}{6}MC_\tau\max\left(C_\kappa,C_{\kappa^*}\right)\rho_n n^{1/s}}\right].
\end{aligned}$$

To complete the proof, we let $R<\infty$ be given and use the bound just obtained to exhibit an M such that (B-3) holds.

First, suppose $\underline{\lim}_{n\to\infty}n^{1-2/s}\left|\mathbf{H}_n\right|/\log n>0$, in which case there exists a $\underline{C}_{\mathbf{H}}>0$ such that

$$\rho_n n^{1/s}=\sqrt{\frac{\log n}{n^{1-2s}\left|\mathbf{H}_n\right|}}\max\left(1,\sqrt{\frac{\log n}{n^{1-2/s}\left|\mathbf{H}_n\right|}}\right)\leq\frac{1}{\underline{C}_{\mathbf{H}}}$$

for all n large enough. For any such n,

$$\frac{n\left|\mathbf{H}_n\right|\rho_n^2 M^2/32}{C_Y\left(2\right)\max\left(C_\kappa,C_{\kappa^*}\right)^2+\frac{1}{6}MC_\tau\max\left(C_\kappa,C_{\kappa^*}\right)\rho_n n^{1/s}}$$
$$\geq\frac{M^2/32}{C_Y\left(2\right)\max\left(C_\kappa,C_{\kappa^*}\right)^2+\frac{1}{6}MC_\tau\max\left(C_\kappa,C_{\kappa^*}\right)/\underline{C}_{\mathbf{H}}}\log n,$$

so if n is large enough and if $M\geq 4C_Y\left(1\right)C_{\kappa^*}$, then

$$\max_{1\leq g\leq G_n}\mathbb{P}\left[\sup_{\mathbf{x}\in\mathcal{X}_{g,n}}\left|\tilde{\Psi}_n(\mathbf{x})-\mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right|>M\rho_n\right]$$
$$\leq 4n^{-M^2/32\left[C_Y(2)\max(C_\kappa,C_{\kappa^*})^2+\frac{1}{6}MC_\tau\max(C_\kappa,C_{\kappa^*})/\underline{C}_{\mathbf{H}}\right]},$$

implying in particular that (B-3) holds if M is large enough.

Next, suppose $\overline{\lim}_{n\to\infty}n^{1-2/s}\left|\mathbf{H}_n\right|/\log n<\infty$, in which case there exists a $\overline{C}_{\mathbf{H}}<\infty$ such that

$$\frac{n^{1-2/s}\left|\mathbf{H}_n\right|}{\log n}\leq\overline{C}_{\mathbf{H}},\qquad\frac{n^{1-2/s}\left|\mathbf{H}_n\right|}{\log n}\rho_n n^{1/s}=\max\left(1,\sqrt{\frac{n^{1-2/s}\left|\mathbf{H}_n\right|}{\log n}}\right)\leq\overline{C}_{\mathbf{H}}$$

for all n large enough. For any such n,

$$\frac{n\left|\mathbf{H}_n\right|\rho_n^2 M^2/32}{C_Y(2)\max\left(C_\kappa,C_{\kappa^*}\right)^2+\frac{1}{6}MC_\tau\max\left(C_\kappa,C_{\kappa^*}\right)\rho_n n^{1/s}}$$
$$\geq\frac{M^2/32}{C_Y(2)\max\left(C_\kappa,C_{\kappa^*}\right)^2\frac{n^{1-2/s}\left|\mathbf{H}_n\right|}{\log n}+\frac{1}{6}MC_\tau\max\left(C_\kappa,C_{\kappa^*}\right)\frac{n^{1-2/s}\left|\mathbf{H}_n\right|}{\log n}\rho_n n^{1/s}}\log n$$
$$\geq\frac{M^2/32}{C_Y(2)\max\left(C_\kappa,C_{\kappa^*}\right)^2\overline{C}_{\mathbf{H}}+\frac{1}{6}MC_\tau\max\left(C_\kappa,C_{\kappa^*}\right)\overline{C}_{\mathbf{H}}}\log n,$$

so if n is large enough and if $M \geq 4C_Y(1)C_{\kappa^*}$, then

$$\max_{1\leq g\leq G_n} \mathbb{P}\left[\sup_{\mathbf{x}\in\mathcal{X}_{g,n}} \left|\tilde{\Psi}_n(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_n(\mathbf{x})]\right| > M\rho_n\right]$$
$$\leq 4n^{-M^2/32\left[C_Y(2)\max(C_\kappa,C_{\kappa^*})^2\overline{C}_{\mathbf{H}}+\frac{1}{6}MC_\tau\max(C_\kappa,C_{\kappa^*})\overline{C}_{\mathbf{H}}\right]},$$

implying once again that (B-3) holds if M is large enough.

Finally, suppose $\overline{\lim}_{n\to\infty} n^{1-2/s}\left|\mathbf{H}_n\right|/\log n = \infty$ and $\underline{\lim}_{n\to\infty} n^{1-2/s}\left|\mathbf{H}_n\right|/\log n = 0$. Suppose that for some $\varepsilon > 0$ and for every M, there exists a subsequence n' with

$$\mathbb{P}\left[\sup_{\mathbf{x}\in\mathcal{X}_{n'}} \left|\tilde{\Psi}_{n'}(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_{n'}(\mathbf{x})]\right| > M\rho_{n'}\right] > \varepsilon$$

for every n'. Given $\varepsilon > 0$, pick an $M \geq 4C_Y(1)C_{\kappa^*}$ satisfying

$$\overline{\lim}_{n\to\infty} G_n n^{-M^2/32\left[C_Y(2)\max(C_\kappa,C_{\kappa^*})^2+\frac{1}{6}MC_\tau\max(C_\kappa,C_{\kappa^*})\right]} < \varepsilon/4.$$

Any subsequence n' contains a further subsubsequence n'' along which

$$\overline{\lim}_{n''\to\infty}\frac{(n'')^{1-2/s}\left|\mathbf{H}_{n''}\right|}{\log n''} = \underline{\lim}_{n''\to\infty}\frac{(n'')^{1-2/s}\left|\mathbf{H}_{n''}\right|}{\log n''} \in [0,\infty].$$

Along such subsubsequences the previous results can be used to show that

$$\overline{\lim}_{n''\to\infty}\mathbb{P}\left[\sup_{\mathbf{x}\in\mathcal{X}_{n''}} \left|\tilde{\Psi}_{n''}(\mathbf{x}) - \mathbb{E}[\tilde{\Psi}_{n''}(\mathbf{x})]\right| > M\rho_{n''}\right] < \varepsilon,$$

a contradiction. ■

2.2. Proof of Lemma B-2. Because $\Psi_{n,i}(\mathbf{x}) = \Psi_n(\mathbf{x})$ and

$$\hat{\Psi}_{n,i}(\mathbf{x}) = \frac{n}{n-1}\hat{\Psi}_n(\mathbf{x}) - \frac{1}{(n-1)}Y_i\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_i),$$

we have the elementary bound

$$\begin{aligned}\left|\hat{\Psi}_{n,i}(\mathbf{x}) - \Psi_{n,i}(\mathbf{x})\right| \;&\leq\; (1-n^{-1})^{-1}\left|\hat{\Psi}_n(\mathbf{x}) - \Psi_n(\mathbf{x})\right| + (n-1)^{-1}\mathbb{E}\left[|\hat{\Psi}_n(\mathbf{x})|\right] \\ &\quad +(n-1)^{-1}\left|Y_{in}\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_i)\right| \\ &\quad +(n-1)^{-1}\left|(Y_i - Y_{in})\,\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_i)\right|,\end{aligned}$$

where $Y_{in} = Y_i\mathbf{1}(|Y_i| \leq \tau_n)$ with $\tau_n = O\left(n^{1/s}\right)$. The first term on the right is covered by Lemma B-1, the second term is $O\left(n^{-1}\right)$, and the third term satisfies

$$\frac{1}{1-n^{-1}}\left|Y_{in}\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_i)\right| \leq \frac{1}{n-1}\left|\mathbf{H}_n\right|^{-1}\tau_n C_\kappa = O\left(n^{1/s-1}\left|\mathbf{H}_n\right|^{-1}\right),$$

where

$$n^{1/s-1}\left|\mathbf{H}_n\right|^{-1} = \sqrt{\frac{1}{n\left|\mathbf{H}_n\right|}}\sqrt{\frac{1}{n^{1-2/s}\left|\mathbf{H}_n\right|}} = o\left(\rho_n\right).$$

Finally, the fourth term is negligible because

$$\mathbb{P}\left[\max_{1\leq i\leq n}\frac{1}{(n-1)}\left|(Y_i - Y_{in})\,\kappa_{\mathbf{H}_n}(\mathbf{x}-\mathbf{X}_i)\right| > 0\right] = \mathbb{P}\left[Y_i \neq Y_{in} \text{ for some } i\right]$$

can be made arbitrarily close to zero. ■

2.3. Proof of Lemma B-3. By Markov's inequality,

$$\mathbb{P}\left[\max_{1\leq i\leq n}\|\mathbf{X}_i\| > n^{2/s_\mathbf{X}}\right] \leq n\mathbb{P}\left[\|\mathbf{X}\|^{s_\mathbf{X}} > n^2\right] \leq n^{-1}\mathbb{E}[\|\mathbf{X}\|^{s_\mathbf{X}}] = o\left(1\right).$$

Setting $C_{\mathbf{X},n} = n^{2/s_{\mathbf{X}}}$, we therefore have

$$\max_{1\le i\le n}\left|\hat{\Psi}_n(\mathbf{X}_i) - \Psi_n(\mathbf{X}_i)\right| \le \sup_{\mathbf{x}\in\mathcal{X}_n}\left|\hat{\Psi}_n(\mathbf{x}) - \Psi_n(\mathbf{x})\right|$$

and

$$\max_{1\le i\le n}\left|\hat{\Psi}_{n,i}(\mathbf{X}_i) - \Psi_{n,i}(\mathbf{X}_i)\right| \le \max_{1\le i\le n}\sup_{\mathbf{x}\in\mathcal{X}_n}\left|\hat{\Psi}_{n,i}(\mathbf{x}) - \Psi_{n,i}(\mathbf{x})\right|$$

with probability approaching one. The result now follows from Lemmas B-1 and B-2. ■

3. Appendix C: ROT Bandwidths Derivation

Recall the two constants of interest:

$$\mathcal{B}_0 = \left(-K(\mathbf{0}_d)\mathbf{I}_d + \int_{\mathbb{R}^d}\left[K(\mathbf{u})^2\mathbf{I}_d + K(\mathbf{u})\dot{\mathbf{K}}(\mathbf{u})\mathbf{u}'\right]\mathrm{d}\mathbf{u}\right)\int_{\mathbb{R}^d} g(\mathbf{x})w(\mathbf{x})\ell(\mathbf{x})\mathrm{d}\mathbf{x},$$

and

$$\mathcal{S}(\mathbf{H}_n) = (-1)^{P+1}\sum_{\mathbf{l}\in\mathbb{Z}_+^d(P)}\frac{\mathbf{h}_n^{\mathbf{l}}}{\mathbf{l}!}\left[\int_{\mathbb{R}^d} w(\mathbf{x})g(\mathbf{x})\left(\partial^{\mathbf{l}}\dot{\mathbf{f}}(\mathbf{x}) + \ell(\mathbf{x})\partial^{\mathbf{l}}f(\mathbf{x})\right)\mathrm{d}\mathbf{x}\right]\left[\int_{\mathbb{R}^d}\mathbf{u}^{\mathbf{l}}K(\mathbf{u})\mathrm{d}\mathbf{u}\right].$$

To derive the ROT bandwidth choices we impose two main assumptions.

Assumption C1. (a) $K(\mathbf{u}) = \prod_{j=1}^d k(u_j)$.

(b) $\lim_{|u|\to\infty}|uk(u)^2| = 0$.

(c) P is even.

Assumption C2. (a) $f(\mathbf{x}) = \prod_{j=1}^d \phi_{\sigma_j}(x_j)$, with $\phi_\sigma(x) = \phi(x/\sigma)/\sigma$ and $\sigma_j > 0$ for all $j = 1, 2, \cdots, d$.

(b) $g(\mathbf{x}) = \mathbf{x}'\boldsymbol{\beta}$, with $\boldsymbol{\beta}\boldsymbol{\beta}'$ positive definite.

(c) $w(\mathbf{x}) = f(\mathbf{x})$.

3.1. Kernel Constants. Assumption C1(a)-(b) implies:

$$\mathcal{B}_0 = C_{\mathcal{B}} \int_{\mathbb{R}^d} g(\mathbf{x}) w(\mathbf{x}) \ell(\mathbf{x}) \mathrm{d}\mathbf{x}, \qquad C_{\mathcal{B}} = -k(0)^d + \frac{1}{2}\left(\int_{\mathbb{R}} k(u)^2 \mathrm{d}u\right)^d,$$

because, using integration by parts,

$$\begin{aligned}
&- K(\mathbf{0}_d)\mathbf{I}_d + \int_{\mathbb{R}^d} \left[K(\mathbf{u})^2 \mathbf{I}_d + K(\mathbf{u})\dot{\mathbf{K}}(\mathbf{u})\mathbf{u}'\right] \mathrm{d}u \\
&= -k(0)^d \mathbf{I}_d + \left[\left(\int_{\mathbb{R}} k(u)^2 \mathrm{d}u\right)^d + \left(\int_{\mathbb{R}} u k(u)\dot{k}(u)\mathrm{d}u\right)\left(\int_{\mathbb{R}} k(u)^2 \mathrm{d}u\right)^{d-1}\right] \mathbf{I}_d \\
&= -k(0)^d \mathbf{I}_d + \left(\int_{\mathbb{R}} k(u)^2 \mathrm{d}u\right)^{d-1} \left[\int_{\mathbb{R}} k(u)^2 \mathrm{d}u + \int_{\mathbb{R}} u k(u)\dot{k}(u)\mathrm{d}u\right] \mathbf{I}_d \\
&= -k(0)^d \mathbf{I}_d + \left(\int_{\mathbb{R}} k(u)^2 \mathrm{d}u\right)^{d-1} \left[\frac{1}{2}\int_{\mathbb{R}} k(u)^2 \mathrm{d}u\right] \mathbf{I}_d \\
&= -k(0)^d \mathbf{I}_d + \frac{1}{2}\left(\int_{\mathbb{R}} k(u)^2 \mathrm{d}u\right)^d \mathbf{I}_d.
\end{aligned}$$

Assumption C1(a) also implies:

$$\mathcal{S}(\mathbf{H}_n) = C_{\mathcal{S}} \sum_{l=1}^d h_{l,n}^P \left[\int_{\mathbb{R}^d} w(\mathbf{x}) g(\mathbf{x}) \left(\frac{\partial^P}{\partial x_l^P}\dot{\mathbf{f}}(\mathbf{x}) + \ell(\mathbf{x})\frac{\partial^P}{\partial x_l^P} f(\mathbf{x})\right) \mathrm{d}\mathbf{x}\right],$$

$$C_{\mathcal{S}} = \frac{(-1)^{P+1}}{P!} \int_{\mathbb{R}} u^P k(u) \mathrm{d}u.$$

Note that $C_{\mathcal{S}} = 0$ if P is odd for the Gaussian-based higher-order kernel; hence Assumption C1(c).

The following table gives some values of $C_{\mathcal{B}}$ and $C_{\mathcal{S}}$ for the Gaussian-based higher-order kernel when $d = 3$:

P	$C_{\mathcal{B}}$	$C_{\mathcal{S}}$
2	$\frac{1-4\sqrt{2}}{16\pi^{3/2}} = -0.0522694$	$-\frac{1}{2} = -0.5$
4	$-\frac{27\left(-729+2048\sqrt{2}\right)}{65536\pi^{3/2}} = -0.160354$	$\frac{1}{8} = 0.125$
6	$-\frac{3375\left(-3442951+8388608\sqrt{2}\right)}{17179869184\pi^{3/2}} = -0.29707$	$-\frac{1}{48} = -0.0208333$
8	$-\frac{42875\left(-1911240521+4294967296\sqrt{2}\right)}{70368744177664\pi^{3/2}} = -0.455492$	$\frac{1}{384} = 0.00260417$
10	$-\frac{31255875\left(-66071557334483+140737488355328\sqrt{2}\right)}{1180591620717411303424\pi^{3/2}} = -0.632168$	$-\frac{1}{3840} = -0.000260417$

Table C1: Some values of kernel constants

($k(u)$ is P-th order Gaussian-based kernel).

3.2. DGP Constants. Assumption C2(a) gives the so-called "normal reference model":

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Omega}|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{x}'\boldsymbol{\Omega}^{-1}\mathbf{x}\right), \qquad \boldsymbol{\Omega} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \cdots, \sigma_d^2),$$

which implies

$$\ell(\mathbf{x}) = -\frac{\dot{\mathbf{f}}(x)}{f(x)} = \boldsymbol{\Omega}^{-1}\mathbf{x} = \left(\frac{x_1}{\sigma_1^2}, \frac{x_1}{\sigma_2^2}, \cdots, \frac{x_1}{\sigma_d^2}\right)'.$$

Thus, Assumptions C2(a)-(b) give

$$\int_{\mathbb{R}^d} g(\mathbf{x})w(\mathbf{x})\ell(\mathbf{x})\mathrm{d}\mathbf{x} = \boldsymbol{\Omega}^{-1}\left(\int_{\mathbb{R}^d} \mathbf{x}\mathbf{x}'w(\mathbf{x})\mathrm{d}\mathbf{x}\right)\boldsymbol{\beta},$$

which gives

$$\mathcal{B}_0 = C_{\mathcal{B}}\boldsymbol{\Omega}^{-1}\left(\int_{\mathbb{R}^d} \mathbf{x}\mathbf{x}'w(\mathbf{x})\mathrm{d}\mathbf{x}\right)\boldsymbol{\beta} = C_{\mathcal{B}}\boldsymbol{\beta},$$

where the last equality uses Assumption C2(c).

Next, we simplify $\mathcal{S}(\mathbf{H}_n)$. For $l = 1, 2, ..., d$, we have

$$\begin{aligned}
&\int_{\mathbb{R}^d} \left(\frac{\partial^P}{\partial x_l^P} \dot{\mathbf{f}}(\mathbf{x}) + \ell(\mathbf{x}) \frac{\partial^P}{\partial x_l^P} f(\mathbf{x}) \right) g(\mathbf{x}) w(\mathbf{x}) \mathrm{d}\mathbf{x} \\
&= \int_{\mathbb{R}^d} \left(\frac{\partial^P}{\partial x_l^P} \left[-\mathbf{\Omega}^{-1} \mathbf{x} f(\mathbf{x}) \right] + \mathbf{\Omega}^{-1} \mathbf{x} \frac{\partial^P}{\partial x_l^P} f(\mathbf{x}) \right) \mathbf{x}' f(\mathbf{x}) \mathrm{d}\mathbf{x} \boldsymbol{\beta} \\
&= \mathbf{\Omega}^{-1} \int_{\mathbb{R}^d} \left(-\frac{\partial^P}{\partial x_l^P} \left[\mathbf{x} f(\mathbf{x}) \right] + \mathbf{x} \frac{\partial^P}{\partial x_l^P} f(\mathbf{x}) \right) \mathbf{x}' f(\mathbf{x}) \mathrm{d}\mathbf{x} \boldsymbol{\beta},
\end{aligned}$$

where, letting $\mathbf{e}_l$ denote the l-th unit vector (i.e., $\mathbf{e}_l = (0, 0, 1, \cdots, 0) \in \mathbb{R}^d$ if $l = 3$),

$$\begin{aligned}
\frac{\partial^P}{\partial x_l^P} \left[\mathbf{x} f(\mathbf{x}) \right] &= \sum_{\ell=1}^{d} \mathbf{e}_\ell \frac{\partial^P}{\partial x_l^P} \left[x_\ell f(\mathbf{x}) \right] = \sum_{\ell=1, \ell \neq l}^{d} \mathbf{e}_\ell x_\ell \frac{\partial^P}{\partial x_l^P} f(\mathbf{x}) + \mathbf{e}_l \frac{\partial^P}{\partial x_l^P} \left[x_l f(\mathbf{x}) \right] \\
&= \mathbf{x} \frac{\partial^P}{\partial x_l^P} f(\mathbf{x}) - \mathbf{e}_l x_l \frac{\partial^P}{\partial x_l^P} f(\mathbf{x}) + \mathbf{e}_l \frac{\partial^P}{\partial x_l^P} \left[x_l f(\mathbf{x}) \right],
\end{aligned}$$

and hence

$$\begin{aligned}
&\int_{\mathbb{R}^d} \left(\frac{\partial^P}{\partial x_l^P} \dot{\mathbf{f}}(\mathbf{x}) + \ell(\mathbf{x}) \frac{\partial^P}{\partial x_l^P} f(\mathbf{x}) \right) g(\mathbf{x}) w(\mathbf{x}) \mathrm{d}\mathbf{x} \\
&= \mathbf{\Omega}^{-1} \int_{\mathbb{R}^d} \mathbf{e}_l \left(x_l \frac{\partial^P}{\partial x_l^P} f(\mathbf{x}) - \frac{\partial^P}{\partial x_l^P} \left[x_l f(\mathbf{x}) \right] \right) \mathbf{x}' f(\mathbf{x}) \mathrm{d}\mathbf{x} \boldsymbol{\beta}.
\end{aligned}$$

Next, recall that

$$\begin{aligned}
\frac{\partial^s}{\partial x^s} \phi_\sigma(x) &= \phi_\sigma^{(s)}(x) = \frac{1}{\sigma^{s+1}} \phi^{(s)} \left(\frac{x}{\sigma} \right) \\
&= \frac{(-1)^s}{\sigma^{s+1}} \mathcal{H}_s \left(\frac{x}{\sigma} \right) \phi \left(\frac{x}{\sigma} \right) = \frac{(-1)^s}{\sigma^s} \mathcal{H}_s \left(\frac{x}{\sigma} \right) \phi_\sigma(x),
\end{aligned}$$

where $\mathcal{H}_s(u)$ is the s-th order Hermite polynomial. Therefore,

$$\begin{aligned}
&\int_{\mathbb{R}^d}\left(\frac{\partial^P}{\partial x_l^P}\dot{\mathbf{f}}(\mathbf{x})+\ell(\mathbf{x})\frac{\partial^P}{\partial x_l^P}f(\mathbf{x})\right)g(\mathbf{x})w(\mathbf{x})\mathrm{d}\mathbf{x}\\
&=\boldsymbol{\Omega}^{-1}\int_{\mathbb{R}^d}\mathbf{e}_l\left(x_l\frac{\partial^P}{\partial x_l^P}f(\mathbf{x})-\frac{\partial^P}{\partial x_l^P}\left[x_lf(\mathbf{x})\right]\right)\mathbf{x}'f(\mathbf{x})\mathrm{d}\mathbf{x}\boldsymbol{\beta}\\
&=\boldsymbol{\Omega}^{-1}\int_{\mathbb{R}^d}\mathbf{e}_l\left(x_l\frac{\partial^P}{\partial x_l^P}f(\mathbf{x})+\sigma_l^2\frac{\partial^P}{\partial x_l^P}\left[-\frac{x_l}{\sigma_l^2}f(\mathbf{x})\right]\right)\mathbf{x}'f(\mathbf{x})\mathrm{d}\mathbf{x}\boldsymbol{\beta}\\
&=\boldsymbol{\Omega}^{-1}\int_{\mathbb{R}^d}\mathbf{e}_l\left(x_l\frac{\partial^P}{\partial x_l^P}f(\mathbf{x})+\sigma_l^2\frac{\partial^{P+1}}{\partial x_l^{P+1}}f(\mathbf{x})\right)\mathbf{x}'f(\mathbf{x})\mathrm{d}\mathbf{x}\boldsymbol{\beta}\\
&=\boldsymbol{\Omega}^{-1}\int_{\mathbb{R}^d}\mathbf{e}_l\left(x_l\frac{(-1)^P}{\sigma_l^P}\mathcal{H}_P\left(\frac{x_l}{\sigma_l}\right)f(\mathbf{x})+\sigma_l^2\frac{(-1)^{P+1}}{\sigma_l^{P+1}}\mathcal{H}_{P+1}\left(\frac{x_l}{\sigma_l}\right)f(\mathbf{x})\right)\mathbf{x}'f(\mathbf{x})\mathrm{d}\mathbf{x}\boldsymbol{\beta}\\
&=\boldsymbol{\Omega}^{-1}\frac{(-1)^P}{\sigma_l^P}\int_{\mathbb{R}^d}\mathbf{e}_l\left[x_l\mathcal{H}_P\left(\frac{x_l}{\sigma_l}\right)-\sigma_l\mathcal{H}_{P+1}\left(\frac{x_l}{\sigma_l}\right)\right]\mathbf{x}'f(\mathbf{x})^2\mathrm{d}\mathbf{x}\boldsymbol{\beta}\\
&=\boldsymbol{\Omega}^{-1}\frac{(-1)^P\beta_l}{\sigma_l^P}\int_{\mathbb{R}^d}\mathbf{e}_l\left[x_l^2\mathcal{H}_P\left(\frac{x_l}{\sigma_l}\right)-\sigma_lx_l\mathcal{H}_{P+1}\left(\frac{x_l}{\sigma_l}\right)\right]f(\mathbf{x})^2\mathrm{d}\mathbf{x},
\end{aligned}$$

because

$$\int_{\mathbb{R}}u\phi(u)^2\mathrm{d}u=0.$$

Therefore, changing variables, we obtain

$$\begin{aligned}
&\int_{\mathbb{R}^d}\left(\frac{\partial^P}{\partial x_l^P}\dot{\mathbf{f}}(\mathbf{x})+\ell(\mathbf{x})\frac{\partial^P}{\partial x_l^P}f(\mathbf{x})\right)g(\mathbf{x})w(\mathbf{x})\mathrm{d}\mathbf{x}\\
&=\boldsymbol{\Omega}^{-1}\frac{(-1)^P\beta_l}{\sigma_l^P}\int_{\mathbb{R}^d}\mathbf{e}_l\left[x_l^2\mathcal{H}_P\left(\frac{x_l}{\sigma_l}\right)-\sigma_lx_l\mathcal{H}_{P+1}\left(\frac{x_l}{\sigma_l}\right)\right]f(\mathbf{x})^2\mathrm{d}\mathbf{x}\\
&=\frac{(-1)^P\beta_l}{\sigma_l^P}\int_{\mathbb{R}^d}\mathbf{e}_l\left[\left(\frac{x_l}{\sigma_l}\right)^2\mathcal{H}_P\left(\frac{x_l}{\sigma_l}\right)-\frac{x_l}{\sigma_l}\mathcal{H}_{P+1}\left(\frac{x_l}{\sigma_l}\right)\right]f(\mathbf{x})^2\mathrm{d}\mathbf{x}\\
&=\mathbf{e}_l\frac{(-1)^P\beta_l}{\sigma_l^P\left|\boldsymbol{\Omega}\right|^{1/2}}\int_{\mathbb{R}^d}\left[u_l^2\mathcal{H}_P(u_l)-u_l\mathcal{H}_{P+1}(u_l)\right]\phi_{\mathbf{I}_d}(\mathbf{u})^2\mathrm{d}\mathbf{u}.
\end{aligned}$$

Now, from Olver et al. (2010; Table 18.17.48) we have

$$\int_{\mathbb{R}}\mathcal{H}_P\left(u\right)\mathcal{H}_Q\left(u\right)\phi(u)^2\mathrm{d}u=\frac{(-1)^Q\mathcal{H}_{P+Q}(0)}{2^{P/2+Q/2+1}\sqrt{\pi}},\qquad\mathcal{H}_{2p}(0)=\left(-\frac{1}{2}\right)^p(p+1)_p\,,$$

where $(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)}$, and also

$$\int_{\mathbb{R}} \phi(u)^2 \mathrm{d}u = \frac{1}{2\sqrt{\pi}}, \qquad \int_{\mathbb{R}} u^2 \phi(u)^2 \mathrm{d}u = \frac{1}{4\sqrt{\pi}}.$$

Using these results, we obtain

$$\begin{aligned}
&\int_{\mathbb{R}^d} \left[u^2 \mathcal{H}_P(u) - u\mathcal{H}_{P+1}(u)\right] \phi_{\mathbf{I}_d}(\mathbf{u})^2 \mathrm{d}\mathbf{u} \\
&= \int_{\mathbb{R}^d} \mathcal{H}_P(u)\mathcal{H}_2(u)\phi_{\mathbf{I}_d}(\mathbf{u})^2\mathrm{d}\mathbf{u} + \int_{\mathbb{R}^d} \mathcal{H}_P(u)\mathcal{H}_0(u)\phi_{\mathbf{I}_d}(\mathbf{u})^2\mathrm{d}\mathbf{u} - \int_{\mathbb{R}^d} \mathcal{H}_{P+1}(u)\mathcal{H}_1(u)\phi_{\mathbf{I}_d}(\mathbf{u})^2\mathrm{d}\mathbf{u} \\
&= \left[\int_{\mathbb{R}} \phi(u)^2 \mathrm{d}u\right]^{d-1} \left[\frac{\mathcal{H}_{P+2}(0)}{2^{P/2+2}\sqrt{\pi}} + \frac{\mathcal{H}_P(0)}{2^{P/2+1}\sqrt{\pi}} + \frac{\mathcal{H}_{P+2}(0)}{2^{P/2+2}\sqrt{\pi}}\right] \\
&= \frac{1}{2^{d+P/2}\pi^{d/2}} \left[\mathcal{H}_{P+2}(0) + \mathcal{H}_P(0)\right] \\
&= \frac{1}{2^{d+P/2}\pi^{d/2}} \frac{\Gamma(P)}{\Gamma(P/2)} \left[-\frac{1}{2}\frac{(P+2)(P+1)P}{(P/2+1)P/2} + 2\right] \\
&= \frac{(-1)^{P/2}}{2^{d+P}\pi^{d/2}} \frac{\Gamma(P)}{\Gamma(P/2)} \left[-\frac{1}{2}\frac{(P+2)(P+1)P}{(P/2+1)P/2} + \frac{P}{P/2}\right] \\
&= \frac{(-1)^{P/2+1}2P}{2^{d+P}\pi^{d/2}} \frac{\Gamma(P)}{\Gamma(P/2)} =: C_{\mathcal{H}}.
\end{aligned}$$

For the case of $d = 3$, the following table gives same values of $C_{\mathcal{H}}$:

P	$C_{\mathcal{H}}$
2	$\frac{1}{8\pi^{3/2}} = 0.0224484$
4	$-\frac{3}{8\pi^{3/2}} = -0.0673452$
6	$\frac{45}{32\pi^{3/2}} = 0.252544$
8	$-\frac{105}{16\pi^{3/2}} = -1.17854$
10	$\frac{4725}{128\pi^{3/2}} = 6.62929$

Table C2: Some values of DGP constant

(P if the order of the kernel used).

As a consequence, we have

$$\mathcal{S}(\mathbf{H}_n) = C_{\mathcal{S}} C_{\mathcal{H}} \sum_{l=1}^{d} \mathbf{e}_l \frac{h_{l,n}^P \beta_l}{\sigma_l^P |\mathbf{\Omega}|^{1/2}}.$$

3.3. Summary of Results. The ROT-based constants are

$$\mathcal{B}_0 = C_{\mathcal{B}} \boldsymbol{\beta} = \sum_{\ell=1}^{d} \mathbf{e}_\ell C_{\mathcal{B}} \beta_\ell \qquad \text{and} \qquad \mathcal{S}(\mathbf{H}_n) = \sum_{\ell=1}^{d} \mathbf{e}_\ell \frac{C_{\mathcal{S}} C_{\mathcal{H}} h_{\ell,n}^P \beta_\ell}{\sigma_\ell^P |\mathbf{\Omega}|^{1/2}},$$

with

$$C_{\mathcal{S}} = \frac{(-1)^{P+1}}{P!} \int_{\mathbb{R}} u^P k(u) \mathrm{d}u, \qquad C_{\mathcal{B}} = -k(0)^d + \frac{1}{2} \left(\int_{\mathbb{R}} k(u)^2 \mathrm{d}u \right)^d,$$

$$C_{\mathcal{H}} = \frac{(-1)^{P/2+1} 2P}{2^{d+P} \pi^{d/2}} \frac{\Gamma(P)}{\Gamma(P/2)}.$$

Recall from above that:

P	$C_{\mathcal{B}}$	$C_{\mathcal{S}}$	$C_{\mathcal{H}}$	
2	-0.0522694	$-\frac{1}{2} = -0.5$	$\frac{1}{8\pi^{3/2}} = 0.0224484$	$\mathrm{sgn}(C_{\mathcal{B}}) = \mathrm{sgn}(C_{\mathcal{S}} C_{\mathcal{H}})$
4	-0.160354	$\frac{1}{8} = 0.125$	$-\frac{3}{8\pi^{3/2}} = -0.0673452$	$\mathrm{sgn}(C_{\mathcal{B}}) = \mathrm{sgn}(C_{\mathcal{S}} C_{\mathcal{H}})$
6	-0.29707	$-\frac{1}{48} = -0.0208333$	$\frac{45}{32\pi^{3/2}} = 0.252544$	$\mathrm{sgn}(C_{\mathcal{B}}) = \mathrm{sgn}(C_{\mathcal{S}} C_{\mathcal{H}})$
8	-0.455492	$\frac{1}{384} = 0.00260417$	$-\frac{105}{16\pi^{3/2}} = -1.17854$	$\mathrm{sgn}(C_{\mathcal{B}}) = \mathrm{sgn}(C_{\mathcal{S}} C_{\mathcal{H}})$
10	-0.632168	$-\frac{1}{3840} = -0.000260417$	$\frac{4725}{128\pi^{3/2}} = 6.62929$	$\mathrm{sgn}(C_{\mathcal{B}}) = \mathrm{sgn}(C_{\mathcal{S}} C_{\mathcal{H}})$

Thus, the AMSE becomes

$$\begin{aligned}\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)] &= \left(\frac{\mathcal{B}_0}{n|\mathbf{H}_n|} + \mathcal{S}(\mathbf{H}_n)\right)\left(\frac{\mathcal{B}_0}{n|\mathbf{H}_n|} + \mathcal{S}(\mathbf{H}_n)\right)' \\ &= \sum_{\ell=1}^{d} \mathbf{e}_\ell \mathbf{e}_\ell' \left(\frac{C_{\mathcal{B}}}{n\prod_{l=1}^{d} h_{l,n}} + \frac{C_{\mathcal{S}} C_{\mathcal{H}} h_{\ell,n}^P}{\sigma_\ell^P \, |\boldsymbol{\Omega}|^{1/2}}\right)^2 \beta_\ell^2.\end{aligned}$$

3.4. Case 1: $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n(h_n\mathbf{I}_d)]$. Recall that in general

$$h_n^* = \begin{cases} \left(\frac{|\mathbf{a}'\mathcal{B}_0|}{|\mathbf{a}'\mathcal{S}(\mathbf{I}_d)|}\frac{1}{n}\right)^{\frac{1}{P+d}} & \text{if } \operatorname{sgn}(\mathbf{a}'\mathcal{B}_0) \neq \operatorname{sgn}(\mathbf{a}'\mathcal{S}(\mathbf{I}_d)) \\ \left(\frac{d|\mathbf{a}'\mathcal{B}_0|}{P|\mathbf{a}'\mathcal{S}(\mathbf{I}_d)|}\frac{1}{n}\right)^{\frac{1}{P+d}} & \text{if } \operatorname{sgn}(\mathbf{a}'\mathcal{B}_0) = \operatorname{sgn}(\mathbf{a}'\mathcal{S}(\mathbf{I}_d)) \end{cases}.$$

Given our ROT calculations above, we have for $\mathbf{a} = (1, 0, 0, \cdots, 0)'$

$$\mathbf{a}'\mathcal{B}_0 = C_{\mathcal{B}}\mathbf{a}'\boldsymbol{\beta} = C_{\mathcal{B}}\beta_1, \qquad \mathbf{a}'\mathcal{S}\left(\mathbf{I}_d\right) = C_{\mathcal{S}} C_{\mathcal{H}} \left|\boldsymbol{\Omega}\right|^{-1/2} \mathbf{a}'\boldsymbol{\Omega}^{-P/2}\boldsymbol{\beta} = \frac{C_{\mathcal{S}} C_{\mathcal{H}} \beta_1}{\sigma_1^P \prod_{l=1}^{d} \sigma_l}.$$

Therefore, the ROT choice becomes

$$h_{\text{ROT-1d},n}^* = \begin{cases} \left(\sigma_1^P \prod_{l=1}^{d} \sigma_l \frac{|C_{\mathcal{B}}|}{|C_{\mathcal{S}} C_{\mathcal{H}}|}\frac{1}{n}\right)^{\frac{1}{P+d}} & \text{if } \operatorname{sgn}(C_{\mathcal{B}}) \neq \operatorname{sgn}(C_{\mathcal{S}} C_{\mathcal{H}}) \\ \left(\sigma_1^P \prod_{l=1}^{d} \sigma_l \frac{d|C_{\mathcal{B}}|}{P|C_{\mathcal{S}} C_{\mathcal{H}}|}\frac{1}{n}\right)^{\frac{1}{P+d}} & \text{if } \operatorname{sgn}(C_{\mathcal{B}}) = \operatorname{sgn}(C_{\mathcal{S}} C_{\mathcal{H}}) \end{cases}.$$

Note that if, in addition, $\sigma = \sigma_1 = \cdots = \sigma_d$, then we obtain $h_{\text{ROT-1d},n}^* \propto (\sigma^{P+d})^{1/(P+d)} = \sigma$.

For our simulations, recall that

$$C_{\mathcal{B}} = -\frac{27\left(-729 + 2048\sqrt{2}\right)}{65536\pi^{3/2}} = -0.160354, \qquad C_{\mathcal{S}} = \frac{1}{8}, \qquad C_{\mathcal{H}} = -\frac{3}{8\pi^{3/2}},$$

and hence $\text{sgn}(C_{\mathcal{B}}) = \text{sgn}(C_{\mathcal{S}}C_{\mathcal{H}})$, which implies that

$$\begin{aligned} h^*_{\text{ROT-1d},n} &= \left(\sigma_1^P \prod\nolimits_{l=1}^d \sigma_l \frac{d\,|C_{\mathcal{B}}|}{P\,|C_{\mathcal{S}}C_{\mathcal{H}}|}\right)^{\frac{1}{P+d}} n^{-1/(P+d)} \\ &= \sigma\left(\frac{d\,|C_{\mathcal{B}}|}{P\,|C_{\mathcal{S}}C_{\mathcal{H}}|}\right)^{\frac{1}{P+d}} n^{-1/(P+d)} \qquad \text{if } \sigma = \sigma_1 = \cdots = \sigma_d. \end{aligned}$$

Assuming $\sigma = 1$ and $n = 700$, we obtain

$$h^*_{\text{ROT-1d},n} = 0.573517.$$

3.5. Case 2: $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n(h_n\mathbf{I}_d)])$. In this case we need to solve:

$$\begin{aligned} \min_{h_n>0} \text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n(h_n\mathbf{I}_d)]) &= \min_{h_n>0} \sum_{\ell=1}^d \left(\frac{C_{\mathcal{B}}}{nh_n^d} + \frac{C_{\mathcal{S}}C_{\mathcal{H}}h_n^P}{\sigma_\ell^P\,|\boldsymbol{\Omega}|^{1/2}}\right)^2 \beta_\ell^2 \\ &= \min_{h_n>0} \sum_{\ell=1}^d \left(\frac{1}{h_n^d} + \frac{\varsigma h_n^P}{\sigma_\ell^P}\right)^2 \beta_\ell^2, \qquad \varsigma = \frac{C_{\mathcal{S}}C_{\mathcal{H}}n}{|\boldsymbol{\Omega}|^{1/2}\,C_{\mathcal{B}}}. \end{aligned}$$

The first-order condition is

$$2\sum_{\ell=1}^d \left(\frac{1}{h_n^d} + \frac{\varsigma h_n^P}{\sigma_\ell^P}\right)\beta_\ell^2\left(-\frac{d}{h_n^{d+1}} + \frac{P\varsigma h_n^{P-1}}{\sigma_\ell^P}\right) = 0$$

$$\begin{aligned} &\Leftrightarrow \sum_{\ell=1}^d \beta_\ell^2\left(1 + \frac{\varsigma}{\sigma_\ell^P}h_n^{P+d}\right)\left(1 - \frac{P\varsigma}{d\sigma_\ell^P}h_n^{P+d}\right) = 0 \\ &\Leftrightarrow \sum_{\ell=1}^d \beta_\ell^2 + \varsigma\left(1 - \frac{P}{d}\right)h_n^{P+d}\sum_{\ell=1}^d \frac{\beta_\ell^2}{\sigma_\ell^P} - \varsigma^2\frac{P}{d}h_n^{2(P+d)}\sum_{\ell=1}^d \frac{\beta_\ell^2}{\sigma_\ell^{2P}} = 0 \\ &\Leftrightarrow a_0 + a_1 z + a_2 z^2 = 0, \end{aligned}$$

with $z = h_n^{P+d}$ and

$$a_0 = \sum_{\ell=1}^{d} \beta_\ell^2, \qquad a_1 = -\varsigma \frac{P-d}{d} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}, \qquad a_2 = -\varsigma^2 \frac{P}{d} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}.$$

The discriminant of the quadratic equation is given by $\Delta = a_1^2 - 4a_0 a_2$. We want

$$\Delta \geq 0 \Leftrightarrow \varsigma^2 \left(\frac{P-d}{d}\right)^2 \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right)^2 + 4 \left(\sum_{\ell=1}^{d} \beta_\ell^2\right) \left(\varsigma^2 \frac{P}{d} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}\right) \geq 0,$$

and therefore we have two real roots since in fact $\Delta > 0$. The roots are given by

$$z_\pm^* = \frac{\varsigma \frac{P-d}{d} \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right) \pm \sqrt{\varsigma^2 \left(\frac{P-d}{d}\right)^2 \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right)^2 + 4 \left(\sum_{\ell=1}^{d} \beta_\ell^2\right) \left(\varsigma^2 \frac{P}{d} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}\right)}}{-2\varsigma^2 \frac{P}{d} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}},$$

and the positive root is given by

$$\begin{aligned}
z_+^* &= \frac{\varsigma \frac{P-d}{d} \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right) - \sqrt{\varsigma^2 \left(\frac{P-d}{d}\right)^2 \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right)^2 + 4 \left(\sum_{\ell=1}^{d} \beta_\ell^2\right) \left(\varsigma^2 \frac{P}{d} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}\right)}}{-2\varsigma^2 \frac{P}{d} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}} \\
&= \frac{-\varsigma (P-d) \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right) + |\varsigma| \sqrt{(P-d)^2 \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right)^2 + 4Pd \left(\sum_{\ell=1}^{d} \beta_\ell^2\right) \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}\right)}}{2\varsigma^2 P \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}\right)} \\
&= \frac{1}{|\varsigma|} \frac{\sqrt{(P-d)^2 \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right)^2 + 4Pd \left(\sum_{\ell=1}^{d} \beta_\ell^2\right) \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}\right)} - \operatorname{sgn}(\varsigma) (P-d) \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P}\right)}{2P \left(\sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}}\right)} \\
&= \frac{1}{|\varsigma|} \frac{\sqrt{(P-d)^2 + 4Pd \frac{\left(\sum_{\ell=1}^{d} \beta_\ell^2\right)\left(\sum_{\ell=1}^{d} \beta_\ell^2/\sigma_\ell^{2P}\right)}{\left(\sum_{\ell=1}^{d} \beta_\ell^2/\sigma_\ell^P\right)^2}} - \operatorname{sgn}(\varsigma) (P-d)}{2P} \frac{\sum_{\ell=1}^{d} \beta_\ell^2/\sigma_\ell^P}{\sum_{\ell=1}^{d} \beta_\ell^2/\sigma_\ell^{2P}},
\end{aligned}$$

which implies that $z_+^* > 0$ (and the other root is negative) because by Holder inequality

$$\frac{\left(\sum_{\ell=1}^{d}\beta_\ell^2\right)\left(\sum_{\ell=1}^{d}\beta_\ell^2/\sigma_\ell^{2P}\right)}{\left(\sum_{\ell=1}^{d}\beta_\ell^2/\sigma_\ell^{P}\right)^2} \geq 1,$$

and recall that $P > d$. So we can write the solution as

$$h^*_{\text{ROT-tr},n} = \left(\frac{|\boldsymbol{\Omega}|^{1/2}\,|C_{\mathcal{B}}|}{|C_{\mathcal{S}}C_{\mathcal{H}}|}\frac{\sqrt{(P-d)^2+4Pd\frac{(\boldsymbol{\beta}'\boldsymbol{\beta})(\boldsymbol{\beta}'\boldsymbol{\Omega}^{-P}\boldsymbol{\beta})}{(\boldsymbol{\beta}'\boldsymbol{\Omega}^{-P/2}\boldsymbol{\beta})^2}}-\operatorname{sgn}(\varsigma)\,(P-d)}{2P}\frac{\boldsymbol{\beta}'\boldsymbol{\Omega}^{-P/2}\boldsymbol{\beta}}{\boldsymbol{\beta}'\boldsymbol{\Omega}^{-P}\boldsymbol{\beta}}\frac{1}{n}\right)^{\frac{1}{P+d}}$$

If $\sigma = \sigma_1 = \cdots = \sigma_d$, then the bandwidth choice simplifies to

$$\begin{aligned} h^*_{\text{ROT-tr},n} &= \left(\frac{\sigma^{P+d}\,|C_{\mathcal{B}}|}{|C_{\mathcal{S}}C_{\mathcal{H}}|}\frac{\sqrt{(P-d)^2+4Pd}-\operatorname{sgn}(\varsigma)\,(P-d)}{2P}\frac{1}{n}\right)^{\frac{1}{P+d}} \\ &= \sigma\left(\frac{|C_{\mathcal{B}}|}{|C_{\mathcal{S}}C_{\mathcal{H}}|}\frac{(P+d)-\operatorname{sgn}(\varsigma)\,(P-d)}{2P}\frac{1}{n}\right)^{\frac{1}{P+d}}. \end{aligned}$$

For our simulations, assuming $\sigma = \sigma_1 = \cdots = \sigma_d = 1$ and $n = 700$, we obtain

$$h^*_{\text{ROT-tr},n} = 0.573517 = h^*_{\text{ROT-1d},n},$$

because recall that $\operatorname{sgn}(C_{\mathcal{B}}) = \operatorname{sgn}(C_{\mathcal{S}}C_{\mathcal{H}})$ (hence $\operatorname{sgn}(\varsigma) = 1$) and therefore

$$\frac{(P+d)-\operatorname{sgn}(\varsigma)\,(P-d)}{2P} = \frac{d}{P}.$$

However, if we assume different variances across covariates then $h_{\text{ROT-1d},n} \neq h_{\text{ROT-tr},n}$ in general.

3.6. Case 3: $\mathrm{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)])$. In this case we need to solve:

$$\begin{aligned}
\min_{\mathbf{H}_n} \mathrm{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)]) &= \min_{\mathbf{H}_n} \sum_{\ell=1}^{d} \left(\frac{C_{\mathcal{B}}}{n \prod_{l=1}^{d} h_{l,n}} + \frac{C_{\mathcal{S}} C_{\mathcal{H}}}{\sigma_\ell^P \, |\boldsymbol{\Omega}|^{1/2}} h_{\ell,n}^P \right)^2 \beta_\ell^2 \\
&= \min_{\mathbf{H}_n} \sum_{\ell=1}^{d} \left(\frac{1}{\prod_{l=1}^{d} h_{l,n}} + \frac{\varsigma h_{\ell,n}^P}{\sigma_\ell^P} \right)^2 \beta_\ell^2, \qquad \varsigma = \frac{C_{\mathcal{S}} C_{\mathcal{H}} n}{|\boldsymbol{\Omega}|^{1/2} C_{\mathcal{B}}}.
\end{aligned}$$

Next, note that

$$\sum_{\ell=1}^{d} \left(\frac{1}{\prod_{l=1}^{d} h_{l,n}} + \frac{\varsigma h_{\ell,n}^P}{\sigma_\ell^P} \right)^2 \beta_\ell^2 = \frac{\sum_{\ell=1}^{d} \beta_\ell^2}{\prod_{l=1}^{d} h_{l,n}^2} + \frac{2\varsigma}{\prod_{l=1}^{d} h_{l,n}} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P} h_{\ell,n}^P + \varsigma^2 \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^{2P}} h_{\ell,n}^{2P},$$

and hence the first-order conditions are, for $k = 1, 2, \cdots, d$,

$$-2 \frac{\sum_{\ell=1}^{d} \beta_\ell^2}{h_{k,n} \prod_{l=1}^{d} h_{l,n}^2} - \frac{2\varsigma}{h_{k,n} \prod_{l=1}^{d} h_{l,n}} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P} h_{\ell,n}^P + \frac{2\varsigma P}{\prod_{l=1}^{d} h_{l,n}} \frac{\beta_k^2}{\sigma_k^P} h_{k,n}^{P-1} + 2\varsigma^2 P \frac{\beta_k^2}{\sigma_k^{2P}} h_{k,n}^{2P-1} = 0$$

$$\begin{aligned}
\Leftrightarrow \quad & -\frac{\sum_{\ell=1}^{d} \beta_\ell^2}{\prod_{l=1}^{d} h_{l,n}^2} - \frac{\varsigma}{\prod_{l=1}^{d} h_{l,n}} \sum_{\ell=1}^{d} \frac{\beta_\ell^2}{\sigma_\ell^P} h_{\ell,n}^P + \frac{\varsigma P}{\prod_{l=1}^{d} h_{l,n}} \frac{\beta_k^2}{\sigma_k^P} h_{k,n}^P + P\varsigma^2 \frac{\beta_k^2}{\sigma_k^{2P}} h_{k,n}^{2P} = 0 \\
\Leftrightarrow \quad & \sum_{\ell=1}^{d} \beta_\ell^2 + \sum_{\ell=1}^{d} \varsigma \frac{\beta_\ell^2}{\sigma_\ell^P} \left(h_{\ell,n}^P \prod_{l=1}^{d} h_{l,n} \right) - \varsigma P \frac{\beta_k^2}{\sigma_k^P} \left(h_{k,n}^P \prod_{l=1}^{d} h_{l,n} \right) - \varsigma^2 P \frac{\beta_k^2}{\sigma_k^{2P}} \left(h_{k,n}^P \prod_{l=1}^{d} h_{l,n} \right)^2 = 0 \\
\Leftrightarrow \quad & \sum_{\ell=1}^{d} \beta_\ell^2 + \sum_{\ell=1}^{d} \mathrm{sgn}(\varsigma) \beta_\ell^2 \left(\frac{|\varsigma|}{\sigma_\ell^P} h_{\ell,n}^P \prod_{l=1}^{d} h_{l,n} \right) - \mathrm{sgn}(\varsigma) P \beta_k^2 \left(\frac{|\varsigma|}{\sigma_k^P} h_{k,n}^P \prod_{l=1}^{d} h_{l,n} \right) \\
& \qquad - P \beta_k^2 \left(\frac{|\varsigma|}{\sigma_k^P} h_{k,n}^P \prod_{l=1}^{d} h_{l,n} \right)^2 = 0 \\
\Leftrightarrow \quad & \sum_{\ell=1}^{d} \beta_\ell^2 + \sum_{\ell=1}^{d} \mathrm{sgn}(\varsigma) \beta_\ell^2 z_\ell - \mathrm{sgn}(\varsigma) P \beta_k^2 z_k - P \beta_k^2 z_k^2 = 0,
\end{aligned}$$

where

$$z_k = \frac{|\varsigma|}{\sigma_k^P} h_{k,n}^P \prod_{l=1}^{d} h_{l,n}.$$

We need to consider two cases:

- $\operatorname{sgn}(\varsigma) = -1$. In this case, a solution is $z_k^* = 1$, $k = 1, 2, \cdots, d$, because

$$\sum_{\ell=1}^{d} \beta_\ell^2 + \sum_{\ell=1}^{d} \operatorname{sgn}(\varsigma)\beta_\ell^2 z_\ell^* - \operatorname{sgn}(\varsigma) P\beta_k^2 z_k^* - P\beta_k^2 (z_k^*)^2 = 0 \Leftrightarrow \sum_{\ell=1}^{d} \beta_\ell^2 - \sum_{\ell=1}^{d} \beta_\ell^2 + P\beta_k^2 - P\beta_k^2 = 0.$$

- $\operatorname{sgn}(\varsigma) = 1$. The first-order conditions reduce to

$$\sum_{\ell=1}^{d} \beta_\ell^2 + \sum_{\ell=1}^{d} \beta_\ell^2 z_\ell - P\beta_k^2 z_k - P\beta_k^2 z_k^2 = 0, \qquad k = 1, 2, \cdots, d.$$

These first-order conditions do not have a closed form solution in general. Nonetheless, a solution can be shown to be $z_k^* = z_k^*(\zeta)$, $k = 1, 2, \cdots, d$, with

$$z_k^*(\zeta) = \frac{1}{2}\left[\sqrt{1 + \frac{4\zeta}{P\beta_k^2}} - 1\right],$$

where the constant ζ is determined by the equation

$$\sum_{l=1}^{d} \beta_l^2 \left[\sqrt{1 + \frac{4\zeta}{P\beta_l^2}} + 1\right] = 2\zeta.$$

A closed form solution is obtained if we assume $\beta_\ell = 1$ (say) for $\ell = 1, 2, \cdots, d$. We then have

$$d + \sum_{\ell=1}^{d} z_\ell - P z_k - P z_k^2 = 0,$$

in which case a solution can be obtained by setting $z_k^* = z^*$ and noting that

$$d + (d - P)\, z^* - P(z^*)^2 = 0$$

and hence (recall that $P > d$)

$$z_{\pm}^{*} = \frac{(P-d) \pm \sqrt{(d-P)^2 + 4Pd}}{-2P}.$$

Thus, taking the positive root, we obtain

$$z_{+}^{*} = \frac{-(P-d) + \sqrt{(d-P)^2 + 4Pd}}{2P} = \frac{2d}{2P} = \frac{d}{P} > 0.$$

The final step is to solve, for some (possibly equal) constants c_l, $l = 1, 2, \cdots, d$, the equations:

$$c_k = \frac{|\varsigma|}{\sigma_k^P} h_{k,n}^P \prod_{l=1}^{d} h_{l,n}, \qquad k = 1, 2, \cdots, d,$$

or, equivalently,

$$1 = \frac{|\varsigma|}{c_k \sigma_k^P} h_{k,n}^P \prod_{l=1}^{d} h_{l,n}, \qquad k = 1, 2, \cdots, d.$$

A solution for the latter is

$$\begin{aligned} h_{\ell,n}^{*} &= \frac{\sigma_\ell c_\ell^{1/P}}{\left(\prod_{l=1}^{d} \sigma_l c_l^{1/P}\right)^{1/(P+d)}} \left(\frac{1}{|\varsigma|}\right)^{\frac{1}{P+d}} = \sigma_\ell \left(\frac{c_\ell^{(P+d)/P}}{|\varsigma|\,|\mathbf{\Omega}|^{1/2} \prod_{l=1}^{d} c_l^{1/P}}\right)^{\frac{1}{P+d}} \\ &= \sigma_\ell \left(\frac{c_\ell^{(P+d)/P}}{\prod_{l=1}^{d} c_l^{1/P}} \frac{|C_{\mathcal{B}}|}{|C_{\mathcal{S}} C_{\mathcal{H}}|} \frac{1}{n}\right)^{\frac{1}{P+d}} \end{aligned}$$

because

$$\begin{aligned}
&\frac{|\varsigma|}{c_k\sigma_k^P}(h_{k,n}^*)^P\prod_{\ell=1}^d h_{\ell,n}^*\\
&=\frac{|\varsigma|}{c_k\sigma_k^P}\left(\frac{\sigma_k c_k^{1/P}}{\left(\prod_{l=1}^d\sigma_l c_l^{1/P}\right)^{1/(P+d)}}\left(\frac{1}{|\varsigma|}\right)^{\frac{1}{P+d}}\right)^P\prod_{\ell=1}^d\frac{\sigma_\ell c_\ell^{1/P}}{\left(\prod_{l=1}^d\sigma_l c_l^{1/P}\right)^{1/(P+d)}}\left(\frac{1}{|\varsigma|}\right)^{\frac{1}{P+d}}\\
&=\frac{1}{c_k\sigma_k^P}\frac{\sigma_k^P c_k}{\prod_{l=1}^d\sigma_l c_l^{1/P}}\prod_{\ell=1}^d\sigma_\ell c_\ell^{1/P}=1.
\end{aligned}$$

Therefore, if $\beta_1=\beta_2=\cdots=\beta_d=1$ (say), we have $c_k=z_+^*=d/P$ and hence

$$h_{\text{ROT-tr},\ell,n}^*=\sigma_\ell\left(\frac{d}{P}\frac{|C_{\mathcal{B}}|}{|C_{\mathcal{S}}C_{\mathcal{H}}|}\frac{1}{n}\right)^{\frac{1}{P+d}}.$$

If, in addition, $\sigma_1=\sigma_2=\cdots=\sigma_d=1$ we obtain for our simulations (recall $\operatorname{sgn}(C_{\mathcal{B}})=\operatorname{sgn}(C_{\mathcal{S}}C_{\mathcal{H}})$) $h_{\text{ROT-tr},\ell,n}^*=0.573517=h_{\text{ROT-1d},n}^*=h_{\text{ROT-tr},n}^*$ for all $\ell=1,2,\cdots,d$.

For the general case (unknown β's and σ's), we solve numerically:

$$\mathbf{H}_{\text{ROT-tr},n}^*=\min_{\mathbf{H}_n}\sum_{\ell=1}^d\left(\frac{C_{\mathcal{B}}}{n\prod_{l=1}^d h_{l,n}}+\frac{C_{\mathcal{S}}C_{\mathcal{H}}}{\sigma_\ell^P\,|\boldsymbol{\Omega}|^{1/2}}h_{\ell,n}^P\right)^2\beta_\ell^2,$$

with $\mathbf{H}_{\text{ROT-tr},n}^*=\operatorname{diag}(h_{\text{ROT-tr},1,n}^*,h_{\text{ROT-tr},2,n}^*,\cdots,h_{\text{ROT-tr},d,n}^*)$.

3.7. Case 4: $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)]$. For $\mathbf{a}=(1,0,0,\cdots,0)'$, under our parametrizations we have

$$\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)]=\left(\frac{C_{\mathcal{B}}}{n\prod_{l=1}^d h_{l,n}}+\frac{C_{\mathcal{S}}C_{\mathcal{H}}}{\sigma_1^P\,|\boldsymbol{\Omega}|^{1/2}}h_{1,n}^P\right)^2\beta_1^2,$$

and hence this optimization problem is not well-defined. A natural approach to this problem is to compute the next higher-order terms for the first element of the vector. This will lead to a valid asymptotic MSE of the form

$$\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)] = \left(\frac{C_{\mathcal{B}}}{n \prod_{l=1}^{d} h_{l,n}} + \frac{C_{\mathcal{S}} C_{\mathcal{H}}}{\sigma_1^P |\boldsymbol{\Omega}|^{1/2}} h_{1,n}^P + \sum_{\ell=2}^{d} \mathcal{S}_\ell h_{\ell,n}^{P+1} \right)^2 \beta_1^2,$$

for some $\mathcal{S}_\ell$'s ("smoothing bias" constants) that should be non-zero. (If they are zero, then we will have to expand even further, leading to higher exponent on the $h_{\ell,n}$'s.) The resulting bandwidth choices will have different rates of convergence.

4. Appendix D: Additional Simulation Evidence

In this section we present the main results from our Monte Carlo experiment, which are collected in 30 tables (3 models, 10 tables per model). Each table reports results for both the classical estimator $\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$ and the generalized jackknife estimator $\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$, where $\mathbf{H}_n$ is either selected from a grid of possible bandwidths around the population MSE "optimal" bandwidth (denoted $\mathbf{H}_n^*$), or estimated using a ROT bandwidth choice (denoted $\hat{\mathbf{H}}_n$). Different tables corresponds to different ways of constructing the generalized jackknife estimator $\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$, as explained in the main text (and further below). For both estimators, each table reports (i) MSE, (ii) squared bias, (iii) variance, (iv) absolute bias divided by square-root of variance, and (v) coverage rates for 95% confidence intervals.

As explained in the paper, for a given bandwidth on the grid, each generalized jackknife estimator is constructed by employing the adjacent bandwidths in the grid as determined by the particular procedure considered. We investigated five different generalized jackknife estimators, which are described as follows:

- $J = 2$, $c_L = 2$, $c_U = 0$.
- $J = 1$, $c_L = 1$, $c_U = 0$.
- $J = 1$, $c_L = 0$, $c_U = 1$.

- $J = 2$, $c_L = 0$, $c_U = 2$.

- $J = 2$, $c_L = 1$, $c_U = 1$.

Each of these procedures describe the number of constants employed and in which direction(s). For example, the procedure $(J = 1, c_L = 0, c_U = 1)$ employs only $J = 1$ and uses the bandwidth above of the bandwidth under consideration on the grid to construct the generalized jackknife procedure. (Note that $J = c_L + c_U$ by construction.) Similarly, the procedure $(J = 2, c_L = 1, c_U = 1)$ uses $J = 2$ and employs the bandwidth below and the bandwidth above (of the bandwidth under consideration on the grid) to construct the generalized jackknife procedure.

Finally, we present results for the three models described in the main text when using either a common bandwidth or different bandwidths. The resulting 30 tables are organized as follows:

- Tables D1 – D5: Model 1, with common bandwidth ($\mathbf{H}_n = h_n \mathbf{I}_d$), for each of the 5 generalized jackknife procedures.

- Tables D6 – D10: Model 1, with different bandwidths ($\mathbf{H}_n = \text{diag}(h_{1,n}, h_{2,n}, h_{3,n})$), for each of the 5 generalized jackknife procedures.

- Tables D11 – D15: Model 2, with common bandwidth ($\mathbf{H}_n = h_n \mathbf{I}_d$), for each of the 5 generalized jackknife procedures.

- Tables D16 – D20: Model 2, with different bandwidths ($\mathbf{H}_n = \text{diag}(h_{1,n}, h_{2,n}, h_{3,n})$), for each of the 5 generalized jackknife procedures.

- Tables D21 – D25: Model 3, with common bandwidth ($\mathbf{H}_n = h_n \mathbf{I}_d$), for each of the 5 generalized jackknife procedures.

- Tables D26 – D30: Model 3, with different bandwidths ($\mathbf{H}_n = \text{diag}(h_{1,n}, h_{2,n}, h_{3,n})$), for each of the 5 generalized jackknife procedures.

Table D-1: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 1, $c_L = 2$, $c_U = 0$.

	$\mathbf{H}_n = h_n \mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	3.620	3.162	0.458	2.626	10.52					
	0.45	0.266	2.815	2.372	0.443	2.313	14.40					
	0.50	0.296	2.179	1.749	0.430	2.018	18.72	3.897	1.709	2.189	0.884	27.74
	0.55	0.325	1.699	1.281	0.418	1.750	23.44	3.382	1.412	1.970	0.847	29.06
	0.60	0.355	1.348	0.939	0.410	1.513	27.84	2.667	0.935	1.732	0.735	31.46
	0.65	0.384	1.098	0.694	0.404	1.310	32.66	1.980	0.494	1.487	0.576	36.20
	0.70	0.414	0.921	0.521	0.400	1.141	37.54	1.474	0.186	1.288	0.380	41.34
	0.75	0.443	0.798	0.401	0.398	1.004	41.42	1.180	0.028	1.152	0.156	44.56
	0.80	0.473	0.713	0.317	0.396	0.894	44.66	1.070	0.008	1.062	0.087	44.82
	0.85	0.502	0.655	0.260	0.396	0.810	47.04	1.116	0.117	0.998	0.343	41.76
	0.90	0.532	0.618	0.221	0.396	0.748	48.72	1.281	0.351	0.930	0.614	37.62
	0.95	0.562	0.594	0.197	0.397	0.705	49.90	1.570	0.704	0.866	0.901	31.88
$\mathbf{H}_n^*$	1.00	0.591	0.582	0.183	0.398	0.679	50.56	1.986	1.162	0.824	1.187	25.44
	1.05	0.621	0.578	0.178	0.400	0.667	50.88	2.508	1.711	0.796	1.466	19.84
	1.10	0.650	0.581	0.179	0.402	0.668	50.68	3.098	2.348	0.750	1.770	14.98
	1.15	0.680	0.590	0.187	0.403	0.681	50.24	3.778	3.063	0.715	2.070	10.72
	1.20	0.709	0.605	0.200	0.405	0.702	49.34	4.520	3.839	0.682	2.373	7.48
	1.25	0.739	0.624	0.218	0.406	0.732	48.30	5.306	4.659	0.647	2.683	5.02
	1.30	0.769	0.649	0.242	0.408	0.770	47.26	6.123	5.507	0.616	2.990	3.86
	1.35	0.798	0.679	0.270	0.409	0.813	45.92	6.952	6.363	0.588	3.289	2.90
	1.40	0.828	0.714	0.305	0.410	0.862	44.30	7.772	7.209	0.563	3.578	1.90
	1.45	0.857	0.754	0.344	0.411	0.915	43.16	8.569	8.029	0.540	3.854	1.42
	1.50	0.887	0.800	0.389	0.411	0.972	41.56	9.328	8.808	0.520	4.115	1.06
	1.55	0.916	0.850	0.438	0.412	1.032	39.94	10.039	9.537	0.502	4.358	0.92
	1.60	0.946	0.905	0.493	0.412	1.095	38.52	10.694	10.207	0.486	4.581	0.72
ROT-1d		0.565	0.593	0.196	0.397	0.703	49.84	1.619	0.748	0.871	0.926	31.26
ROT-tr		0.564	0.593	0.196	0.397	0.704	49.82	1.615	0.747	0.868	0.928	31.34

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-2: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 1, $c_L = 2$, $c_U = 0$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	0.236	0.236	3.620	3.162	0.458	2.626	10.52					
	0.45	0.266	0.266	0.266	2.815	2.372	0.443	2.313	14.40					
	0.50	0.296	0.296	0.296	2.179	1.749	0.430	2.018	18.72	3.897	1.709	2.189	0.884	27.74
	0.55	0.325	0.325	0.325	1.699	1.281	0.418	1.750	23.44	3.382	1.412	1.970	0.847	29.06
	0.60	0.355	0.355	0.355	1.348	0.939	0.410	1.513	27.84	2.667	0.935	1.732	0.735	31.46
	0.65	0.384	0.384	0.384	1.098	0.694	0.404	1.310	32.66	1.980	0.494	1.487	0.576	36.20
	0.70	0.414	0.414	0.414	0.921	0.521	0.400	1.141	37.54	1.474	0.186	1.288	0.380	41.34
	0.75	0.443	0.443	0.443	0.798	0.401	0.398	1.004	41.42	1.180	0.028	1.152	0.156	44.56
	0.80	0.473	0.473	0.473	0.713	0.317	0.396	0.894	44.66	1.070	0.008	1.062	0.087	44.82
	0.85	0.502	0.502	0.502	0.655	0.260	0.396	0.810	47.04	1.116	0.117	0.998	0.343	41.76
	0.90	0.532	0.532	0.532	0.618	0.221	0.396	0.748	48.72	1.281	0.351	0.930	0.614	37.62
	0.95	0.562	0.562	0.562	0.594	0.197	0.397	0.705	49.90	1.570	0.704	0.866	0.901	31.88
$\mathbf{H}_n^*$	1.00	0.591	0.591	0.591	0.582	0.183	0.398	0.679	50.56	1.986	1.162	0.824	1.187	25.44
	1.05	0.621	0.621	0.621	0.578	0.178	0.400	0.667	50.88	2.508	1.711	0.796	1.466	19.84
	1.10	0.650	0.650	0.650	0.581	0.179	0.402	0.668	50.68	3.098	2.348	0.750	1.770	14.98
	1.15	0.680	0.680	0.680	0.590	0.187	0.403	0.681	50.24	3.778	3.063	0.715	2.070	10.72
	1.20	0.709	0.709	0.709	0.605	0.200	0.405	0.702	49.34	4.520	3.839	0.682	2.373	7.48
	1.25	0.739	0.739	0.739	0.624	0.218	0.406	0.732	48.30	5.306	4.659	0.647	2.683	5.02
	1.30	0.769	0.769	0.769	0.649	0.242	0.408	0.770	47.26	6.123	5.507	0.616	2.990	3.86
	1.35	0.798	0.798	0.798	0.679	0.270	0.409	0.813	45.92	6.952	6.363	0.588	3.289	2.90
	1.40	0.828	0.828	0.828	0.714	0.305	0.410	0.862	44.30	7.772	7.209	0.563	3.578	1.90
	1.45	0.857	0.857	0.857	0.754	0.344	0.411	0.915	43.16	8.569	8.029	0.540	3.854	1.42
	1.50	0.887	0.887	0.887	0.800	0.389	0.411	0.972	41.56	9.328	8.808	0.520	4.115	1.06
	1.55	0.916	0.916	0.916	0.850	0.438	0.412	1.032	39.94	10.039	9.537	0.502	4.358	0.92
	1.60	0.946	0.946	0.946	0.905	0.493	0.412	1.095	38.52	10.694	10.207	0.486	4.581	0.72
ROT-tr		0.565	0.565	0.565	0.593	0.196	0.397	0.703	49.84	1.619	0.748	0.871	0.926	31.26

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-3: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 1, $c_L = 1$, $c_U = 0$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	3.620	3.162	0.458	2.626	10.52					
	0.45	0.266	2.815	2.372	0.443	2.313	14.40	1.435	0.957	0.478	1.414	36.84
	0.50	0.296	2.179	1.749	0.430	2.018	18.72	1.001	0.545	0.456	1.092	45.62
	0.55	0.325	1.699	1.281	0.418	1.750	23.44	0.749	0.308	0.442	0.835	52.86
	0.60	0.355	1.348	0.939	0.410	1.513	27.84	0.611	0.179	0.432	0.643	56.74
	0.65	0.384	1.098	0.694	0.404	1.310	32.66	0.536	0.111	0.425	0.510	58.44
	0.70	0.414	0.921	0.521	0.400	1.141	37.54	0.497	0.076	0.420	0.426	59.24
	0.75	0.443	0.798	0.401	0.398	1.004	41.42	0.477	0.060	0.417	0.380	59.40
	0.80	0.473	0.713	0.317	0.396	0.894	44.66	0.471	0.055	0.416	0.365	58.78
	0.85	0.502	0.655	0.260	0.396	0.810	47.04	0.475	0.059	0.416	0.375	58.34
	0.90	0.532	0.618	0.221	0.396	0.748	48.72	0.487	0.069	0.418	0.405	57.56
	0.95	0.562	0.594	0.197	0.397	0.705	49.90	0.506	0.086	0.420	0.452	55.96
$\mathbf{H}^*_n$	1.00	0.591	0.582	0.183	0.398	0.679	50.56	0.533	0.111	0.422	0.513	54.42
	1.05	0.621	0.578	0.178	0.400	0.667	50.88	0.570	0.145	0.424	0.585	51.84
	1.10	0.650	0.581	0.179	0.402	0.668	50.68	0.615	0.189	0.426	0.665	49.54
	1.15	0.680	0.590	0.187	0.403	0.681	50.24	0.671	0.243	0.428	0.754	46.66
	1.20	0.709	0.605	0.200	0.405	0.702	49.34	0.737	0.308	0.429	0.848	44.08
	1.25	0.739	0.624	0.218	0.406	0.732	48.30	0.815	0.385	0.429	0.947	40.94
	1.30	0.769	0.649	0.242	0.408	0.770	47.26	0.903	0.474	0.429	1.051	38.14
	1.35	0.798	0.679	0.270	0.409	0.813	45.92	1.003	0.574	0.429	1.157	35.46
	1.40	0.828	0.714	0.305	0.410	0.862	44.30	1.113	0.685	0.428	1.265	32.32
	1.45	0.857	0.754	0.344	0.411	0.915	43.16	1.233	0.806	0.427	1.375	29.64
	1.50	0.887	0.800	0.389	0.411	0.972	41.56	1.362	0.937	0.425	1.485	27.22
	1.55	0.916	0.850	0.438	0.412	1.032	39.94	1.500	1.077	0.423	1.595	24.74
	1.60	0.946	0.905	0.493	0.412	1.095	38.52	1.645	1.224	0.421	1.705	22.66
ROT-1d		0.565	0.593	0.196	0.397	0.703	49.84	0.510	0.089	0.421	0.459	55.78
ROT-tr		0.564	0.593	0.196	0.397	0.704	49.82	0.510	0.088	0.421	0.458	55.80

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}^{**}_n(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}^{**}_n(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-4: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 1, $c_L = 1$, $c_U = 0$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	0.236	0.236	3.620	3.162	0.458	2.626	10.52					
	0.45	0.266	0.266	0.266	2.815	2.372	0.443	2.313	14.40	1.435	0.957	0.478	1.414	36.84
	0.50	0.296	0.296	0.296	2.179	1.749	0.430	2.018	18.72	1.001	0.545	0.456	1.092	45.62
	0.55	0.325	0.325	0.325	1.699	1.281	0.418	1.750	23.44	0.749	0.308	0.442	0.835	52.86
	0.60	0.355	0.355	0.355	1.348	0.939	0.410	1.513	27.84	0.611	0.179	0.432	0.643	56.74
	0.65	0.384	0.384	0.384	1.098	0.694	0.404	1.310	32.66	0.536	0.111	0.425	0.510	58.44
	0.70	0.414	0.414	0.414	0.921	0.521	0.400	1.141	37.54	0.497	0.076	0.420	0.426	59.24
	0.75	0.443	0.443	0.443	0.798	0.401	0.398	1.004	41.42	0.477	0.060	0.417	0.380	59.40
	0.80	0.473	0.473	0.473	0.713	0.317	0.396	0.894	44.66	0.471	0.055	0.416	0.365	58.78
	0.85	0.502	0.502	0.502	0.655	0.260	0.396	0.810	47.04	0.475	0.059	0.416	0.375	58.34
	0.90	0.532	0.532	0.532	0.618	0.221	0.396	0.748	48.72	0.487	0.069	0.418	0.405	57.56
	0.95	0.562	0.562	0.562	0.594	0.197	0.397	0.705	49.90	0.506	0.086	0.420	0.452	55.96
$\mathbf{H}_n^*$	1.00	0.591	0.591	0.591	0.582	0.183	0.398	0.679	50.56	0.533	0.111	0.422	0.513	54.42
	1.05	0.621	0.621	0.621	0.578	0.178	0.400	0.667	50.88	0.570	0.145	0.424	0.585	51.84
	1.10	0.650	0.650	0.650	0.581	0.179	0.402	0.668	50.68	0.615	0.189	0.426	0.665	49.54
	1.15	0.680	0.680	0.680	0.590	0.187	0.403	0.681	50.24	0.671	0.243	0.428	0.754	46.66
	1.20	0.709	0.709	0.709	0.605	0.200	0.405	0.702	49.34	0.737	0.308	0.429	0.848	44.08
	1.25	0.739	0.739	0.739	0.624	0.218	0.406	0.732	48.30	0.815	0.385	0.429	0.947	40.94
	1.30	0.769	0.769	0.769	0.649	0.242	0.408	0.770	47.26	0.903	0.474	0.429	1.051	38.14
	1.35	0.798	0.798	0.798	0.679	0.270	0.409	0.813	45.92	1.003	0.574	0.429	1.157	35.46
	1.40	0.828	0.828	0.828	0.714	0.305	0.410	0.862	44.30	1.113	0.685	0.428	1.265	32.32
	1.45	0.857	0.857	0.857	0.754	0.344	0.411	0.915	43.16	1.233	0.806	0.427	1.375	29.64
	1.50	0.887	0.887	0.887	0.800	0.389	0.411	0.972	41.56	1.362	0.937	0.425	1.485	27.22
	1.55	0.916	0.916	0.916	0.850	0.438	0.412	1.032	39.94	1.500	1.077	0.423	1.595	24.74
	1.60	0.946	0.946	0.946	0.905	0.493	0.412	1.095	38.52	1.645	1.224	0.421	1.705	22.66
ROT-tr		0.565	0.565	0.565	0.593	0.196	0.397	0.703	49.84	0.510	0.089	0.422	0.459	55.78

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-5: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 1, $c_L = 0$, $c_U = 1$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	3.620	3.162	0.458	2.626	10.52	1.435	0.957	0.478	1.414	40.12
	0.45	0.266	2.815	2.372	0.443	2.313	14.40	1.001	0.545	0.456	1.092	49.40
	0.50	0.296	2.179	1.749	0.430	2.018	18.72	0.749	0.308	0.442	0.835	56.48
	0.55	0.325	1.699	1.281	0.418	1.750	23.44	0.611	0.179	0.432	0.643	59.60
	0.60	0.355	1.348	0.939	0.410	1.513	27.84	0.536	0.111	0.425	0.510	60.38
	0.65	0.384	1.098	0.694	0.404	1.310	32.66	0.497	0.076	0.420	0.426	61.04
	0.70	0.414	0.921	0.521	0.400	1.141	37.54	0.477	0.060	0.417	0.380	60.50
	0.75	0.443	0.798	0.401	0.398	1.004	41.42	0.471	0.055	0.416	0.365	59.56
	0.80	0.473	0.713	0.317	0.396	0.894	44.66	0.475	0.059	0.416	0.375	58.66
	0.85	0.502	0.655	0.260	0.396	0.810	47.04	0.487	0.069	0.418	0.405	57.72
	0.90	0.532	0.618	0.221	0.396	0.748	48.72	0.506	0.086	0.420	0.452	56.12
	0.95	0.562	0.594	0.197	0.397	0.705	49.90	0.533	0.111	0.422	0.513	54.32
$\mathbf{H}_n^*$	1.00	0.591	0.582	0.183	0.398	0.679	50.56	0.570	0.145	0.424	0.585	51.80
	1.05	0.621	0.578	0.178	0.400	0.667	50.88	0.615	0.189	0.426	0.665	49.44
	1.10	0.650	0.581	0.179	0.402	0.668	50.68	0.671	0.243	0.428	0.754	46.62
	1.15	0.680	0.590	0.187	0.403	0.681	50.24	0.737	0.308	0.429	0.848	43.94
	1.20	0.709	0.605	0.200	0.405	0.702	49.34	0.815	0.385	0.429	0.947	40.82
	1.25	0.739	0.624	0.218	0.406	0.732	48.30	0.903	0.474	0.429	1.051	37.98
	1.30	0.769	0.649	0.242	0.408	0.770	47.26	1.003	0.574	0.429	1.157	35.22
	1.35	0.798	0.679	0.270	0.409	0.813	45.92	1.113	0.685	0.428	1.265	32.14
	1.40	0.828	0.714	0.305	0.410	0.862	44.30	1.233	0.806	0.427	1.375	29.44
	1.45	0.857	0.754	0.344	0.411	0.915	43.16	1.362	0.937	0.425	1.485	26.94
	1.50	0.887	0.800	0.389	0.411	0.972	41.56	1.500	1.077	0.423	1.595	24.58
	1.55	0.916	0.850	0.438	0.412	1.032	39.94	1.645	1.224	0.421	1.705	22.40
	1.60	0.946	0.905	0.493	0.412	1.095	38.52					
ROT-1d		0.565	0.593	0.196	0.397	0.703	49.84	0.537	0.113	0.424	0.517	53.78
ROT-tr		0.564	0.593	0.196	0.397	0.704	49.82	0.536	0.113	0.424	0.516	54.00

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-6: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 1, $c_L = 0$, $c_U = 1$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	0.236	0.236	3.620	3.162	0.458	2.626	10.52	1.435	0.957	0.478	1.414	40.12
	0.45	0.266	0.266	0.266	2.815	2.372	0.443	2.313	14.40	1.001	0.545	0.456	1.092	49.40
	0.50	0.296	0.296	0.296	2.179	1.749	0.430	2.018	18.72	0.749	0.308	0.442	0.835	56.48
	0.55	0.325	0.325	0.325	1.699	1.281	0.418	1.750	23.44	0.611	0.179	0.432	0.643	59.60
	0.60	0.355	0.355	0.355	1.348	0.939	0.410	1.513	27.84	0.536	0.111	0.425	0.510	60.38
	0.65	0.384	0.384	0.384	1.098	0.694	0.404	1.310	32.66	0.497	0.076	0.420	0.426	61.04
	0.70	0.414	0.414	0.414	0.921	0.521	0.400	1.141	37.54	0.477	0.060	0.417	0.380	60.50
	0.75	0.443	0.443	0.443	0.798	0.401	0.398	1.004	41.42	0.471	0.055	0.416	0.365	59.56
	0.80	0.473	0.473	0.473	0.713	0.317	0.396	0.894	44.66	0.475	0.059	0.416	0.375	58.66
	0.85	0.502	0.502	0.502	0.655	0.260	0.396	0.810	47.04	0.487	0.069	0.418	0.405	57.72
	0.90	0.532	0.532	0.532	0.618	0.221	0.396	0.748	48.72	0.506	0.086	0.420	0.452	56.12
	0.95	0.562	0.562	0.562	0.594	0.197	0.397	0.705	49.90	0.533	0.111	0.422	0.513	54.32
$\mathbf{H}_n^*$	1.00	0.591	0.591	0.591	0.582	0.183	0.398	0.679	50.56	0.570	0.145	0.424	0.585	51.80
	1.05	0.621	0.621	0.621	0.578	0.178	0.400	0.667	50.88	0.615	0.189	0.426	0.665	49.44
	1.10	0.650	0.650	0.650	0.581	0.179	0.402	0.668	50.68	0.671	0.243	0.428	0.754	46.62
	1.15	0.680	0.680	0.680	0.590	0.187	0.403	0.681	50.24	0.737	0.308	0.429	0.848	43.94
	1.20	0.709	0.709	0.709	0.605	0.200	0.405	0.702	49.34	0.815	0.385	0.429	0.947	40.82
	1.25	0.739	0.739	0.739	0.624	0.218	0.406	0.732	48.30	0.903	0.474	0.429	1.051	37.98
	1.30	0.769	0.769	0.769	0.649	0.242	0.408	0.770	47.26	1.003	0.574	0.429	1.157	35.22
	1.35	0.798	0.798	0.798	0.679	0.270	0.409	0.813	45.92	1.113	0.685	0.428	1.265	32.14
	1.40	0.828	0.828	0.828	0.714	0.305	0.410	0.862	44.30	1.233	0.806	0.427	1.375	29.44
	1.45	0.857	0.857	0.857	0.754	0.344	0.411	0.915	43.16	1.362	0.937	0.425	1.485	26.94
	1.50	0.887	0.887	0.887	0.800	0.389	0.411	0.972	41.56	1.500	1.077	0.423	1.595	24.58
	1.55	0.916	0.916	0.916	0.850	0.438	0.412	1.032	39.94	1.645	1.224	0.421	1.705	22.40
	1.60	0.946	0.946	0.946	0.905	0.493	0.412	1.095	38.52					
ROT-tr		0.565	0.565	0.565	0.593	0.196	0.397	0.703	49.84	0.537	0.113	0.424	0.517	53.78

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-7: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 1, $c_L = 0$, $c_U = 2$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	3.620	3.162	0.458	2.626	10.52	3.897	1.709	2.189	0.884	32.64
	0.45	0.266	2.815	2.372	0.443	2.313	14.40	3.382	1.412	1.970	0.847	33.46
	0.50	0.296	2.179	1.749	0.430	2.018	18.72	2.667	0.935	1.732	0.735	35.98
	0.55	0.325	1.699	1.281	0.418	1.750	23.44	1.980	0.494	1.487	0.576	39.48
	0.60	0.355	1.348	0.939	0.410	1.513	27.84	1.474	0.186	1.288	0.380	44.44
	0.65	0.384	1.098	0.694	0.404	1.310	32.66	1.180	0.028	1.152	0.156	46.32
	0.70	0.414	0.921	0.521	0.400	1.141	37.54	1.070	0.008	1.062	0.087	45.88
	0.75	0.443	0.798	0.401	0.398	1.004	41.42	1.116	0.117	0.998	0.343	42.48
	0.80	0.473	0.713	0.317	0.396	0.894	44.66	1.281	0.351	0.930	0.614	37.58
	0.85	0.502	0.655	0.260	0.396	0.810	47.04	1.570	0.704	0.866	0.901	31.86
	0.90	0.532	0.618	0.221	0.396	0.748	48.72	1.986	1.162	0.824	1.187	25.30
	0.95	0.562	0.594	0.197	0.397	0.705	49.90	2.508	1.711	0.796	1.466	19.46
$\mathbf{H}_n^*$	1.00	0.591	0.582	0.183	0.398	0.679	50.56	3.098	2.348	0.750	1.770	14.70
	1.05	0.621	0.578	0.178	0.400	0.667	50.88	3.778	3.063	0.715	2.070	10.50
	1.10	0.650	0.581	0.179	0.402	0.668	50.68	4.520	3.839	0.682	2.373	7.46
	1.15	0.680	0.590	0.187	0.403	0.681	50.24	5.306	4.659	0.647	2.683	4.94
	1.20	0.709	0.605	0.200	0.405	0.702	49.34	6.123	5.507	0.616	2.990	3.70
	1.25	0.739	0.624	0.218	0.406	0.732	48.30	6.952	6.363	0.588	3.289	2.74
	1.30	0.769	0.649	0.242	0.408	0.770	47.26	7.772	7.209	0.563	3.578	1.82
	1.35	0.798	0.679	0.270	0.409	0.813	45.92	8.569	8.029	0.540	3.854	1.34
	1.40	0.828	0.714	0.305	0.410	0.862	44.30	9.328	8.808	0.520	4.115	0.98
	1.45	0.857	0.754	0.344	0.411	0.915	43.16	10.039	9.537	0.502	4.358	0.82
	1.50	0.887	0.800	0.389	0.411	0.972	41.56	10.694	10.207	0.486	4.581	0.68
	1.55	0.916	0.850	0.438	0.412	1.032	39.94					
	1.60	0.946	0.905	0.493	0.412	1.095	38.52					
ROT-1d		0.565	0.593	0.196	0.397	0.703	49.84	2.506	1.717	0.789	1.475	19.84
ROT-tr		0.564	0.593	0.196	0.397	0.704	49.82	2.516	1.713	0.803	1.460	19.94

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-8: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 1, $c_L = 0$, $c_U = 2$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	0.236	0.236	3.620	3.162	0.458	2.626	10.52	3.897	1.709	2.189	0.884	32.64
	0.45	0.266	0.266	0.266	2.815	2.372	0.443	2.313	14.40	3.382	1.412	1.970	0.847	33.46
	0.50	0.296	0.296	0.296	2.179	1.749	0.430	2.018	18.72	2.667	0.935	1.732	0.735	35.98
	0.55	0.325	0.325	0.325	1.699	1.281	0.418	1.750	23.44	1.980	0.494	1.487	0.576	39.48
	0.60	0.355	0.355	0.355	1.348	0.939	0.410	1.513	27.84	1.474	0.186	1.288	0.380	44.44
	0.65	0.384	0.384	0.384	1.098	0.694	0.404	1.310	32.66	1.180	0.028	1.152	0.156	46.32
	0.70	0.414	0.414	0.414	0.921	0.521	0.400	1.141	37.54	1.070	0.008	1.062	0.087	45.88
	0.75	0.443	0.443	0.443	0.798	0.401	0.398	1.004	41.42	1.116	0.117	0.998	0.343	42.48
	0.80	0.473	0.473	0.473	0.713	0.317	0.396	0.894	44.66	1.281	0.351	0.930	0.614	37.58
	0.85	0.502	0.502	0.502	0.655	0.260	0.396	0.810	47.04	1.570	0.704	0.866	0.901	31.86
	0.90	0.532	0.532	0.532	0.618	0.221	0.396	0.748	48.72	1.986	1.162	0.824	1.187	25.30
	0.95	0.562	0.562	0.562	0.594	0.197	0.397	0.705	49.90	2.508	1.711	0.796	1.466	19.46
$\mathbf{H}_n^*$	1.00	0.591	0.591	0.591	0.582	0.183	0.398	0.679	50.56	3.098	2.348	0.750	1.770	14.70
	1.05	0.621	0.621	0.621	0.578	0.178	0.400	0.667	50.88	3.778	3.063	0.715	2.070	10.50
	1.10	0.650	0.650	0.650	0.581	0.179	0.402	0.668	50.68	4.520	3.839	0.682	2.373	7.46
	1.15	0.680	0.680	0.680	0.590	0.187	0.403	0.681	50.24	5.306	4.659	0.647	2.683	4.94
	1.20	0.709	0.709	0.709	0.605	0.200	0.405	0.702	49.34	6.123	5.507	0.616	2.990	3.70
	1.25	0.739	0.739	0.739	0.624	0.218	0.406	0.732	48.30	6.952	6.363	0.588	3.289	2.74
	1.30	0.769	0.769	0.769	0.649	0.242	0.408	0.770	47.26	7.772	7.209	0.563	3.578	1.82
	1.35	0.798	0.798	0.798	0.679	0.270	0.409	0.813	45.92	8.569	8.029	0.540	3.854	1.34
	1.40	0.828	0.828	0.828	0.714	0.305	0.410	0.862	44.30	9.328	8.808	0.520	4.115	0.98
	1.45	0.857	0.857	0.857	0.754	0.344	0.411	0.915	43.16	10.039	9.537	0.502	4.358	0.82
	1.50	0.887	0.887	0.887	0.800	0.389	0.411	0.972	41.56	10.694	10.207	0.486	4.581	0.68
	1.55	0.916	0.916	0.916	0.850	0.438	0.412	1.032	39.94					
	1.60	0.946	0.946	0.946	0.905	0.493	0.412	1.095	38.52					
ROT-tr		0.565	0.565	0.565	0.593	0.196	0.397	0.703	49.84	2.506	1.717	0.789	1.475	19.84

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-9: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 1, $c_L = 1$, $c_U = 1$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	3.620	3.162	0.458	2.626	10.52					
	0.45	0.266	2.815	2.372	0.443	2.313	14.40	3.897	1.709	2.189	0.884	29.98
	0.50	0.296	2.179	1.749	0.430	2.018	18.72	3.382	1.412	1.970	0.847	30.96
	0.55	0.325	1.699	1.281	0.418	1.750	23.44	2.667	0.935	1.732	0.735	33.52
	0.60	0.355	1.348	0.939	0.410	1.513	27.84	1.980	0.494	1.487	0.576	37.86
	0.65	0.384	1.098	0.694	0.404	1.310	32.66	1.474	0.186	1.288	0.380	42.58
	0.70	0.414	0.921	0.521	0.400	1.141	37.54	1.180	0.028	1.152	0.156	45.16
	0.75	0.443	0.798	0.401	0.398	1.004	41.42	1.070	0.008	1.062	0.087	45.12
	0.80	0.473	0.713	0.317	0.396	0.894	44.66	1.116	0.117	0.998	0.343	42.04
	0.85	0.502	0.655	0.260	0.396	0.810	47.04	1.281	0.351	0.930	0.614	37.58
	0.90	0.532	0.618	0.221	0.396	0.748	48.72	1.570	0.704	0.866	0.901	31.82
	0.95	0.562	0.594	0.197	0.397	0.705	49.90	1.986	1.162	0.824	1.187	25.44
$\mathbf{H}_n^*$	1.00	0.591	0.582	0.183	0.398	0.679	50.56	2.508	1.711	0.796	1.466	19.56
	1.05	0.621	0.578	0.178	0.400	0.667	50.88	3.098	2.348	0.750	1.770	14.84
	1.10	0.650	0.581	0.179	0.402	0.668	50.68	3.778	3.063	0.715	2.070	10.60
	1.15	0.680	0.590	0.187	0.403	0.681	50.24	4.520	3.839	0.682	2.373	7.44
	1.20	0.709	0.605	0.200	0.405	0.702	49.34	5.306	4.659	0.647	2.683	4.98
	1.25	0.739	0.624	0.218	0.406	0.732	48.30	6.123	5.507	0.616	2.990	3.80
	1.30	0.769	0.649	0.242	0.408	0.770	47.26	6.952	6.363	0.588	3.289	2.84
	1.35	0.798	0.679	0.270	0.409	0.813	45.92	7.772	7.209	0.563	3.578	1.82
	1.40	0.828	0.714	0.305	0.410	0.862	44.30	8.569	8.029	0.540	3.854	1.40
	1.45	0.857	0.754	0.344	0.411	0.915	43.16	9.328	8.808	0.520	4.115	1.00
	1.50	0.887	0.800	0.389	0.411	0.972	41.56	10.039	9.537	0.502	4.358	0.86
	1.55	0.916	0.850	0.438	0.412	1.032	39.94	10.694	10.207	0.486	4.581	0.68
	1.60	0.946	0.905	0.493	0.412	1.095	38.52					
ROT-1d		0.565	0.593	0.196	0.397	0.703	49.84	2.020	1.189	0.831	1.197	24.84
ROT-tr		0.564	0.593	0.196	0.397	0.704	49.82	2.017	1.189	0.828	1.198	25.00

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\mathrm{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-10: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 1, $c_L = 1$, $c_U = 1$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.236	0.236	0.236	3.620	3.162	0.458	2.626	10.52					
	0.45	0.266	0.266	0.266	2.815	2.372	0.443	2.313	14.40	3.897	1.709	2.189	0.884	29.98
	0.50	0.296	0.296	0.296	2.179	1.749	0.430	2.018	18.72	3.382	1.412	1.970	0.847	30.96
	0.55	0.325	0.325	0.325	1.699	1.281	0.418	1.750	23.44	2.667	0.935	1.732	0.735	33.52
	0.60	0.355	0.355	0.355	1.348	0.939	0.410	1.513	27.84	1.980	0.494	1.487	0.576	37.86
	0.65	0.384	0.384	0.384	1.098	0.694	0.404	1.310	32.66	1.474	0.186	1.288	0.380	42.58
	0.70	0.414	0.414	0.414	0.921	0.521	0.400	1.141	37.54	1.180	0.028	1.152	0.156	45.16
	0.75	0.443	0.443	0.443	0.798	0.401	0.398	1.004	41.42	1.070	0.008	1.062	0.087	45.12
	0.80	0.473	0.473	0.473	0.713	0.317	0.396	0.894	44.66	1.116	0.117	0.998	0.343	42.04
	0.85	0.502	0.502	0.502	0.655	0.260	0.396	0.810	47.04	1.281	0.351	0.930	0.614	37.58
	0.90	0.532	0.532	0.532	0.618	0.221	0.396	0.748	48.72	1.570	0.704	0.866	0.901	31.82
	0.95	0.562	0.562	0.562	0.594	0.197	0.397	0.705	49.90	1.986	1.162	0.824	1.187	25.44
$\mathbf{H}_n^*$	1.00	0.591	0.591	0.591	0.582	0.183	0.398	0.679	50.56	2.508	1.711	0.796	1.466	19.56
	1.05	0.621	0.621	0.621	0.578	0.178	0.400	0.667	50.88	3.098	2.348	0.750	1.770	14.84
	1.10	0.650	0.650	0.650	0.581	0.179	0.402	0.668	50.68	3.778	3.063	0.715	2.070	10.60
	1.15	0.680	0.680	0.680	0.590	0.187	0.403	0.681	50.24	4.520	3.839	0.682	2.373	7.44
	1.20	0.709	0.709	0.709	0.605	0.200	0.405	0.702	49.34	5.306	4.659	0.647	2.683	4.98
	1.25	0.739	0.739	0.739	0.624	0.218	0.406	0.732	48.30	6.123	5.507	0.616	2.990	3.80
	1.30	0.769	0.769	0.769	0.649	0.242	0.408	0.770	47.26	6.952	6.363	0.588	3.289	2.84
	1.35	0.798	0.798	0.798	0.679	0.270	0.409	0.813	45.92	7.772	7.209	0.563	3.578	1.82
	1.40	0.828	0.828	0.828	0.714	0.305	0.410	0.862	44.30	8.569	8.029	0.540	3.854	1.40
	1.45	0.857	0.857	0.857	0.754	0.344	0.411	0.915	43.16	9.328	8.808	0.520	4.115	1.00
	1.50	0.887	0.887	0.887	0.800	0.389	0.411	0.972	41.56	10.039	9.537	0.502	4.358	0.86
	1.55	0.916	0.916	0.916	0.850	0.438	0.412	1.032	39.94	10.694	10.207	0.486	4.581	0.68
	1.60	0.946	0.946	0.946	0.905	0.493	0.412	1.095	38.52					
ROT-tr		0.565	0.565	0.565	0.593	0.196	0.397	0.703	49.84	2.020	1.189	0.831	1.197	24.84

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-11: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 2, $c_L = 2$, $c_U = 0$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.232	2.024	1.560	0.464	1.834	26.60					
	0.45	0.261	1.522	1.082	0.440	1.568	31.54					
	0.50	0.290	1.162	0.740	0.423	1.323	36.28	3.234	1.186	2.049	0.761	30.06
	0.55	0.319	0.916	0.504	0.411	1.107	40.60	2.438	0.753	1.685	0.668	32.72
	0.60	0.348	0.751	0.346	0.405	0.925	44.12	1.801	0.381	1.420	0.518	37.28
	0.65	0.377	0.643	0.242	0.401	0.778	47.36	1.357	0.141	1.216	0.340	41.28
	0.70	0.406	0.572	0.174	0.399	0.660	50.14	1.082	0.025	1.056	0.155	43.98
	0.75	0.435	0.527	0.129	0.398	0.569	52.44	0.954	0.001	0.954	0.025	45.94
	0.80	0.464	0.499	0.100	0.399	0.500	53.80	0.914	0.034	0.880	0.198	46.32
	0.85	0.493	0.481	0.081	0.400	0.450	54.86	0.926	0.114	0.813	0.374	44.48
	0.90	0.522	0.471	0.069	0.402	0.415	55.58	1.002	0.235	0.767	0.554	41.64
	0.95	0.551	0.466	0.062	0.403	0.393	55.96	1.128	0.399	0.729	0.740	37.70
$\mathbf{H}_n^*$	1.00	0.580	0.464	0.059	0.405	0.382	56.22	1.298	0.604	0.694	0.933	33.82
	1.05	0.609	0.466	0.059	0.407	0.380	56.04	1.516	0.850	0.666	1.130	28.56
	1.10	0.638	0.470	0.061	0.409	0.387	55.76	1.775	1.133	0.643	1.327	24.80
	1.15	0.667	0.476	0.066	0.410	0.400	55.16	2.073	1.450	0.623	1.526	20.30
	1.20	0.696	0.484	0.072	0.412	0.419	54.30	2.405	1.798	0.607	1.722	17.28
	1.25	0.725	0.494	0.081	0.413	0.443	53.48	3.421	2.139	1.282	1.292	14.16
	1.30	0.754	0.506	0.092	0.414	0.472	52.72	3.427	2.590	0.836	1.760	11.80
	1.35	0.783	0.521	0.106	0.415	0.505	52.16	4.812	3.029	1.784	1.303	9.58
	1.40	0.812	0.537	0.121	0.416	0.540	50.84	4.417	3.352	1.066	1.774	8.10
	1.45	0.841	0.556	0.140	0.417	0.579	49.80	4.348	3.801	0.547	2.635	6.88
	1.50	0.870	0.577	0.160	0.417	0.620	48.64	4.745	4.214	0.531	2.816	5.72
	1.55	0.899	0.601	0.183	0.417	0.663	47.12	5.135	4.616	0.520	2.981	4.92
	1.60	0.928	0.627	0.209	0.418	0.708	45.72	5.512	5.004	0.508	3.138	4.12
ROT-1d		0.549	0.467	0.063	0.404	0.396	55.92	1.113	0.380	0.734	0.719	38.44
ROT-tr		0.516	0.473	0.072	0.401	0.422	55.40	1.000	0.220	0.780	0.531	42.12

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-12: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 2, $c_L = 2$, $c_U = 0$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.212	0.221	0.212	2.402	1.919	0.483	1.994	24.00					
	0.45	0.238	0.248	0.238	1.845	1.387	0.458	1.741	29.00					
	0.50	0.265	0.276	0.265	1.420	0.984	0.437	1.501	33.06	3.802	1.426	2.376	0.775	29.66
	0.55	0.291	0.303	0.291	1.112	0.690	0.422	1.280	37.70	3.102	1.092	2.011	0.737	30.62
	0.60	0.318	0.331	0.318	0.895	0.484	0.411	1.085	41.28	2.360	0.692	1.668	0.644	33.40
	0.65	0.344	0.358	0.344	0.746	0.342	0.405	0.919	44.68	1.786	0.363	1.423	0.505	37.40
	0.70	0.371	0.386	0.371	0.646	0.245	0.401	0.781	47.44	1.378	0.147	1.231	0.345	41.16
	0.75	0.397	0.413	0.397	0.578	0.179	0.399	0.670	50.16	1.113	0.035	1.078	0.181	43.82
	0.80	0.423	0.441	0.423	0.533	0.135	0.398	0.581	52.36	0.974	0.000	0.974	0.020	45.86
	0.85	0.450	0.469	0.450	0.503	0.104	0.398	0.512	53.88	0.918	0.016	0.902	0.134	46.38
	0.90	0.476	0.496	0.476	0.484	0.084	0.399	0.459	55.14	0.905	0.070	0.835	0.290	46.20
	0.95	0.503	0.524	0.503	0.472	0.071	0.401	0.420	55.56	0.945	0.160	0.785	0.451	43.58
$\mathbf{H}_n^*$	1.00	0.529	0.551	0.529	0.465	0.062	0.402	0.393	56.02	1.031	0.283	0.748	0.615	41.16
	1.05	0.556	0.579	0.556	0.461	0.057	0.404	0.377	56.46	1.155	0.441	0.714	0.786	37.40
	1.10	0.582	0.606	0.582	0.461	0.055	0.406	0.369	56.68	1.316	0.633	0.684	0.962	33.12
	1.15	0.609	0.634	0.609	0.462	0.055	0.407	0.368	56.46	1.518	0.858	0.661	1.139	28.46
	1.20	0.635	0.662	0.635	0.466	0.057	0.409	0.374	56.08	1.754	1.114	0.641	1.318	24.90
	1.25	0.662	0.689	0.662	0.471	0.061	0.410	0.386	55.56	2.022	1.399	0.624	1.498	20.72
	1.30	0.688	0.717	0.688	0.478	0.067	0.412	0.402	54.78	2.319	1.710	0.609	1.676	18.00
	1.35	0.715	0.744	0.715	0.488	0.074	0.414	0.422	54.32	43.887	1.793	42.103	0.206	15.02
	1.40	0.741	0.772	0.741	0.497	0.083	0.414	0.448	53.34	180.579	3.016	177.598	0.130	12.88
	1.45	0.767	0.799	0.767	0.509	0.094	0.415	0.476	52.76	71.537	2.389	69.162	0.186	10.74
	1.50	0.794	0.827	0.794	0.523	0.107	0.416	0.508	51.86	7.045	3.231	3.814	0.920	9.06
	1.55	0.820	0.854	0.820	0.538	0.122	0.417	0.541	50.84	4.179	3.501	0.678	2.273	7.70
	1.60	0.847	0.882	0.847	0.556	0.139	0.417	0.577	49.78	4.438	3.897	0.541	2.683	6.66
ROT-tr		0.563	0.484	0.564	0.484	0.079	0.405	0.442	54.30	1.252	0.486	0.766	0.797	35.30

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-13: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 2, $c_L = 1$, $c_U = 0$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.232	2.024	1.560	0.464	1.834	26.60					
	0.45	0.261	1.522	1.082	0.440	1.568	31.54	0.757	0.299	0.458	0.809	57.48
	0.50	0.290	1.162	0.740	0.423	1.323	36.28	0.581	0.141	0.440	0.566	61.56
	0.55	0.319	0.916	0.504	0.411	1.107	40.60	0.496	0.066	0.430	0.392	62.94
	0.60	0.348	0.751	0.346	0.405	0.925	44.12	0.457	0.033	0.424	0.278	63.94
	0.65	0.377	0.643	0.242	0.401	0.778	47.36	0.439	0.019	0.421	0.211	62.90
	0.70	0.406	0.572	0.174	0.399	0.660	50.14	0.432	0.013	0.419	0.177	62.52
	0.75	0.435	0.527	0.129	0.398	0.569	52.44	0.432	0.012	0.420	0.168	61.90
	0.80	0.464	0.499	0.100	0.399	0.500	53.80	0.434	0.013	0.421	0.175	61.24
	0.85	0.493	0.481	0.081	0.400	0.450	54.86	0.439	0.016	0.423	0.195	60.34
	0.90	0.522	0.471	0.069	0.402	0.415	55.58	0.447	0.022	0.425	0.226	59.46
	0.95	0.551	0.466	0.062	0.403	0.393	55.96	0.457	0.030	0.427	0.266	58.32
$\mathbf{H}_n^*$	1.00	0.580	0.464	0.059	0.405	0.382	56.22	0.470	0.042	0.429	0.313	56.94
	1.05	0.609	0.466	0.059	0.407	0.380	56.04	0.487	0.057	0.430	0.366	55.40
	1.10	0.638	0.470	0.061	0.409	0.387	55.76	0.508	0.077	0.431	0.424	53.30
	1.15	0.667	0.476	0.066	0.410	0.400	55.16	0.533	0.102	0.431	0.486	51.64
	1.20	0.696	0.484	0.072	0.412	0.419	54.30	0.563	0.131	0.432	0.552	50.16
	1.25	0.725	0.494	0.081	0.413	0.443	53.48	0.600	0.166	0.434	0.619	48.50
	1.30	0.754	0.506	0.092	0.414	0.472	52.72	0.639	0.207	0.432	0.692	46.16
	1.35	0.783	0.521	0.106	0.415	0.505	52.16	0.685	0.253	0.432	0.766	43.94
	1.40	0.812	0.537	0.121	0.416	0.540	50.84	0.735	0.304	0.431	0.840	42.06
	1.45	0.841	0.556	0.140	0.417	0.579	49.80	0.791	0.361	0.430	0.915	39.84
	1.50	0.870	0.577	0.160	0.417	0.620	48.64	0.851	0.422	0.430	0.991	37.58
	1.55	0.899	0.601	0.183	0.417	0.663	47.12	0.917	0.488	0.429	1.067	35.60
	1.60	0.928	0.627	0.209	0.418	0.708	45.72	0.986	0.559	0.428	1.143	33.84
ROT-1d		0.549	0.467	0.063	0.404	0.396	55.92	0.457	0.030	0.428	0.263	58.50
ROT-tr		0.516	0.473	0.072	0.401	0.422	55.40	0.447	0.021	0.426	0.222	59.76

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-14: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 2, $c_L = 1$, $c_U = 0$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.212	0.221	0.212	2.402	1.919	0.483	1.994	24.00					
	0.45	0.238	0.248	0.238	1.845	1.387	0.458	1.741	29.00	0.953	0.474	0.478	0.996	53.46
	0.50	0.265	0.276	0.265	1.420	0.984	0.437	1.501	33.06	0.694	0.241	0.453	0.729	59.56
	0.55	0.291	0.303	0.291	1.112	0.690	0.422	1.280	37.70	0.557	0.119	0.438	0.521	63.18
	0.60	0.318	0.331	0.318	0.895	0.484	0.411	1.085	41.28	0.489	0.059	0.430	0.370	63.80
	0.65	0.344	0.358	0.344	0.746	0.342	0.405	0.919	44.68	0.455	0.031	0.424	0.269	64.26
	0.70	0.371	0.386	0.371	0.646	0.245	0.401	0.781	47.44	0.439	0.018	0.421	0.206	63.44
	0.75	0.397	0.413	0.397	0.578	0.179	0.399	0.670	50.16	0.432	0.012	0.419	0.172	63.02
	0.80	0.423	0.441	0.423	0.533	0.135	0.398	0.581	52.36	0.430	0.011	0.420	0.159	62.50
	0.85	0.450	0.469	0.450	0.503	0.104	0.398	0.512	53.88	0.432	0.011	0.421	0.161	61.72
	0.90	0.476	0.496	0.476	0.484	0.084	0.399	0.459	55.14	0.436	0.013	0.423	0.175	61.00
	0.95	0.503	0.524	0.503	0.472	0.071	0.401	0.420	55.56	0.441	0.017	0.425	0.198	60.10
$\mathbf{H}_n^*$	1.00	0.529	0.551	0.529	0.465	0.062	0.402	0.393	56.02	0.449	0.022	0.426	0.230	59.32
	1.05	0.556	0.579	0.556	0.461	0.057	0.404	0.377	56.46	0.458	0.031	0.428	0.268	58.40
	1.10	0.582	0.606	0.582	0.461	0.055	0.406	0.369	56.68	0.471	0.042	0.429	0.311	57.02
	1.15	0.609	0.634	0.609	0.462	0.055	0.407	0.368	56.46	0.486	0.056	0.430	0.360	55.60
	1.20	0.635	0.662	0.635	0.466	0.057	0.409	0.374	56.08	0.504	0.073	0.431	0.412	53.60
	1.25	0.662	0.689	0.662	0.471	0.061	0.410	0.386	55.56	0.526	0.095	0.432	0.469	52.30
	1.30	0.688	0.717	0.688	0.478	0.067	0.412	0.402	54.78	0.553	0.121	0.432	0.528	50.76
	1.35	0.715	0.744	0.715	0.488	0.074	0.414	0.422	54.32	0.643	0.148	0.495	0.547	49.26
	1.40	0.741	0.772	0.741	0.497	0.083	0.414	0.448	53.34	0.679	0.188	0.491	0.619	47.08
	1.45	0.767	0.799	0.767	0.509	0.094	0.415	0.476	52.76	0.659	0.223	0.435	0.716	45.28
	1.50	0.794	0.827	0.794	0.523	0.107	0.416	0.508	51.86	0.700	0.268	0.432	0.788	43.40
	1.55	0.820	0.854	0.820	0.538	0.122	0.417	0.541	50.84	0.747	0.316	0.431	0.856	41.54
	1.60	0.847	0.882	0.847	0.556	0.139	0.417	0.577	49.78	0.799	0.368	0.431	0.925	39.68
ROT-tr		0.563	0.484	0.564	0.484	0.079	0.405	0.442	54.30	0.472	0.041	0.431	0.310	56.60

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-15: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 2, $c_L = 0$, $c_U = 1$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.232	2.024	1.560	0.464	1.834	26.60	0.757	0.299	0.458	0.809	62.42
	0.45	0.261	1.522	1.082	0.440	1.568	31.54	0.581	0.141	0.440	0.566	66.92
	0.50	0.290	1.162	0.740	0.423	1.323	36.28	0.496	0.066	0.430	0.392	67.12
	0.55	0.319	0.916	0.504	0.411	1.107	40.60	0.457	0.033	0.424	0.278	66.64
	0.60	0.348	0.751	0.346	0.405	0.925	44.12	0.439	0.019	0.421	0.211	65.22
	0.65	0.377	0.643	0.242	0.401	0.778	47.36	0.432	0.013	0.419	0.177	63.86
	0.70	0.406	0.572	0.174	0.399	0.660	50.14	0.432	0.012	0.420	0.168	62.62
	0.75	0.435	0.527	0.129	0.398	0.569	52.44	0.434	0.013	0.421	0.175	61.62
	0.80	0.464	0.499	0.100	0.399	0.500	53.80	0.439	0.016	0.423	0.195	60.78
	0.85	0.493	0.481	0.081	0.400	0.450	54.86	0.447	0.022	0.425	0.226	59.48
	0.90	0.522	0.471	0.069	0.402	0.415	55.58	0.457	0.030	0.427	0.266	58.54
	0.95	0.551	0.466	0.062	0.403	0.393	55.96	0.470	0.042	0.429	0.313	57.16
$\mathbf{H}_n^*$	1.00	0.580	0.464	0.059	0.405	0.382	56.22	0.487	0.057	0.430	0.366	55.46
	1.05	0.609	0.466	0.059	0.407	0.380	56.04	0.508	0.077	0.431	0.424	53.24
	1.10	0.638	0.470	0.061	0.409	0.387	55.76	0.533	0.102	0.431	0.486	51.68
	1.15	0.667	0.476	0.066	0.410	0.400	55.16	0.563	0.131	0.432	0.552	50.28
	1.20	0.696	0.484	0.072	0.412	0.419	54.30	0.600	0.166	0.434	0.619	48.62
	1.25	0.725	0.494	0.081	0.413	0.443	53.48	0.639	0.207	0.432	0.692	46.30
	1.30	0.754	0.506	0.092	0.414	0.472	52.72	0.685	0.253	0.432	0.766	44.04
	1.35	0.783	0.521	0.106	0.415	0.505	52.16	0.735	0.304	0.431	0.840	42.06
	1.40	0.812	0.537	0.121	0.416	0.540	50.84	0.791	0.361	0.430	0.915	39.92
	1.45	0.841	0.556	0.140	0.417	0.579	49.80	0.851	0.422	0.430	0.991	37.76
	1.50	0.870	0.577	0.160	0.417	0.620	48.64	0.917	0.488	0.429	1.067	35.56
	1.55	0.899	0.601	0.183	0.417	0.663	47.12	0.986	0.559	0.428	1.143	33.82
	1.60	0.928	0.627	0.209	0.418	0.708	45.72					
ROT-1d		0.549	0.467	0.063	0.404	0.396	55.92	0.469	0.040	0.429	0.306	57.38
ROT-tr		0.516	0.473	0.072	0.401	0.422	55.40	0.455	0.028	0.428	0.255	58.86

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-16: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 2, $c_L = 0$, $c_U = 1$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.212	0.221	0.212	2.402	1.919	0.483	1.994	24.00	0.953	0.474	0.478	0.996	59.40
	0.45	0.238	0.248	0.238	1.845	1.387	0.458	1.741	29.00	0.694	0.241	0.453	0.729	64.64
	0.50	0.265	0.276	0.265	1.420	0.984	0.437	1.501	33.06	0.557	0.119	0.438	0.521	67.48
	0.55	0.291	0.303	0.291	1.112	0.690	0.422	1.280	37.70	0.489	0.059	0.430	0.370	67.64
	0.60	0.318	0.331	0.318	0.895	0.484	0.411	1.085	41.28	0.455	0.031	0.424	0.269	66.88
	0.65	0.344	0.358	0.344	0.746	0.342	0.405	0.919	44.68	0.439	0.018	0.421	0.206	65.82
	0.70	0.371	0.386	0.371	0.646	0.245	0.401	0.781	47.44	0.432	0.012	0.419	0.172	64.38
	0.75	0.397	0.413	0.397	0.578	0.179	0.399	0.670	50.16	0.430	0.011	0.420	0.159	63.32
	0.80	0.423	0.441	0.423	0.533	0.135	0.398	0.581	52.36	0.432	0.011	0.421	0.161	62.38
	0.85	0.450	0.469	0.450	0.503	0.104	0.398	0.512	53.88	0.436	0.013	0.423	0.175	61.54
	0.90	0.476	0.496	0.476	0.484	0.084	0.399	0.459	55.14	0.441	0.017	0.425	0.198	60.48
	0.95	0.503	0.524	0.503	0.472	0.071	0.401	0.420	55.56	0.449	0.022	0.426	0.230	59.44
$\mathbf{H}_n^*$	1.00	0.529	0.551	0.529	0.465	0.062	0.402	0.393	56.02	0.458	0.031	0.428	0.268	58.46
	1.05	0.556	0.579	0.556	0.461	0.057	0.404	0.377	56.46	0.471	0.042	0.429	0.311	57.20
	1.10	0.582	0.606	0.582	0.461	0.055	0.406	0.369	56.68	0.486	0.056	0.430	0.360	55.52
	1.15	0.609	0.634	0.609	0.462	0.055	0.407	0.368	56.46	0.504	0.073	0.431	0.412	53.92
	1.20	0.635	0.662	0.635	0.466	0.057	0.409	0.374	56.08	0.526	0.095	0.432	0.469	52.30
	1.25	0.662	0.689	0.662	0.471	0.061	0.410	0.386	55.56	0.553	0.121	0.432	0.528	50.78
	1.30	0.688	0.717	0.688	0.478	0.067	0.412	0.402	54.78	0.643	0.148	0.495	0.547	49.24
	1.35	0.715	0.744	0.715	0.488	0.074	0.414	0.422	54.32	0.679	0.188	0.491	0.619	47.22
	1.40	0.741	0.772	0.741	0.497	0.083	0.414	0.448	53.34	0.659	0.223	0.435	0.716	45.24
	1.45	0.767	0.799	0.767	0.509	0.094	0.415	0.476	52.76	0.700	0.268	0.432	0.788	43.42
	1.50	0.794	0.827	0.794	0.523	0.107	0.416	0.508	51.86	0.747	0.316	0.431	0.856	41.62
	1.55	0.820	0.854	0.820	0.538	0.122	0.417	0.541	50.84	0.799	0.368	0.431	0.925	39.66
	1.60	0.847	0.882	0.847	0.556	0.139	0.417	0.577	49.78					
ROT-tr		0.563	0.484	0.564	0.484	0.079	0.405	0.442	54.30	0.488	0.055	0.433	0.357	55.14

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-17: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 2, $c_L = 0$, $c_U = 2$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.232	2.024	1.560	0.464	1.834	26.60	3.234	1.186	2.049	0.761	35.84
	0.45	0.261	1.522	1.082	0.440	1.568	31.54	2.438	0.753	1.685	0.668	38.36
	0.50	0.290	1.162	0.740	0.423	1.323	36.28	1.801	0.381	1.420	0.518	42.06
	0.55	0.319	0.916	0.504	0.411	1.107	40.60	1.357	0.141	1.216	0.340	44.80
	0.60	0.348	0.751	0.346	0.405	0.925	44.12	1.082	0.025	1.056	0.155	46.78
	0.65	0.377	0.643	0.242	0.401	0.778	47.36	0.954	0.001	0.954	0.025	47.60
	0.70	0.406	0.572	0.174	0.399	0.660	50.14	0.914	0.034	0.880	0.198	47.36
	0.75	0.435	0.527	0.129	0.398	0.569	52.44	0.926	0.114	0.813	0.374	44.84
	0.80	0.464	0.499	0.100	0.399	0.500	53.80	1.002	0.235	0.767	0.554	41.72
	0.85	0.493	0.481	0.081	0.400	0.450	54.86	1.128	0.399	0.729	0.740	37.96
	0.90	0.522	0.471	0.069	0.402	0.415	55.58	1.298	0.604	0.694	0.933	33.68
	0.95	0.551	0.466	0.062	0.403	0.393	55.96	1.516	0.850	0.666	1.130	28.36
$\mathbf{H}_n^*$	1.00	0.580	0.464	0.059	0.405	0.382	56.22	1.775	1.133	0.643	1.327	24.54
	1.05	0.609	0.466	0.059	0.407	0.380	56.04	2.073	1.450	0.623	1.526	20.14
	1.10	0.638	0.470	0.061	0.409	0.387	55.76	2.405	1.798	0.607	1.722	17.02
	1.15	0.667	0.476	0.066	0.410	0.400	55.16	3.421	2.139	1.282	1.292	14.02
	1.20	0.696	0.484	0.072	0.412	0.419	54.30	3.427	2.590	0.836	1.760	11.64
	1.25	0.725	0.494	0.081	0.413	0.443	53.48	4.812	3.029	1.784	1.303	9.52
	1.30	0.754	0.506	0.092	0.414	0.472	52.72	4.417	3.352	1.066	1.774	7.98
	1.35	0.783	0.521	0.106	0.415	0.505	52.16	4.348	3.801	0.547	2.635	6.72
	1.40	0.812	0.537	0.121	0.416	0.540	50.84	4.745	4.214	0.531	2.816	5.64
	1.45	0.841	0.556	0.140	0.417	0.579	49.80	5.135	4.616	0.520	2.981	4.68
	1.50	0.870	0.577	0.160	0.417	0.620	48.64	5.512	5.004	0.508	3.138	4.04
	1.55	0.899	0.601	0.183	0.417	0.663	47.12					
	1.60	0.928	0.627	0.209	0.418	0.708	45.72					
ROT-1d		0.549	0.467	0.063	0.404	0.396	55.92	1.457	0.789	0.668	1.087	29.74
ROT-tr		0.516	0.473	0.072	0.401	0.422	55.40	1.236	0.528	0.708	0.864	35.12

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-18: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 2, $c_L = 0$, $c_U = 2$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.212	0.221	0.212	2.402	1.919	0.483	1.994	24.00	3.802	1.426	2.376	0.775	35.02
	0.45	0.238	0.248	0.238	1.845	1.387	0.458	1.741	29.00	3.102	1.092	2.011	0.737	36.00
	0.50	0.265	0.276	0.265	1.420	0.984	0.437	1.501	33.06	2.360	0.692	1.668	0.644	39.04
	0.55	0.291	0.303	0.291	1.112	0.690	0.422	1.280	37.70	1.786	0.363	1.423	0.505	41.88
	0.60	0.318	0.331	0.318	0.895	0.484	0.411	1.085	41.28	1.378	0.147	1.231	0.345	44.56
	0.65	0.344	0.358	0.344	0.746	0.342	0.405	0.919	44.68	1.113	0.035	1.078	0.181	46.52
	0.70	0.371	0.386	0.371	0.646	0.245	0.401	0.781	47.44	0.974	0.000	0.974	0.020	47.92
	0.75	0.397	0.413	0.397	0.578	0.179	0.399	0.670	50.16	0.918	0.016	0.902	0.134	47.86
	0.80	0.423	0.441	0.423	0.533	0.135	0.398	0.581	52.36	0.905	0.070	0.835	0.290	46.90
	0.85	0.450	0.469	0.450	0.503	0.104	0.398	0.512	53.88	0.945	0.160	0.785	0.451	43.90
	0.90	0.476	0.496	0.476	0.484	0.084	0.399	0.459	55.14	1.031	0.283	0.748	0.615	41.40
	0.95	0.503	0.524	0.503	0.472	0.071	0.401	0.420	55.56	1.155	0.441	0.714	0.786	37.20
$\mathbf{H}_n^*$	1.00	0.529	0.551	0.529	0.465	0.062	0.402	0.393	56.02	1.316	0.633	0.684	0.962	33.08
	1.05	0.556	0.579	0.556	0.461	0.057	0.404	0.377	56.46	1.518	0.858	0.661	1.139	28.34
	1.10	0.582	0.606	0.582	0.461	0.055	0.406	0.369	56.68	1.754	1.114	0.641	1.318	24.88
	1.15	0.609	0.634	0.609	0.462	0.055	0.407	0.368	56.46	2.022	1.399	0.624	1.498	20.62
	1.20	0.635	0.662	0.635	0.466	0.057	0.409	0.374	56.08	2.319	1.710	0.609	1.676	17.74
	1.25	0.662	0.689	0.662	0.471	0.061	0.410	0.386	55.56	43.887	1.793	42.103	0.206	14.96
	1.30	0.688	0.717	0.688	0.478	0.067	0.412	0.402	54.78	180.579	3.016	177.598	0.130	12.90
	1.35	0.715	0.744	0.715	0.488	0.074	0.414	0.422	54.32	71.537	2.389	69.162	0.186	10.56
	1.40	0.741	0.772	0.741	0.497	0.083	0.414	0.448	53.34	7.045	3.231	3.814	0.920	9.00
	1.45	0.767	0.799	0.767	0.509	0.094	0.415	0.476	52.76	4.179	3.501	0.678	2.273	7.56
	1.50	0.794	0.827	0.794	0.523	0.107	0.416	0.508	51.86	4.438	3.897	0.541	2.683	6.48
	1.55	0.820	0.854	0.820	0.538	0.122	0.417	0.541	50.84					
	1.60	0.847	0.882	0.847	0.556	0.139	0.417	0.577	49.78					
ROT-tr		0.563	0.484	0.564	0.484	0.079	0.405	0.442	54.30	1.668	0.971	0.697	1.180	26.78

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-19: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 2, $c_L = 1$, $c_U = 1$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.232	2.024	1.560	0.464	1.834	26.60					
	0.45	0.261	1.522	1.082	0.440	1.568	31.54	3.234	1.186	2.049	0.761	32.86
	0.50	0.290	1.162	0.740	0.423	1.323	36.28	2.438	0.753	1.685	0.668	35.48
	0.55	0.319	0.916	0.504	0.411	1.107	40.60	1.801	0.381	1.420	0.518	39.44
	0.60	0.348	0.751	0.346	0.405	0.925	44.12	1.357	0.141	1.216	0.340	42.76
	0.65	0.377	0.643	0.242	0.401	0.778	47.36	1.082	0.025	1.056	0.155	45.04
	0.70	0.406	0.572	0.174	0.399	0.660	50.14	0.954	0.001	0.954	0.025	46.38
	0.75	0.435	0.527	0.129	0.398	0.569	52.44	0.914	0.034	0.880	0.198	46.82
	0.80	0.464	0.499	0.100	0.399	0.500	53.80	0.926	0.114	0.813	0.374	44.68
	0.85	0.493	0.481	0.081	0.400	0.450	54.86	1.002	0.235	0.767	0.554	41.62
	0.90	0.522	0.471	0.069	0.402	0.415	55.58	1.128	0.399	0.729	0.740	37.68
	0.95	0.551	0.466	0.062	0.403	0.393	55.96	1.298	0.604	0.694	0.933	33.66
$\mathbf{H}_n^*$	1.00	0.580	0.464	0.059	0.405	0.382	56.22	1.516	0.850	0.666	1.130	28.44
	1.05	0.609	0.466	0.059	0.407	0.380	56.04	1.775	1.133	0.643	1.327	24.66
	1.10	0.638	0.470	0.061	0.409	0.387	55.76	2.073	1.450	0.623	1.526	20.18
	1.15	0.667	0.476	0.066	0.410	0.400	55.16	2.405	1.798	0.607	1.722	17.22
	1.20	0.696	0.484	0.072	0.412	0.419	54.30	3.421	2.139	1.282	1.292	14.12
	1.25	0.725	0.494	0.081	0.413	0.443	53.48	3.427	2.590	0.836	1.760	11.68
	1.30	0.754	0.506	0.092	0.414	0.472	52.72	4.812	3.029	1.784	1.303	9.56
	1.35	0.783	0.521	0.106	0.415	0.505	52.16	4.417	3.352	1.066	1.774	8.04
	1.40	0.812	0.537	0.121	0.416	0.540	50.84	4.348	3.801	0.547	2.635	6.82
	1.45	0.841	0.556	0.140	0.417	0.579	49.80	4.745	4.214	0.531	2.816	5.68
	1.50	0.870	0.577	0.160	0.417	0.620	48.64	5.135	4.616	0.520	2.981	4.74
	1.55	0.899	0.601	0.183	0.417	0.663	47.12	5.512	5.004	0.508	3.138	4.08
	1.60	0.928	0.627	0.209	0.418	0.708	45.72					
ROT-1d		0.549	0.467	0.063	0.404	0.396	55.92	1.264	0.566	0.697	0.901	34.20
ROT-tr		0.516	0.473	0.072	0.401	0.422	55.40	1.099	0.358	0.741	0.695	39.02

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\mathrm{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-20: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 2, $c_L = 1$, $c_U = 1$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.212	0.221	0.212	2.402	1.919	0.483	1.994	24.00					
	0.45	0.238	0.248	0.238	1.845	1.387	0.458	1.741	29.00	3.802	1.426	2.376	0.775	32.04
	0.50	0.265	0.276	0.265	1.420	0.984	0.437	1.501	33.06	3.102	1.092	2.011	0.737	33.02
	0.55	0.291	0.303	0.291	1.112	0.690	0.422	1.280	37.70	2.360	0.692	1.668	0.644	35.80
	0.60	0.318	0.331	0.318	0.895	0.484	0.411	1.085	41.28	1.786	0.363	1.423	0.505	39.56
	0.65	0.344	0.358	0.344	0.746	0.342	0.405	0.919	44.68	1.378	0.147	1.231	0.345	42.54
	0.70	0.371	0.386	0.371	0.646	0.245	0.401	0.781	47.44	1.113	0.035	1.078	0.181	44.98
	0.75	0.397	0.413	0.397	0.578	0.179	0.399	0.670	50.16	0.974	0.000	0.974	0.020	46.56
	0.80	0.423	0.441	0.423	0.533	0.135	0.398	0.581	52.36	0.918	0.016	0.902	0.134	47.12
	0.85	0.450	0.469	0.450	0.503	0.104	0.398	0.512	53.88	0.905	0.070	0.835	0.290	46.34
	0.90	0.476	0.496	0.476	0.484	0.084	0.399	0.459	55.14	0.945	0.160	0.785	0.451	43.76
	0.95	0.503	0.524	0.503	0.472	0.071	0.401	0.420	55.56	1.031	0.283	0.748	0.615	41.28
$\mathbf{H}_n^*$	1.00	0.529	0.551	0.529	0.465	0.062	0.402	0.393	56.02	1.155	0.441	0.714	0.786	37.34
	1.05	0.556	0.579	0.556	0.461	0.057	0.404	0.377	56.46	1.316	0.633	0.684	0.962	33.14
	1.10	0.582	0.606	0.582	0.461	0.055	0.406	0.369	56.68	1.518	0.858	0.661	1.139	28.36
	1.15	0.609	0.634	0.609	0.462	0.055	0.407	0.368	56.46	1.754	1.114	0.641	1.318	24.82
	1.20	0.635	0.662	0.635	0.466	0.057	0.409	0.374	56.08	2.022	1.399	0.624	1.498	20.66
	1.25	0.662	0.689	0.662	0.471	0.061	0.410	0.386	55.56	2.319	1.710	0.609	1.676	17.88
	1.30	0.688	0.717	0.688	0.478	0.067	0.412	0.402	54.78	43.887	1.793	42.103	0.206	15.02
	1.35	0.715	0.744	0.715	0.488	0.074	0.414	0.422	54.32	180.579	3.016	177.598	0.130	12.90
	1.40	0.741	0.772	0.741	0.497	0.083	0.414	0.448	53.34	71.537	2.389	69.162	0.186	10.66
	1.45	0.767	0.799	0.767	0.509	0.094	0.415	0.476	52.76	7.045	3.231	3.814	0.920	9.04
	1.50	0.794	0.827	0.794	0.523	0.107	0.416	0.508	51.86	4.179	3.501	0.678	2.273	7.60
	1.55	0.820	0.854	0.820	0.538	0.122	0.417	0.541	50.84	4.438	3.897	0.541	2.683	6.54
	1.60	0.847	0.882	0.847	0.556	0.139	0.417	0.577	49.78					
ROT-tr		0.563	0.484	0.564	0.484	0.079	0.405	0.442	54.30	1.437	0.709	0.728	0.987	31.02

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-21: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 3, $c_L = 2$, $c_U = 0$.

	$\mathbf{H}_n = h_n \mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.186	1.585	1.340	0.245	2.339	26.60					
	0.45	0.210	1.236	1.008	0.228	2.103	32.56					
	0.50	0.233	0.959	0.747	0.212	1.876	38.26	2.278	0.665	1.613	0.642	37.38
	0.55	0.256	0.751	0.552	0.199	1.664	44.72	1.833	0.459	1.374	0.578	38.80
	0.60	0.280	0.602	0.412	0.190	1.474	50.52	1.374	0.228	1.146	0.446	42.34
	0.65	0.303	0.496	0.313	0.183	1.310	55.12	1.015	0.072	0.943	0.275	47.66
	0.70	0.326	0.423	0.245	0.178	1.173	59.12	0.782	0.005	0.777	0.079	52.30
	0.75	0.349	0.372	0.197	0.175	1.063	62.30	0.669	0.012	0.657	0.136	54.30
	0.80	0.373	0.338	0.165	0.173	0.976	64.84	0.646	0.073	0.573	0.357	53.32
	0.85	0.396	0.315	0.143	0.172	0.912	66.22	0.683	0.169	0.514	0.573	49.64
	0.90	0.419	0.301	0.129	0.172	0.866	67.60	0.758	0.288	0.470	0.782	46.36
	0.95	0.443	0.292	0.120	0.172	0.835	68.36	0.863	0.426	0.437	0.988	41.56
$\mathbf{H}_n^*$	1.00	0.466	0.289	0.116	0.173	0.819	68.92	1.001	0.589	0.412	1.195	35.08
	1.05	0.489	0.289	0.115	0.174	0.814	69.00	1.176	0.781	0.394	1.408	29.72
	1.10	0.512	0.292	0.117	0.175	0.818	68.64	1.394	1.012	0.382	1.627	23.74
	1.15	0.536	0.297	0.122	0.176	0.832	68.08	1.663	1.287	0.376	1.850	18.40
	1.20	0.559	0.306	0.129	0.177	0.854	67.30	1.986	1.612	0.374	2.076	13.54
	1.25	0.582	0.316	0.139	0.178	0.882	66.30	2.366	1.992	0.374	2.309	9.64
	1.30	0.606	0.330	0.151	0.179	0.917	64.88	2.802	2.429	0.373	2.551	6.66
	1.35	0.629	0.345	0.165	0.180	0.958	63.30	3.290	2.920	0.371	2.807	4.32
	1.40	0.652	0.364	0.183	0.181	1.004	61.60	3.822	3.456	0.366	3.073	2.72
	1.45	0.675	0.385	0.203	0.182	1.055	59.54	4.384	4.025	0.359	3.348	1.44
	1.50	0.699	0.409	0.226	0.183	1.110	57.46	4.960	4.611	0.350	3.632	0.84
	1.55	0.722	0.436	0.252	0.184	1.169	55.02	5.533	5.195	0.338	3.919	0.56
	1.60	0.745	0.466	0.281	0.185	1.231	52.58	6.085	5.759	0.326	4.205	0.22
ROT-1d		0.517	0.292	0.117	0.175	0.819	68.68	1.378	1.007	0.371	1.649	23.40
ROT-tr		0.506	0.291	0.116	0.175	0.816	68.98	1.298	0.916	0.382	1.548	25.98

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-22: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 3, $c_L = 2$, $c_U = 0$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.196	0.186	0.183	1.548	1.311	0.237	2.353	24.90					
	0.45	0.221	0.210	0.205	1.203	0.983	0.221	2.109	30.70					
	0.50	0.246	0.233	0.228	0.933	0.727	0.207	1.876	36.92	2.113	0.621	1.493	0.645	37.84
	0.55	0.270	0.256	0.251	0.733	0.538	0.195	1.661	43.38	1.663	0.403	1.260	0.566	39.46
	0.60	0.295	0.279	0.274	0.590	0.403	0.187	1.470	49.16	1.232	0.181	1.051	0.416	43.96
	0.65	0.319	0.303	0.297	0.489	0.309	0.181	1.307	54.26	0.912	0.044	0.868	0.226	49.36
	0.70	0.344	0.326	0.319	0.420	0.243	0.177	1.173	58.04	0.722	0.000	0.722	0.010	52.72
	0.75	0.368	0.349	0.342	0.373	0.199	0.174	1.068	61.14	0.646	0.031	0.616	0.224	53.48
	0.80	0.393	0.373	0.365	0.341	0.168	0.173	0.987	63.84	0.656	0.115	0.541	0.462	51.46
	0.85	0.417	0.396	0.388	0.321	0.148	0.172	0.928	65.48	0.724	0.236	0.488	0.695	47.80
	0.90	0.442	0.419	0.411	0.308	0.136	0.172	0.888	66.50	0.832	0.381	0.451	0.920	42.98
	0.95	0.466	0.442	0.434	0.302	0.129	0.173	0.864	67.10	0.976	0.554	0.423	1.145	36.56
$\mathbf{H}_n^*$	1.00	0.491	0.466	0.456	0.300	0.126	0.174	0.853	67.28	1.162	0.759	0.403	1.373	30.32
	1.05	0.516	0.489	0.479	0.302	0.128	0.175	0.855	67.30	1.395	1.006	0.389	1.608	24.02
	1.10	0.540	0.512	0.502	0.307	0.132	0.176	0.866	66.94	1.684	1.304	0.380	1.851	18.12
	1.15	0.565	0.536	0.525	0.316	0.139	0.177	0.887	66.08	2.034	1.658	0.376	2.099	12.96
	1.20	0.589	0.559	0.548	0.327	0.149	0.178	0.916	64.70	2.447	2.073	0.374	2.354	8.92
	1.25	0.614	0.582	0.571	0.341	0.162	0.179	0.953	63.48	2.923	2.551	0.372	2.619	5.94
	1.30	0.638	0.606	0.593	0.358	0.178	0.180	0.996	61.74	3.455	3.087	0.368	2.895	3.58
	1.35	0.663	0.629	0.616	0.378	0.197	0.181	1.045	60.04	4.031	3.669	0.362	3.183	2.04
	1.40	0.687	0.652	0.639	0.402	0.220	0.182	1.099	57.82	4.636	4.282	0.354	3.477	1.14
	1.45	0.712	0.675	0.662	0.428	0.245	0.183	1.158	55.42	5.247	4.904	0.344	3.778	0.66
	1.50	0.737	0.699	0.685	0.458	0.274	0.184	1.221	52.92	5.843	5.512	0.331	4.079	0.24
	1.55	0.761	0.722	0.707	0.492	0.307	0.185	1.287	49.96	6.404	6.086	0.318	4.376	0.16
	1.60	0.786	0.745	0.730	0.529	0.342	0.186	1.356	46.90	6.911	6.607	0.304	4.663	0.14
ROT-tr		0.506	0.488	0.555	0.288	0.112	0.176	0.796	69.52	1.291	0.909	0.382	1.542	26.36

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-23: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 3, $c_L = 1$, $c_U = 0$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.186	1.585	1.340	0.245	2.339	26.60					
	0.45	0.210	1.236	1.008	0.228	2.103	32.56	0.654	0.411	0.243	1.302	60.90
	0.50	0.233	0.959	0.747	0.212	1.876	38.26	0.460	0.238	0.221	1.037	68.74
	0.55	0.256	0.751	0.552	0.199	1.664	44.72	0.350	0.142	0.207	0.829	73.62
	0.60	0.280	0.602	0.412	0.190	1.474	50.52	0.290	0.092	0.198	0.681	76.68
	0.65	0.303	0.496	0.313	0.183	1.310	55.12	0.258	0.066	0.192	0.588	77.72
	0.70	0.326	0.423	0.245	0.178	1.173	59.12	0.242	0.054	0.187	0.539	77.88
	0.75	0.349	0.372	0.197	0.175	1.063	62.30	0.235	0.050	0.185	0.523	77.58
	0.80	0.373	0.338	0.165	0.173	0.976	64.84	0.236	0.052	0.184	0.531	76.58
	0.85	0.396	0.315	0.143	0.172	0.912	66.22	0.241	0.057	0.184	0.557	75.18
	0.90	0.419	0.301	0.129	0.172	0.866	67.60	0.250	0.065	0.185	0.595	73.74
	0.95	0.443	0.292	0.120	0.172	0.835	68.36	0.262	0.077	0.186	0.643	72.42
$\mathbf{H}_n^*$	1.00	0.466	0.289	0.116	0.173	0.819	68.92	0.278	0.091	0.187	0.699	70.14
	1.05	0.489	0.289	0.115	0.174	0.814	69.00	0.298	0.109	0.188	0.762	68.40
	1.10	0.512	0.292	0.117	0.175	0.818	68.64	0.321	0.131	0.190	0.832	66.10
	1.15	0.536	0.297	0.122	0.176	0.832	68.08	0.348	0.157	0.191	0.907	63.80
	1.20	0.559	0.306	0.129	0.177	0.854	67.30	0.380	0.188	0.192	0.989	60.52
	1.25	0.582	0.316	0.139	0.178	0.882	66.30	0.418	0.224	0.194	1.076	56.96
	1.30	0.606	0.330	0.151	0.179	0.917	64.88	0.461	0.266	0.195	1.168	53.46
	1.35	0.629	0.345	0.165	0.180	0.958	63.30	0.511	0.315	0.197	1.265	49.08
	1.40	0.652	0.364	0.183	0.181	1.004	61.60	0.568	0.370	0.198	1.366	45.18
	1.45	0.675	0.385	0.203	0.182	1.055	59.54	0.631	0.432	0.200	1.471	40.94
	1.50	0.699	0.409	0.226	0.183	1.110	57.46	0.702	0.501	0.201	1.579	36.72
	1.55	0.722	0.436	0.252	0.184	1.169	55.02	0.778	0.576	0.202	1.689	32.94
	1.60	0.745	0.466	0.281	0.185	1.231	52.58	0.861	0.658	0.203	1.801	29.48
ROT-1d		0.517	0.292	0.117	0.175	0.819	68.68	0.322	0.132	0.190	0.831	66.16
ROT-tr		0.506	0.291	0.116	0.175	0.816	68.98	0.313	0.123	0.190	0.805	66.86

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-24: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 3, $c_L = 1$, $c_U = 0$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.196	0.186	0.183	1.548	1.311	0.237	2.353	24.90					
	0.45	0.221	0.210	0.205	1.203	0.983	0.221	2.109	30.70	0.629	0.395	0.234	1.300	59.46
	0.50	0.246	0.233	0.228	0.933	0.727	0.207	1.876	36.92	0.446	0.230	0.216	1.033	67.74
	0.55	0.270	0.256	0.251	0.733	0.538	0.195	1.661	43.38	0.343	0.140	0.204	0.828	72.56
	0.60	0.295	0.279	0.274	0.590	0.403	0.187	1.470	49.16	0.288	0.093	0.196	0.688	75.20
	0.65	0.319	0.303	0.297	0.489	0.309	0.181	1.307	54.26	0.259	0.069	0.190	0.604	76.12
	0.70	0.344	0.326	0.319	0.420	0.243	0.177	1.173	58.04	0.246	0.059	0.187	0.563	76.58
	0.75	0.368	0.349	0.342	0.373	0.199	0.174	1.068	61.14	0.242	0.057	0.185	0.555	76.02
	0.80	0.393	0.373	0.365	0.341	0.168	0.173	0.987	63.84	0.244	0.060	0.184	0.571	74.94
	0.85	0.417	0.396	0.388	0.321	0.148	0.172	0.928	65.48	0.252	0.067	0.184	0.604	73.54
	0.90	0.442	0.419	0.411	0.308	0.136	0.172	0.888	66.50	0.264	0.078	0.185	0.651	71.80
	0.95	0.466	0.442	0.434	0.302	0.129	0.173	0.864	67.10	0.279	0.093	0.186	0.707	69.86
$\mathbf{H}_n^*$	1.00	0.491	0.466	0.456	0.300	0.126	0.174	0.853	67.28	0.299	0.112	0.187	0.772	67.96
	1.05	0.516	0.489	0.479	0.302	0.128	0.175	0.855	67.30	0.323	0.134	0.189	0.844	65.56
	1.10	0.540	0.512	0.502	0.307	0.132	0.176	0.866	66.94	0.352	0.162	0.190	0.924	62.92
	1.15	0.565	0.536	0.525	0.316	0.139	0.177	0.887	66.08	0.387	0.195	0.191	1.010	59.44
	1.20	0.589	0.559	0.548	0.327	0.149	0.178	0.916	64.70	0.428	0.235	0.193	1.103	55.72
	1.25	0.614	0.582	0.571	0.341	0.162	0.179	0.953	63.48	0.475	0.281	0.194	1.202	52.10
	1.30	0.638	0.606	0.593	0.358	0.178	0.180	0.996	61.74	0.530	0.334	0.196	1.306	47.52
	1.35	0.663	0.629	0.616	0.378	0.197	0.181	1.045	60.04	0.592	0.395	0.197	1.415	43.24
	1.40	0.687	0.652	0.639	0.402	0.220	0.182	1.099	57.82	0.662	0.464	0.199	1.527	38.66
	1.45	0.712	0.675	0.662	0.428	0.245	0.183	1.158	55.42	0.740	0.540	0.200	1.643	34.52
	1.50	0.737	0.699	0.685	0.458	0.274	0.184	1.221	52.92	0.824	0.623	0.201	1.760	30.54
	1.55	0.761	0.722	0.707	0.492	0.307	0.185	1.287	49.96	0.915	0.713	0.202	1.879	27.38
	1.60	0.786	0.745	0.730	0.529	0.342	0.186	1.356	46.90	1.011	0.808	0.203	1.998	24.22
ROT-tr		0.506	0.488	0.555	0.288	0.112	0.176	0.796	69.52	0.313	0.120	0.193	0.790	67.26

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-25: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 3, $c_L = 0$, $c_U = 1$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.186	1.585	1.340	0.245	2.339	26.60	0.654	0.411	0.243	1.302	65.08
	0.45	0.210	1.236	1.008	0.228	2.103	32.56	0.460	0.238	0.221	1.037	72.92
	0.50	0.233	0.959	0.747	0.212	1.876	38.26	0.350	0.142	0.207	0.829	76.82
	0.55	0.256	0.751	0.552	0.199	1.664	44.72	0.290	0.092	0.198	0.681	79.00
	0.60	0.280	0.602	0.412	0.190	1.474	50.52	0.258	0.066	0.192	0.588	79.78
	0.65	0.303	0.496	0.313	0.183	1.310	55.12	0.242	0.054	0.187	0.539	79.62
	0.70	0.326	0.423	0.245	0.178	1.173	59.12	0.235	0.050	0.185	0.523	78.66
	0.75	0.349	0.372	0.197	0.175	1.063	62.30	0.236	0.052	0.184	0.531	77.34
	0.80	0.373	0.338	0.165	0.173	0.976	64.84	0.241	0.057	0.184	0.557	75.66
	0.85	0.396	0.315	0.143	0.172	0.912	66.22	0.250	0.065	0.185	0.595	74.06
	0.90	0.419	0.301	0.129	0.172	0.866	67.60	0.262	0.077	0.186	0.643	72.64
	0.95	0.443	0.292	0.120	0.172	0.835	68.36	0.278	0.091	0.187	0.699	70.28
$\mathbf{H}_n^*$	1.00	0.466	0.289	0.116	0.173	0.819	68.92	0.298	0.109	0.188	0.762	68.46
	1.05	0.489	0.289	0.115	0.174	0.814	69.00	0.321	0.131	0.190	0.832	66.18
	1.10	0.512	0.292	0.117	0.175	0.818	68.64	0.348	0.157	0.191	0.907	63.84
	1.15	0.536	0.297	0.122	0.176	0.832	68.08	0.380	0.188	0.192	0.989	60.50
	1.20	0.559	0.306	0.129	0.177	0.854	67.30	0.418	0.224	0.194	1.076	56.90
	1.25	0.582	0.316	0.139	0.178	0.882	66.30	0.461	0.266	0.195	1.168	53.48
	1.30	0.606	0.330	0.151	0.179	0.917	64.88	0.511	0.315	0.197	1.265	49.08
	1.35	0.629	0.345	0.165	0.180	0.958	63.30	0.568	0.370	0.198	1.366	45.08
	1.40	0.652	0.364	0.183	0.181	1.004	61.60	0.631	0.432	0.200	1.471	40.82
	1.45	0.675	0.385	0.203	0.182	1.055	59.54	0.702	0.501	0.201	1.579	36.78
	1.50	0.699	0.409	0.226	0.183	1.110	57.46	0.778	0.576	0.202	1.689	32.86
	1.55	0.722	0.436	0.252	0.184	1.169	55.02	0.861	0.658	0.203	1.801	29.52
	1.60	0.745	0.466	0.281	0.185	1.231	52.58					
ROT-1d		0.517	0.292	0.117	0.175	0.819	68.68	0.353	0.161	0.192	0.916	63.40
ROT-tr		0.506	0.291	0.116	0.175	0.816	68.98	0.341	0.150	0.191	0.884	64.44

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-26: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 3, $c_L = 0$, $c_U = 1$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.196	0.186	0.183	1.548	1.311	0.237	2.353	24.90	0.629	0.395	0.234	1.300	63.62
	0.45	0.221	0.210	0.205	1.203	0.983	0.221	2.109	30.70	0.446	0.230	0.216	1.033	71.42
	0.50	0.246	0.233	0.228	0.933	0.727	0.207	1.876	36.92	0.343	0.140	0.204	0.828	75.60
	0.55	0.270	0.256	0.251	0.733	0.538	0.195	1.661	43.38	0.288	0.093	0.196	0.688	77.62
	0.60	0.295	0.279	0.274	0.590	0.403	0.187	1.470	49.16	0.259	0.069	0.190	0.604	78.18
	0.65	0.319	0.303	0.297	0.489	0.309	0.181	1.307	54.26	0.246	0.059	0.187	0.563	78.16
	0.70	0.344	0.326	0.319	0.420	0.243	0.177	1.173	58.04	0.242	0.057	0.185	0.555	77.26
	0.75	0.368	0.349	0.342	0.373	0.199	0.174	1.068	61.14	0.244	0.060	0.184	0.571	75.46
	0.80	0.393	0.373	0.365	0.341	0.168	0.173	0.987	63.84	0.252	0.067	0.184	0.604	73.88
	0.85	0.417	0.396	0.388	0.321	0.148	0.172	0.928	65.48	0.264	0.078	0.185	0.651	72.14
	0.90	0.442	0.419	0.411	0.308	0.136	0.172	0.888	66.50	0.279	0.093	0.186	0.707	69.96
	0.95	0.466	0.442	0.434	0.302	0.129	0.173	0.864	67.10	0.299	0.112	0.187	0.772	68.04
$\mathbf{H}_n^*$	1.00	0.491	0.466	0.456	0.300	0.126	0.174	0.853	67.28	0.323	0.134	0.189	0.844	65.50
	1.05	0.516	0.489	0.479	0.302	0.128	0.175	0.855	67.30	0.352	0.162	0.190	0.924	62.94
	1.10	0.540	0.512	0.502	0.307	0.132	0.176	0.866	66.94	0.387	0.195	0.191	1.010	59.50
	1.15	0.565	0.536	0.525	0.316	0.139	0.177	0.887	66.08	0.428	0.235	0.193	1.103	55.76
	1.20	0.589	0.559	0.548	0.327	0.149	0.178	0.916	64.70	0.475	0.281	0.194	1.202	52.04
	1.25	0.614	0.582	0.571	0.341	0.162	0.179	0.953	63.48	0.530	0.334	0.196	1.306	47.48
	1.30	0.638	0.606	0.593	0.358	0.178	0.180	0.996	61.74	0.592	0.395	0.197	1.415	43.12
	1.35	0.663	0.629	0.616	0.378	0.197	0.181	1.045	60.04	0.662	0.464	0.199	1.527	38.68
	1.40	0.687	0.652	0.639	0.402	0.220	0.182	1.099	57.82	0.740	0.540	0.200	1.643	34.54
	1.45	0.712	0.675	0.662	0.428	0.245	0.183	1.158	55.42	0.824	0.623	0.201	1.760	30.50
	1.50	0.737	0.699	0.685	0.458	0.274	0.184	1.221	52.92	0.915	0.713	0.202	1.879	27.40
	1.55	0.761	0.722	0.707	0.492	0.307	0.185	1.287	49.96	1.011	0.808	0.203	1.998	24.16
	1.60	0.786	0.745	0.730	0.529	0.342	0.186	1.356	46.90					
ROT-tr		0.506	0.488	0.555	0.288	0.112	0.176	0.796	69.52	0.341	0.147	0.194	0.869	64.58

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-27: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 3, $c_L = 0$, $c_U = 2$.

	$\mathbf{H}_n = h_n\mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.186	1.585	1.340	0.245	2.339	26.60	2.278	0.665	1.613	0.642	41.98
	0.45	0.210	1.236	1.008	0.228	2.103	32.56	1.833	0.459	1.374	0.578	43.20
	0.50	0.233	0.959	0.747	0.212	1.876	38.26	1.374	0.228	1.146	0.446	47.16
	0.55	0.256	0.751	0.552	0.199	1.664	44.72	1.015	0.072	0.943	0.275	51.72
	0.60	0.280	0.602	0.412	0.190	1.474	50.52	0.782	0.005	0.777	0.079	55.74
	0.65	0.303	0.496	0.313	0.183	1.310	55.12	0.669	0.012	0.657	0.136	56.64
	0.70	0.326	0.423	0.245	0.178	1.173	59.12	0.646	0.073	0.573	0.357	54.64
	0.75	0.349	0.372	0.197	0.175	1.063	62.30	0.683	0.169	0.514	0.573	50.94
	0.80	0.373	0.338	0.165	0.173	0.976	64.84	0.758	0.288	0.470	0.782	47.18
	0.85	0.396	0.315	0.143	0.172	0.912	66.22	0.863	0.426	0.437	0.988	42.04
	0.90	0.419	0.301	0.129	0.172	0.866	67.60	1.001	0.589	0.412	1.195	35.32
	0.95	0.443	0.292	0.120	0.172	0.835	68.36	1.176	0.781	0.394	1.408	29.72
$\mathbf{H}_n^*$	1.00	0.466	0.289	0.116	0.173	0.819	68.92	1.394	1.012	0.382	1.627	23.72
	1.05	0.489	0.289	0.115	0.174	0.814	69.00	1.663	1.287	0.376	1.850	18.26
	1.10	0.512	0.292	0.117	0.175	0.818	68.64	1.986	1.612	0.374	2.076	13.48
	1.15	0.536	0.297	0.122	0.176	0.832	68.08	2.366	1.992	0.374	2.309	9.56
	1.20	0.559	0.306	0.129	0.177	0.854	67.30	2.802	2.429	0.373	2.551	6.66
	1.25	0.582	0.316	0.139	0.178	0.882	66.30	3.290	2.920	0.371	2.807	4.28
	1.30	0.606	0.330	0.151	0.179	0.917	64.88	3.822	3.456	0.366	3.073	2.68
	1.35	0.629	0.345	0.165	0.180	0.958	63.30	4.384	4.025	0.359	3.348	1.40
	1.40	0.652	0.364	0.183	0.181	1.004	61.60	4.960	4.611	0.350	3.632	0.86
	1.45	0.675	0.385	0.203	0.182	1.055	59.54	5.533	5.195	0.338	3.919	0.56
	1.50	0.699	0.409	0.226	0.183	1.110	57.46	6.085	5.759	0.326	4.205	0.22
	1.55	0.722	0.436	0.252	0.184	1.169	55.02					
	1.60	0.745	0.466	0.281	0.185	1.231	52.58					
ROT-1d		0.517	0.292	0.117	0.175	0.819	68.68	2.048	1.694	0.355	2.186	12.02
ROT-tr		0.506	0.291	0.116	0.175	0.816	68.98	1.907	1.540	0.368	2.047	14.34

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\mathrm{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-28: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 3, $c_L = 0$, $c_U = 2$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.196	0.186	0.183	1.548	1.311	0.237	2.353	24.90	2.113	0.621	1.493	0.645	41.90
	0.45	0.221	0.210	0.205	1.203	0.983	0.221	2.109	30.70	1.663	0.403	1.260	0.566	43.88
	0.50	0.246	0.233	0.228	0.933	0.727	0.207	1.876	36.92	1.232	0.181	1.051	0.416	48.02
	0.55	0.270	0.256	0.251	0.733	0.538	0.195	1.661	43.38	0.912	0.044	0.868	0.226	53.34
	0.60	0.295	0.279	0.274	0.590	0.403	0.187	1.470	49.16	0.722	0.000	0.722	0.010	56.02
	0.65	0.319	0.303	0.297	0.489	0.309	0.181	1.307	54.26	0.646	0.031	0.616	0.224	55.56
	0.70	0.344	0.326	0.319	0.420	0.243	0.177	1.173	58.04	0.656	0.115	0.541	0.462	52.70
	0.75	0.368	0.349	0.342	0.373	0.199	0.174	1.068	61.14	0.724	0.236	0.488	0.695	48.64
	0.80	0.393	0.373	0.365	0.341	0.168	0.173	0.987	63.84	0.832	0.381	0.451	0.920	43.38
	0.85	0.417	0.396	0.388	0.321	0.148	0.172	0.928	65.48	0.976	0.554	0.423	1.145	36.76
	0.90	0.442	0.419	0.411	0.308	0.136	0.172	0.888	66.50	1.162	0.759	0.403	1.373	30.38
	0.95	0.466	0.442	0.434	0.302	0.129	0.173	0.864	67.10	1.395	1.006	0.389	1.608	23.98
$\mathbf{H}_n^*$	1.00	0.491	0.466	0.456	0.300	0.126	0.174	0.853	67.28	1.684	1.304	0.380	1.851	18.14
	1.05	0.516	0.489	0.479	0.302	0.128	0.175	0.855	67.30	2.034	1.658	0.376	2.099	12.82
	1.10	0.540	0.512	0.502	0.307	0.132	0.176	0.866	66.94	2.447	2.073	0.374	2.354	8.82
	1.15	0.565	0.536	0.525	0.316	0.139	0.177	0.887	66.08	2.923	2.551	0.372	2.619	5.88
	1.20	0.589	0.559	0.548	0.327	0.149	0.178	0.916	64.70	3.455	3.087	0.368	2.895	3.56
	1.25	0.614	0.582	0.571	0.341	0.162	0.179	0.953	63.48	4.031	3.669	0.362	3.183	1.98
	1.30	0.638	0.606	0.593	0.358	0.178	0.180	0.996	61.74	4.636	4.282	0.354	3.477	1.12
	1.35	0.663	0.629	0.616	0.378	0.197	0.181	1.045	60.04	5.247	4.904	0.344	3.778	0.64
	1.40	0.687	0.652	0.639	0.402	0.220	0.182	1.099	57.82	5.843	5.512	0.331	4.079	0.26
	1.45	0.712	0.675	0.662	0.428	0.245	0.183	1.158	55.42	6.404	6.086	0.318	4.376	0.16
	1.50	0.737	0.699	0.685	0.458	0.274	0.184	1.221	52.92	6.911	6.607	0.304	4.663	0.12
	1.55	0.761	0.722	0.707	0.492	0.307	0.185	1.287	49.96					
	1.60	0.786	0.745	0.730	0.529	0.342	0.186	1.356	46.90					
ROT-tr		0.506	0.488	0.555	0.288	0.112	0.176	0.796	69.52	1.900	1.531	0.369	2.036	14.22

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.

Table D-29: Classical and Generalized Jackknife Estimators, Common Bandwidth, Model 3, $c_L = 1$, $c_U = 1$.

	$\mathbf{H}_n = h_n \mathbf{I}_3$		$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	h_n	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.186	1.585	1.340	0.245	2.339	26.60					
	0.45	0.210	1.236	1.008	0.228	2.103	32.56	2.278	0.665	1.613	0.642	39.78
	0.50	0.233	0.959	0.747	0.212	1.876	38.26	1.833	0.459	1.374	0.578	40.86
	0.55	0.256	0.751	0.552	0.199	1.664	44.72	1.374	0.228	1.146	0.446	44.54
	0.60	0.280	0.602	0.412	0.190	1.474	50.52	1.015	0.072	0.943	0.275	49.48
	0.65	0.303	0.496	0.313	0.183	1.310	55.12	0.782	0.005	0.777	0.079	53.60
	0.70	0.326	0.423	0.245	0.178	1.173	59.12	0.669	0.012	0.657	0.136	55.16
	0.75	0.349	0.372	0.197	0.175	1.063	62.30	0.646	0.073	0.573	0.357	53.82
	0.80	0.373	0.338	0.165	0.173	0.976	64.84	0.683	0.169	0.514	0.573	50.18
	0.85	0.396	0.315	0.143	0.172	0.912	66.22	0.758	0.288	0.470	0.782	46.58
	0.90	0.419	0.301	0.129	0.172	0.866	67.60	0.863	0.426	0.437	0.988	41.72
	0.95	0.443	0.292	0.120	0.172	0.835	68.36	1.001	0.589	0.412	1.195	35.14
$\mathbf{H}_n^*$	1.00	0.466	0.289	0.116	0.173	0.819	68.92	1.176	0.781	0.394	1.408	29.68
	1.05	0.489	0.289	0.115	0.174	0.814	69.00	1.394	1.012	0.382	1.627	23.68
	1.10	0.512	0.292	0.117	0.175	0.818	68.64	1.663	1.287	0.376	1.850	18.34
	1.15	0.536	0.297	0.122	0.176	0.832	68.08	1.986	1.612	0.374	2.076	13.56
	1.20	0.559	0.306	0.129	0.177	0.854	67.30	2.366	1.992	0.374	2.309	9.60
	1.25	0.582	0.316	0.139	0.178	0.882	66.30	2.802	2.429	0.373	2.551	6.64
	1.30	0.606	0.330	0.151	0.179	0.917	64.88	3.290	2.920	0.371	2.807	4.30
	1.35	0.629	0.345	0.165	0.180	0.958	63.30	3.822	3.456	0.366	3.073	2.72
	1.40	0.652	0.364	0.183	0.181	1.004	61.60	4.384	4.025	0.359	3.348	1.40
	1.45	0.675	0.385	0.203	0.182	1.055	59.54	4.960	4.611	0.350	3.632	0.86
	1.50	0.699	0.409	0.226	0.183	1.110	57.46	5.533	5.195	0.338	3.919	0.56
	1.55	0.722	0.436	0.252	0.184	1.169	55.02	6.085	5.759	0.326	4.205	0.22
	1.60	0.745	0.466	0.281	0.185	1.231	52.58					
ROT-1d		0.517	0.292	0.117	0.175	0.819	68.68	1.679	1.318	0.361	1.912	17.28
ROT-tr		0.506	0.291	0.116	0.175	0.816	68.98	1.571	1.199	0.372	1.794	19.50

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-1d and ROT-tr correspond to ROT bandwidth estimates based on, respectively, $\mathsf{AMSE}[\mathbf{a}'\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)]$ and $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(h_n\mathbf{I}_3)])$, and average of estimated bandwidths are reported in bandwidth column.

Table D-30: Classical and Generalized Jackknife Estimators, Different Bandwidths, Model 3, $c_L = 1$, $c_U = 1$.

	$\mathbf{H}_n = \mathbf{diag}(h_{1,n}, h_{2,n}, h_{3,n})$				$\hat{\boldsymbol{\theta}}_n(\mathbf{H}_n)$					$\tilde{\boldsymbol{\theta}}_n(\mathbf{H}_n, \mathbf{c})$				
	ϑ	$h_{1,n}$	$h_{2,n}$	$h_{3,n}$	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI	MSE	BIAS^2	VAR	$\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$	95% CI
	0.40	0.196	0.186	0.183	1.548	1.311	0.237	2.353	24.90					
	0.45	0.221	0.210	0.205	1.203	0.983	0.221	2.109	30.70	2.113	0.621	1.493	0.645	39.78
	0.50	0.246	0.233	0.228	0.933	0.727	0.207	1.876	36.92	1.663	0.403	1.260	0.566	41.62
	0.55	0.270	0.256	0.251	0.733	0.538	0.195	1.661	43.38	1.232	0.181	1.051	0.416	45.78
	0.60	0.295	0.279	0.274	0.590	0.403	0.187	1.470	49.16	0.912	0.044	0.868	0.226	51.16
	0.65	0.319	0.303	0.297	0.489	0.309	0.181	1.307	54.26	0.722	0.000	0.722	0.010	54.02
	0.70	0.344	0.326	0.319	0.420	0.243	0.177	1.173	58.04	0.646	0.031	0.616	0.224	54.36
	0.75	0.368	0.349	0.342	0.373	0.199	0.174	1.068	61.14	0.656	0.115	0.541	0.462	51.94
	0.80	0.393	0.373	0.365	0.341	0.168	0.173	0.987	63.84	0.724	0.236	0.488	0.695	48.20
	0.85	0.417	0.396	0.388	0.321	0.148	0.172	0.928	65.48	0.832	0.381	0.451	0.920	43.18
	0.90	0.442	0.419	0.411	0.308	0.136	0.172	0.888	66.50	0.976	0.554	0.423	1.145	36.58
	0.95	0.466	0.442	0.434	0.302	0.129	0.173	0.864	67.10	1.162	0.759	0.403	1.373	30.32
$\mathbf{H}_n^*$	1.00	0.491	0.466	0.456	0.300	0.126	0.174	0.853	67.28	1.395	1.006	0.389	1.608	23.96
	1.05	0.516	0.489	0.479	0.302	0.128	0.175	0.855	67.30	1.684	1.304	0.380	1.851	18.10
	1.10	0.540	0.512	0.502	0.307	0.132	0.176	0.866	66.94	2.034	1.658	0.376	2.099	12.82
	1.15	0.565	0.536	0.525	0.316	0.139	0.177	0.887	66.08	2.447	2.073	0.374	2.354	8.88
	1.20	0.589	0.559	0.548	0.327	0.149	0.178	0.916	64.70	2.923	2.551	0.372	2.619	5.92
	1.25	0.614	0.582	0.571	0.341	0.162	0.179	0.953	63.48	3.455	3.087	0.368	2.895	3.58
	1.30	0.638	0.606	0.593	0.358	0.178	0.180	0.996	61.74	4.031	3.669	0.362	3.183	2.04
	1.35	0.663	0.629	0.616	0.378	0.197	0.181	1.045	60.04	4.636	4.282	0.354	3.477	1.14
	1.40	0.687	0.652	0.639	0.402	0.220	0.182	1.099	57.82	5.247	4.904	0.344	3.778	0.66
	1.45	0.712	0.675	0.662	0.428	0.245	0.183	1.158	55.42	5.843	5.512	0.331	4.079	0.26
	1.50	0.737	0.699	0.685	0.458	0.274	0.184	1.221	52.92	6.404	6.086	0.318	4.376	0.16
	1.55	0.761	0.722	0.707	0.492	0.307	0.185	1.287	49.96	6.911	6.607	0.304	4.663	0.12
	1.60	0.786	0.745	0.730	0.529	0.342	0.186	1.356	46.90					
ROT-tr		0.506	0.488	0.555	0.288	0.112	0.176	0.796	69.52	1.564	1.191	0.374	1.785	19.74

Notes: (i) columns MSE, BIAS^2, VAR, $\frac{\text{BIAS}}{\sqrt{\text{VAR}}}$ and 95% CI report, respectively, mean square error, square bias, variance, absolute bias divided by square root of variance and coverage rate of 95% confidence intervals for each estimator; (ii) ROT-tr corresponds to ROT bandwidth estimate based on $\text{tr}(\mathsf{AMSE}[\hat{\boldsymbol{\theta}}_n^{**}(\mathbf{H}_n)])$, and average of estimated bandwidths are reported in bandwidths columns.