Ecom 240B   Spring 2009
Problem Set 2

This problem set is due in class on Monday, April 13th

1. Let $\{X_n\}$ be a sequence of random variables. Assume that

$$\sqrt{n}X_n \xrightarrow{d} N(0,1)$$

   (a) Find the asymptotic distribution of $\sqrt{n}\left(e^{X_n} - \text{plim}e^{X_n}\right)$ and $\sqrt{n}\left(X_n^2 - \text{plim}X_n^2\right)$.

   (b) Find the asymptotic distribution of $e^{\sqrt{n}X_n}$ and $\sqrt{n}X_n^2$.

2. Suppose that $X_1, \ldots, X_n$ are independent and that it is known that $(X_i)^\lambda - 10$ has a standard normal distribution, $i = 1, \ldots, n$. This is called the *Box-Cox transformation.*

   - Derive the second-round estimator $\hat{\lambda}_2$ of the Newton-Raphson iteration, starting from an initial guess that $\hat{\lambda}_1 = 1$.
   - For the following data, compute $\hat{\lambda}_2$:
     96, 125, 146, 76, 114, 69, 130, 119, 85, 106
   - Write a computer program to iterate to convergence or to 100 times.

3. Consider a discrete random variable N having probability mass function

$$p_N(n;\theta^0) = \text{Prob}\left(N = n;\theta^0\right) = \frac{-(\theta^0)^n}{n\log\left(1-\theta^0\right)} \qquad n = 1, 2, \ldots, \;\; 0 < \theta^0 < 1$$

   which is often referred to as the *logarithmic series* distribution for reasons that will become clear later in the problem.

   (a) Prove that

$$\sum_{n=1}^{\infty} p_N(n;\theta^0) = 1.$$

   (Hint: consider the infinite order taylor series expansion of $\log(1+x)$ and substitute in $x = -\theta^0$.)

   (b) Find the expected value of N, $E(N)$. (Hint: $\sum_{n=1}^{\infty} \rho^n = \frac{\rho}{1-\rho}$.)

   (c) Find the variance of N, $V(N)$. (Hint: remember that the derivative of a sum is the sum of the derivatives of each of the sum's parts.)

   (d) Define the maximum-likelihood estimator $\hat{\theta}_{mle}$ of θ^0.

   (e) After considerable effort, a researcher has obtained a random sample of one thousand measurements on N. These data are summarized in Table 1.

Table 1
Observed Frequency Distribution of N

N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Frequency	700	205	50	26	10	6	1	1	1

(f) Write a matlab program that implements Newton's method to calculate the maximum-likelihood estimate of θ^0 using the above data.

(g) Write another matlab program to implement the bisection method to calculate the maximum likelihood estimate of θ^0 using the above data. An introduction to the bisection method for solving a nonlinear equation of one variable can be found at:

`http://www.library.cornell.edu/nr/bookcpdf/c9-1.pdf`