Economics 280C Problem
Home Currency Preference in Portfolio Choice

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An investor can hold two assets, home and foreign-currency bonds. Money price levels in the two countries are fixed at P and P^*, respectively, and E denotes the exchange rate, the home price of foreign currency. We normalize so that $P = P^* = 1$.

The home investor enters the asset market with beginning of period wealth (in home currency), W_0. He/she must divide it among home and foreign bonds, and consume the proceeds at the end of the period. The utility function dependent on consumption of home and foreign goods, and is

$$U = \frac{\left(C_H^{\gamma} C_F^{1-\gamma}\right)^{1-R}}{1-R},$$

where $R > 0$ is the degree of risk aversion. At the end of the period, the consumer will allocate his/her wealth W (a random variable from the perspective of the period's beginning) among the two goods according to the budget constraint $W = C_H + EC_F$. Importantly, E, the end-of-period exchange rate, also is random—and in this context, assumed to be the only random component of relative asset returns. The beginning-of-period exchange rate is denoted E_0.

(a) Write realized end-of-period utility in terms of W and E.

(b)Let i be the nominal interest rate on home currency and i^* the nominal interest rate on foreign. Explain the constraint:

$$W = (1+i)I + \frac{E(1+i^*)}{E_0}(W_0 - I), \tag{0.1}$$

where I is the initial investment in home bonds.

(c) Suppose the investor maximizes expected end-of-period utility, EU. Show, using eq. (0.1), that the first-order condition for optimal investment has the form

$$\frac{\mathrm{dE}U}{\mathrm{d}I} = \mathrm{E}\left\{W^{-R}E^{-(1-\gamma)(1-R)}\left[(1+i) - \frac{E(1+i^*)}{E_0}\right]\right\} = 0. \tag{0.2}$$

(d) Define

$$\rho \equiv \frac{E(1+i^*)}{E_0(1+i)}.$$

Show how to rewrite eq. (0.2) as

$$\begin{aligned} &\frac{\mathrm{dE}U}{\mathrm{d}I} \\ = \; &\underbrace{(1+i)\left[\frac{E_0(1+i)}{1+i^*}\right]^{-(1-\gamma)(1-R)}}_{K} \mathrm{E}\left\{W^{-R}\rho^{-(1-\gamma)(1-R)}(1-\rho)\right\} = 0, \end{aligned} \tag{0.3}$$

where K is simply a constant from the investor's perspective.

(e) The portfolio balance model suggests that as γ rises (greater home preference for home goods), λ (share of wealth invested in home-currency bonds should rise). In terms of eq. (0.2), we'd expect, if the theory is right, that

$$\frac{\mathrm{d}^2 \mathrm{E}U}{\mathrm{d}I \mathrm{d}\gamma} > 0. \tag{0.4}$$

(This inequality means that, at a higher level of home-goods preference γ, expected utility is still rising as I, the holding of home bonds, rises, so that the investor will have to raise I further to reach the point where $\frac{\mathrm{dE}U}{\mathrm{d}I} = 0$.) By differentiating eq. (0.3), check for conditions under which inequality (0.4) indeed holds true. (You will need to use the calculus fact that if $y = a^x$, $\frac{\mathrm{d}y}{\mathrm{d}x} = a^x \log a$, where the log is the natural log.) Does the portfolio-balance effect require an implausibly high degree of risk aversion, in your opinion?