Should Central Banks Reveal Expected Future Interest Rates?

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Abstract

We examine the conditions under which a central bank raises welfare by revealing its expected future interest rate in a simple two-period model with heterogeneous information. The release of this information fully aligns central bank and private sector expectations about future shocks, therefore about future inflation and interest rates, which determine the current long-term interest rate. Transparency, therefore, tends to raise welfare because it reduces the impact of expectation errors on inflation volatility. Yet, it may be desirable for the central bank not to release the expected interest rate. This possibility arises because of how the private sector interprets the latest interest rate decision. The less msitaken it is, the more transparency is desirable. Conditions that favor the case for transparency are a high degree of precision of central bank relative to private sector information, reasonably good early information and a high elasticity of current to expected inflation.

1 Introduction

Recently, the Reserve Bank of New Zealand, the Bank of Norway and the Swedish Riksbank have started to publish their expected interest rate paths. One reason for doing so is purely logical. Inflation-targeting central banks publish the expected inflation rate, typically over a two or three-year horizon, but what assumptions underlie their expectation? Obviously, they make a large number of assumptions about the likely evolution of exogenous variables. One of these is the policy interest rate. The central banks usually provide information about their expectations about the exogenous variables but, with the two exceptions previously mentioned, they are unwilling to do the same regarding the interest rate.

Most banks report that they assume a constant policy interest rate. If, however, the resulting expected rate of inflation exceeds the inflation target, the central bank is bound to raise the policy rate, which is inconsistent with one assumption used to forecast inflation, which is the key policy signal. Other banks report that their inflation forecasting procedure relies on the interest rate implicit in the yield curve set by the market (this is the case with the Bank of England; the Riksbank did so until its recent switch). As long as the markets correctly anticipates future policy moves, there is no inconsistency, but what if not? Then, again, the inflation forecast is not consistent with both policy intentions and with the central bank's true expectations.

It may seem surprising that central banks behave inconsistently. Their answer is that their forecasts are conditional on the assumed interest rate path. Then, indeed, the inflation forecasts are not what the central banks expect to see. As they show where inflation would be if the interest rate were to follow the assumed path, they reveal implicitly that the actual interest rate will differ from the assumed path whenever the conditional inflation forecast does not converge over the chosen horizon towards the target. Put differently, the central bank practice is not inconsistent as far as conditional forecasts are concerned. The inconsistency charge only concerns unconditional inflation forecasts, those that would take into account the actually projected interest rate path. Yet, why do these central banks conceal their unconditional inflation forecasts and, therefore, their expected interest rate paths?

Goodhart (2006) offers a number of answers. He first argues that central bank decisions are normally made by committees - the Reserve Bank of New Zealand is an exception among inflation-targeting central banks - which are typically unable to agree on future interest rates. This explanation is not borne out by the case of the Bank of Norway. Two other arguments, which revolve about the obvious fact that the policy interest rate is not exogenous for a central bank, it is its instrument, are nicely put as follows:

"If, as I suggest, the central bank has very little extra (private, unpublished) information beyond that in the market, [releasing the expected interest rate path forces the bank to chose between] the Scilla of the market attaching excess credibility to the central bank's forecast (the argument advanced by Stephen Morris and Hyun Song Shin), or the Charybdis of losing credibility from erroneous forecasts."

A first concern, then, is that publishing the expected interest rate path fully reveals the bank intentions. A central bank could become unwillingly committed to its earlier announcements even though the state of the economy has changed in ways that were then unpredictable. The risk is that either the central bank validates the pre-announced path, and enacts suboptimal policies, or that it chooses a previously unexpected path and loses credibility since it does not do what it earlier said it would be doing. This argument is a reminder of the familiar debate on time inconsistency. The debate has shown that full discretion is not desirable. Blinder *et al.* (2001) and Woodford (2005) argue instead in favor of a strategy that is clearly explained and shown to the public to guide policy decisions.

The second concern is related to the result by Morris and Shin (2002) that the public tends to attribute too much weight to central bank announcements, not because central banks are better informed, but because these announcements are common knowledge. A related argument is that central bank announcements receive too much weight because, in contrast with private sector information, central bank information guides interest rate decisions. It becomes possible, then, that the private sector sets future interest rate expectations, and therefore long-term rates that directly affect the economy, to closely match central bank announcements even though those announcements are based on imprecise information.

As expected, we find that publishing the policy interest rate path may lead to welfare losses when central bank information is relatively imprecise. This result, which echoes Goodhart's reservation, reflects a subtle exchange of information between the central bank and the public. In our model, when it announces current and future policy rates, the central bank effectively reveals all its information set. This, in turn, allows the central bank to recover the private sector information set simply by observing the long-term interest rate (over the same horizon). As a consequence, the central bank can use the current short-term rate to achieve the optimal long-term rate, even if that means choosing a policy rate that is optimal for the current period viewpoint. Obviously, given the central bank's power to decide single-handedly on the policy rate, this 'monopoly' power can misused when the central bank own information is imprecise. Put differently, by exchanging information, the private sector and the central bank make it possible to carry out a monetary policy strategy that looks beyond discretion, exactly what Blinder *et al.* (2001) and Woodford (2005) have advocated.

This result can be seen as an application of second best theory. Hellwig (2005) has shown that the Morris and Shin (2002) result occurs because the combination of asymmetric information and incomplete markets implies that more information is not necessarily always welfare-increasing. Much the same occurs here. The welfare effect of revealing the interest rate path may increase or reduce welfare depending on the precision of central bank information.

The literature on the revelation of expected future policy interest rates is so far limited. Archer (2005) and Qvigstad (2005) present, respectively, the approach followed by the Royal Bank of New Zealand and the Bank of Norway. Svensson (2005) presents a detailed discussion of the shortcomings of central bank forecasts based on the constant interest rate assumption or on market rates to build up the case for using and revealing the policy interest rate path. Faust and Leeper (2005) emphasize the distinction between conditional and unconditional forecasts. They assume that the central bank holds an information advantage over the private sector, which in their model implies that sharing that information is welfare-enhancing. They show that conditional forecasts - i.e. not revealing the policy interest path - provide little information on the more valuable unconditional forecasts, for which they find some supporting empirical evidence.

Like Faust and Leeper (2005), Rudebusch and Williams (2006) assume the presence of an information asymmetry between the central bank and the private sector regarding both policy preferences and targets.[1] The private sector learns about these factors by running regressions on past information, which may include the expected interest rate path. They allow for a "transmission noise" that distorts its communication. Through simulations, they find that revealing the expected path improves the estimation process and welfare, but the gain declines as the transmission noise increases. Additionally, they explore the case when the accuracy of the central bank signals is not known by the public. They find that accuracy underestimation limits the gains from releasing the expected interest path while overestimation may be counterproductive. This result is not of the Morris-Shin variety, however, because what is at stake is not the precision of information but the size of the transmission noise, a very different phenomenon.

Our contribution is complementary to the works of Faust and Leeper (2005) and Rudebusch and Williams (2006) who assume the existence of an information *asymmetry.* Instead we focus on information *heterogeneity.* An important further difference is that in the previous papers more information is always better, at least if it is credible, while in our model poor-quality information may be welfare-reducing. In that sense, we do not assume that transparency is desirable *per se.* This is not because of the beauty contest emphasized by Morris and Shin (2002), where information is heterogeneous among private agents. Here, the assumed information heterogeneity occurs between the central bank and the private sector, considered as a whole.

Transparency may be welfare-inferior because of the way the private sector interprets the latest central bank interest rate decision. The more accurately the private sector infers the underlying central bank information, the more transparency is desirable. It might seem paradoxical that central bank opacity can be welfare improving when the private sector is poorly informed, but it is not. The more accurate is the private sector, the closer opacity comes to transparency but opacity can never beat transparency on its own turf. We show how opacity may raise welfare when it leads the private sector to so misinterpret the central bank that its own expectations are negatively correlated to those of the central bank, in effect stabilizing inflation.

The next section presents the model, a simple two-period version of the standard Neo-Keynesian log-linear model. Section 3 looks at the case when the central bank optimally chooses the interest rate and announces its expected future interest rate. In Section 4, the

[1] Rudebusch and Williams (2006) also offer an excellent overview of the policy debate about how central banks signal their intentions regarding future policy actions.

central bank follows the same rule as in Section 3, but does not reveal its expected future interest rate. Section 5 compares the welfare outcomes of the two policy regimes and the last section concludes with a discussion of arguments frequently presented to reject the release of interest rate expectations by central banks.

2 The Model

We adopt the now-standard Neo-Keynesian log-linear model, as in Woodford (2003). It includes a Phillips curve:

$$\pi_t = \beta E_t^P \pi_{t+1} + \kappa_1 y_t + \varepsilon_t \tag{1}$$

where y_t is the output gap and ε_t is a random disturbance, which is assumed to be uniformly distributed over the real line, therefore with an improper distribution and a zero unconditional mean. In what follows, without loss of generality, we assume a zero rate of time preference so that $\beta = 1$. The output gap is given by the forward-looking IS curve:

$$y_t = E_t^P y_{t+1} - \kappa_2(r_t - E_t\pi_{t+1} - r^*) + \eta_t \tag{2}$$

where r_t is the nominal interest rate and η_t is a random disturbance. We assume that the natural interest rate $r^* = 0$. Note that all expectations E^P are those of the private sector, which sets prices and decides on output.

We limit our horizon to two periods by assuming that the economy is in steady state at $t = 0$ and $t \geq 3$, with zero inflation, output gap and shocks. This assumption is meant to describe a situation where past disturbances have been absorbed so that today's central bank action is looked upon as concerned with the current situation ($t = 1$) given expectations about the near future ($t = 2$) - say two to three years ahead - while too little is known about the very long run ($t \geq 3$) to bring into consideration. Consequently, (1) and (2) imply:

$$\pi_1 = E_1^P \pi_2 - \kappa(r_1 - E_1^P \pi_2 + E_1^P r_2 - E_2^P \pi_3) + \kappa E_1^P y_3 + \varepsilon_1$$

where $\kappa = \kappa_1\kappa_2$. Since the economy is known to return to steady state in period 3, this simplifies to:

$$\pi_1 = (1+\kappa)E_1^P \pi_2 - \kappa(r_1 + E_1^P r_2) + \varepsilon_1 \tag{3}$$

where $r_1 + E_1^P r_2$ is the long-run (two period) interest rate, and similarly:

$$\pi_2 = -\kappa r_2 + \varepsilon_2 \tag{4}$$

where we also assume that the central bank sets $r_t = r^*$ for $t \geq 3$, which is indeed optimal as will soon be clear.

The loss function usually assumes that society is concerned with stabilizing both inflation and the output gap around some target levels, which allows for an interesting but well-known inflation-output trade-off. Much of the literature on central bank transparency additionally focuses on the idea that the public at large may not know how the central bank weighs these two objectives. This assumption creates an information asymmetry, which makes transparency generally desirable, as shown in Rudebusch and Williams (2005). Here, instead, we ignore this issue by assuming that the weight on the output gap is zero and that the target inflation rate is also nil. Since the rate of time preference is zero, the loss function is, therefore, evaluated as the unconditional expectation:

$$L = E(\pi_1^2 + \pi_2^2) \tag{5}$$

and this is known to everyone.

The information structure is crucial. At the beginning of period 0, both the central bank and the private sector receive a signal on the shock ε_1. These signals have known variances $(k\alpha)^{-1}$ and $(k\beta)^{-1}$ for the central bank and the private sector, respectively. At the beginning of period 1, new signals on ε_{1n}^{CB}, and ε_{1n}^{P} with variances $[(1-k)\alpha]^{-1}$ and $[(1-k)\beta]^{-1}$,respectively, are received by the central bank and the private sector. Using both signals through Bayesian updating, the central bank and the private sector infer expectations ε_1^{CB} and ε_1^{P} with variances α^{-1} and β^{-1}. We assume that $0 < k \leq 1/2$, which implies that the signal newly received in period 1 is more precise than the one received in period 0.

Much the same occurs concerning the period 2 disturbance ε_2. At the beginning of period 1, the central bank and the private sector receive, respectively, the signals ε_2^{CB} and ε_2^{P} with variances $(k\alpha)^{-1}$ and $(k\beta)^{-1}$. At the beginning of period 2, they receive new signals about ε_2, ε_{2n}^{CB} and ε_{2n}^{P} with lower variances $[(1-k)\alpha]^{-1}$ and $[(1-k)\beta]^{-1}$, respectively, so that the variances of the updated forecasts $E_2^{CB}\varepsilon_2$ and $E_2^{P}\varepsilon_2$ have variances α^{-1} and β^{-1}.

In addition, we assume that, at the beginning of period 2, the realized values of π_1 and ε_1 become known to both the central bank and the private sector. After the signals have been received at the beginning of each period t, the central bank decides on its interest rate r_t for that period in order to minimize $E_t^{CB}L$. Finally, after the central bank decision and potential signaling, the private sector decides on output and prices.

The focus of the paper is whether, in addition to choosing and announcing r_t, the central bank should also reveal its expectation of the interest rates in the following periods r_{t+i}. This issue is made simpler once we recognized that $r_t = 0$ for all $t \geq 3$, so that we will only need to consider the choice of r_1 and r_1 and whether the central bank reveals $E_1^{CB} r_2$.

3 The Central Bank Reveals its Interest Rate Forecast

We first look at the case where the central banks reveals $E_1^{CB} r_2$, which we refer to as the transparency case. In period 2, the central bank sets the interest rate in order to minimize $E_2^{CB}(\pi_2)^2$ conditional on the information available at the beginning of this period, i.e. after it has received the signal ε_{2n}. The central bank seeks to offset the perceived shock and sets:

$$r_2 = \frac{1}{\kappa} E_2^{CB} \varepsilon_2 \tag{6}$$

The simplicity of this choice is a consequence of our assumption that the economy will return to the steady state in period $t = 3$. It can be viewed as a rule or, alternatively, as discretionary action given the new information received at the beginning of the period.

Moving to period 1, when the central bank publishes $E_1^{CB} r_2 = \frac{1}{\kappa} E_1^{CB} \varepsilon_2 = \frac{1}{\kappa} \varepsilon_2^{CB}$ it fully reveals its own signal ε_2^{CB}. As a consequence, the private sector receives two signals about ε_2: its own signal ε_2^{P} with precision $k\beta$ and, as just noted, the central bank signal ε_2^{CB} with known precision $k\alpha$. Denoting the relative precision of the central bank and private sector signals as $z = \frac{\alpha}{\beta}$, the optimal period 1 forecast of ε_2 by the private sector is:

$$E_1^{P} \varepsilon_2 = \gamma_1^{tr} \varepsilon_2^{P} + (1 - \gamma_1^{tr}) \varepsilon_2^{CB} = \frac{1}{1+z} \varepsilon_2^{P} + \frac{z}{1+z} \varepsilon_2^{CB} \tag{7}$$

Similarly, conjecture that:

$$E_1^P E_2^{CB} \varepsilon_2 = \gamma_2^{tr} \varepsilon_2^P + (1 - \gamma_2^{tr}) \varepsilon_2^{CB} \tag{8}$$

with unknown coefficient γ_2^{tr} to be determined.

When period 2 starts, π_1 and ε_1 become known. As a consequence, (3) and (6) show that $\pi_1 + \kappa r_1 - \varepsilon_1 = (1+\kappa)(E_1^P \varepsilon_2 - E_1^P E_2^{CB} \varepsilon_2) - E_1^P E_2^{CB} \varepsilon_2$, which is known. Using (7) and (8) we have:

$$\pi_1 + \kappa r_1 - \varepsilon_1 = [(1+\kappa)(\gamma_1^{tr} - \gamma_2^{tr}) - \gamma_2^{tr}](\varepsilon_2^P - \varepsilon_2^{CB}) + \gamma_2^{tr} \varepsilon_2^{CB}$$

This implies that, at the beginning of period 2, when π_1 and ε_1 become known, the central bank can recover the private signal ε_2^P. We have a mirror effect: by revealing the expected future interest rate, the central gives out its period 1 information ε_2^{CB} and gets in return, in period 2, the private information ε_2^P. Put differently, by observing how its own information was previously used, the central bank now recovers the signal previously received by the private sector: the mirror image is not identical to the original, it adds information to both the central bank and the private sector.

Consequently, at the time of setting the interest rate r_2, the central bank has received three signals about ε_2: ε_2^{CB} received in period 1 with precision $k\alpha$, ε_{2n}^{CB} received in period 2 with precision $(1-k)\alpha$ and now ε_2^P with known precision $k\beta$. Applying Bayes' rule we have:

$$E_2^{CB} \varepsilon_2 = \frac{z[k\varepsilon_2^{CB} + (1-k)\varepsilon_{2n}^{CB}] + k\varepsilon_2^P}{z+k}$$

Noting that $E_1^P \varepsilon_{2n}^{CB} = E_1^P \varepsilon_2$, it follows that $\gamma_1^{tr} = \gamma_2^{tr}$ and therefore:[2]

$$E_1^P E_2^{CB} \varepsilon_2 = \frac{1}{1+z} \varepsilon_2^P + \frac{z}{1+z} \varepsilon_2^{CB} = E_1^P \varepsilon_2$$

As they swap signals, both the central bank and the private sector learn from each other. The key result is that the private sector's own forecast of the future shock is perfectly aligned with its perception of the future central bank estimate of this shock which, it knows, will lead to the choice of the future interest rate. The private sector knows that its own forecast will be taken into account by the central bank.

Proposition 1 *When the central bank reveals its expected future interest rate, the private sector and the central bank exchange information about their signals received in period 1 about the period 2 shock:*

- in period 1, the central fully reveals its signal, which the private sector uses to improve its own forecast.

- in period 2, the central bank can identify the signal previously received by the private sector.,

- as a result, central bank and private expectations are fully aligned and, in period 1, both expect future inflation to be zero.

[2] Proof:

$$\begin{aligned} E_1^P E_2^{CB} \varepsilon_2 &= \frac{z E_1^P [k\varepsilon_2^{CB} + (1-k)\varepsilon_{2n}^{CB}] + k\varepsilon_2^p}{z+k} \\ &= \frac{z E_1^P \left[k\varepsilon_2^{CB} + (1-k)\left(\frac{\varepsilon_2^p}{1+z} + \frac{z\varepsilon_2^{CB}}{1+z}\right)\right] + k\varepsilon_2^p}{z+k} \\ &= \frac{\varepsilon_2^P}{1+z} + \frac{z\varepsilon_2^{CB}}{1+z} \end{aligned}$$

The last statement in the proposition is readily established. In period 2, the interest rate is set by the central bank according to (6) and inflation is set by the private sector based on its own information set available at the beginning of the period, which includes the interest rate r_2 and therefore $E_2^{CB}\varepsilon_2$, the central bank updated information about the shock ε_2. According to (4), this is:

$$\pi_2 = \varepsilon_2 - E_2^{CB}\varepsilon_2 \tag{9}$$

As a consequence $E_1^P\pi_2 = E_1^P\varepsilon_2 - E_1^P E_2^{CB}\varepsilon_2 = 0 = E_1^{CB}\pi_2$.

In period 1, the central bank sets the interest rates in order to minimize $E_1^{CB}(\pi_1^2 + \pi_2^2)$ conditional on available information. It follows from (9) that r_1 does not affect π_2, so in period 1 the central can simply minimize $E_1^{CB}\pi_1^2$. With $E_1^P\pi_2 = 0$, from (3) we see that the central bank chooses the short-term interest rate r_1 such that the long-term interest rate - which matters for aggregate demand - fully offsets the expected shock:

$$\kappa r_1 + \kappa E_1^{CB} E_1^P r_2 = E_1^{CB}\varepsilon_1$$

Since the central bank has released $E_1^{CB}r_2$, $E_1^P r_2 = E_1^{CB} r_2$, from (6) we know that $\kappa E_1^{CB} r_2 = \varepsilon_2^{CB}$, and the optimal policy decision is:

$$r_1 = \frac{1}{\kappa}\left(\varepsilon_1^{CB} - \varepsilon_2^{CB}\right) \tag{10}$$

The first term is obvious: the central bank offsets the inflation impact of the first-period shock. As for the second term, a high signal ε_2^{CB} indicates that inflation will be high in period 2. Consequently, the central bank anticipates that it will set the period 2 interest rate high. In period 1, since it cares about the long-term interest rate, *ceteris paribus* the central bank will offset this expected rise in the future short-term rate by setting the current short-term rate lower.

Collecting the previous results, we obtain:

$$\pi_1 = \left(\varepsilon_1 - \varepsilon_1^{CB}\right) + \frac{1}{1+z}\left(\varepsilon_2^{CB} - \varepsilon_2^P\right) \tag{11}$$

Period 1 inflation depends on two forecasting errors: the period 1 central bank forecasting error and the discrepancy between the central bank and the private sector signals regarding period 2 shock. Note that the impact of this last discrepancy is less than one for one ($\frac{1}{1+z} < 1$) because the revelation of ε_2^{CB} by the central bank leads the private sector to discount its own signal ε_2^P and to bring its forecast $E_1^P\varepsilon_2$ in the direction of ε_2^{CB}.

It is worth emphasizing that the private sector is well aware that the central interest rate forecast is bound to be revealed inaccurate. Indeed, in general, there is no reason for $E_1^P E_2^{CB}\varepsilon_2$ to be equal to ε_2, but the eventual realization of this difference is irrelevant. The private sector fully understands that the future interest rate will usually differ from what was announced since the central bank will then respond to newly received information ε_{2n}^{CB}, see (6). This eventual discrepancy is fully anticipated by the private sector because the central bank strategy - in other words, its loss function - is public knowledge, so credibility is not an issue here. The difference between the pre-announced rate $E_1^{CB}r_2$ and the actual rate r_2 is well understood to be purely random and therefore uninformative. Importantly, this result holds independently of the degree of precision of the signals received by the central bank and the private sector. What matters is that signal precision be known.[3]

[3]The case when the signal precisions are not known is left for further research. For a study of this case in a different setting, see Gosselin et al. (2006).

Finally, for future reference, in this case of transparency the unconditional loss function is:

$$L^{tr} = E\left(\pi_1\right)^2 + E\left(\pi_2\right)^2 = \frac{1}{\beta}\left[\frac{1}{z} + \frac{1}{k}\left(\frac{1}{1+z}\right)^2\left(\frac{1}{z} + \frac{1}{k}\right) + \frac{1}{z+k}\right]$$

4 The Central Bank Does not Reveal its Interest Rate Forecast

We consider now the case when the central bank does not announce its expectation of the future interest rate. We call this the opacity case. The optimal interest rate in period 2 remains given by (6), formally unchanged from Section 3. The resulting inflation rate is also the same as in (9), although the information available to the central bank is different from that in the previous case, as will be emphasized below.

In period 1, the central bank still reveals the current interest rate, which is set on the basis of the information available to the central bank, i.e. $E_1^{CB}\varepsilon_1 = \varepsilon_1^{CB}$ and $E_1^{CB}\varepsilon_2 = \varepsilon_2^{CB}$. We restrict our attention to the following policy linear rule which optimally uses all available information:[4]

$$r_1 = \mu E_1^{CB}\varepsilon_1 + \nu E_1^{CB}\varepsilon_2 \tag{12}$$

Having observed r_1, the private sector sets the inflation rate according to (3). To that effect, it needs to forecast future inflation, which by (9) depends on $E_2^{CB}\varepsilon_2 = k\varepsilon_2^{CB} + (1-k)\varepsilon_{2n}^{CB}$. In contrast to the previous case, ε_2^{CB} is now unknown to the private sector. As a consequence $E_1^P\varepsilon_2$ no longer coincides with $E_1^P E_2^{CB}\varepsilon_2$. In order to form its forecast $E_1^P E_2^{CB}\varepsilon_2$, following Bayes' rule, the private sector uses its three available signals ε_1^P, ε_2^P and r_1. It can use ε_2^P directly. In addition, the interest rate rule (12) implies that $E_1^{CB}\varepsilon_2 = (r_1 - \mu E_1^{CB}\varepsilon_1)/\nu$. However the private sector does not know $E_1^{CB}\varepsilon_1 = \varepsilon_1^{CB}$, it only knows $E_1^P\varepsilon_1 = \varepsilon_1^P$. Still, taking ε_1^P as a signal for ε_1^{CB} the private sector can use the linear combination $(r_1 - \mu\varepsilon_1^P)/\nu$ of the two signals r_1 and ε_1^P to improve its forecast $E_1^P E_2^{CB}\varepsilon_2$.

However, doing so introduces an error $\varepsilon_1^{CB} - \varepsilon_1^P$. In order to correct for this error, the private sector must forecast $E_1^P(\varepsilon_1^{CB} - \varepsilon_1^P)$ and adjust Bayes' rule accordingly. Using again the interest rate rule (12), we see that $(r_1 - \nu E_1^{CB}\varepsilon_2)/\mu = \varepsilon_1^{CB}$ so that $E_1^P(\varepsilon_1^{CB} - \varepsilon_1^P) = (r_1 - \nu\varepsilon_2^P)/\mu - \varepsilon_1^P$. The optimal forecast is therefore of the form:

$$\begin{aligned}
E_1^P E_2^{CB}\varepsilon_2 &= \gamma_2^{op}\varepsilon_2^P + (1-\gamma_2^{op})\left(\frac{r_1 - \mu\varepsilon_1^P}{\nu}\right) + \gamma_3^{op}\left(\frac{r_1 - \nu\varepsilon_2^P}{\mu} - \varepsilon_1^P\right) \\
&= \gamma_2^{op}\varepsilon_2^P + (1-\gamma_2^{op})\left[\varepsilon_2^{CB} - \frac{\mu}{\nu}(\varepsilon_1^P - \varepsilon_1^{CB})\right] \\
&\quad + \gamma_3^{op}\left[\varepsilon_1^P - \varepsilon_1^{CB} + \frac{\nu}{\mu}(\varepsilon_2^P - \varepsilon_2^{CB})\right]
\end{aligned} \tag{13}$$

with γ_2^{op} and γ_3^{op} to be determined. Note that the two first terms are signals about ε_2, so their weights add up to unity, while the third term corresponds to the adjustment $E_1^P(\varepsilon_1^{CB} - \varepsilon_1^P)$ and is zero mean. The same reasoning can be applied to $E_1^P\varepsilon_2$ to obtain:

$$E_1^P\varepsilon_2 = \gamma_1^{op}\varepsilon_2^P + (1-\gamma_1^{op})\left[\varepsilon_2^{CB} - \frac{\mu}{\nu}\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)\right] \tag{14}$$

[4] There is no reason to presume that a linear rule is optimal. This restrictive assumption, required to carry through the calculations that follow, can be seen as a form of Taylor rule that approximates the optimal policy. This introduces some asymmetry between the transparency and opacity cases: in the former, the rule is optimal, in the latter it may not be. Unfortunately, we are not able to derive the optimal policy choice under opacity.

where $\gamma_1^{op} = \frac{k(1+z)+\left(\frac{\nu}{\mu}\right)^2}{(1+z)\left[k+\left(\frac{\nu}{\mu}\right)^2\right]}$.

As in the transparency case, the unknown weighting coefficients γ_2^{op} and γ_3^{op}can be found by identification. In this case, there is no analytical solution. The Appendix shows that:

$$\gamma_1^{op} - \gamma_2^{op} + \gamma_3^{op} \theta \frac{\nu}{\mu} = 0 \tag{15}$$

where $\theta = 1 + \dfrac{1}{(1+\kappa)\left(\gamma_1^{op} - \gamma_2^{op}\right) - \gamma_2^{op} + (2+\kappa)\dfrac{\nu}{\mu}\gamma_3^{op}}$.

In comparison with the case where the central bank publishes its expected future interest rate, (15) implies that, in general, $\gamma_1^{op} \neq \gamma_2^{op}$ so that $E_1^P \varepsilon_2 \neq E_1^P E_2^{CB} \varepsilon_2$. The private sector is now well aware that its own period 1 forecast of the disturbance ε_2 differs from that of the central bank . Since it is $E_1^{CB}\pi_2$ that will inform the choice of r_1, $E_1^P \pi_2$ is no longer nil. This is the key difference between transparency and opacity. The gap between central bank and private sector expectations is conveniently captured by $\theta = \dfrac{\gamma_1^{op} - \gamma_2^{op}}{\gamma_3^{op}\left(-\dfrac{\nu}{\mu}\right)}$.

The Appendix shows that the optimum interest rate rule in period 1 requires $\mu = -\nu = \kappa^{-1}$. Note that this implies that the monetary policy rule is formally identical to (10) obtained in the case of transparency. This will not imply the same inflation rates since the information sets of the central bank and of the private sector are not the same. The above results can be summarized as follows:

Proposition 2 *When the central bank follows the linear rule (12), formally it makes the same interest rate decision irrespective of whether it announces or not its expected future interest rate. Private and central bank expectations are no longer aligned.*

To understand why $\mu = -\nu = \kappa^{-1}$ is optimal, note that (3) and (6) imply:

$$\pi_1 = (1+\kappa)E_1^P \pi_2 - \kappa r_1 - E_1^P E_2^{CB} \varepsilon_2 + \varepsilon_1 \tag{16}$$

The central bank chooses r_1 to minimize $E_1^{CB}\pi_1^2$. While $E_1^P \pi_2 \neq 0$, we still have $E_1^{CB} E_1^P \pi_2 = 0$ since the deviation is the result of an expectation discrepancy unknown to the central bank. As in the transparency case, therefore, the central bank chooses a short-term interest rate r_1 such that the long-term interest rate fully offsets the expected shock ε_1^{CB}. Using (12), and noting that $E_1^{CB}\varepsilon_1 = \varepsilon_1^{CB}$ and $E_1^{CB}\varepsilon_2 = \varepsilon_2^{CB}$, (16) can be rewritten as:

$$\pi_1 = (1+\kappa)E_1^P \pi_2 - \left(E_1^P E_2^{CB} \varepsilon_2 + \kappa\nu\varepsilon_2^{CB}\right) + \left(\varepsilon_1 - \kappa\mu\varepsilon_1^{CB}\right) \tag{17}$$

Noting that $E_1^{CB} E_1^P E_2^{CB} \varepsilon_2 = \varepsilon_2^{CB}$, we have $E_1^{CB}\pi_1 = -(\varepsilon_2^{CB} + \kappa\nu\varepsilon_2^{CB}) + (\varepsilon_1^{CB} - \kappa\mu\varepsilon_1^{CB})$, and it becomes clear why the optimal policy rules requires $\kappa\mu = 1$ and $\kappa\nu = -1$. The resulting inflation rate is:

$$\pi_1 = \frac{1}{\theta - 1}\left(\varepsilon_2^P - \varepsilon_2\right) - \frac{\theta}{\theta - 1}\left(\varepsilon_1^P - \varepsilon_1\right) + \frac{1}{\theta - 1}\left[\left(\varepsilon_1^{CB} - \varepsilon_1\right) - \left(\varepsilon_2^{CB} - \varepsilon_2\right)\right] \tag{18}$$

which combines all forecast errors of both the private sector and the central bank. Note that the expectation alignment discrepancy parameter becomes $\theta = \dfrac{\gamma_1^{op} - \gamma_2^{op}}{\gamma_3^{op}}$.

The loss function under central bank opacity is then:

$$
\begin{aligned}
L^{op} &= E(\pi_1)^2 + E(\pi_2)^2 \\
&= \frac{1}{\beta}\left[\left(\frac{1}{\theta-1}\right)^2\left(\frac{1}{z}+\frac{1}{k}+\frac{1}{kz}\right)+\left(\frac{\theta}{\theta-1}\right)^2+\frac{1}{z}\frac{z+k\theta^2(1+z)}{z+k+k\theta^2(1+z)}\right]
\end{aligned}
$$

5 Welfare Analysis

In this section, we compare welfare when the central bank reveals its expected interest rate - labeled transparency - and when it does not - labeled opacity: $\Delta L = L^{op} - L^{tr}$.

5.1 Preliminary observation

We first compare the welfare losses separately period by period. Starting with period 2, we have:

$$
\beta\Delta L_2 = L_2^{op} - L_2^{tr} = \frac{k^2\theta^2(1+z)}{z(z+k)\left[z+k+k\theta^2(1+z)\right]} > 0
$$

Proposition 3 *Transparency is always welfare-increasing in period 2.*

The reason is that the central bank is better informed when it can recover the private sector signal ε_2^P, see (9).

Thus, a sufficient condition for transparency to be welfare-improving is that the period 1 welfare difference $\Delta L_1 = L_1^{op} - L_1^{tr} \geq 0$. We have:

$$
\beta\Delta L_1(\theta) = \left(\frac{1}{\theta-1}\right)^2\frac{k+1+z}{kz} + \left(\frac{\theta}{\theta-1}\right)^2 - \frac{1+k(1+z)}{kz(1+z)} \tag{19}
$$

A study of this expression as a function of θ, presented in the Appendix, yields the following sufficient condition for transparency to be welfare improving:

Proposition 4 *A sufficient condition for the release by the central bank of its expected future interest rate to be welfare-improving is that* $z > \frac{1+k}{\sqrt{k}}$.

The more precise is the central bank signal α relative to the private sector signal β - the higher is z - the more likely it is that transparency pays off. By releasing relatively precise information, the central bank allows the private sector to improve its forecasts. Conversely, if central bank information is of poor quality, it can reduce the precision of private sector inference and, when $z < \frac{1+k}{\sqrt{k}}$, the situation becomes ambiguous.[5] We now turn to a direct study of the general case.

5.2 Why may opacity raise welfare?

We first ask why opacity could reduce welfare. We know that this must be by reducing inflation volatility in Period 1. Recalling from (3) that $\pi_1 = (1+\kappa)E_1^P\pi_2 - \left[\kappa(r_1 + E_1^P r_2) - \varepsilon_1\right]$, we see that inflation volatility depends on the variability of future inflation, as expected by the private sector, and on how accurately the nominal long-term interest rate $r_1 + E_1^P r_2$ tracks the first period disturbance ε_1 that the central bank aims at offsetting.

[5] Note that for k ranging from 0 to 0.5, the threshold value of z ranges from ∞ (when $k = 0$ the early signals are useless) to 2.1.

Under transparency we know that the private sector trusts the central bank to do the right thing in period 2, so it forecasts $E_1^P\pi_2 = 0$. Under opacity, by contrast, $E_1^P\pi_2 \neq 0$: by being opaque, the central bank allows a source of inflation volatility to creep in. Opacity, therefore, is welfare reducing, unless expected inflation is sufficiently positively correlated with the second term, which is rewritten using (6), (10) and (9):

$$\kappa(r_1 + E_1^P r_2) - \varepsilon_1 = (\varepsilon_1^{CB} - \varepsilon_1) + (E_1^P \varepsilon_2 - \varepsilon_2^{CB}) - E_1^P \pi_2$$

Note first that expected inflation now appears on the right hand-side with a negative sign. Indeed, $E_1^P\pi_2 = E_1^P\varepsilon_2 - E_1^P E_1^{CB}\varepsilon_2$ is, say, positive when the private sector believes that the central bank underestimates the future disturbance. In this case the private sector expects the central bank to set the interest rate low. *Ceteris paribus*, this terms tends to make the correlation between $E_1^P\pi_2$ and $\kappa(r_1 + E_1^P r_2) - \varepsilon_1$ negative, hence to reinforce the negative impact of opacity on welfare. For opacity to stand a chance of being welfare improving, therefore, we need a (strongly) positive correlation between $E_1^P\pi_2$ and the remaining terms in the above equation:

$$\psi = \kappa(r_1 + E_1^P r_2) - \varepsilon_1 + E_1^P \pi_2 = (\varepsilon_1^{CB} - \varepsilon_1) + \left(E_1^P \varepsilon_2 - \varepsilon_2^{CB}\right) \quad (20)$$

We call ψ the "policy miss" since it incorporates the discrepancy between the intended effect of the long-term interest rate as well as the distortionary effect of opacity on private sector inflationary expectations. Note that some policy miss is unavoidable, even under transparency, because the central bank sets the interest rate before it can observe the relevant disturbance.

The question now is whether it is possible that expected inflation be negatively correlated with ψ. Using (14) with $\mu/\nu = -1$ we have:

$$E_1^P \varepsilon_2 - \varepsilon_2^{CB} = \gamma_1^{op}(\varepsilon_2^P - \varepsilon_2^{CB}) + (1 - \gamma_1^{op})\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)$$

Then from (13) we have:

$$E_1^P \pi_2 = E_1^P \varepsilon_2 - E_1^P E_1^{CB} \varepsilon_2 = (\gamma_1^{op} - \gamma_2^{op} - \gamma_3^{op})\left[(\varepsilon_2^P - \varepsilon_2^{CB}) - \left(\varepsilon_1^P - \varepsilon_1^{CB}\right)\right] \quad (21)$$

Noting that (15) implies $\gamma_1^{op} - \gamma_2^{op} - \gamma_3^{op} = \gamma_3^{op}(\theta - 1)$, and using the definition of γ_1^{op} in (14), we find:

$$Cov(E_1^P \pi_2, \psi) = \gamma_3^{op}(\theta - 1)\frac{k+1}{\alpha k}$$

Thus opacity may reduce inflation volatility if $\gamma_3^{op}(\theta - 1) > 0$.

In order to understand the issue, consider for instance the situation when the central bank signal ε_2^{CB} is very low, lower than the private sector signal ε_2^P. Independently of whether it is transparent or not, the central bank uses the same rules (6) and (10). *Ceteris paribus*, therefore, a low ε_2^{CB} means a high interest rate r_1 and a large ψ. The question, then, is what does it mean for the expected exchange rate $E_1^P\pi_2$?

It all depends on how the private sector reacts to the low signal ε_2^{CB}. This reaction differs according to the policy regime. Under transparency, this signal is known to the private sector. It aligns its expectations accordingly and remains confident that the central bank will achieve its aim, subject to an unavoidable forecast error, so $E_1^P\pi_2 = 0$ and there is no correlation.[6] Under opacity, the signal is not known to the private sector anymore. All that it can do is to observe a surprisingly high interest rate r_1 and try to draw the best inference that it can from (10). The private sector may correctly infer

[6]Formally $\gamma_3^{op} = 0$.

that the central bank signal ε_2^{CB} is low. As a result it will forecast a low inflation rate and $Cov(E_1^P \pi_2, \psi) < 0$. Alternatively, the private sector may interpret a high r_1 as an indication that the central bank signal ε_1^{CB} is large. This possibility leads the private sector to diverge from the central bank and forecast a higher period 2 inflation rate, in which case $Cov(E_1^P \pi_2, \psi) > 0$.

It all depends, therefore, on the weight that the private sector puts on the two central bank signals that determine the current interest rate. If the private gets is mostly right, opacity is not desirable because it just adds volatility in the current period. Opacity may be welfare-enhancing if the private sector draws the wrong inference, in which case its expectation error offsets expected inflation in driving current inflation. The overall effect is ambiguous, as reflected in the correlation condition. It depends on the quality of information (z, k) and of the effect of expected future inflation on current inflation (κ).

The link between these parameters and the sign of $Cov(E_1^P \pi_2, \psi)$ is studied in the Appendix. The interpretation of these results is fairly intuitive. A few observations can be made. We have seen that, paradoxically, opacity can be desirable when the private sector is misled because its expectation errors tend to offset each other. This is the case when the the relative precision z of central bank signals and when the precision of the early signals is not too large. Conversely, ceteris paribus, a high relative precision of central bank signals or of early signals enhances the case for transparency. In addition, since opacity raises current inflation volatility by making future inflation expectations more volatile, it will be less desirable the higher is the effect κ of the latter on the former.

The condition $Cov(E_1^P \pi_2, \psi) > 0$ is necessary, not sufficient, for three reasons. First, as we have seen, there is a natural tendency toward a negative correlation because the private sector forecasts a positive inflation rate when it believes that the central bank underestimates the disturbance ε_2 and will therefore set the future interest rate r_2 too low. Second, even if the correlation is positive, it may be too small to offset the detrimental effect of opacity, which comes in the form of volatility of private sector forecasts of future inflation. Finally, it is not enough that opacity reduces first period inflation; it must reduce it by a lot to offset the fact that inflation in period 2 is always less volatile under transparency than under opacity. Thus the odds seem pretty much stacked in favor of revealing the expected future interest rate.

5.3 When does opacity raise welfare?

We now look at how welfare comparisons relate to the three model parameters z, κ and k. Figures 1 and 2 illustrate their combined effects on the desirability of transparency. These figures are based on a detailed analysis presented in the Appendix. Figure 1 presents the situation when κ is large, Figure 2 to the case of a lower κ. The shaded area corresponds to the case where $\Delta L < 0$, i.e. when opacity welfare-dominates transparency. The following draws some general implications, which are further detailed and interpreted below.

Proposition 5 *A central bank that follows an optimal linear interest rule (12) raises welfare by revealing the future interest rate in the following cases:*

- when the central bank signal is high enough relative to the private signal precision (a high z).

- when the elasticity of current to expected inflation is large (high κ) and the relative signal and early signal precision are not too low (z and k not too low).

[Figures 1 and 2 about here]

Consider first the case when the central bank precision is high relative to the private sector precision, i.e. when z is large. From Proposition 4 we already know that, in this case, irrespective of the values of κ and k, transparency dominates, including in period 1. Under opacity, as $z \to \infty$, $\gamma_3^{op} \to 0$, and the expected inflation term in (3) vanishes; the dominating source of welfare loss is private sector forecast errors, the second term. Indeed then the private sector increasingly disregards its own signals, which tends to eliminate the expected inflation term ($\gamma_1^{op} - \gamma_2^{op} \to 0$). There is little to gain from opacity and transparency welfare-dominates because it provides for more information. Conversely, as z becomes smaller, the benefit from information disclosure declines while the possibility that forecast errors offset each other rises, which implies that opacity may increase welfare.

Now consider the role of the elasticity of current to expected inflation κ. This is the channel through which future expected inflation and interest rates affect current inflation. It determines, therefore, the choice of the current interest rate by the central bank. Its effect is illustrated by the two figures, which show that transparency generally dominates at lower values of z while opacity dominates for a wider range of higher values of k. As κ increases, so does the the expected inflation term in (3):[7] because the expected future real interest rate increasingly becomes a source of inflation volatility in period 1, it is crucially important to align expectations. This has two effects. First, it generally makes transparency more desirable; second, the precision of the central bank signal becomes more important. Conversely, when κ is smaller, it may pay for the central bank not to reveal its expected future interest rate if the cost of non-alignment of expectations helps offset its own expectation errors. This, in turn, depends less on the relative precision of early signals, which does little to align expectations, and more on the relative central bank information precision z.

Finally, we look at the role of early information precision k. Remember that for each period disturbance, a first signal is received one period ahead, with precision $k\alpha$ for the central bank and $k\beta$ for the private sector, and a second signal is received at the beginning of the period, with precision $(1-k)\alpha$ for the central bank and $(1-k)\beta$ for the private sector. Thus a low k means that early information is highly imprecise. We further assume that $k < 1/2$ to insure that precision increases over time. The role of k is ambiguous but, in general, we find that transparency is more desirable the more precise are the early signals.

To understand why the welfare effect of k is ambiguous, remember that, when it releases its interest rate forecast, the central bank reveals to the private sector its early signal for the next period and then captures back the private sector's own early forecast. Thus, on the one hand, when k is low, early signals are imprecise and therefore transparency is not particularly helpful.[8] Thus a larger k tends to favor transparency because it provides more useful information. On the other hand, as k increases, more attention is paid by both the central bank and the private sectors to their early signals, which increases expectation discrepancy. If the relative precision z of the central bank signal is not very large, the offsetting effect of the signal alignment discrepancy becomes large, which makes opacity more desirable. As k further rises, the value of exchanging increasingly precise signal exchanges comes to dominate and transparency becomes optimal again.

Another way to understand the combined roles of the model's parameters is to remember Proposition 4, which states that transparency is desirable whenever the central bank relative signal precision z exceeds a threshold $\widehat{z} = \frac{1+k}{\sqrt{k}}$. This threshold declines as k rises. Thus the more precise are the early signals, the less precise needs to be the relative precision of the central bank signals. Similarly, we have seen that a larger elasticity of

[7] It can be shown that as $\kappa \to \infty$ $(2+\kappa)\gamma_3^{op}(\theta-1) \sim \sqrt{\kappa+1} + 0(\frac{1}{\kappa})$.

[8] More precisely, as $k \to 0$, $\gamma_3^{op} \to 0$ and $\gamma_1^{op} - \gamma_2^{op} \to 0$. At the limit there is no expectation misalignment. Still, transparency provides more information than opacity. Graphically, we see that transparency always dominate when k is close to zero.

current to expected inflation κ increases the role of the expectations alignment discrepancy. As k rises, the threshold level of beyond which transparency is desirable becomes lower.[9] The reason is that high early signal precision makes transparency more effective in eliminating the alignment discrepancy which is more desirable when the elasticity of current to expected inflation is high.

6 Conclusions

The result that the release of interest rate expectations may be desirable is not generally held, especially by practitioners. An articulate presentation of the opposite view is provided by Goodhart (2005):

> If an MPC's non-constant forecast was to be published, there is a widespread view, in most central banks, that it would be taken by the public as more of a commitment, and less of a rather uncertain forecast than should be the case, (though that could be mitigated by producing a fan chart of possible interest rate paths, rather than a point estimate: no doubt, though, measuring rulers and magnifying glasses would be used to extract the central tendency). Once there was a published central tendency, then this might easily influence the private sector's own forecasts more than its own inherent uncertainty warranted, along lines analysed by Morris and Shin (1998, 2002, 2004). Likewise when new, and unpredicted, events occurred, and made the MPC want to adjust the prior forecast path for interest rates, this might give rise to criticisms, ranging from claims that the MPC had made forecasting errors to accusations that they had reneged on a (partial) commitment.

Part of the argument directly refers on Morris and Shin's common knowledge effect. We do not address this issue here because it has been shown to rest on highly unlikely assumptions. Indeed, it assumes that the central bank is relatively poorly informed (z is low) and that the central bank does not even reveal the current interest rate.[10]

Another part of the argument is that releasing the expected interest rate might box in central banks into setting its interest rate in the future at levels no longer desirable given newly available information. One justification is the classic rules versus discretion argument in the presence of time inconsistency. This issue is discussed in Woodford (2005). Another justification is that the private sector will not realize that the central bank's forecast is imprecise and will badly interpret any discrepancy between the preannounced and the realized interest rate decision. This does not happen here as we do not allow for time inconsistency: there is no inflation bias and the central bank preferences are well known. Under these conditions, when it explicitly recognizes that the central bank forecast is imprecise, the private sector can still improve on both public and private signals by combining them. In our model, it is this Bayesian signal extraction mechanism that is the source of welfare gain, and the gain is larger the more precise are the early (i.e. one period ahead) signals. Transparency raises welfare because it fully aligns the central bank and the private sector forecasts of the future shock.

The effect is further enhanced when the real long-term interest rate has a strong impact on aggregate demand. Through this channel enter private sector expectations of future nominal interest rate and inflation. Under transparency, these expectations reflect only forecasting errors and therefore average out to zero, so the size of the elasticity of

[9] This is most easily seen by noting that $\Delta L \to 2\dfrac{2+k}{k\kappa+k-1}$ when $z \to 0$. Then $\Delta L > 0$ when $\kappa > \widehat{\kappa} = \dfrac{1-k}{k}$ with $\dfrac{\partial\widehat{\kappa}}{\partial k} < 0$.

[10] See Svensson (2005b), Hellwig (2005) and Gosselin et al. (2006).

current to expected inflation is irrelevant. Under opacity, these expectations also depend on the gap between private and central bank expectations, which does not average out to zero. As a consequence, when the elasticity of current to expected inflation rises, so does the variance of inflation under opacity but not under transparency. The reason is that, under transparency, the private sector does not expect future inflation to deviate from its target. Intuitively, opacity raises the volatility of expected future inflation and therefore the volatility of current inflation.

While these results broadly support the release of central bank interest rate expectations, the support is not general. Proposition 5 states that transparency reduces welfare when three conditions are satisfied: aggregate demand is relatively insensitive to the long-term real interest rate (low κ), early signals are imprecise relative to contemporary signals (k not too large), and the central bank signal precision is not too high relative to the private sector signal precision (z not too high). When these three conditions are jointly satisfied, opacity becomes welfare-improving because the expectations alignment discrepancy is negatively correlated with the central bank forecast errors.

Note that the three conditions must be jointly satisfied for opacity to be welfare improving. In contrast transparency is desirable when either the central bank relative signal precision is high or the elasticity of current to expected inflation is large. Relatively precise early signals are not enough to give transparency a hedge irrespective of the two other parameters, but precise early signals lower the thresholds beyond which central bank relative signal precision or the elasticity of current to expected inflation are large enough to favor transparency.

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Appendix

Proof of (14) and (15)

Using (12) note that $\varepsilon_1^{CB} = (r_1 - \nu\varepsilon_2^{CB})/\mu$ is a signal about ε_1. In period 1, the private sector observes $\dfrac{r_1 - \nu\varepsilon_2^P}{\mu} = \varepsilon_1^{CB} + \dfrac{\nu}{\mu}(\varepsilon_2^{CB} - \varepsilon_2^P)$, which is therefore also a signal about ε_1 available for the private sector with variance $\dfrac{1}{\alpha} + (\dfrac{\nu}{\mu})^2(\dfrac{1}{k\alpha} + \dfrac{1}{k\beta})$. Similarly, in period 1, the private sector observes $\dfrac{r_1 - \mu\varepsilon_1^P}{\nu} = \varepsilon_2^{CB} + \dfrac{\mu}{\nu}(\varepsilon_1^{CB} - \varepsilon_1^P)$ which is a signal about ε_2 with variance $\dfrac{1}{\beta}\left[\dfrac{1}{kz} + \left(\dfrac{\mu}{\nu}\right)^2 (1 + \dfrac{1}{z})\right]$. Using these signals, we can apply Bayes Theorem to obtain:

$$E_1^P\varepsilon_1 = \frac{\left[k + (1+z)\left(\frac{\nu}{\mu}\right)^2\right]\varepsilon_1^P + kz\left[\varepsilon_1^{CB} - \frac{\nu}{\mu}\left(\varepsilon_2^P - \varepsilon_2^{CB}\right)\right]}{k(1+z) + (1+z)\left(\frac{\nu}{\mu}\right)^2}$$

$$E_1^P\varepsilon_1^{CB} = \frac{\left(\frac{\nu}{\mu}\right)^2\varepsilon_1^P + k\left[\varepsilon_1^{CB} - \frac{\nu}{\mu}\left(\varepsilon_2^p - \varepsilon_2^{CB}\right)\right]}{k + \left(\frac{\nu}{\mu}\right)^2}$$

$$\begin{aligned} E_1^P\varepsilon_2 &= \frac{\left[k(1+z) + \left(\frac{\nu}{\mu}\right)^2\right]\varepsilon_2^P + z\left(\frac{\nu}{\mu}\right)^2\left[\varepsilon_2^{CB} - \frac{\mu}{\nu}\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)\right]}{(1+z)\left(k + \left(\frac{\nu}{\mu}\right)^2\right)} \\ &= \gamma_1\varepsilon_2^P + (1-\gamma_1)\left[\varepsilon_2^{CB} - \frac{\mu}{\nu}\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)\right] \end{aligned}$$

which defines $\gamma_1^{op} = \frac{k(1+z) + \left(\frac{\nu}{\mu}\right)^2}{(1+z)\left(k + \left(\frac{\nu}{\mu}\right)^2\right)}$.

$$E_1^P\varepsilon_2^{CB} = \frac{k\varepsilon_2^P + \left(\frac{\nu}{\mu}\right)^2\left[\varepsilon_2^{CB} - \frac{\mu}{\nu}\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)\right]}{k + \left(\frac{\nu}{\mu}\right)^2}$$

It follows that:

$$E_1^P\varepsilon_1 - E_1^P\varepsilon_1^{CB} = \frac{\varepsilon_1^P - \varepsilon_1^{CB} + \frac{\nu}{\mu}(\varepsilon_2^P - \varepsilon_2^{CB})}{(1+z)\left(k + \left(\frac{\nu}{\mu}\right)^2\right)}$$

Recall (13):

$$E_1^P E_2^{CB}\varepsilon_2 = \gamma_2^{op}\varepsilon_2^P + (1-\gamma_2^{op})\left[\varepsilon_2^{CB} - \frac{\mu}{\nu}\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)\right] - \gamma_3^{op}\left[\varepsilon_1^P - \varepsilon_1^{CB} + \frac{\nu}{\mu}\left(\varepsilon_2^P - \varepsilon_2^{CB}\right)\right]$$

Using (3), (6) and (9), we can now compute π_1, which is necessary to obtain the signal extracted by the central bank at time 2 :

$$\begin{aligned}
\pi_1 &= (1+\kappa)\left(E_1^P\varepsilon_2 - E_1^P E_2^{CB}\varepsilon_2\right) - E_1^P E_2^{CB}\varepsilon_2 - \kappa r_1 + \varepsilon_1 \\
&= (1+\kappa)\left(\gamma_1^{op} - \gamma_2^{op}\right)\left[\varepsilon_2^P - \left(\varepsilon_2^{CB} - \frac{\mu}{\nu}\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)\right)\right] \\
&\quad +\gamma_3^{op}(1+\kappa)\left[\varepsilon_1^P - \varepsilon_1^{CB} + \frac{\nu}{\mu}\left(\varepsilon_2^P - \varepsilon_2^{CB}\right)\right] \\
&\quad -\kappa r_1 - \left[\gamma_2^{op}\varepsilon_2^P + (1-\gamma_2^{op})\left(\varepsilon_2^{CB} - \frac{\mu}{\nu}\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)\right)\right] \\
&\quad -\gamma_3^{op}\left[\left(\varepsilon_1^P - \varepsilon_1^{CB}\right) + \frac{\nu}{\mu}\left(\varepsilon_2^P - \varepsilon_2^{CB}\right)\right] + \varepsilon_1
\end{aligned}$$

This expression can be rewritten as:

$$\frac{\pi_1 + \kappa r_1 - \varepsilon_1}{(1+\kappa)\left(\gamma_1^{op} - \gamma_2^{op}\right) - \gamma_2^{op} + (2+\kappa)\gamma_3^{op}\frac{\nu}{\mu}} + \theta\varepsilon_2^{CB} = \varepsilon_2^P + \theta\frac{\mu}{\nu}\left(\varepsilon_1^P - \varepsilon_1^{CB}\right)$$

where $\theta = 1 + \frac{1}{(1+\delta)\left(\gamma_1^{op} - \gamma_2^{op}\right) - \gamma_2^{op} + (2+\delta)\gamma_3^{op}\frac{\nu}{\mu}}$. Now note that π_1 and ε_1 become known in period 2 (and r_1 is always known). It follows that the right hand-side in the previous expression is known to the central bank when period 2 starts and it can be used as a signal about ε_2.

We can now use Bayes rule to find $E_1^P E_2^{CB}\varepsilon_2$. Some computations lead to:

$$E_2^{CB}\varepsilon_2 = \frac{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(z+1)k + \alpha\right)\left(k\varepsilon_2^{CB} + (1-k)\varepsilon_{2n}^{CB}\right) + k\left(\varepsilon_2^p + \frac{\mu}{\nu}\theta\left(\varepsilon_1^p - \varepsilon_1^{CB}\right)\right)}{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(z+1)k + z + k\right)}$$

so that the composed expectation is given by:

$$E_1^P E_2^{CB}\varepsilon_2 = \frac{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(\alpha+\beta)k + \alpha\right)\left(kE_1^P\varepsilon_2^{CB} + (1-k)E_1^P\varepsilon_{2n}^{CB}\right) + k\beta\left(\varepsilon_2^p + \frac{\mu}{\nu}\theta E_1^P\left(\varepsilon_1^p - \varepsilon_1^{CB}\right)\right)}{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(\alpha+\beta)k + \alpha + k\beta\right)}$$

Using the expressions for the various private expectations, we can deduce by identification:

$$\begin{aligned}
\gamma_2^{op} &= \frac{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(z+1)k + z\right)\left(\frac{k^2}{k+\left(\frac{\nu}{\mu}\right)^2} + (1-k)\frac{\left(k(z+1)+\left(\frac{\nu}{\mu}\right)^2\right)}{k(z+1)+(z+1)\left(\frac{\nu}{\mu}\right)^2}\right) + k}{\left(\left(\frac{\mu}{\nu}\right)^2\theta^2(z+1)k + z + k\right)} \\
\gamma_3^{op} &= -\frac{\frac{\mu}{\nu}\theta\frac{k^2}{k+\left(\frac{\nu}{\mu}\right)^2}}{\left(\left(\frac{\mu}{\nu}\right)^2(z+1)\theta^2 k + z + k\right)}
\end{aligned}$$

from which we find:

$$\gamma_1^{op} - \gamma_2^{op} + \gamma_3^{op}\theta\frac{\nu}{\mu} = 0$$

Proof of Proposition 2

The parameters for r_1 are found by minimizing the unconditional loss function $E(\pi_1)^2 + E(\pi_2)^2$.[11] Using (15), the previous expression for π_1 can be rewritten as:

$$\begin{aligned}\pi_1 &= \frac{1}{\theta-1}\left(\varepsilon_2^P-\varepsilon_2\right)+\frac{\mu}{\nu}\frac{\theta}{\theta-1}\left(\varepsilon_1^P-\varepsilon_1\right)\\&+(\delta\nu+\frac{\theta}{\theta-1})\left[(\varepsilon_2-\varepsilon_2^{CB})+\frac{\mu}{\nu}(\varepsilon_1-\varepsilon_1^{CB})\right]+(1-\kappa\mu)\,\varepsilon_1-(1+\kappa\nu)\,\varepsilon_2\end{aligned}$$

which implies that:

$$E(\pi_1)^2=(1-\kappa\mu)^2\,E(\varepsilon_1)^2+(1+\kappa\nu)^2\,E(\varepsilon_2)^2\ \text{ + other terms}$$

where the other terms depend on k, α, β, μ and ν.

Similarly, note that $\pi_2=\varepsilon_2-E_2^{CB}\varepsilon_2$ and that $E_2^{CB}\varepsilon_2$ is optimally found by the central bank by using the signals ε_2^{CB}, ε_{2n}^{CB}, and $\varepsilon_2^P+\theta\frac{\mu}{\nu}\left(\varepsilon_1^P-\varepsilon_1^{CB}\right)$ as indicated above, which gives:

$$E_2^{CB}\varepsilon_2=\frac{\left[\alpha+(\alpha+\beta)\,k\theta^2\left(\frac{\mu}{\nu}\right)^2\right]\left[k\varepsilon_2^{CB}+(1-k)\varepsilon_{2n}^{CB}\right]+k\beta\left[\varepsilon_2^P+\theta\frac{\mu}{\nu}\left(\varepsilon_1^P-\varepsilon_1^{CB}\right)\right]}{\alpha+k\beta+(\alpha+\beta)\,k\theta^2\left(\frac{\mu}{\nu}\right)^2}$$

so that:

$$\pi_2=\frac{\left[\alpha+(\alpha+\beta)\,k\theta^2\left(\frac{\mu}{\nu}\right)^2\right]\left[k(\varepsilon_2-\varepsilon_2^{CB})+(1-k)(\varepsilon_2-\varepsilon_{2n}^{CB})\right]+k\beta\left[\varepsilon_2^P+\frac{\mu}{\nu}\theta\left(\varepsilon_1^P-\varepsilon_1^{CB}\right)\right]}{\alpha+k\beta+(\alpha+\beta)\,k\theta^2\left(\frac{\mu}{\nu}\right)^2}$$

and $E(\pi_2)^2$ only includes terms in k, α, β, μ and ν. It follows that the total unconditionally expected loss under opacity can be written as:

$$L^{op}=(1-\kappa\mu)^2\,E(\varepsilon_1)^2+(1+\kappa\nu)^2\,E(\varepsilon_2)^2\ \text{ + other terms}$$

Since both ε_1 and ε_2 are assumed to be uniformly distributed, $E(\varepsilon_1)^2$ and $E(\varepsilon_2)^2$ are arbitrarily large relative to the other terms, in particular the variances α^{-2} and β^{-2}. It follows that the rule that minimizes L^{op} sets these terms equal to zero.

Proof of Proposition 4

The study of (19) shows that $\beta\Delta L_1(\theta)$ reaches a minimum of $\dfrac{kz^2-(1+k)^2}{kz(1+k)(1+z)}$ when $\theta=-\dfrac{1+k+z}{kz}$. This minimum is positive when $z>\dfrac{1+k}{\sqrt{k}}$.

[11] Unconditional because, if it were conditional on central bank information, the coefficients μ and ν would be nonlinear functions of $E_1^{CB}\varepsilon_1$ and $E_1^{CB}\varepsilon_2$ so the rule would not be linear - and impossible to derive in closed form.

Study of the of sign of $Cov(E_1^P\pi_2, \psi) = \gamma_3^{op}\,(\theta - 1)\,\dfrac{1+k}{\alpha k}$

Using the optimality condition $\frac{\mu}{\nu} = -1$, the parameters γ_2^{op}, γ_3^{op} and θ are jointly determined by the following conditions:

$$\gamma_3^{op} = \frac{k^2\theta}{(1+k)\left[k + z + k\theta^2(1+z)\right]}$$

$$\gamma_1^{op} - \gamma_2^{op} + \gamma_3^{op}\theta\frac{\nu}{\mu} = 0$$

$$\theta = 1 + \frac{1}{(1+\delta)\left(\gamma_1^{op} - \gamma_2^{op}\right) - \gamma_2^{op} - (2+\delta)\,\gamma_3^{op}}$$

Noting that the value of γ_1^{op} is given in the text, we can use these equations to compute γ_2^{op} and γ_3^{op} as a function of θ, but we cannot explicitly solve for θ, which is determined by the following condition (found by computing γ_3^{op} from the above):

$$\frac{k^2\theta}{(1+k)\left[k + z + k\theta^2(1+z)\right]} = \frac{\gamma_1^{op}\,(\theta-1)+1}{(\theta-1)^2\,(2+\kappa)}$$

This is a third-degree equation in θ. Examining graphically this equation, we find that the roots are positive if and only if $kz + 1 - k\kappa - k < 0$, and negative in the opposite case.[12] This is the curve is labeled "*Sign of* θ" in Figures 1 and 2. Moreover, when they are positive, the roots are greater than unity when $z > \widetilde{z}(k,\,\kappa)$. Since γ_3^{op} has the same sign as θ, $\gamma_3^{op}(\theta-1)$ has the same sign as $\theta(\theta-1)$. It follows that $\gamma_3^{op}(\theta-1) > 0$ when the roots are either negative or positive and larger than unity. We conclude graphically that $Cov(E_1^P\pi_2, \psi) > 0$ when either $kz + 1 - k\kappa - k > 0$ or when $kz + 1 - k\kappa - k < 0$ and $z > \widetilde{z}(k,\,\kappa)$.

Study of ΔL and explanation of Figures 1 and 2

Using the equations that jointly determine the parameters γ_2^{op}, γ_3^{op} and θ, we can write $\Delta L = L^{op} - L^{tr}$ as:

$$\beta\Delta L = \frac{A(\theta, k, z, \kappa)}{zk\,(\theta-1)^2\,(z+k)\,(2+\kappa)\,(1+z)}$$

where $A\,(\theta, k, z, \kappa) = a\,(k, z, \kappa)\,\theta^2 + b\,(k, z, \kappa)\,\theta + c\,(k, z, \kappa)$ is a second-order polynomial in θ with determinant $\Delta\,(k, z, \kappa)$ and roots $\theta_1\,(k, z, \kappa)$ and $\theta_2\,(k, z, \kappa)$, with $\theta_1\,(k, z, \kappa) < \theta_2\,(k, z, \kappa)$. Obviously $sign\ \Delta L = sign\ A\,(\theta, k, z, \kappa)$. which determines the sign of ΔL. Although we cannot compute analytically θ we note that $A\,(\theta, k, z, \kappa) > 0$ when either $\Delta\,(\theta, k, z, \kappa) < 0$ and $a\,(\theta, k, z, \kappa) > 0$ or when $\Delta\,(\theta, k, z, \kappa) > 0$, $a\,(\theta, k, z, \kappa) > 0$ and either $\theta < \theta_1\,(k, z, \kappa)$ or $\theta > \theta_2\,(k, z, \kappa)$, or when $\Delta\,(\theta, k, z, \kappa) > 0$, $a\,(\theta, k, z, \kappa) < 0$ and $\theta_1\,(k, z, \kappa) < \theta < \theta_2\,(k, z, \kappa)$. These are the conditions that, along with the curve "*Sign of* θ" discussed in the previous section, lie behind the graphical analysis in Figures 1 and 2. More precisely, the figures are based on the following reasoning.

Define $z_1(k, \kappa)$ and $z_2(k, \kappa)$ such that $\Delta\,(k, z_1, \kappa) = 0$ and $a\,(k, z_2, \kappa) = 0$, respectively. It can be shown graphically that $z_1(k, \kappa) > z_2(k, \kappa)\ \forall k,\ \kappa$. It follows that $\Delta L > 0$ when:

- $z > z_1(k, \kappa)$, $\Delta\,(k, z, \kappa) < 0$ and $a\,(k, z, \kappa) > 0$.

[12] When κ is very large, there might be negative roots when $kz + 1 - k\kappa - k < 0$, but this does not invalidate the conclusions that follow.

•$z_1(k,\kappa) > z > z_2(k,\kappa)$, $\Delta(k,z,\kappa) > 0$, $a(k,z,\kappa) > 0$ and either $\theta < \theta_1(k,z,\kappa)$ or $\theta > \theta_2(k,z,\kappa)$.

•$z < z_2(k,\kappa)$, $\Delta(k,z,\kappa) > 0$, $a(k,z,\kappa) < 0$ and $\theta_1(k,z,\kappa) < \theta < \theta_2(k,z,\kappa)$.

In the last two cases, we need to check where θ lies with respect to the roots of $A(\theta,k,z,\kappa)$. As already mentioned, this cannot be done analytically. The shape of the opacity zone in Figures 1 and 2 has been determined from a graphical three-dimensional analysis using MathLab and is therefore not precisely known. The figures are also informed by the study of the following limit cases:

•When $k \to 0$, $\Delta L \sim \dfrac{z}{1+z} > 0$. Thus there is exist along the vertical axis a (possibly) thin vertical zone where transparency dominates $\forall\ \kappa$, z.

•When $z \to 0$, there exists a function $k_1(\kappa)$ with $\partial k/\partial\kappa < 0$, such that $\Delta L > 0\ \forall k > k_1(\kappa)$. Thus along the horizontal axis, opacity zone shrinks to the left as κ increases.

•When $\kappa \to \infty$, then $\theta \to 0$ and $\Delta L \sim \dfrac{2+z}{k(1+z)} > 0$. For a high value of κ transparency always dominates. Graphically in Figure 1, as κ becomes larger the opacity zone shrinks against the vertical transparency zone along the k axis described above in the limit case $k \to 0$.

•When $\kappa \to 0$, for a given value of $z < z_1(k,\kappa)$, $\Delta L > 0$ when k is not too large. Graphically, in Figure 2, the opacity zone shrinks down and spreads along the horizontal axis, except for the narrow band that corresponds to the limit case $k \to 0$.

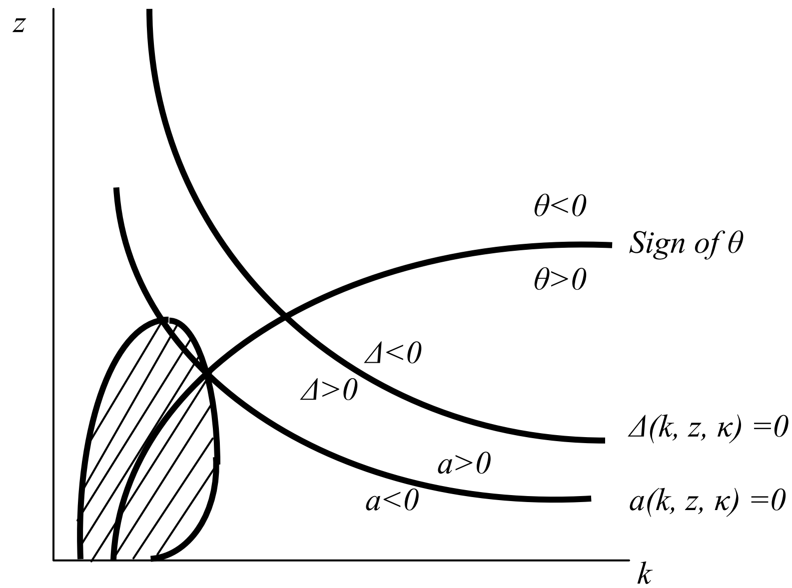

Figure 1. Higher κ

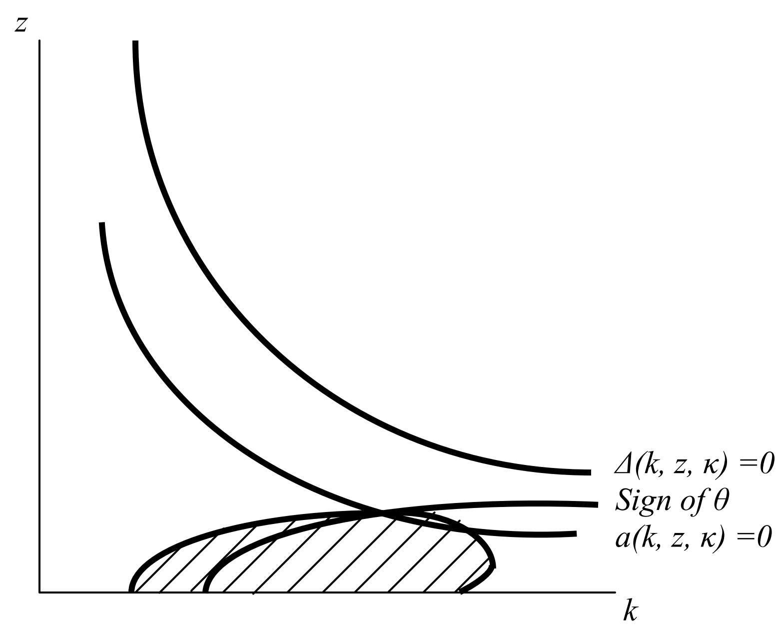

Figure 2. Lower κ