Stabilization Policy in a Model of Endogenous Growth*

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Abstract

In his famous monograph, Lucas (1987) put forth an argument that the welfare gains from reducing the volatility of aggregate consumption are negligible. Subsequent work that has revisited Lucas' calculation has continued to find only small benefits from reducing the volatility of consumption, further reinforcing the perception that business cycles don't matter. This paper argues instead that fluctuations could affect the growth process, which could have much larger effects than consumption volatility. I present an argument for why stabilization could increase growth without a reduction in current consumption, which could imply substantial welfare effects as Lucas (1987) already observed in his calculation. I establish upper bounds on the welfare effects of stabilization that suggest they could be quite sizable.

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Introduction

In his famous monograph, Lucas (1987) put forth an elegant argument that the welfare effects of business cycles in the United States are negligible. The logic of argument is as follows. Consider a representative consumer with a conventional time-separable constant-relative risk aversion (CRRA) utility function

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

where $\gamma \geq 0$. This consumer is given a consumption stream C_t defined by

$$C_t = (1+\lambda)^t (1+\varepsilon_t) C_0$$

where λ reflects the average growth of consumption over time, and ε_t is an i.i.d. random variable with mean zero and variance σ_ε^2 that captures deviations of consumption from its trend growth rate. The parameters λ and σ_ε^2 can be estimated from data on log per-capita consumption for the United States over the post-war period. To determine the costs of aggregate fluctuations, Lucas asks what fraction of his initial consumption C_0 this consumer would be willing to sacrifice in order to stabilize his consumption stream, i.e. to replace ε_t with its mean $E(\varepsilon_t) = 0$. For reasonable estimates of risk aversion γ, the answer turns out to be astonishingly small: less than 0.1%. By contrast, consumers would be willing to sacrifice a much larger fraction of current consumption, about 20% when $\gamma = 1$, in order to increase the growth rate λ by one percentage point.

Various authors have since revisited Lucas' calculation, but for the most part have continued to find only small effects from stabilization, further reinforcing the perception that business cycles have only a negligible impact on welfare. One line of attack has focused on the shock process ε_t. The first to make this argument was Imrohoroglu (1989). She criticizes Lucas' calculation for its implicit assumption that agents have access to full insurance, which guarantees them the average level of consumption each period. If aggregate shocks do not affect agents uniformly and consumption insurance is imperfect, the volatility of each individual's consumption stream would be greater than the volatility of per capita consumption σ_ε^2. But Imrohoruglu's calculations yield a cost of aggregate fluctuations that is not that much larger than Lucas' original estimate; she finds a cost of only 0.3% when $\gamma = 1.5$. Moreover, subsequent papers by Atkeson and Phelan (1994) and Krusell and Smith (1999) argue that her notion of stabilization removes too much idiosyncratic risk faced by agents along with aggregate risk. They find zero and even negative benefits to stabilization when aggregate risk is stabilized in a way that leaves the idiosyncratic risk faced by agents unaffected.[1] Another criticism focuses on the fact that the ε_t process assumed in Lucas' calculation is not sufficiently persistent. If agents had to bear the consequences of aggregate shocks for extended periods, they would be more averse to fluctuations. The calculations when ε_t follows a random walk with drift are reported in Obstfeld (1994a). For preference parameters that are similar to those used by Lucas, he finds costs of business cycles that are again about 0.3%.[2]

A second line of attack has focused on preferences, arguing the CRRA utility function is too restrictive and could underestimate the benefits of stabilization. Obstfeld (1994a) considers non-expected utility preferences that can separate between risk-aversion and intertemporal substitution. But once again, by Obstfeld's own characterization, this raises the welfare effect from a microscopic level to merely small, and the estimated cost remains below 1%.[3] Pemberton (1996b) and Dolmas (1998) consider a different class of non-expected utility preferences that exhibit 'first-order' risk aversion, arguing that such preferences capture observed attitudes towards large and small bets that are inconsistent with expected utility. Some of their estimates are very large, but only under the assumption that ε_t is very close to a unit root. Even when ε_t is autocorrelated with $\rho = .98$, Dolmas reports that the costs of fluctuations do not exceed

[1] In a parallel debate, Clark, Leslie, and Symons (1994) estimate the volatility of individual income from panel data and estimate a cost of cycles around nine-tenths of one percent for the United Kingdom. This estimate is criticized in Pemberton (1996a) for counting the reduction of purely idiosyncratic risk as a benefit of stabilization.

[2] Beaudry and Pages (1999) combine the two criticisms by studying highly persistent idiosyncratic shocks. They obtain costs that range between 1 and 4%. This approach seems a promising way to generate more costly business cycles, especially since the way in which they introduction stabilization is immune to the critique that it eliminates idiosyncratic risk. However, it is vulnerable to Lucas' observation that if the costs of fluctuations stem from incomplete insurance, policymakers should aim to promote insurance rather than stabilization.

[3] Tallarini (1999) argues that Obstfeld's parameters are inconsistent with the equity premium. He generates a large cost of business cycles using a coefficient of risk aversion that is orders of magnitude greater than Obstfeld. A similar point is made in Campbell and Cochrane (1995) who use non time-separable preferences to explain the equity premium and report a large cost of business cycles, although Otrok (1999) argues that a more "disciplined" calibration of non-separable preferences generates only small welfare effects. These arguments notwithstanding, the equity premium does not necessarily imply large costs of business cycles. First, the equity premium could be due to market frictions. Second, as Alvarez and Jermann (1999) point out, the equity premium and the cost of consumption fluctuations are distinct. They estimate a factor model for the marginal utility of consumption using financial data, and put an upper bound on the benefits of stabilization of 0.3%.

1% for reasonable parameterizations.

This paper pursues a different approach that could potentially generate much larger costs of aggregate fluctuations than the previous work cited above. It is motivated by Lucas' original observation that small changes in the growth rate λ have very large welfare consequences. As such, if aggregate fluctuations somehow affected the average growth rate, the costs of business cycles could potentially be orders of magnitude larger. This possibility is explicitly ruled out by Lucas' thought experiment; he treats the growth rate λ as an exogenous parameter that is unaffected by changes in ε. But there are models of endogenous growth where the level of economic activity affects the incentives of agents to engage in growth-enhancing activities, so that $\lambda = \lambda(\varepsilon_t)$. If this growth rate $\lambda(\varepsilon)$ is concave, stabilization would increase the long-run growth rate of consumption. Since even small changes in λ have very large welfare consequences, stabilization policy could potentially generate welfare effects that eclipse those described in previous work. In other words, the major cost of aggregate fluctuations comes not from volatility in consumption, as has been stressed in previous work, but because fluctuations impede the process of growth. While this idea has been occasionally discussed in previous work, it has yet to be incorporated into the original Lucas framework in a satisfactory way.[4] This is unfortunate, because the notion that stabilization can significantly raise growth is far from obvious. Under what assumptions will the growth rate be concave in the level of economic activity? How much additional growth should we expect from the elimination of cyclical fluctuations? Is it enough to generate larger welfare effects than what is calculated in previous work?

To address these questions, I develop a model of endogenous growth in which shocks affect the level of economic activity. The model generates a growth rate of the form $\lambda = \phi(n(\varepsilon))$, where $n(\cdot)$ is the amount of resources allocated to innovation activity and $\phi(\cdot)$ is the rate at

[4]There is a rather diffuse literature which argues that stabilization yields benefits other than reduced consumption volatility. For example, DeLong and Summers (1988) and Ramey and Ramey (1991) argue stabilization increases the level of output. This is discounted by Romer (1996), who cites evidence that stabilizing aggregate demand is unlikely to affect average output. More recent work studies the effects of stabilization on growth using models of factor accumulation with linear production technologies. This includes Aizenman and Marion (1993), Hopenhayn and Muniagurria (1996), de Hek (1999), and Jones, Manuelli, and Stacchetti (1999), as well as related work by Obstfeld (1994b) on the growth benefits of globalization. Except for Obstfeld, these papers are concerned with growth *per se* rather than welfare; however, as I argue below, these models imply only modest welfare gains from stabilization for reasonable prameters. Finally, Ramey and Ramey (1995) argue that volatility and growth are negatively related based on empirical evidence from a cross country evidence. But without a model, they cannot interpret the parameters they estimate structurally for welfare calculations.

which the innovation sector develops better technologies for producing goods. The reduced form function $n(\varepsilon)$ can be either concave or convex in equilibrium, which mirrors the well known ambiguity in the relationship between investment and uncertainty. Diminishing returns in the innovation sector imply $\phi(\cdot)$ is concave. This identifies a force which unambiguously works in the direction of making stabilization conducive for long-run growth, and distinguishes my model from earlier work on the growth effects of stabilization. In those papers, growth effects occur through changes in average investment. By contrast, my model implies that even when average investment is unaffected, stabilization increases long-run growth because it smooths innovation over time rather than having it concentrated in particular periods. Since this effect implies an increase in growth that does not require in increase in the average amount of resources employed in innovation, it could have a substantial impact on welfare; a calibration suggests that as an upper bound, consumers might be willing to give up as much as $3-7\%$ of their current consumption to stabilize aggregate fluctuations, nearly two orders of magnitude greater than Lucas' original calculation. The impact on welfare could be even bigger under alternative assumptions that involve occasionally binding constraints on the entrepreneurs who engage in innovation. An upper bound allows the possibility that individuals would be willing to sacrifice as much as $25-35\%$ of their current consumption, although this is surely an overly generous estimate of the gains from stabilization.

The paper is organized as follows. Section 1 develops the basic model of R&D with diminishing returns to innovation. Section 2 relates this model to previous work on stabilization and endogenous growth. Section 3 discusses the role of occasionally binding constraints. Section 4 estimates upper bounds on the effect of stabilization on long-run growth and welfare. Section 5 concludes.

1. A Model of Diminishing Returns

To study how fluctuations affect the long-run growth rate of the economy, I need a model of endogenous growth that admits fluctuations in the level of economic activity. Models of technological innovation such as Grossman and Helpman (1991) and Aghion and Howitt (1992) satisfy this criterion. These models have two important features. First, they assume a monopolistic competition framework in which demand shocks can change the level of economic activity. Second, changes in the level of economic activity affect the incentives to innovate by changing the size of the market a monopolist could capture if he develops a superior production technology. The particular source of fluctuations in my model are shocks to the composition of government spending. This choice is made for convenience, *not* because I wish to argue that government spending is an important source of aggregate fluctuations. Any rigorous model of stabilization would require a government authority, and allowing government to also act as the source of shocks helps minimize notation. The source of the shock does not play an important role in the following analysis, and the particular shock I investigate can be viewed as a proxy for demand shocks more generally.[5] The first part of this section sets up the model, and the second characterizes its equilibrium.[6]

1.1. Setup

The economy contains three agents:

1. A representative agent, who consumes goods and supplies labor
2. A government, which taxes the agent and spends the revenue it collects
3. Entrepreneurs, who hire labor to produce goods and to develop new production methods

Consider the three agents in turn. First, the representative agent has standard CRRA preferences over a consumption aggregate, i.e. his utility at a date t is given by

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

where $\gamma \geq 0$ and C_t is a Cobb-Douglas aggregator over consumption goods c_{jt} for $j \in [0, 1]$:

$$C_t = \exp\left[\int_0^1 \ln c_{jt}\ dj\right]$$

[5] While the source of aggregate fluctuations has no bearing on the question of whether stabilization affects growth, the source of aggregate fluctuations is crucial for determining the costs of stabilization, which ultimately determines whether stabilization is desirable. But the point of this paper to challenge the notion that stabilization has no first order welfare effects, not to argue that stabilization is optimal.

[6] Since writing this paper, I have become aware of a related model by Fatas (1998). Aside from differing in certain details from this model, he focuses on different questons.

Time is continuous, and the agent cares about his expected discounted utility

$$\int_0^\infty U(C_t)\, e^{-\rho t} dt$$

where $\rho > 0$ is his discount rate. I choose CRRA utility despite its drawbacks which are discussed in the Introduction; these concern attitude towards risk, which is not important for my analysis. For this reason, γ should be interpreted as the inverse of the elasticity of intertemporal substitution rather than a coefficient of risk-aversion.

The agent is endowed with L units of labor each instant. Leisure does not enter his utility function, so he supplies all of his labor whenever the wage is positive. The agent also owns all claims on the profits of entrepreneurs in this economy, positive and negative. Following Lucas, I abstract from savings; the agent must consume all of his disposable income, which is the sum of labor income and profits net of any tax liabilities. The only choice he must make is how to allocate his income across different goods. Since C_t is a Cobb-Douglas aggregator, he will spend an equal amount on each good $j \in [0, 1]$. Integrating over all goods implies the spending rate on each good j is the same as disposable income,

$$p_j c_j = \int_0^1 p_j c_j dj = \Pi + W - T$$

where Π denotes aggregate profits, W denotes labor income, and T denotes tax liabilities. Let $Y \equiv \Pi + W$ denote gross nominal income.

Next, consider the government sector. Government taxes the labor income of the agent and uses the funds it raises to finance various expenditures. The tax rate is a constant $\tau \in (0, 1)$. Since labor supply is inelastic, this tax is equivalent to a lump-sum tax. However, the fact that the tax is proportional to labor income is important, since it implies government revenue grows as labor resources become more productive. Without this assumption, government share of total output would eventually vanish. In what follows, I will use labor as the numeraire good. The tax revenue in each instant is therefore $T = \tau L$. The budget is always balanced, i.e. the entire tax revenue T is spent at each instant. The expenditures of the government pay for goods and government workers:

$$T = G_t + N_t$$

where G_t denotes government spending on goods, and N_t denotes the total wage bill of government employees. Given the normalization that labor is the numeraire, the latter is also the number of workers employed by the government. This notation might be a little confusing at first, especially since G_t is a nominal quantity while C_t is a real quantity. However, it is convenient to work with nominal quantities in solving the model, even though we ultimately care about real consumption. I denote nominal consumption by $P_t C_t \equiv \int_0^1 p_{jt} c_{jt}$.

Government expenditures on goods are assumed to be allocated equally across the different commodities, i.e. spending on each good j at date t is the same amount g_t regardless of the price of that good. It follows that aggregate government spending on goods is equal to g_t, since

$$G_t = \int_0^1 g_t dj = g_t$$

Following Matsuyama (1995), I assume the government maintains a constant tax collection T but periodically shifts the composition of spending between goods and labor. Specifically, aggregate government expenditures G_t can assume one of two values, $G_1 > G_0$, and switches between these two values at a rate μ per unit of time. These shifts in the composition of government expenditures act as the source of fluctuations in the economy. Denote the associated number of government workers employed in each regime by $N_0 = T - G_0$ and $N_1 = T - G_1$, respectively. Budget balance implies $N_0 > N_1$, i.e. more workers can work in the private sector when government shifts its spending towards goods.

Finally, I turn to entrepreneurs. They play two roles in the economy: they produce the various goods $j \in [0, 1]$, and they search for better ways of producing these goods. In both capacities, they are driven by profit maximization. The nature of production in this economy is as follows. Each good j is associated with a number m_j that reflects the highest generation of technology available for producing that good. Put another way, m_j is the number of times the technology for producing good j has been improved upon since date $t = 0$. Each generation converts labor into output at a linear rate, but successive generations are more productive. Specifically, the m-th generation technology allows one unit of labor to produce λ^m units of output, where $\lambda > 1$ is the rate of progress associated with each improvement. Technologies are protected by indefinite patents, so only the creator of the m-th generation technology can use it to produce goods. Still, the development of the m-th generation allows other entrepreneurs to develop the

$m+1$-th generation, so discovering a new production technique helps both the innovator who discovers it and his competitors.

I first describe entrepreneurs in their capacity as producers. Each entrepreneur will produce using the highest generation he has access to. His only real decision is what price to charge for his good. Since both consumer and government spending on each good are independent of the price, demand for each good is unit elastic. Each monopolist will therefore want to set as high a price as possible; this way, he earns the same revenue but can produce fewer goods. But he cannot post a price that is too high; if his price exceeds the cost of producing a single unit of his next most efficient competitor, the latter will post a slightly lower price and steal away all of his business. Hence, producers quote a price equal to the marginal cost of their most efficient competitor, and the entrepreneur with the highest generation of technology will be the one supplying goods to the market. This implies that an entrepreneur with the m-th generation technology will set his price p_j to $\lambda^{-(m-1)}$: his next most efficient competitor needs $\lambda^{-(m-1)}$ workers to produce one unit of output, and each worker is paid a wage normalized to 1. At this price, the number of units the monopolist will sell is given by

$$\frac{Y-T+G}{p_j}=\lambda^{m-1}(Y-T+G) \tag{1.1}$$

To produce each of these units, he needs to hire λ^{-m} units of labor. Hence, his total labor requirement is given by $\lambda^{-1}(Y-T+G)$. With labor as the numeraire, this is also the total cost of producing the quantity in (1.1). His profits will then equal his revenue net of costs, or

$$\begin{aligned}\pi &= p_j c_j - \frac{1}{\lambda}(Y-T+G)\\ &= \frac{\lambda-1}{\lambda}(Y-T+G)\end{aligned} \tag{1.2}$$

In what follows, I restrict attention to Markov equilibria in which nominal income Y depends only on the level of government spending G. In such an equilibrium, profits π will depend only on the level of government spending. Let π_0 denote profits when $G=G_0$ and π_1 when $G=G_1$.

Finally, I describe entrepreneurs in their capacity as innovators. I assume there are only two entrepreneurs in sector j at any point in time: one who owns the patent to the best available technology and produces output, and the other who can work on developing a better technology for producing this good. Previous work has avoided having to make such an explicit assumption by imposing constant returns to scale in research along with free entry. In this case, the number of potential entrants does not matter, and an incumbent monopolist will never choose to engage in research in equilibrium. Precisely because I want to allow for diminishing returns, I need to add these assumptions.[7] If the researcher-entrepreneur employs n workers for research, he discovers the next generation technology at a rate $\phi(n)$ per unit of time. The function $\phi(n)$ is strictly increasing and concave, with $\phi(0) = 0$, and satisfies the usual boundary conditions $\lim_{n \to 0} \phi'(n) = \infty$ and $\lim_{n \to \infty} \phi'(n) = 0$. Concavity in $\phi(n)$ is associated with diminishing returns in research and development: the contribution of the marginal worker to the probability of success decreases with each additional worker. Denoting the value of a successful innovation by v_t, the innovator's problem is to choose n to maximize $\phi(n) v_t - n$, so

$$\phi'(n) v_t = 1 \tag{1.3}$$

1.2. Equilibrium

With the description of the economy complete, I can proceed to characterize its equilibrium. As usual, an equilibrium is a set of prices and a set of quantities at each instant such that (1) agents choose prices and quantities optimally; (2) the government budget is balanced; and (3) supply and demand are equal in all output and factor markets. As noted before, I focus on Markov equilibria in which nominal variables vary only with the level of government spending; it is an open issue as to whether other equilibria also exist. In what follows, I reduce the problem of solving for an equilibrium to one of solving only for n, the number of workers employed in innovation. All remaining parts of the equilibrium can be reconstructed from this variable. To do this, I make use of the first order condition (1.3), which expresses n as a function of the value of a patent v. Thus, a necessary first step is to express v in terms of n.

In solving for v, I use the approach of Lucas (1978). That is, suppose this economy has a market for patents. Since the representative agent owns all patents in equilibrium, the value of a patent must be such that he is indifferent between buying one and selling one. The expected utility from buying a patent is the marginal value of consumption one could afford with the

[7] An alternative approach would be to allow for diminishing returns at the aggregate level rather than the individual level. See, for example, Stokey (1995).

profits it yields. This has an expected utility value of

$$E_t\left[\int_0^{\infty} U'\left(C_{t+s}\right)\frac{dC_{t+s}}{dY_{t+s}}\pi_{t+s}e^{-\rho(t+s)}ds\right]$$

This has to be the same as the utility from selling the patent. In that case, the consumer can increase his income by v, which allows him to increase his current consumption. The additional utility from this option is given by

$$U'\left(C_t\right)\frac{dC_t}{dY_t}v_t$$

Indifference between the two yields the value of a successful innovation:

$$v_t = E_t\left[\int_t^{\infty}\frac{U'\left(C_{t+s}\right)}{U'(C_t)}\frac{dC_{t+s}/dY_{t+s}}{dC_t/dY_t}\pi_{t+s}e^{-\rho(t+s)}ds\right]$$

The following lemma establishes that v_t exists when $\gamma \geq 1$, and that it is a linear function of profits under the two regimes, π_0 and π_1. Its proof, as well as those of all other claims in the paper, is delegated to the Appendix.

Lemma 1: Suppose $\gamma \geq 1$. In a Markov stationary equilibrium, the value of a successful innovation is given by

$$v_t = \begin{cases} \dfrac{1}{\omega\left(n_0\right)\omega\left(n_1\right)-\mu^2}\left(\omega\left(n_0\right)\pi_0+\mu\pi_1\right) & \text{if } G_t = G_0 \\ \\ \dfrac{1}{\omega\left(n_0\right)\omega\left(n_1\right)-\mu^2}\left(\omega\left(n_1\right)\pi_1+\mu\pi_0\right) & \text{if } G_t = G_1 \end{cases} \tag{1.4}$$

where

$$\omega\left(n\right) = \mu+\rho+\phi\left(n\right)\left[1+\left(\gamma-1\right)\ln\lambda\right]$$

Note that if $n_0 = n_1$, the value of a patent v would be higher in whichever regime offers higher profits; this is because a patent is more valuable if it pays out high dividends today rather than in the discounted future. If $n_0 \neq n_1$, the patent could be more valuable in the low profit regime, since v depends not only on the timing of payouts but also on the innovation rate $\phi\left(n\right)$. This rate affects the value of a patent in two distinct ways. First, ϕ reflects the rate at which an entrepreneur's technology is rendered obsolete by his competitor; a higher ϕ reduces expected future profits, making the patent less valuable. Second, ϕ reflects the rate at which other goods are innovated. Faster innovation in other sectors could make a patent either more or less valuable. With more rapid growth in labor productivity, real income and hence demand for each good grow more rapidly. This increases the expected profits of an incumbent, making patents more valuable. On the other hand, the more rapidly income grows, the more uneven is the anticipated future profile of consumption. This makes current consumption more valuable than future consumption, reducing the value of the patent. Which of these effects dominates depends on how much the consumer is willing to substitute intertemporally. When $\gamma \geq 1$ so the agent is unwilling to substitute intertemporally, the value of a patent unambiguously decreases with ϕ.[8]

To express v solely in terms of n, I need to express profits π in terms of n. Here, I use the fact that aggregate profits Π are the sum of profits of entrepreneurs in all sectors net the costs of innovation. That is,

$$\Pi = \int_0^1 \pi_j dj - \int_0^1 n_j dj = \pi - n$$

Substituting this into (1.2) gives profits π as a function of n:

$$\pi = (\lambda - 1)(L - n - N)$$

which allows us to rewrite (1.3) as a system of two equations with n_0 and n_1 as the only endogenous variables:

$$\phi'(n_i) \frac{\lambda - 1}{\omega(n_1)\omega(n_0) - \mu^2} \left[(\omega(n_j) + \mu) L - \omega(n_j)(n_i + N_i) - \mu(n_j + N_j) \right] = 1 \tag{1.5}$$

While obtaining a closed form solution for n_0 and n_1 is not possible in general, I can still characterize how employment, profits, the value of patents, and output vary across the two regimes.

Proposition 1: Suppose $\gamma \geq 1$. Then in equilibrium,

1. Employment in the innovation sector is increasing in G, i.e. $n_1 > n_0$. Hence, the growth rate $\phi(n) \ln \lambda$ is increasing in G.

[8] Values of γ that are greater than 1 are also consistent with empirical evidence. See, for example, Epstein and Zinn (1991).

2. The value of patents is increasing in G, i.e. $v_1 > v_0$.

3. Nominal profits are increasing in G, i.e. $\pi_1 > \pi_0$.

4. Nominal income Y and consumption PC could be either increasing or decreasing in G; however, both are related to G in the same way.

Proposition 1 implies that innovation is accelerated when profits, and thus patents, are more valuable. It further establishes that patents are more valuable when government spending on goods is high. To rephrase Proposition 1 in terms of more conventional business cycle terminology, though, it will be useful to relate the model to the original Lucas setup. Recall that he posits the representative agent receives a consumption stream of the form

$$C_t = \left[\prod_{s=0}^{t} (1+\lambda_s) \right] (1+\varepsilon_t) C_0$$

where the growth rate λ_s is assumed to be constant. The term $\left[\prod_{s=0}^{t} (1+\lambda_s)\right] C_0$ reflects expected consumption given cumulative growth up to period t, but the actual level of consumption differs from this expectation by a stochastic term $1+\varepsilon_t$ that reflects cyclical fluctuations. A little algebra shows that my model admits an analogous continuous-time representation for the consumption aggregate,

$$C_t = \exp\left[\int_0^t \phi(n_s) \ln \lambda \right] (1+\varepsilon_t) C_0 \tag{1.6}$$

where the deviations from trend ε_t depend on the level of government spending, i.e. $\varepsilon_t = \varepsilon(G_t)$. The virtue of using labor as the numeraire good is that nominal quantities correspond to detrended real quantities. This is because aggregate real variables grow at the same rate as labor productivity: expressing the value of quantities in terms of units of labor effectively removes the growth term $\exp\left[\int_0^t \phi_s \ln \lambda\right]$ in (1.6). As a result, the nominal value of consumption is proportional to $1+\varepsilon_t$. This allows us to recover deviations from trend consumption using the level of nominal consumption, and similarly for other real macroeconomic series in the model.

Armed with this observation, we can interpret Proposition 1 as a statement about the cyclical properties of various variables in this economy. It is natural to define the cycle in terms of real gross income, i.e. the economy is said to be in a boom if real income is above its expected growth rate. This occurs when nominal gross income Y assumes its higher value. Proposition 1 implies consumption is procyclical, i.e. it attains its higher value when nominal income attains its higher value. However, it does not pin down how government spending, and thus profits, patent values, and the economy's growth rate move over the cycle. The fact that the growth rate could be procyclical or countercyclical should not be entirely surprising. Aghion and Saint Paul (1998) find that growth could be either procyclical or countercyclical, depending on the specification for R&D technology. My model makes a similar prediction, but for a different reason: with monopoly power in the production of goods, diverting workers out of production and into innovation where profit margins are lower will reduce total income. If the increase in G diverts enough resources to innovation, it is possible that gross income falls even as each producer's profits rise. This is similar to effect described in Helpman and Trajtenberg (1998) where the arrival of a more productive technology causes an initial fall in income as resources are diverted to innovation. Empirically, government spending, profits, and R&D expenditures are all procyclical, which would tend to favor procyclical growth. This is also plausible given the fact that innovation is such a small share of the aggregate economy. Still, the question of whether growth is procyclical or countercyclical is irrelevant for the question of how stabilization affects growth. This depends on whether $\lambda(\varepsilon)$ is concave in ε, not on whether λ is increasing or decreasing in ε. It is to this question that I now turn.

Although it is a useful heuristic device to pose the question of whether stabilization increases growth in terms of whether the growth rate λ is concave in the level of economic activity ε, the fact that λ and ε are determined simultaneously in equilibrium means λ is not, strictly speaking, a function of ε. To settle whether stabilization increases long-run growth, then, we must introduce a stabilization policy directly into the model and compare the new equilibrium growth rate with the growth rate that prevails when aggregate fluctuations are allowed. Since government spending is the source of aggregate fluctuations, and since its level is set by policymakers, it seems natural to define a stabilization policy as one which sets government spending constant at the average of government spending across the two regimes, i.e. $G_t = \frac{1}{2}(G_0 + G_1) \cong \overline{G}$ for all t. Let $n(G)$ denote the number of workers employed in innovation when government spending is constant and equal to G, and $n^*(G)$ the number of workers employed in innovation when government spending fluctuates but is currently equal to G. Stabilization will increase long-run growth if

$$\phi\left(n\left(\overline{G}\right)\right)\ln\lambda > \tfrac{1}{2}\left[\phi\left(n^*(G_0)\right)\ln\lambda + \phi\left(n^*(G_1)\right)\ln\lambda\right] \tag{1.7}$$

The notation is meant to suggest that stabilizing government spending in this economy will increase growth if the growth rate $\phi(n(\cdot))\ln\lambda$ is "effectively concave" in government spending. This is only a modified notion of concavity, since (1.7) compares the average realized growth rate under two different stochastic processes for G_t, i.e. it compares the function $\phi(n(\cdot))$ at $\overline{G}$ with the average of a different function $\phi(n^*(\cdot))$. The (true) concavity of $\phi(\cdot)$ establishes the following sufficient condition for stabilization to increase growth:

Proposition 2: If n is effectively concave in G, i.e. if

$$n\left(\overline{G}\right) \geq \tfrac{1}{2}\left[n^*(G_0) + n^*(G_1)\right] \tag{1.8}$$

then (1.7) will be satisfied, i.e. stabilization increases average growth.

(1.8) is only a sufficient condition. Even if n is effectively convex in G so that (1.8) is reversed, it is still possible that (1.7) will be satisfied provided $\phi(\cdot)$ is sufficiently concave. Whether condition (1.8) is satisfied or not depends on third derivative properties of $\phi(\cdot)$. To see why, recall that n is determined by the first order condition $\phi'(n)v = 1$. To determine whether n depends on v in a concave or convex manner, we need to differentiate this Euler equation twice, which involves the third derivative ϕ''' whose sign and magnitude is ambiguous. This ambiguity between investment and uncertainty reflects a general result: depending on the production function, the resources devoted to investment can either increase or decrease in response to a mean preserving spread.[9] Since this model imposes no restrictions on the relationship between investment and volatility, it does not deliver a sharp prediction as to how stabilization affects growth. However, it identifies an important force which unambiguously works towards making stabilization conducive to growth, namely diminishing returns to innovation. Intuitively, diminishing returns implies that taking away resources from the innovation sector reduces growth more than adding a commensurate amount of resources to the innovation sector raises it.

Note that the model predicts stabilization will increase growth is the borderline case in which stabilization has no significant effect on the average level of investment, i.e. where n is effectively

[9] For example, Hartman (1972) and Abel (1983) develop models where investment increases under uncertainty, while Bernanke (1983) and Dixit and Pindyck (1994) develop models where investment decreases under uncertainty. Caballero (1991) presents a unified framework in which investment can either increase or decrease with uncertainty, depending on the parameters of the production function and the cost of investment function.

linear:

$$n\left(\overline{G}\right) = \tfrac{1}{2}\left[n^*\left(G_0\right) + n^*\left(G_1\right)\right] \tag{1.9}$$

There are several reasons why (1.9) is an important case. First, there is some evidence that investment is effectively linear, i.e. changes in volatility do not appear to affect average investment. This claim is based on work by Ramey and Ramey (1995), who examine the effects of volatility in growth using cross-country evidence. They find increased volatility in growth is associated with lower average growth but with no implication for average investment.[10] They find this fact puzzling given conventional wisdom that equates growth effects with changes in investment. However, this pattern is consistent with the notion that it is diminishing returns to innovation rather than changes in average investment that is the primary channel by which stabilization affects growth.[11] Second, even if we believe stabilization does affect the average investment rate, shutting down the role of changes in the level of investment allows us to isolate the effect of diminishing returns from the effect of changes in average investment. This turns out to be important, particularly for welfare implications. Finally, the case where n is effectively linear is important because it directly maps into the original Lucas framework. Recall that Lucas considers a stabilization policy that sets consumption equal to its average value. By contrast, this model defined stabilization as setting government spending equal to its average value. As the next lemma illustrates, stabilizing government spending is equivalent to stabilizing consumption if and only if n is effectively linear:

Lemma 2: Stabilizing government consumption also stabilizes consumption if and only if (1.9) is satisfied, i.e.

$$PC\left(\overline{G}\right) = \tfrac{1}{2}\left[PC^*\left(G_0\right) + PC^*\left(G_1\right)\right] \quad \Leftrightarrow \quad n\left(\overline{G}\right) = \tfrac{1}{2}\left[n^*\left(G_0\right) + n^*\left(G_1\right)\right]$$

This result comes from the assumption that labor resources are fixed in this economy: any change in resources allocated to innovation must be offset by an equal and opposite change in

[10] Aizenman and Marion (1999) revisit the same evidence and argue that, at least among developing countries, private investment is lower in countries where growth is more volatile. However, private and public investment combined are not significantly correlated with volatility in growth.

[11] It should be noted that Ramey and Ramey look at gross investment rather than R&D investment. Thus, it is not clear how relevant their evidence is for the model developed in this section. In the next section, I consider a factor accumulation model in which investment contributes to the capital stock at a diminishing rate, which generates similar results, and where the relevant measure is in fact a broader measure of investment.

consumption. Hence, average consumption will be unaffected if and only if the average level of innovation is unaffected. This lemma illustrates a more fundamental principle, namely that we can only talk about a general stabilization policy if there is enough linearity in the model; otherwise, it would be impossible to simultaneously set all of the relevant macroeconomic series equal to their average. Allowing for (1.9) makes it possible to compare this model with the original calculation carried out by Lucas. In particular, it shows that when consumption is stabilized at its average value, the rate at which consumption will grow from this point on will be higher than the average growth rate of consumption prior to stabilization. By ignoring this possibility, Lucas' calculation abstracts from one benefit of stabilization, even though it could potentially be significant.

2. Alternative Models of Endogenous Growth

As noted in the Introduction, previous authors have already raised the possibility that stabilization could affect long-run growth. In large part, this literature relies on models of factor accumulation with linear production technologies to generate endogenous growth. Notably, this literature has not calculated the welfare effects of stabilization or discussed how their findings relate to those of Lucas. This section reviews this class of models and highlights key differences between my results and theirs.

Consider the following model, which is taken from Jones, Manuelli, and Stacchetti (1999).[12] Time is discrete. There are now two agents: the representative agent and the government. The agent has no labor resources, but is endowed with an initial amount K_0 of capital. He also has access to a linear production technology that converts a unit of capital into $1+\lambda$ units of output, where $\lambda > 0$. Each period, the agent uses all of his capital to produce output. He then decides how much of this output to consume and how much to leave as capital for the subsequent production. Capital depreciates in full during production, so the only capital available for production in period $t+1$ is that which was set aside in period t. The agent has a

[12] Aside from Jones, Manuelli, and Stachetti, various authors have used this model to study endogenous growth under uncertainty. It was first developed as a model of saving under uncertainty by Levhari and Srinivasan (1969). They solve an infinite horizon version of a problem first studied by Phelps (1962). Leland (1974) subsequently reinterpreted their model in terms of growth, and Eaton (1981) demonstrated how to introduce government policy into this framework.

conventional CRRA utility function over consumption in each period, and discounts the future at a rate β.

Just as before, government spending acts as the source of fluctuations in the model. Since there is no labor, government spending is devoted entirely to purchasing output. To finance its expenditures, the government taxes the income of the agent, so it takes a fraction τ of the output produced. As before, the budget is balanced every period, i.e. $G_t = \tau_t Y_t$. In contrast to the previous section, G_t now denotes real government spending. Once again, the share of government spending can assume only two values, and suppose this share is i.i.d. over time.[13] From budget balance, this implies the tax rate τ_t can take on two values $\tau_1 < \tau_0$, each equally likely. This leaves the agent with $(1-\tau_t)(1+\lambda) K_t \equiv (1+\lambda_t) K_t$ units of output each period.[14] The problem facing the agent is given by

$$\max_{C_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}-1}{1-\gamma} \right]$$

subject to

$$K_{t+1} = (1+\lambda_t) K_t - C_t$$

where K_0 is given and $0 \leq C_t \leq (1+\lambda_t) K_t$. Applying a perturbation argument to the first order conditions of the agent, it can be shown that the solution to this problem is given by

$$\begin{aligned} C_t &= c(1+\lambda_t) K_t \\ K_{t+1} &= i(1+\lambda_t) K_t \end{aligned}$$

where i and c are constants, i is given by

$$i = \left(\beta E \left[(1+\lambda_t)^{1-\gamma} \right] \right)^{\frac{1}{\gamma}} \tag{2.1}$$

and $c = 1 - i$. The growth rate of consumption is given by

$$\frac{\Delta C_{t+1}}{C_t} = i(1+\lambda_t) - 1 \tag{2.2}$$

[13] By contrast, the previous section assumed G_t was Poisson, implying persistent shocks. Jones, Manuelli, and Stacchetti (1999) discuss solving a version of the model with some persistence.

[14] I restrict τ_0 and τ_1 so that $\beta E\left[(1-\tau_t)(1+\lambda)\right]^{1-\gamma} < 1$, which insures the individual's maximization problem is well-defined.

which in equilibrium will be the same as the growth rate of output and of the capital stock. The growth rate is proportional to the investment rate i, and thus depends on the behavior of the agent.

We can study the effects of stabilization by setting the share of government spending, and thus the tax rate τ, to its expected value. This amounts to setting λ_t equal to its average value. The only way this will affect the average growth rate in (2.2) is by affecting the propensity to save i. From (2.1), it follows that stabilization increases average growth if and only if $\gamma < 1$. Since estimates of γ are typically 1 or greater, stabilization should either has no effect on growth or causes it to fall, in contrast with the previous model in which stabilization could increase growth when $\gamma \geq 1$.[15] However, the welfare implications of this observation are not immediately clear. In particular, lower growth in this model occurs because the agent chooses voluntarily to consume a greater fraction of his output and allocate less capital to future production. Given that this action is voluntary, it no longer follows that lower growth necessarily reduces welfare. How does this square with my observation in the Introduction that faster growth generates substantial improvements in welfare? That result is based on Lucas' calculation which examines an increase in the growth rate of consumption that leaves the original level of consumption unchanged. This model, by contrast, generates growth effects that occur at the expense of current consumption.

The above remarks hint at the fact that models of factor accumulation introduce endogenous growth into the Lucas framework in a conceptually different way than the model of the previous section. First and foremost, the model of factor accumulation above generates a consumption path that is difference stationary rather than trend stationary, i.e. consumption is given by

$$C_t = \left[\prod_{s=1}^{t} i \left(1 + \lambda_s\right) \right] C_0 \tag{2.3}$$

where C_0 is determined endogenously. There is no analogous ε_t term that represents deviations of consumption from its average value. Thus, it is not the level of consumption which is

[15] Jones, Manuelli, and Stacchetti (1999) make a stronger claim that with less than full depreciation, stabilization reduces growth for all $\gamma \geq 1$. However, this is a particular feature of the way they introduce stabilization and whether it involves setting the growth rate equal to its arithmetic or geometric average. In the continuous time analog of the model where the capital stock does not fully depreciate and the value of λ_t is an increment from a Weiner process, as in Eaton (1981), stabilization reduces growth only for $\gamma > 1$.

stabilized, but the growth rate of consumption for a fixed investment level. Growth effects arise because changes in stabilization affect the investment decisions of the agent, and, by construction, stabilizing the level of consumption will have no effect on i.

This discussion points to the fact that the standard model of factor accumulation critiques Lucas' assumption that the agent cannot save along with the assumption that the growth rate λ is determined exogenously. One could certainly compute the welfare effects of stabilization under this alternative set of assumptions, but there is no reason to expect that they would be similar to those of the model described in the previous section. In fact, for reasonable parameter estimates, the factor accumulation model is not likely to generate much larger benefits from stabilization than Lucas. Recall that reasonable estimates of γ are on the order of magnitude of 1. For the particular case where $\gamma = 1$, stabilization has no effect on growth, and the model reduces to the one considered in Obstfeld (1994a). But as reported in the Introduction, Obstfeld computes a welfare gain of only 0.3%.

As noted above, the reason endogenous growth in standard factor accumulation models does not yield greater benefits from stabilization is that any additional growth must come out of current consumption. This is different from the effect of diminishing returns, in which stabilization raises average growth by smoothing investment over time rather than by increasing average investment. The latter raises growth without requiring a sacrifice in current expected consumption, which is why it could potentially generate very large welfare benefits. This is not to say that factor accumulation models are inconsistent with diminishing returns. We could easily introduce diminishing returns in investment into the model and recover the same result.[16] Following Uzawa (1969), suppose that the capital stock in period $t+1$ is given by

$$K_{t+1} = \Phi\left(I_t, K_t\right)$$

where $I_t = \left(1+\lambda_t\right) K_t - C_t$ is the part of disposable income that is not consumed. The function $\Phi\left(\cdot,\cdot\right)$ is increasing and concave in I. This functional form implies that only Φ percent of investment actually turns into capital, while the remainder of investment is eaten up in the process of installing capital. I further assume the installation function exhibits constant returns

[16] Diminishing returns to investment should not be confused with the more conventional notion of diminishing returns to capital. The former refers to the assumption that investment adds to the capital stock at a diminishing rate, while the latter refers to the assumption that capital contributes to final output at a diminishing rate.

to scale, so

$$\Phi(I_t, K_t) = \phi\left(\frac{I}{K}\right) K_t$$

The growth rate of capital in this economy, which will be equal to the growth rate of output and consumption in equilibrium, is given by

$$\begin{aligned}\frac{\Delta K_t}{K_t} &= \frac{K_{t+1} - K_t}{K_t} \\ &= \phi\left(\frac{I_t}{K_t}\right) - 1 \\ &= \phi\left[\, i(\lambda_t)(1+\lambda_t)\right] - 1\end{aligned}$$

where $i(\lambda_t)$ is the fraction of after-tax income that is devoted to investment. Just as in the previous section, stabilizing λ_t has ambiguous effects on growth, since the average level of i can either increase or decrease, depending on conditions governing the third derivative ϕ'''. When $i(\lambda_t)$ is effectively linear, stabilization will increase growth without affecting average investment, just as in the previous section. Models which use factor accumulation to generate endogenous growth are therefore not incompatible with the notion of diminishing returns presented in the previous section; they simply fail to acknowledge its role.

3. Occasionally Binding Constraints

So far, I have abstracted from the effects of stabilization on the level of investment by focusing on the case where investment is effectively linear. The justification for this assumption was evidence from cross-country comparisons by Ramey and Ramey (1995) and Aizenman and Marion (1999) that aggregate volatility does not appear to be correlated with total investment. However, there might be legitimate concern whether cross-country evidence is appropriate for making inference on the effects of stabilization within a country. This allows for the possibility that the average level of investment may be affected by stabilization, which raises the question of whether stabilization will in fact increase growth, and what implications changes in average investment have on welfare. This section discusses a modification of the model from Section 1 which insures stabilization increases average investment, and thus average growth, regardless of the size and magnitude of the third derivative $\phi'''(\cdot)$. This result is important in two respects. First, it provides a set of assumptions that insure stabilization is growth-increasing which do not rely on such subtle properties of the innovation function. Second, in contrast with the previous section which argued that endogenous growth effects which stem from changes in average investment have only small welfare effects, this section suggests a channel by which stabilization might generate significant welfare effects beyond those that accrue from smoothing innovation over time.

My modification relies on the notion of occasionally binding constraints on the innovation process, i.e. constraints that bind for some levels of economic activity but not others.[17] Suppose that certain underlying frictions limit the number of workers that can be hired in the innovation sector for a given state of economic conditions. For example, if innovators have to borrow funds to cover their expenses, there might be a limit on how much they can borrow without creating an incentive to default on their debt. This limit could depend on prevailing economic conditions. Thus, suppose that if government spending was constant at a level G, employment in the innovation sector is bounded above by $n \leq \overline{n}(G)$. In addition, define $\widetilde{n}(G)$ as the amount of resources that would be employed in innovation if government spending is constant at G if the constraint is removed. In what follows, I assume that $\overline{n}(\cdot)$ and $\widetilde{n}(\cdot)$ are both differentiable. The fact that constraints bind for some levels of government spending but not others implies $\overline{n}(G)$ and $\widetilde{n}(G)$ will cross at unequal slopes for some level of government spending $\overline{G}$; this level is where the constraint is exactly binding. Figure 1 illustrates the case where the constraint is binding at high levels of government spending. For G close to $\overline{G}$, continuity implies employment will equal the minimum of these two curves, i.e. there exists an $\varepsilon > 0$ such that

$$n = \min\left[\widetilde{n}(G), \overline{n}(G)\right]$$

for all $G \in \left(\overline{G} - \varepsilon, \overline{G} + \varepsilon\right)$. Since $\widetilde{n}(G)$ or $\overline{n}(G)$ intersect to form a concave kink at $\overline{G}$, the amount of resources n employed in innovation will be locally concave, even if $\widetilde{n}(G)$ or $\overline{n}(G)$ themselves are convex, as illustrated in Figure 1. If we allow government spending to fluctuate within a small neighborhood of $\overline{G}$, continuity insures $n^*(G) \approx n(G)$, implying $n^*(G)$ will also be effectively concave. Thus, stabilization of small shocks near the point at which constraints become binding will increase average investment. This result is robust to the shape of $\phi(\cdot)$. It also does not depend on whether constraints on innovation bind at high values or low values of G. Thus, if we have evidence of occasionally binding constraints, we can be assured that a small reduction in volatility will increase average investment, and consequently average growth.

[17]Note that constraints are assumed to operate in the innovation sector as opposed to production, which is important for my result. However, since research is inherently difficult to monitor, innovation seems much more vulnerable to incentive problems than production.

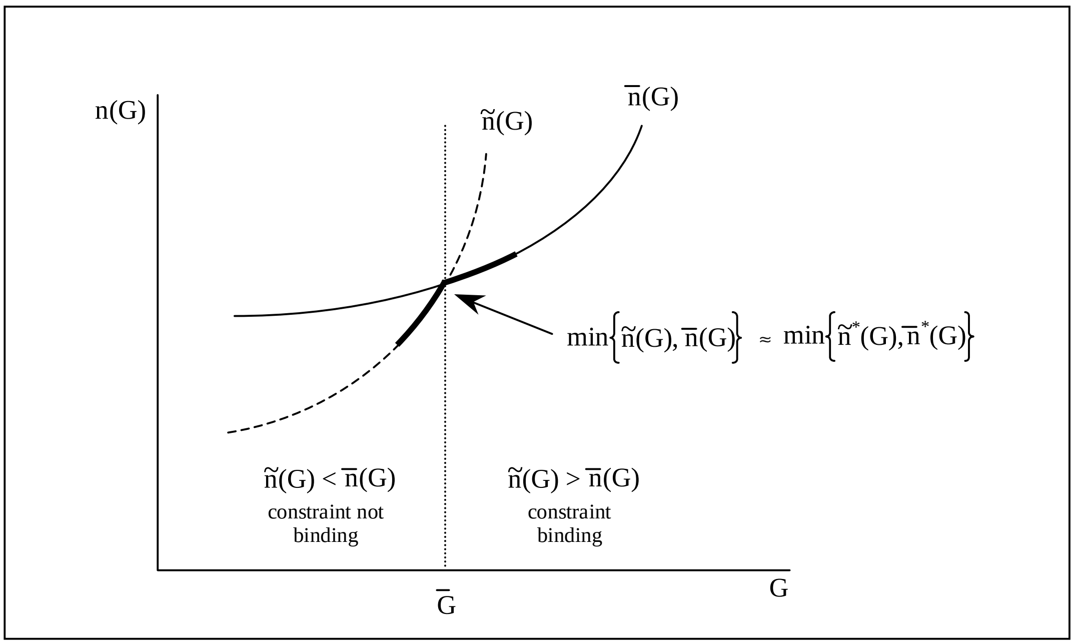


Figure 1: Occasionally Binding Constraints

To make this intuition concrete, consider the following modification of the model in Section 1. Recall that in that model, an innovator who did not succeed in discovery charged his losses to his owners. Suppose instead there is limited liability, so entrepreneurs engaged in innovation must borrow in order to pay their wage bill. Since innovators who fail in research have no resources which they can use to repay their loans, debt contracts will stipulate that an innovator only repay his debt if he succeeds in innovation. If there is a banking sector that can take the resources of the representative agent and extend credit to a diversified portfolio of entrepreneurs, it can be certain that a fraction $\phi(n)\,dt$ of all firms it lends to will successfully innovate. Hence, banks face no aggregate risk, and if the banking sector is competitive, the interest rate on debt repayments will be such that the expected profit from any loan is exactly zero. This implies a gross interest rate of $\phi(n)^{-1}$. Given this debt contract, the problem of an entrepreneur engaging in research remains unchanged. In particular, he will maximize

$$\phi(n)\,dt\left(v - \phi(n)^{-1} n\right) + (1 - \phi(n)\,dt) \cdot 0 = \phi(n)\,v - n$$

Without additional restrictions, the equilibrium will be just as before, with the exception that the representative agent receives his non-labor income as both debt and equity. For debt contracts to have an effect on equilibrium, we need to introduce a friction into the model. Suppose, then, that when an entrepreneur makes a discovery, he has the option to secretly meet with somebody else and make a take-it-or-leave it offer to this other party to sell the blueprints of his discovery. However, keeping this activity a secret requires a cost K. This way, he can receive the full value of the patent, v_t, but avoids repaying his loan since he can represent to his creditor that somebody else has made a successful discovery. Thus, the creditor knows that he will not be paid back if

$$v - K > v - \frac{n}{\phi(n)}$$

or if the total debt burden is sufficiently large, i.e. $\frac{n}{\phi(n)} > K$. We now make use of the following result:

Lemma 3: $\frac{n}{\phi(n)}$ is increasing in n, i.e. an entrepreneur will not default if and only if $n \leq \overline{n}$, where $\overline{n}$ solves

$$\frac{\overline{n}}{\phi(\overline{n})} = K$$

To rule out uninteresting cases, suppose that $L \gg \overline{n}$. Consider the constant level of government spending $\overline{G}$ at which the constraint just binds, i.e. $n\left(\overline{G}\right) = \overline{n}$ where $\overline{n}$ is the same as in Lemma 3. Solving for $\overline{G}$ as a function of $\overline{n}$ yields

$$\overline{G} = T + \overline{n} + \frac{\rho + \phi\left(\overline{n}\right)\left[1 + (\gamma - 1)\ln\lambda\right]}{(\lambda - 1)\,\phi'\left(\overline{n}\right)} - L \tag{3.1}$$

This expression is well defined for L sufficiently large. Now, consider a small perturbation where government spending fluctuates between $G_0 = \overline{G} - \varepsilon$ and $G_1 = \overline{G} + \varepsilon$. The next Proposition confirms that, regardless of the functional form of $\phi(n)$, stabilization of government spending will increase investment.

Proposition 3: For $\overline{G}$ given by (3.1),

1. $n^*(G_1) = \overline{n}$
2. $n^*(G_0) < \overline{n}$
3. $\frac{1}{2}\left[n^*(G_0) + n^*(G_1)\right] < n\left(\overline{G}\right) \equiv \overline{n}$, so n is effectively concave.

In this particular instance, n is effectively concave for all values of ε. That is, for any fluctuations around the constant level of government spending where the constraint starts to bind, stabilization will be growth-increasing, regardless of the third derivative of ϕ. However, as the previous discussion illustrates, this result will not hold more generally. Instead, it will typically be the case that occasionally binding constraints can only guarantee effective concavity in a local region around the level of economic activity where the constraint becomes binding.

As emphasized in previous discussion, the fact that stabilization increases growth by increasing average investment yields ambiguous welfare implications, since faster growth occurs at the expense of lower current consumption. Thus, the mere fact that Proposition 3 insures stabilization increases growth does not insure the agent is necessarily better off. However, with binding constraints, the increase in average investment will be welfare-enhancing. This is because with constraints, growth will generally be suboptimally low, since the value of an additional worker in the innovation sector is higher than his value in the production sector. Thus, although stabilization shifts resources out of current consumption, on the margin the agent would generally prefer to increase average investment. Thus, occasionally binding constraints introduce an additional dimension through which stabilization can increase welfare. This channel is distinct from the observation Lucas makes that individuals are made better off by increased growth; his calculation assumes that average consumption is unchanged, while the increase in welfare here is due precisely to an increase in average investment at the expense of current consumption. However, Lucas' calculation still provides an upper bound on the benefits of stabilization — although potentially a very loose upper bound — since it represents an upper bound on how valuable additional growth is to the agent. I make use of this observation in the next section.

4. Quantitative Analysis

Although the preceding analysis shows that business cycles could generate additional welfare effects through endogenous growth channels that have been ignored in previous work, the most interesting question yet remains: are these benefits large enough to make business cycles important for welfare? One approach is to choose a specific function $\phi(n)$ and estimate the implied welfare effects for this function. For example, Stokey (1995) studies the effects of diminishing returns in generating a wedge between the equilibrium rate of growth and the optimal rate of growth, and parameterizes the probability of success by $\phi(n) = \Phi n^{\xi}$. However, finding the right functional form to match first, second, and third derivatives as this model requires is generally quite difficult. For example, the constant elasticity function Stokey uses is inconsistent with condition (1.9). Thus, instead of pursuing a particular functional form, I settle on a less ambitious question, but which is very much in the spirit of Lucas' original calculation: what is the largest increase in growth that diminishing returns could generate for which the model remains consistent with empirical observations on consumption from the post-War period? At the very least, this can tell us whether there is any hope that stabilization can produce large welfare effects.

The question of how stabilization effects growth can be broken down into two parts. The first involves the effect of diminishing returns: how much will stabilization increase growth holding average employment in innovation fixed? This growth effect corresponds to an increase in welfare as implied by Lucas' original calculation, since it raises the rate of consumption growth without reducing average current consumption. The magnitude of this effect is given by the difference between the average growth rate $\frac{1}{2}[\phi(n_0)+\phi(n_1)]\ln\lambda$ and the constant growth rate $\phi\left(\frac{1}{2}[n_0+n_1]\right)\ln\lambda$ under stabilization. This difference clearly depends on the concavity of the function $\phi(\cdot)$. After I estimate a bound for the effect of diminishing returns, I turn to the potential welfare gains when stabilization also raises the average level of innovation, as discussed in Section 3, i.e. the difference between the average growth rate $\frac{1}{2}[\phi(n_0)+\phi(n_1)]\ln\lambda$ and the constant growth rate $\phi(\overline{n})\ln\lambda$ for some $\overline{n}>\frac{1}{2}[n_0+n_1]$.

In taking the model to the data, I use a discrete version of (1.6). The model implies consumption between period $t-1$ and t will change for two reasons. First, technological innovation improves productivity and allows more consumption goods to be produced from a given amount of resources. This growth rate is equal to $\phi(n_t)\ln\lambda$, which depends on the underlying growth regime in the current period. Second, regime changes trigger changes in the amount of resources employed in production and thus the level of consumption. The change in log consumption between two periods is therefore given by

$$\Delta\ln C_t=\phi(n_t)\ln\lambda+(\varepsilon_t-\varepsilon_{t-1}) \tag{4.1}$$

In fitting this stochastic process to actual consumption data, I assume there are two regimes, denoted by $s_t\in\{0,1\}$. Without loss of generality, I designate $s_t=0$ as the low productivity growth regime, i.e. $\lambda_0\leq\lambda_1$. Let $\varepsilon=\varepsilon_1-\varepsilon_0$ denote the difference in the level of consumption across the two regimes. With this notation, we can rewrite (4.1) as $\lambda_0(1-s_t)+\lambda_1 s_t+\varepsilon(s_t-s_{t-1})$. I then estimate λ_0, λ_1, and ε from annual data by minimizing the mean square error over all possible sequences $\{s_t\}$:

$$\min_{\lambda_0,\lambda_1,\varepsilon,\{s_t\}}\sum_{t=1951}^{1998}\left(\Delta\ln C_t-[\lambda_0(1-s_t)+\lambda_1 s_t+\varepsilon(s_t-s_{t-1})]\right)^2 \tag{4.2}$$

We can think of this minimization problem in two stages. First, for each sequence $\{s_t\}$, I look for the vector $(\lambda_0,\lambda_1,\varepsilon)$ which minimizes mean square error. I then look for the sequence of realizations s_t for which this minimum mean square error is lowest. The problem with solving (4.2) is that the number of possible sequences involves $2^{49}=5.6\times10^{14}$ combinations. To get at the minimum, I follow a routine that is similar to simulated annealing. That is, I guess an initial sequence of s_t and then allow for small stochastic perturbations around the original sequence. If a lower value is achieved, I use the new minimum as my initial sequence. I repeated this procedure from several different initial conditions and also allowed for larger perturbations to check against local optima. The estimates at the minimum were $\lambda_0 = .002$, $\lambda_1 = .038$, and $\varepsilon = -.007$. This implies that the growth rate in consumption because of technological progress fluctuates between 0.2% and 3.8% per year, for an average of 2% per year. This estimate lines up with conventional estimates of average growth in GDP per capita for the U.S. The fact that the point estimate for ε is negative implies that consumption growth is countercyclical, since consumption grows more rapidly if it is below its expected level. This finding is somewhat disturbing, since I previously argued procyclical growth is more consistent with the empirical evidence on procyclical profits and equity values. However, this is not important for calculating the effects of stabilization, and, as I discuss further below, this finding is not very robust.

Without any additional restrictions, we could in principle make $\phi(n)$ arbitrarily concave. Hence, stabilization could conceivably raise growth from an average of 2.0% all the way to 3.8% per year. However, the model imposes additional discipline which allows us to significantly improve upon this upper bound. In particular, the first order condition (1.3) imposes a restriction on the ratio of the slope of $\phi(\cdot)$ at the two regimes, namely

$$\frac{\phi'(n_0)}{\phi'(n_1)} = \frac{v_1}{v_0} > 1 \tag{4.3}$$

This restriction significantly limits the amount of concavity that $\phi(\cdot)$ can exhibit, since it limits the rate at which $\phi'(\cdot)$ can change with n. In particular, we have

Proposition 4: If $\phi'(n_0) = \frac{v_1}{v_0}\phi'(n_1)$, then

$$\phi\left(\tfrac{1}{2}[n_0 + n_1]\right)\ln\lambda \leq \frac{1}{v_1/v_0 + 1}\phi(n_0)\ln\lambda + \frac{v_1/v_0}{v_1/v_0 + 1}\phi(n_1)\ln\lambda \tag{4.4}$$

Figure 2 illustrates the intuition behind the Proposition 4. Consumption data pins down the values of $\phi(n_0)\ln\lambda$ and $\phi(n_1)\ln\lambda$ at 0.002 and 0.038, respectively. Now, set the derivative of the growth rate at $n = n_1$ to some positive constant a where $(n_1 - n_0)\,a < \phi(n_1)\ln\lambda - \phi(n_0)\ln\lambda$. From the first order condition, we know that the slope at $n = n_0$ must be equal to $\phi'(n_1)\ln\lambda = \frac{v_1}{v_0}a$. The two lines originating at n_0 and n_1 form an upper envelope for any function $\lambda\phi(n)$ that are consistent with the restriction in (4.3). Proposition 4 reports the results of maximizing

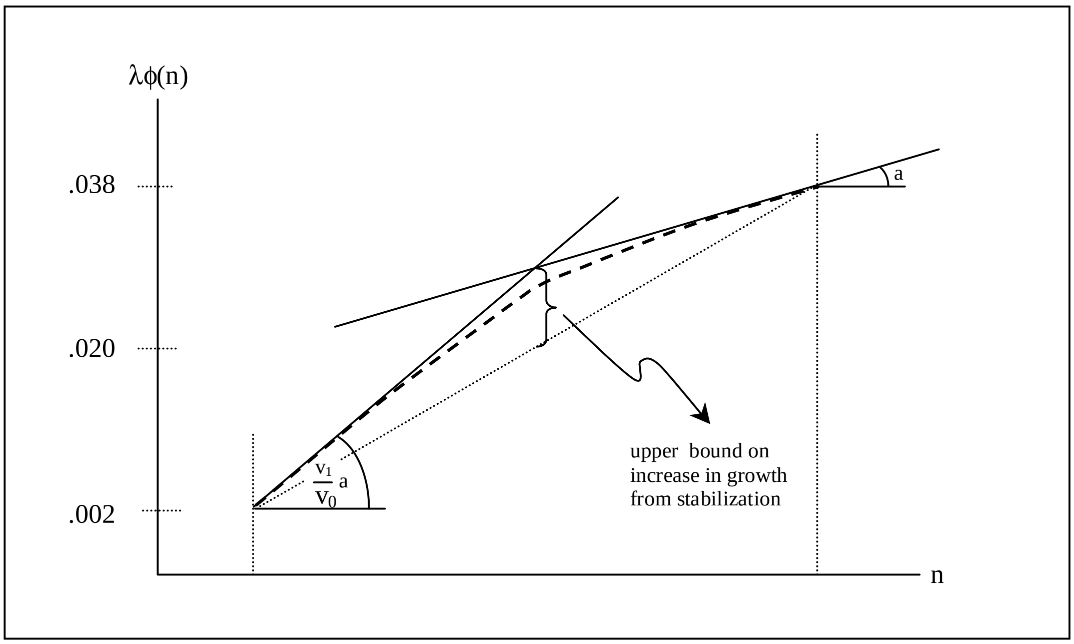


Figure 2: Computing Upper Bounds on the Effect of Stabilization

this upper envelope at the point $n = \frac{1}{2}[n_0 + n_1]$, i.e. maximizing $\phi\left(\frac{1}{2}[n_0 + n_1]\right)\ln\lambda$ over all relevant values of a.

From (4.4), the upper bound on the growth potential from diminishing returns depends on the ratio $\frac{v_1}{v_0}$. One approach to estimating this ratio is to use stock market data as a measure of the value of firms over the cycle. Conventional estimates, such as Christiano and Fisher (1998), suggest the standard deviation of the S&P 500 is on the order of 10%. However, since the prices of new firms are known to be volatile, and new firms are listed in exchanges and counted in stock indices such as the S&P only with a lag, this is likely to be an underestimate of true volatility in profit opportunities over the cycle. As an alternative measure, recall that v is a weighted discounted flow of profits across the two regimes. Hence, the ratio of profits in the two regimes will serve as an upper bound on the ratio $\frac{v_1}{v_0}$. Data on after-tax corporate profits in the U.S. suggests a standard deviation for $\Delta\ln(\text{profits})$ of about $10 - 15\%$.[18] A standard deviation of 15% would imply that the ratio $\frac{v_1}{v_0} = \frac{1.15}{0.85} = 1.35$. If the standard deviation of v is allowed to be as large as 20%, which is a reasonable upper bound, $\frac{v_1}{v_0}$ will equal 1.5. Substituting this into (4.4) yields

$$\phi\left(\tfrac{1}{2}[n_0 + n_1]\right)\ln\lambda \leq \begin{cases} .0227 & \text{if } \dfrac{v_1}{v_0} = \dfrac{1.15}{0.85} = 1.35 \\ \\ .0236 & \text{if } \dfrac{v_1}{v_0} = \dfrac{1.20}{0.80} = 1.50 \end{cases}$$

Thus, the model is consistent with an increase in consumption growth as large as 0.36 percentage points. Of course, it must be stressed that this is only an upper bound; it only tells us that such an increase in growth is possible, not that it is likely. Still, we can compare these upper bounds with the empirical estimates in Ramey and Ramey (1995) on the effects of stabilization on growth from cross-country evidence. They find that after controlling for the average investment share, a one point reduction in the standard deviation of output growth is typically associated with an increased growth rate of $0.1 - 0.2\%$. Since the standard deviation of output growth in the U.S. is 2.5%, this implies stabilization should increase the growth rate of between 0.25% and 0.5%. This compares reasonably with the upper bound estimate of 0.36%.[19] The fact that

[18] This covers the same period $1951 - 1998$. This estimate accounts for detrending implied productivity growth using consumption growth. Detrending has only a minor impact on the estimate of the standard deviation, since $\Delta\ln(\text{profits})$ is far more volatile than $\Delta\ln(C)$.

[19] It should be noted that Ramey and Ramey estimates are unstable, particularly when they restrict attention

these estimates are so close to the upper bound need not be entirely surprising given previous evidence of fairly large diminishing returns to innovation.[20]

To interpret the above increase in growth in terms of welfare, we can apply Lucas' original calculation for how much the individual would value an unconditional increase in his growth. This is because diminishing returns involves an increase in growth without a change in the average level of consumption, which is precisely what Lucas calculates when he examines the welfare effects of increased growth. For $\gamma = 1$, Lucas shows that the consumer would be willing to sacrifice approximately 20% of C_0 for each percentage point in the growth rate. This implies an upper bound on the benefits from stabilization of $0.27 \times 20 = 5.4\%$ of initial consumption when the standard deviation of v is set to 15%, and 7.2% when the standard deviation is equal to 20%. For the case where $\gamma = 2$, a similar approximation from Obstfeld (1994a) suggests the consumer would be willing to sacrifice approximately 13% of C_0 for each percentage point in the growth rate, which implies the benefits of stabilization amount can be as much as 3.5% and 4.6% of initial consumption, respectively. Hence, even though the model imposes relatively tight restrictions on the effects of stabilization on growth — no more than a few basis points — the fact that increased growth does not come at the expense of present consumption allows for very large welfare effects, much larger than those reported in the previous literature. My calculations allow the possibility of welfare effects that nearly 100 times as large as Lucas' original estimate, and more than 20 times as large as revised estimates that place the value of stabilization at 0.3% of consumption. Thus, after accounting for endogenous growth effects, it seems that business cycles could matter, and if cross-country evidence is reliable for inferring the effects of stabilization, it seems that they do matter.

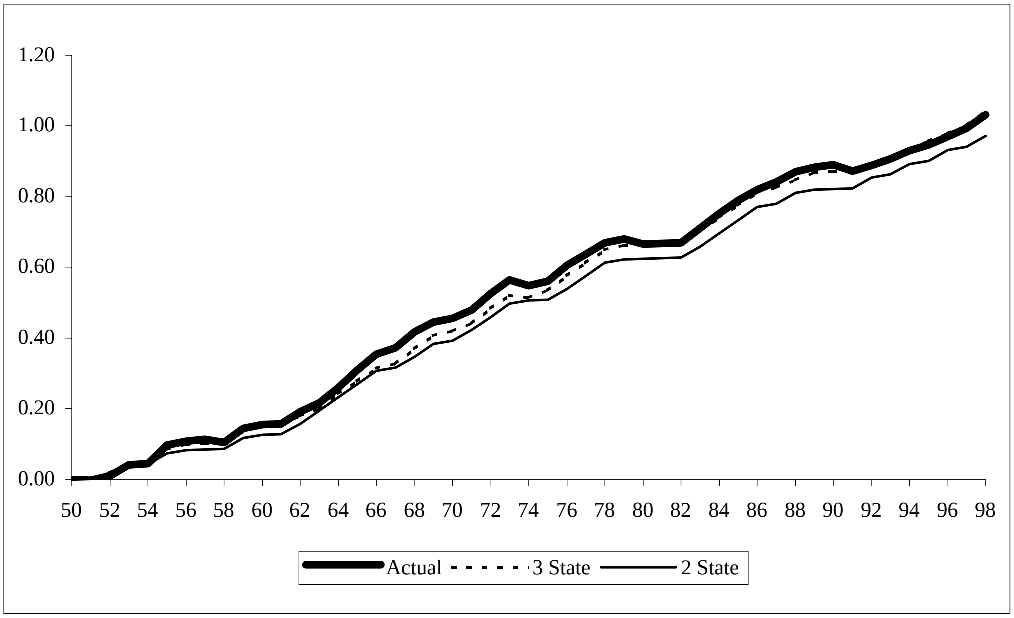
To determine whether my results hinge on my assumption of two-states, I experimented with estimating a three-state regime. That is, I repeated the maximization problem in (4.2) allowing for three growth rates $\lambda_0 \leq \lambda_1 \leq \lambda_2$ and for two changes in levels $\varepsilon_2 - \varepsilon_1$ and $\varepsilon_1 - \varepsilon_0$. Part of the motivation for this is that the sequence $\{s_t\}$ associated with the best fit is quite volatile, in part because consumption growth during the 1990s was steady at around 2% which required

to only OECD countries, and some of their estimates imply stabilization would increase growth from 2% to 4%. These appear wildly inconsistent with the upper bounds derived here.

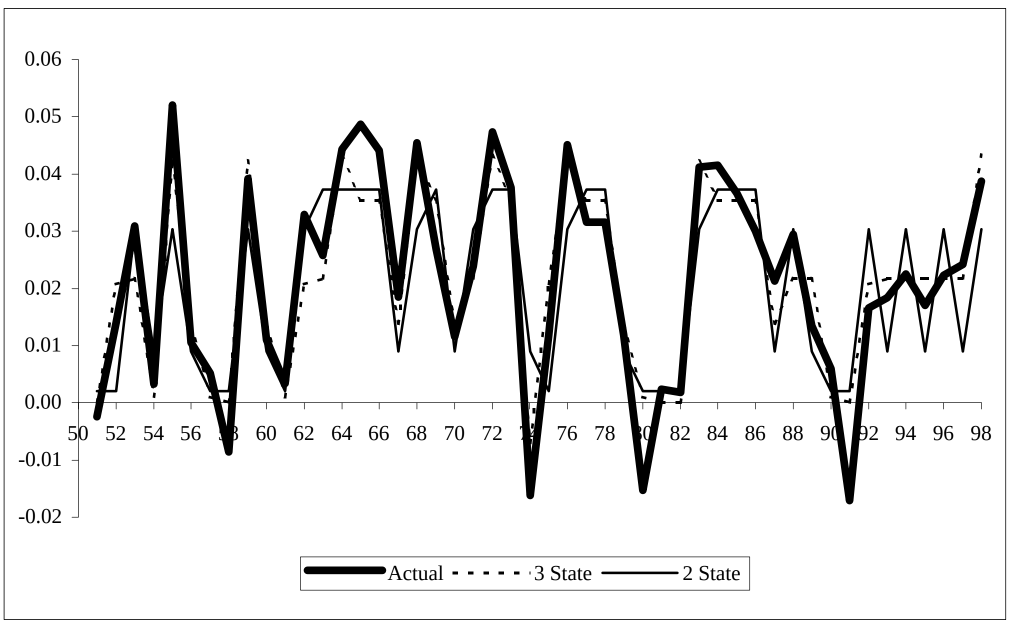
[20] For example, in calibrating the extent of diminishing returns, Stokey relies on Kortum (1993) who reports that estimates of ξ range between 0.1 and 0.6.

consumption growth to oscillate constantly during this period. In estimating the three-state model, I obtained a negative point estimate for the lowest growth rate λ_0. Since technological regress is incompatible with the model, I repeated the minimization under the constraint that $\lambda_0 \geq 0$. The estimates then were $\lambda_0 = 0.000$, $\lambda_1 = 0.022$ and $\lambda_2 = 0.036$, $\varepsilon_2 - \varepsilon_1 = .008$, and $\varepsilon_1 - \varepsilon_0 = -.001$ and $\varepsilon_2 - \varepsilon_1 = .009$. A few things are worth noting about these estimates. First, the difference between the lowest and highest growth rate is the same as in the two-state model. Thus, stabilizing employment in the innovation sector to the average of employment in the two extreme points will have the same welfare effect as in the two-state model. Second, unlike the two-state model, the three state model no longer predicts countercyclical growth: $\lambda_2 - \lambda_1$ and $\varepsilon_2 - \varepsilon_1$ are both positive, suggesting that more rapid growth is associated with higher consumption levels. The estimate for $\varepsilon_1 - \varepsilon_0$ is still negative, but it is essentially indistinguishable from 0. Third, the three state regime provides some evidence in favor of diminishing returns, since $\varepsilon_2 - \varepsilon_1 > \varepsilon_1 - \varepsilon_0$ while $\lambda_2 - \lambda_1 < \lambda_1 - \lambda_0$. This suggests that an increase in the level of economic activity at low levels generates a larger increase in growth than an increase in economic activity at intermediate levels of economic activity. Lastly, the sequence $\{s_t\}$ that achieves the smallest mean square error in the three state model accords quite well with conventional measures of the business cycle. For example, it assigns the lowest possible state in almost perfect accordance with NBER business cycle dates. Figure 3 shows the simulated consumption paths and changes in consumption for both the two regime model and the three regime model, and shows that the three regime model provides a good fit to the actual data.

Finally, Section 3 raises the possibility that stabilization increases growth not only because of the concavity of $\phi(n)$, but also by increasing average investment. In general, increased investment need not increase welfare, since more rapid growth comes at the expense of current consumption. However, if the increase in average consumption is due to occasionally binding constraints, stabilization allows the agent to allocate more resources to innovation that he is effectively prevented from doing because of constraints. In this case, stabilization yields additional welfare benefits above and beyond those that are due to diminishing returns. However, without more information on $\phi(n)$, it is impossible to determine the size of this effect. At most, we know that the value of additional growth is what is computed by Lucas; his calculation reports the value of growth when no consumption is sacrificed. Since stabilization could conceivably increase growth up to 3.8%, an increase in growth of 1.8%, the upper bound is given by 36.0% of current consumption when $\gamma = 1$ and 23.4% when $\gamma = 2$. These allow for incredibly large welfare gains



(a) Log Consumption - Actual vs. Simulated



(b) Δ Log Consumption - Actual vs. Simulated

Figure 3

from stabilization, although they are also very likely to be gross overestimates of the actual benefit.

5. Conclusion

This paper has explored the welfare effects of business cycles that arise from endogenous growth effects. While these effects have been occasionally mentioned in previous work, they have yet to be analyzed formally. Previous authors have not provided a discussion of when stabilization will increase growth, nor have they attempted to assess how the welfare benefits associated with such growth effects would compare with those that have been previously calculated by Lucas and others for consumption volatility. This paper identifies forces that suggest aggregate fluctuations retard the long-run growth process, and shows that they could generate substantial welfare effects. However, I was only able to establish an upper bounds on how large this effect could potentially be. This leaves the door open for future work to establish how tight these bounds are. Still, empirical estimates on the effects of stabilization on growth from cross-country comparisons are quite close to these upper bound estimates, suggesting a welfare gain of about $3 - 5\%$.

Since there is already an extensive literature that examines growth under uncertainty, it would seem fitting to end with a recap of what this paper contributes to this literature. First and foremost, this paper emphasizes diminishing returns to investment as a source of growth effects, rather than changes in average investment which previous work has studied. The reason this distinction is important is because diminishing returns generates an increase in growth without an offsetting decline in current consumption. This point is crucial: Lucas derives huge welfare effects from an increase in the growth rate only because he considers an increase in the growth rate for a given level of initial consumption. Previous models that emphasize endogenous growth effects from changes in average accumulation do not imply such large welfare gains, precisely because more rapid growth is offset by a reduction in current consumption. In fact, there is nothing to assure us in these models that welfare is higher at higher rates of consumption growth, and so such models cannot get at the intuition that the large welfare effects from increased growth that Lucas reports could imply large welfare effects for business cycles. This paper also offers the case of occasionally binding constraints as one where faster growth enhances welfare even when growth effects operate through changes in average investment. The reason is that constraints preclude resources from being allocated efficiently to growth, so an increase in investment should make the agent better off. However, there is no counterpart to Lucas' calculations that tell us precisely how big of an impact this would have on welfare.

Second, this paper examines the welfare effects of stabilization rather than just the effects of stabilization on the growth rate per se. Thus, even if previous authors already argued that stabilization could affect growth by changing the growth rate of consumption, and even estimated empirically the relationship between volatility and growth, they did not provide a framework which could interpret these observations in terms of welfare and compare them with the numbers generated by Lucas. Carrying out these welfare calculations explicitly shows that business cycles can matter. This conclusion stands in contrast to the large body of work which computes only small welfare gains when only accounting for costs due to consumption volatility. As the discussion in the Introduction reveals, the initial observation by Lucas that consumption volatility from aggregate fluctuations has negligible welfare implications seems to have held up to subsequent scrutiny, except perhaps when such shocks are both very persistent and highly volatile. But this does not imply that stabilization cannot produce first order welfare effects: aggregate fluctuations could still have important effects by retarding the process of long-run growth.[21] In a sense, this paper closes the circle that Lucas originally began: he used his calculation to argue that because consumption volatility has negligible welfare effects, only growth matters. But if business cycles affect the process of long-run growth in a way that is consistent with his thought experiment, which this paper argues, they will matter as well.

[21] This statement applies to stabilization policies which set macroeconomic variables equal to their average, rather than policies that stimulate macroeconomic activity. The former emphasizes the benefits from reduced volatility, while the latter emphasizes "filling in business cycle troughs without shaving off business cycle peaks" to quote DeLong and Summers (1988). This distinction was recently examined in Chatterjee and Corbae (1999), who argue that much of the welfare gains from moving to the business cycle environment after the second World War were quite large were due to a reduction in average unemployment rather than changes in its duration or volatility. They conclude that individuals are better off under today's cycles than prior to WWII, but are ambivalent whether to attribute this to stabilization policy.

Appendix

Proof of Lemma 1: Recall that the value of a patent is given by

$$v_t = E_t\left[\int_t^\infty \frac{U'\left(C_{t+s}\right)}{U'(C_t)}\frac{dC_{t+s}}{d\Pi_{t+s}}\pi_{t+s}e^{-\rho(t+s)}ds\right] \equiv E_t\left[\int_0^\infty \delta_{t+s}\pi_{t+s}e^{-\rho(t+s)}ds\right]$$

I begin by characterizing the evolution of the discount factor δ_{t+s} and of expected profits $E_t\left[\pi_{t+s}\right]$ assuming government spending remains constant between t and $t+s$, i.e. where for all $\tau \in [t, t+s]$, $G_\tau = G_i$:

1. From (1.1), we have

$$c_{j,t+s} = \frac{Y_{t+s} - T}{p_{j,t+s}}$$

which implies

$$\begin{aligned}
\int_0^1 \ln c_{j,t+s} dj &= \int_0^1 \ln\left(\frac{Y_{t+s} - T}{p_{j,t+s}}\right) dj \\
&= \ln\left(Y_{t+s} - T\right) - \int_0^1 \ln p_{jt}\ dj \\
&= \ln\left(Y_{t+s} - T\right) - \int_0^1 \ln \lambda^{m_s(j)-1}\ dj \\
&= \ln\left(Y_{t+s} - T\right) - \int_0^1 \left[m_s\left(j\right) - 1\right] dj \cdot \ln\lambda \\
&= \ln\left(Y_{t+s} - T\right) - \left(\ln\lambda\right)\left(m_t + \phi_i s - 1\right)
\end{aligned}$$

where

$$m_{t+s} = \int_0^1 \left[m_s\left(j\right) - 1\right] = E\left[m_s\left(j\right)\right] - 1$$

The last step uses the fact that $\left(m_{t+s} - m_t\right) \sim \text{Poisson}(\phi_i s)$. This implies as long as the same regime i prevails between dates t and $t+s$,

$$C_{t+s} = \exp\left[\int_0^1 \ln c_{j,t+s} dj\right] = e^{(\ln\lambda)(m_t - 1 + \phi_i s)}\left(Y_{t+s} - T\right)$$

so that

$$\frac{dC_{t+s}}{dY_{t+s}} = e^{(\ln\lambda)(m_t - 1 + \phi_i s)}$$

2. Using the fact that $\frac{U'(C_{t+s})}{U'(C_t)} = \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma}$, I compute the ratio of consumption $\frac{C_{t+s}}{C_t}$. The growth of consumption at a given instant is given by

$$\begin{aligned} d\ln C_t &= \int_0^1 d(\ln c_{jt})\,dj \\ &= \int_0^1 [(1-\phi_i dt)\cdot 0 + \phi_i (\Delta \ln c_j)\,dt]\,dj \\ &= \int_0^1 [(1-\phi_i dt)\cdot 0 + \phi_i (\ln\lambda)\,dt]\,dj \\ &= \phi_i (\ln\lambda)\,dt \end{aligned}$$

so that as long as regime i prevails between time t and $t+s$, we have

$$C_{t+s} = C_t e^{(\ln\lambda)\phi_i s}$$

3. Profits π_{t+s} are either equal to π_i if a better technology was not invented and 0 if it was. For a constant level of government spending, the probability that no new discovery is made between dates t and $t+s$ is given by $e^{-\phi_i s}$. Hence,

$$E_t[\pi_{t+s}] = \pi_i e^{-\phi_i s}$$

It follows that

$$E_t\left[\frac{U'(C_{t+s})}{U'(C_t)}\frac{dC_t}{dY_t}\pi_{t+s}\right]e^{-\rho t} = \text{constant}\cdot\ e^{-(\rho+\phi_i[1+(\gamma-1)(\ln\lambda)])s}$$

For the value v_i to be defined, the exponent above must be negative, i.e.

$$\rho + [1+(\gamma-1)\ln\lambda]\,\phi > 0$$

This will be true as long as $\gamma \geq 1$.

From above, the evolution of average generation of technology m_t exhibits the following law of motion within a given regime:

$$\frac{\dot{m}_t}{m_t} = -\phi(\gamma-1)\ln\lambda$$

Using this fact, the value of owning a patent is characterized by the asset equation

$$\begin{aligned} \rho v_i(m) &= m\pi_i + \mu(v_i - v_j) + v'(m)\,\dot{m} - \phi_i v_i \\ &= m\pi_i + \mu(v_i - v_j) - m\phi_i(\gamma-1)(\ln\lambda)\,v_i'(m) - \phi_i v_i \end{aligned}$$

so we have a system of equation

$$\begin{aligned} (\rho+\phi_0+\mu)\,v_0(m) - \mu v_1(m) + m\phi_0(\gamma-1)(\ln\lambda)\,v_0'(m) &= m\pi_0 \\ (\rho+\phi_1+\mu)\,v_1(m) - \mu v_0(m) + m\phi_1(\gamma-1)(\ln\lambda)\,v_1'(m) &= m\pi_1 \end{aligned}$$

To solve the above system, we guess

$$\begin{aligned} v_0 &= \beta_0 m \\ v_1 &= \beta_1 m \end{aligned}$$

Matching coefficients yields the coefficients ω_0 and ω_1 reported in the text. ■

Proof of Proposition 1: I begin by showing that $n_1 > n_0$. For suppose not, i.e. $n_1 \leq n_0$. Then the fact that $N_0 > N_1$ implies

$$L - N_1 - n_1 > L - N_0 - n_0$$

so that $\pi_1 > \pi_0$. Since $n_1 > n_0$ and $\phi'(n) > 0$, it follows that $\phi(n_0) > \phi(n_1)$. In this case,

$$\begin{aligned} \frac{v_0}{v_1} &= \frac{[\rho + \mu + (1 + (\gamma - 1)\ln\lambda)\phi(n_1)]\pi_0 + \mu\pi_1}{[\rho + \mu + (1 + (\gamma - 1)\ln\lambda)\phi(n_0)]\pi_1 + \mu\pi_0} \\ &< \frac{[\rho + \mu + (1 + (\gamma - 1)\ln\lambda)\phi(n_0)]\pi_0 + \mu\pi_1}{[\rho + \mu + (1 + (\gamma - 1)\ln\lambda)\phi(n_0)]\pi_1 + \mu\pi_0} \\ &= \frac{\mu(\pi_0 + \pi_1) + [\rho + (1 + (\gamma - 1)\ln\lambda)\phi(n_0)]\pi_0}{\mu(\pi_0 + \pi_1) + [\rho + (1 + (\gamma - 1)\ln\lambda)\phi(n_0)]\pi_1} \\ &< 1 \end{aligned}$$

However, from the first order condition (1.3), $n_0 > n_1 \Rightarrow v_0 > v_1$, which is a contradiction. Hence, $n_1 > n_0$. This establishes part (1) of the Proposition. From the first order condition (1.3), this also implies $v_1 > v_0$, which establishes part (2).

To establish (3), suppose that $n_1 - n_0 > N_0 - N_1 > 0$, so

$$L - N_0 - n_0 > L - N_1 - n_1$$

and $\pi_0 > \pi_1$. But then,

$$\begin{aligned} \frac{v_0}{v_1} &= \frac{(\rho + \mu + (1 + (\gamma - 1)\ln\lambda)\phi(n_1))\pi_0 + \mu\pi_1}{(\rho + \mu + (1 + (\gamma - 1)\ln\lambda)\phi(n_0))\pi_1 + \mu\pi_0} \\ &> \frac{(\rho + \mu + (1 + (\gamma - 1)\ln\lambda)\phi(n_0))\pi_0 + \mu\pi_1}{(\rho + \mu + (1 + (\gamma - 1)\ln\lambda)\phi(n_0))\pi_1 + \mu\pi_0} \\ &= \frac{\mu(\pi_0 + \pi_1) + (\rho + (1 + (\gamma - 1)\ln\lambda)\phi(n_0))\pi_0}{\mu(\pi_0 + \pi_1) + (\rho + (1 + (\gamma - 1)\ln\lambda)\phi(n_0))\pi_1} \\ &> 1 \end{aligned}$$

which contradicts the fact that $v_1 > v_0$. Hence, $\pi_1 > \pi_0$, establishing (3).

To show (4), note that nominal income is given by

$$Y = \Pi + W = \pi + L - n$$

so that $Y_1 - Y_0$ is given by

$$\begin{aligned} \pi_1 - \pi_0 - (n_1 - n_0) &= (\lambda - 1)(N_0 + n_0 - N_1 - n_1) - (n_1 - n_0) \\ &= -\lambda (n_1 - n_0) - (\lambda - 1)(N_1 - N_0) \end{aligned}$$

whose sign is ambiguous since we can only establish that

$$N_0 - N_1 > n_1 - n_0 > 0$$

whereas $Y_1 > Y_0$ only if

$$N_0 - N_1 > \frac{\lambda}{\lambda - 1}(n_1 - n_0) > n_1 - n_0$$

It is possible to generate results whether the first inequality is and is not satisfied. The fact that $C = Y - T$ establishes (4). ■

Proof of Lemma 2: Applying my previous results, we have that for each regime $i \in \{0, 1\}$,

$$\begin{aligned} (PC)_i &= Y_i - T \\ &= \Pi_i + L - n_i - T \\ &= (\lambda - 1)(L - N_i - n_i) + L - n_i - T \\ &= \lambda (L - n_i) - (\lambda - 1) N_i - T \\ &= \lambda (L - n_i) + (\lambda - 1) G_i - \lambda T \end{aligned}$$

Thus, comparing nominal consumption under stabilization is given by

$$\lambda L - \lambda T - \lambda n \left(\overline{G}\right) + (\lambda - 1)\overline{G}$$

while average nominal consumption in a world of stochastic shocks is given by

$$\lambda L - \lambda T - \frac{\lambda}{2}(n^* (G_0) + n^* (G_1)) + \frac{\lambda - 1}{2}(G_0 + G_1)$$

Rearranging these two equations and using the fact that $\overline{G} - \frac{1}{2}(G_0 + G_1)$, it follows that

$$\left. \begin{array}{l} PC\left(\overline{G}\right) > \dfrac{PC^* (G_0) + PC^* (G_1)}{2} \\[2ex] PC\left(\overline{G}\right) = \dfrac{PC^* (G_0) + PC^* (G_1)}{2} \\[2ex] PC\left(\overline{G}\right) < \dfrac{PC^* (G_0) + PC^* (G_1)}{2} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} n\left(\overline{G}\right) < \dfrac{n^* (G_0) + n^* (G_1)}{2} \\[2ex] n\left(\overline{G}\right) = \dfrac{n^* (G_0) + n^* (G_1)}{2} \\[2ex] n\left(\overline{G}\right) > \dfrac{n^* (G_0) + n^* (G_1)}{2} \end{array} \right.$$

establishing the claim. ■

Proof of Lemma 3: Simple differentiation yields

$$\frac{d}{dn}\left(\frac{n}{\phi(n)}\right)=\frac{\phi(n)-\phi'(n)\,n}{n^2}$$

By concavity we know that

$$\phi(n)>\phi(0)+\phi'(n)\,n$$

Since $\phi(0)=0$ by assumption, it follows that the derivative is positive. ■

Proof of Proposition 3: The problem of an entrepreneur engaging in innovation now becomes

$$\max_{n_i}\phi(n_i)\,v_i-n_i$$

subject to the constraint that $n_i\leq\overline{n}$. Using the Lagrange multiplier, we have

$$\begin{aligned}\phi'(n_i)\,v_i-1&\geq 0\\ \lambda_i(\overline{n}-n_i)&=0\end{aligned}$$

so that $n(G_1)=\min(n_1',\overline{n})$ where n_1' solves

$$\phi'(n_1')\,v_1=1$$

Suppose $n(G_1)\neq\overline{n}$. Then $n(G_1)<\overline{n}$, and $\lambda_1=0$. We can use the same argument as in Proposition 1 to establish that $n_0<n_1<\overline{n}$. Next,

$$\begin{aligned}v_1&=\frac{\omega(n_1)\,\pi_1+\mu\pi_0}{\omega(n_0)\,\omega(n_1)-\mu^2}\\
&=(\lambda-1)\frac{\omega(n_1)\left(L-n_1-\overline{N}+\varepsilon\right)+\mu\left(L-n_0-\overline{N}-\varepsilon\right)}{\omega(n_0)\,\omega(n_1)-\mu^2}\\
&=(\lambda-1)\frac{\omega(n_1)\left(L-n_1-\overline{N}\right)+\mu\left(L-n_0-\overline{N}\right)+(\omega(n_1)-\mu)\,\varepsilon}{\omega(n_0)\,\omega(n_1)-\mu^2}\\
&>(\lambda-1)\frac{\omega(n_1)\left(L-n_1-\overline{N}\right)+\mu\left(L-n_0-\overline{N}\right)}{\omega(n_0)\,\omega(n_1)-\mu^2}\\
&>(\lambda-1)\frac{\omega(n_1)\left(L-\overline{n}-\overline{N}\right)+\mu\left(L-\overline{n}-\overline{N}\right)}{\omega(n_0)\,\omega(n_1)-\mu^2}\\
&>(\lambda-1)\frac{\omega(\overline{n})\left(L-\overline{n}-\overline{N}\right)+\mu\left(L-\overline{n}-\overline{N}\right)}{\omega(\overline{n})\,\omega(\overline{n})-\mu^2}\equiv\overline{v}\end{aligned}$$

However, by construction, $\phi'(\overline{n})\,\overline{v}=1$, which implies that when $G=G_1$, entrepreneurs would wish to hire at least $\overline{n}$ workers, which is a contradiction. Hence, $n(G_1)=\overline{n}$.

Next, to show $n(G_0) < \overline{n}$, suppose not. Since $n(G_1) = \overline{n}$, we have

$$\begin{aligned} v_0 &= \frac{\omega(\overline{n})\pi_1 + \mu\pi_0}{\omega(\overline{n})\omega(\overline{n}) - \mu^2} \\ &= (\lambda - 1)\frac{\omega(\overline{n})\left(L - \overline{n} - \overline{N} - \widetilde{\varepsilon}\right) + \mu\left(L - \overline{n} - \overline{N} + \widetilde{\varepsilon}\right)}{\omega(\overline{n})\omega(\overline{n}) - \mu^2} \\ &= (\lambda - 1)\frac{(\omega(\overline{n}) + \mu)\left(L - \overline{n} - \overline{N}\right) - (\omega(\overline{n}) - \mu)\widetilde{\varepsilon}}{\omega(\overline{n})\omega(\overline{n}) - \mu^2} \\ &< \overline{v} \end{aligned}$$

Since $\phi'(\overline{n})\overline{v} = 1$, it follows that there exists an $n < \overline{n}$ for which $\phi'(n)v_0 = 1$, which contradicts the fact the constrained optimum is $n = \overline{n}$. The last claim follows trivially from the first two. ■

Proof of Proposition 4: The maximization problem can be expressed as

$$\max_a \left[\min\left(\phi(n_0)\ln\lambda + \frac{v_1}{v_0}a\frac{\Delta n}{2}, \phi(n_1)\ln\lambda - a\frac{\Delta n}{2}\right)\right]$$

At the maximum, the two expressions must be equal, since if they are not equal, it would always be possible to increase this expression either by increasing a or decreasing a, depending on which expression is larger. Solving for a at the point of equality and substituting back in yields the desired result. ■

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