Economics 280B Take-Home Final

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Due: Thursday, December 12, 2 pm, 695 Evans

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Instructions: Answer all questions. Working in groups is NOT permitted. Exam is open book.

1. (Trade costs and international risk sharing) Consider a version of the Lucas (1982) model in which we abstract from dynamics and consider a one-period portfolio problem. There are two countries, Home and Foreign, and individuals in the two countries trade Arrow-Debreu contracts over states of nature in the (one) period that follows. States of nature depend entirely on the realized values (Y_H, Y_F) of Home and Foreign (exogenous, stochastic) output. Assume a completely symmetric joint distribution for the national outputs (Y_H, Y_F).

A Home or Foreign individual chooses state-contingent consumptions C_H and C_F of the home and foreign goods in order to maximize

$$\mathrm{E}U = \mathrm{E}\left\{ \frac{1}{1-\rho} \left[\left(C_H^{\frac{\theta-1}{\theta}} + C_F^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{1-\rho} \right\} = \mathrm{E}\left\{ \frac{C^{1-\rho}}{1-\rho} \right\},$$

Above, C is the index of total real consumption, θ is consumers' elasticity of substitution between the two goods, and ρ is the coefficient of relative risk aversion.

There is free and costless international trade in the Arrow-Debreu securities. (Imagine that the securities' payoffs are made in a costlessly tradable international monetary unit of account.) However there are "iceberg" costs of trade, such that only a fraction $1-\tau$ of a unit of good shipped abroad reaches its destination. With competitive markets, goods prices would have to be $P_F = P_F^*/(1-\tau)$ and $P_H = (1-\tau)P_H^*$, where variables pertaining to the Foreign country are denoted by an asterisk.

(a) Derive (using the fact that the two countries are ex ante symmetric) the Backus-Smith conditions,

$$\frac{1}{P_H}\frac{\partial U}{\partial C_H} = \frac{1}{P_H^*}\frac{\partial U^*}{\partial C_H^*},$$

and

$$\frac{1}{P_F}\frac{\partial U}{\partial C_F} = \frac{1}{P_F^*}\frac{\partial U^*}{\partial C_F^*},$$

for every state of nature. (Without symmetry, these would be proportionality conditions rather than equalities.) Show that they imply

$$C_H^{-\frac{1}{\theta}} C^{\frac{1}{\theta}-\rho} = (1-\tau)\left(C_H^*\right)^{-\frac{1}{\theta}} C^{*\frac{1}{\theta}-\rho}, \tag{1}$$

$$(1-\tau) C_F^{-\frac{1}{\theta}} C^{\frac{1}{\theta}-\rho} = \left(C_F^*\right)^{-\frac{1}{\theta}} C^{*\frac{1}{\theta}-\rho}, \tag{2}$$

and the ex post consumption efficiency condition

$$\left(\frac{P_F}{P_H}\right)^{\theta} = \frac{C_H}{C_F} = (1-\tau)^{-2\theta}\frac{C_H^*}{C_F^*} = (1-\tau)^{-2\theta}\left(\frac{P_F^*}{P_H^*}\right)^{\theta}.$$

(b) Specialize to the case $\rho = 1/\theta$ [in which the Arrow-Debreu conditions (1) and (2) simplify considerably]. Suppose that instead of trading Arrow-Debreu contracts, agents can trade only shares in Home or Foreign outputs. Let x_H (x_F) denotes the Home (representative) agent's share of total equity in the Home (Foreign) industry, and x_H^* (x_F^*) denotes the Foreigner's optimal equity shares. Show that the Arrow-Debreu risk-sharing allocation can still be attained with this more restricted menu of assets. Also show that, given the assumption of symmetry of output distributions, the equilibrium portfolio shares are

$$x_H = \frac{1}{1+(1-\tau)^{\theta-1}} Y_H,$$

$$x_H^* = \frac{(1-\tau)^{\theta-1}}{1+(1-\tau)^{\theta-1}} Y_H,$$

$$x_F = \frac{(1-\tau)^{\theta-1}}{1+(1-\tau)^{\theta-1}} Y_F^*,$$

$$x_F^* = \frac{1}{1+(1-\tau)^{\theta-1}} Y_F^*.$$

2. (The Dornbusch model with an interest-rate rule) Most central banks operate by setting a nominal short-term interest rate rather than by setting the level of the money supply. These days, the interest-rate rule is designed so as to offset inflationary shocks. Accordingly, consider the following discrete-time, stochastic version of the Dornbusch model with rational expectations, where u_t is a conditional mean-zero random "monetary" shock to the domestic interest rate:

$$\text{interest-rate policy rule:} \quad i_t = i^* + \beta\left(\mathrm{E}_t\{p_{t+1} - p_t\}\right) + u_t, \quad \mathrm{E}_t\left\{u_{t+1}\right\} = 0$$

$$\text{aggregate output:} \quad y_t = \delta(e_t - p_t)$$

$$\text{price adjustment:} \quad p_{t+1} - p_t = \theta y_t$$

$$\text{uncovered interest rate parity:} \quad i_t = i^* + \mathrm{E}_t\{e_{t+1} - e_t\}.$$

(Think of the variables above as deviations from trends, and assume the foreign price level p^* is constant and normalized to equal zero.) Noticethat we do not need to write down the money-market equilibrium condition, since, given the price level and output, a central bank setting the interest rate has no choice but simply to supply the amount of money necessary in order that the money market clear at the desired rate (i.e., the money supply becomes endogenous). As usual, the price level is a predetermined (non-jumping) variable (and thus it is perfectly predictable, $\mathrm{E}_t\left\{p_{t+1}\right\} = p_{t+1}$).

(a) Express the model as a single difference equation describing the expected change in the real exchange rate $q = e - p$. Hint: Start by thinking about the date t expected values of the date $t+1$ equilibrium values.

(b) Solve that difference equation for q_t by successive forward substitutions. That is, your expression for $\mathrm{E}_t\{q_{t+1}\}$ derived in part (a), if pushed one period ahead, gives an equation for $\mathrm{E}_t\{q_{t+2}\}$ in terms of $\mathrm{E}_t\{q_{t+1}\}$ and $\mathrm{E}_t\{u_{t+1}\}$. Solve this second equation for $\mathrm{E}_t\{q_{t+1}\}$ and substitute the result into your expression from (a). Now repeat, using the expression for $\mathrm{E}_t\{q_{t+3}\}$ to eliminate $\mathrm{E}_t\{q_{t+2}\}$ from what you have just derived; and continue.

(c) On the assumption that $\beta > 1$, what is your solution for q_t? Intuitively, what does $\beta > 1$ mean in terms of the interest-rate policy rule above?

(d) Solve for the stochastic process driving the price level, p_t, in terms of the lagged price level and lagged interest-rate shock. Is the price level a stationary (finite-variance) random variable or not?

(e) Give the intuition behind your answer to (d) by explaining (in words) the expected adjustment path to a negative realization of the random variable u_t. (You may assume $\theta\delta < 1$.) Note that you need only worry about what happens in period t and what is expected to happen in period $t+1$.

3. (The trade balance and the intertemporal budget constraint) Suppose that a one-good small endowment economy with *negative* net foreign assets adopts a policy of running a trade balance surplus sufficient to repay a constant small fraction of the interest due on its debt each period. It continually rolls over the remaining interest by fresh borrowing. That is, the country sets its date s trade balance $TB_s = Y_s - C_s$ according to the rule,

$$TB_s = -\xi r B_s,$$

where r is the exogenously given (constant) world real interest rate and ξ is a small number less than 1 (and $B_s < 0$ for all s).

(a) Using the current account identity and the definition of the trade balance, show that under the preceding policy net foreign assets evolve according to the equation:

$$B_{s+1} = [1 + (1 - \xi)r] \, B_s.$$

(b) Show directly that the intertemporal budget constraint is satisfied for *any* $\xi > 0$, no matter how tiny. [Hint: Show that

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} TB_s = -\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \xi r B_s = -(1+r)B_t.]$$

(c) Does part (b) prove that a sustainable current account (i.e., national intertemporal solvency) requires only that an arbitrarily small fraction of the interest owed to foreigners be paid each period (with the remaining interest due rolled over into new foreign debt)? [Hint: Consider an economy with $G = I = 0$ and constant output Y. How big can the foreign debt $-B$ get before the country owes all of its future output to foreigners? Will this bound be violated if ξ isn't big enough? If so, how could the intertemporal budget constraint have held in part (b)?]