Economics 280B Take-Home Final

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Instructions: Answer all questions. Working in groups is NOT permitted. Exam is open book.

1. (Partially complete asset markets) Consider a two-period, two-country model with exogenous output. There is uncertainty over the second-period levels of output *and* of (exogenous) government spending: for each country there are two possible levels of output on date 2 and two possible levels of government spending, and there are four distinct possible date 2 states of nature in all indexed by $s = 1, 2, 3, 4$.

We let y (y^*), c (c^*), and g (g^*) denote Home (Foreign) levels of output, consumption, and government spending on date 1. On date 2, the possible combinations of Home output and government spending levels are:

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
Date 2 output	y_1	y_2	y_1	y_2
Date 2 govt. spending	g_1	g_1	g_2	g_2

with the corresponding "starred" levels for Foreign (e.g., in state $s = 2$ Foreign output is y_2^* and its government spends g_1^*).

The probability that state s occurs is $\pi(s)$.

The key assumption we will make is that people can trade only two state-contingent securities: a security that pays 1 output unit if state 1 *or* state 3 occurs (and 0 otherwise), and a security that pays 1 output unit in state 2 or state 4 (and 0 otherwise). Call these the *odd* and *even* securities, respectively, and denote their prices in terms of sure (i.e., certain) date 2 output by $p(o)$ and $p(e)$. Looking at the table, you can see that the assumed asset structure makes output shocks insurable, but government spending shocks uninsurable.

Let $b(o)$ and $b(e)$ be a Home individual's purchase of odd and even securities on date 1. Clearly, if r is the world risk-free real interest rate, then because total asset purchases equal savings,

$$\frac{p(o)}{1+r}b(o) + \frac{p(e)}{1+r}b(e) = y - g - c. \tag{1}$$

(a) Given $b(o)$ and $b(e)$, write down the constraints determining the date 2 contingent consumption levels $c(s)$ for $s \in \{1, 2, 3, 4\}$.

(b) Individuals maximize expected lifetime utility,

$$u(c)+\beta\sum_{s=1}^{4}\pi(s)u[c(s)].$$

Using (1) and your answers to (a), which are the constraints for the problem, derive first-order conditions for maximizing expected utility.

(c) Show how to derive Euler equations for the two securities; for example, that for the odd security is

$$\frac{p(o)}{1+r}u'(c)=\beta\left\{\pi(1)u'[c(1)]+\pi(3)u'[c(3)]\right\}.$$

Interpret these Euler equations. How do we know that $c\,(1)\neq c(3)$ and $c(2)\neq c(4)$?

(d) Under free international trade in output and assets, show that the following two equalities hold:

$$\pi(1)\left\{\frac{\beta u'[c(1)]}{u'(c)}-\frac{\beta u'[c^*(1)]}{u'(c^*)}\right\}+\pi(3)\left\{\frac{\beta u'[c(3)]}{u'(c)}-\frac{\beta u'[c^*(3)]}{u'(c^*)}\right\}=0, \tag{2}$$

$$\pi(2)\left\{\frac{\beta u'[c(2)]}{u'(c)}-\frac{\beta u'[c^*(2)]}{u'(c^*)}\right\}+\pi(4)\left\{\frac{\beta u'[c(4)]}{u'(c)}-\frac{\beta u'[c^*(4)]}{u'(c^*)}\right\}=0. \tag{3}$$

What set of four stronger equalities would hold under complete asset markets?

(e) Show that (2) and (3) imply that expected intertemporal marginal rates of substitution are equalized:

$$\mathrm{E}\left\{\frac{\beta u'[c(s)]}{u'(c)}-\frac{\beta u'[c^*(s)]}{u'(c^*)}\right\}=0.$$

Conclude from this and from (2) and (3) that

$$\begin{aligned}\mathrm{Cov}\left\{\frac{\beta u'[c(s)]}{u'(c)}-\frac{\beta u'[c^*(s)]}{u'(c^*)},y(s)\right\} &= \mathrm{Cov}\left\{\frac{\beta u'[c(s)]}{u'(c)}-\frac{\beta u'[c^*(s)]}{u'(c^*)},y^*(s)\right\}\\ &= 0, \end{aligned}\tag{4}$$

where $y(s)$ is the value of output on date 2 in state s. Can you *interpret* this condition?

(f) Let V be the date 1 price of a claim to Home's date 2 output. (Think of this as an equity share in Home product.) Explain why, for a Home consumer,

$$Vu'(c)=\beta\mathrm{E}\left\{u'[c(s)]\,y(s)\right\}.$$

Given that this Euler equation holds, show that V can be written in terms of: (i) the expected "dividend" $\mathrm{E}\{y(s)\}$, (ii) the risk-free real interest rate, r, and

(iii) the covariance between a Home resident's intertemporal marginal rate of substitution and date 2 Home output $y(s)$. (Hint: You'll need to use the Euler equation for risk-free bonds.) Let $\tilde{V}$ be the corresponding price for the same asset (i.e., a claim to Home output), calculated by a Foreign resident. Give the expression for $\tilde{V}$ corresponding to the one for V.

(g) Using (4), show that $V = \tilde{V}$. This proves that fully complete markets are not necessary for heterogeneous agents to agree on asset values. Here, output-contingent securities suffice.

2. (Stabilizing properties of floating exchange rates) Consider the following discrete-time, stochastic version of the Dornbusch model with rational expectations, where u_t and v_t are mean-zero random "real" and "monetary" shocks, respectively, and w_t is the pre-set (set one period in advance) nominal domestic wage:

$$\text{aggregate demand:} \quad y_t^d = \delta(e_t + p^* - p_t) + u_t$$

$$\text{aggregate supply:} \quad y_t^s = \alpha(p_t - w_t)$$

$$\text{money-market equilibrium:} \quad m_t - p_t = y_t - \lambda i_t + v_t$$

$$\text{uncovered interest rate parity:} \quad i_t = i^* + \mathrm{E}_t(e_{t+1} - e_t)$$

$$\text{sticky wage determination:} \quad w_t = \mathrm{E}_{t-1} p_t.$$

This last equation means that wages for date t are set on date $t-1$ so as to maintain constant purchasing power on date t; that is, we have "one-period" wage stickiness rather than the differential equation for price adjustment that characterizes the Dornbusch model in its "classic" form. (Think of the variables above as deviations from trends.)

(a) Assume that u_t and v_t are white noise processes, that is, $\mathrm{E}_t\{u_{t+1}\} = \mathrm{E}_t\{v_{t+1}\} = 0$. Suppose that we have a floating exchange rate and that m_t is constant over time at $m_t = m$. (Variables without time subscripts are constants.) Solve for the equilibrium values of p_t, y_t, i_t, and e_t. (Of course, you should impose $y^d = y^s = y$ in order to solve.) Hint: Start by thinking about the date t expected values of the date $t+1$ equilibrium values.

(b) Assuming $\mathrm{Cov}(u, v) = 0$ (for simplicity), calculate the variance of y in terms of the underlying shock variances $\mathrm{Var}(u)$ and $\mathrm{Var}(v)$.

(c) Now assume the exchange rate is *fixed*, so that $e_t = e_{t+1} = e$ but m_t becomes an endogenous variable due to intervention in support of the exchange rate. Solve the endogenous variables of the model.

(d) Continuing with the hypothesis of a fixed exchange rate, and still assuming $\mathrm{Cov}(u, v) = 0$, calculate the variance of y in terms of the underlying shock variances $\mathrm{Var}(u)$ and $\mathrm{Var}(v)$.

(e) Can you say anything about which exchange-rate regime, floating or fixed, is preferable from the standpoint of minimizing output variance?

3. (Equilibrium interest rates) A representative individual maximizes

$$\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} u(C_s) \right\}$$

subject to budget constraints.

(a) Explain intuitively why the relationship linking the real date t interest rate r_t to the domestic nominal date t interest rate i_t (both are interest rates between dates t and $t+1$) is

$$1 + r_t = \frac{P_t(1+i_t)\mathrm{E}_t\left\{u'(C_{t+1})/P_{t+1}\right\}}{\mathrm{E}_t\left\{u'(C_{t+1})\right\}}.$$

(b) Let P_t^* be the Foreign price level and $\mathcal{E}_t$ the price of Foreign currency in terms of Home currency. How would you write the arbitrage relationship, corresponding to the one in part (a), between r_t and i_t^*, the nominal interest rate on Foreign-currency loans? (Again, do this from the perspective of a Home agent.)

(c) What is the implied relationship between $1 + i_t$ and $1 + i_t^*$?

(d) Let $u'(C) = C^{-\rho}$. Assuming purchasing power parity, and that all endogenous random variables have lognormal distributions, what is the relationship between $\log(1 + i_t)$ and $\log(1 + i_t^*)$? Explain the resulting "deviation from uncovered interest parity" intuitively. (Use the standard notation $c = \log C$, $e = \log \mathcal{E}$, etc.) (We may assume that, because of complete markets or some other mechanism, $c = c^*$, that is, consumption is equalized for Home and Foreign.)