

# Reputations and Sovereign Debt

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## Abstract

Why do countries repay their debts? If countries in default have sufficient opportunities to save, Bulow and Rogoff [6] have shown that the answer *cannot* stem from a country's desire to preserve a reputation for repayment. As a result, researchers have explained the existence of sovereign debt by either placing restrictions on the deposit contracts banks can offer, or by looking outside the credit market for alternative means of enforcement, including the imposition of trade embargoes, or spillovers to other reputational relationships of the country. In contrast, in this paper we demonstrate that a country's concern for its reputation can work to enforce repayment without placing *any* technological restrictions on the ability of banks to offer contracts, and without appealing to *any* mechanisms outside of the credit market itself, as long as there are incentives for banks to tacitly collude in punishing a country in default. Such incentives exist as long as the number of banks is not too large, even if banks make zero profits.

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# 1 Introduction

In the event of a sovereign default, investors typically have limited access to legal remedies<sup>1</sup>. This begs the question: Why do countries ever repay their debts? A common answer, first articulated by Eaton and Gersovitz [12], is that countries repay in order to preserve a reputation for repayment, and so retain the ability to borrow in the future<sup>2</sup>. However, in a provocative paper, Bulow and Rogoff [6] have shown that a sufficient condition for reputations to fail to enforce repayment is that countries in default have access to a rich set of deposit contracts. Intuitively, when expected future repayments are large, a country can default, invest the resources saved in deposit contracts, and enjoy a higher level of consumption thereafter.

In order to explain the existence of sovereign debt, researchers have explored a number of alternative mechanisms for enforcing repayment. One approach revives the notion of credit market reputations by placing restrictions on the ability of banks to offer deposit contracts (for example, Kletzer and B. Wright [20]). A second looks beyond the credit market for alternative means of enforcement based on the imposition of direct sanctions, ranging all the way from military intervention to trade embargoes (for example, Bulow and Rogoff [5] and Fernandez and Rosenthal [14]). A third invokes the possibility that other relationships of the borrowing country, outside of the credit market, will deteriorate as a result of a default in credit markets (Cole and Kehoe [7], [8], and [9]). Clearly, each of these mechanisms has the ability to enforce repayment in principle. And direct sanctions, such as military intervention to force repayment, have been used in the past. However, it is not evident that these mechanisms can explain the (small, but not insignificant) level of emerging market sovereign lending that is observed today.

The aim of this paper is to re-examine the question of whether there is a role for credit market reputations in the enforcement of sovereign lending. In particular, it demonstrates that credit market reputations can work to enforce repayment without placing *any* technological restrictions on the ability of banks to offer contracts, and without appealing to *any* mechanisms outside of the credit market itself. Instead, the approach of this paper is to focus upon the incentives for banks to tacitly collude in punishing a country in default. To see why this can work to support repayment, note that for a bank to voluntarily lend to a country, it is necessary that they expect repayment in at least one future state of the world.

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<sup>1</sup>Since the 1970's, the extent of sovereign immunity has diminished as a result of legislation such as the US Foreign Sovereign Immunity Act of 1976 and in the UK, the State Immunity Act of 1978. However, attempts to attach sovereign assets against such claims have met with limited success. Appendix A illustrates the difficulties with a case study.

<sup>2</sup>The approach has attracted a large number of advocates, a non-exhaustive sample of which includes Atkeson [2], Grossman and van Huyck [15], Manuelli [27], and Worrall [34]. Similar models have also been applied to questions of intra-national lending by Alvarez and Jermann [1], Kocherlakota [21], Kehoe and Levine [19], and Krueger [24] amongst others.

If loan contracts are enforceable by law, these payments will always be made. However, when loan contract are not enforceable by law, lenders must have some ability to prevent other creditors from competing for these profits *ex post*. The approaches noted above all describe ways in which competition can be restricted: models of direct sanctions and general reputations grant creditors local monopoly power by limiting the ability of a country to switch creditors; models with restrictions on deposit contracts allow banks to preserve their rents through credible threats to disrupt any other lending relationship that might be formed. In this paper, we take the more direct approach of adopting a richer specification of the credit market in which the potential for tacit collusion can be analyzed.

Specifically, the paper begins by outlining a version of the widely used complete information model of sovereign lending, first introduced by Eaton and Gersovitz [12], in which a risk averse country trades intertemporally with a risk neutral bank in order to both smooth, and shift intertemporally, it's consumption profile. Although the country cannot commit to honoring it's contracts, it is assumed that the bank can commit, so that deposit contracts of the Bulow-Rogoff type are feasible. In such a model, with only one creditor, lending to the country can be supported by the threat of exclusion from future intertemporal trade. Importantly, the existence of lending is independent of the distribution of surplus between the country and the bank: competition *ex ante* has no effect on the ability to support debt.

The model is then modified by the introduction of another identical bank, and an analogue to the Bulow and Rogoff result is derived: no borrowing can be supported in equilibrium. More precisely, it is shown that if the country is restricted to deal only with one bank, the presence of another bank is sufficient to undermine lending. In the light of the above discussion, this result can be interpreted as stating that there are no mechanisms by which tacit collusion between creditors can be supported in the canonical model. In particular, no collusion can be supported *even with as few as two banks*. This should not be surprising: under the restriction that the country can only trade with one bank, the surplus derived from the banking relationship cannot be used to coordinate banks, in the absence of some system of side payments. We go on to show that, if this assumption is relaxed, a consolidated loan that supports the constrained efficient level of lending can be formed as long as the number of banks is not too large. Interestingly, it is now a bank's own reputation that is important: banks cooperate in lending to the country, and in punishing a country in default, as long as each has maintained a reputation for cooperation in the past.

Before continuing, it is useful to stress five points. First, the idea that some form of monopoly power in the credit market could explain reputation mechanisms is not new: in their original paper, Bulow and Rogoff [6] conjecture that the existence of small costs of establishing relationships would be sufficient to overturn their non-existence result<sup>3</sup>. Further,

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<sup>3</sup>Recently, Krueger and Uhlig [23] have provided a formal justification of this claim in a model with a continuum of competing banks. Although their model is not explicitly couched in terms of sovereign borrowing, the authors motivate an application of the model to sovereign borrowing by interpreting the costs as arising from some form of trade embargo, like in the direct sanctions models listed above.

as noted above, all of the other suggested resolutions to the problem of sovereign debt work, in effect, by postulating some mechanism through which competition can be deterred. But in these examples, monopoly power arises from the inability of new lenders to replicate the terms of the old relationship. Where the model of this paper differs is it they come at the competition issue directly: banks are able to replicate the activity of other banks, but chose not to do so. Moreover, in this paper, tacit collusion can be supported without the existence of monopoly rents.

Second, the mechanism we describe must do more than simply explain why Bulow-Rogoff deposit contracts are not available. In the simplest version of our model, these contracts are *sufficient* to disrupt the reputation mechanism. As a result, establishing their non-existence is merely *necessary* to support lending. To prove the existence of equilibria with lending, it is necessary to show that there are no contracts *of any type* that a creditor will choose to offer a country in default which would undermine the punishment of that country. The creditor reputation mechanisms we describe ensure that this is the case. Similar issues arise in the papers cited above. For example, in contrast to the model of this paper, Kletzer and B. Wright [20] rule out the Bulow-Rogoff deposit contracts by the assumption that banks cannot commit to honoring them. However, it is also a creditor reputation mechanism, although of a very different (and much more devious) sort to the one in this paper, that establishes that creditors will not compete in any other form of contract.

Third, it is important to emphasize that we restore credit market reputations as an enforcement mechanism in a way that leaves the deposit contracts of Bulow-Rogoff *feasible*, both in terms of available resources and aggregate incentives. Concern for ones reputation induces banks not to offer these contracts to countries in default because they are *unprofitable*. Fourth, our mechanisms are able to rationalize the existence of deposit contracts in equilibrium; lending is enforced in part because they will not be offered out of equilibrium to countries in default. Fifth, the effect of allowing commitment by banks has markedly differing effects depending upon the number of banks. Specifically, allowing banks to commit increases the ability of the parties to exploit potential gains from trade, and in the bilateral lending case always weakly increases the value of the value of trade between the parties. This remains true as long as the number of banks is small. However, because the ability to commit also expands the range of possible defections from the contract, allowing commitment when the number of banks is large can strictly reduce the amount of trade in equilibrium.

The rest of this paper is organized as follows. Section 2 outlines the environment of the model in which a risk averse country borrows from one or more international banks to smooth fluctuations in it's exogenously given endowment. It is assumed that banks are risk neutral and have access to a commitment technology that allows them to credibly commit to honoring any contracts that they sign, so that banks can offer Bulow-Rogoff contracts. Special mention is made of a particular deterministic example, which is used to illustrate the results of the various sections below. Section 3 begins by analyzing the optimal

contract between one bank and the country, which serves also to establish the constrained efficient level of borrowing in the model. Section 4 introduces the presence of multiple banks in this framework and establishes the extent to which competition undermines reputation mechanisms: one extra bank is sufficient to undermine lending to the country, because there are no incentives for tacit collusion. Sections 5 then modifies the canonical model to allow for syndicated lending and shows that as long as the potential gains from trade here are sufficiently positive, lending to the developing country can be supported in equilibrium. Section 6 deals with some extensions to and variations on the above framework while Section 7 concludes with a discussion of some of the implications of the model, and a technical appendix collects proofs of all results in the text.

## 2 Model Environment

Our environment is a multiple bank version of the complete information model first introduced by Eaton and Gersovitz [12]. The environment has two elements. The first is a risk averse developing country. Each period, the country receives an exogenous endowment of the single non-storable consumption good. Time is discrete, and the only uncertainty surrounds the level of the country's endowment, which in each period  $t = 0, 1, 2, \dots$ , is denoted by  $\theta_t$ . We assume that for all  $t$  the *state*  $\theta_t$  belongs to the finite set  $\Theta$ , with  $\theta_1 \equiv \min \Theta > 0$ . Information about states forms a Markov chain, and the transition probability from  $\theta$  to  $\theta'$  is given by  $\pi(\theta'|\theta)$  with the initial state  $\theta_0$  given. We let  $\theta^t \equiv (\theta_0, \theta_1, \dots, \theta_t) \in \Theta^t$  denote a history of the country's endowment levels up to date  $t$ . The notation  $\theta^s | \theta^t$  for  $s > t$  refers to a history  $\theta^s$  that *continues*  $\theta^t$  in the sense that  $\theta^s = (\theta^t, \theta_{t+1}, \theta_{t+2}, \dots, \theta_s)$ . The probability of observing history  $\theta^t$  is denoted by  $\pi(\theta^t)$ , and that of observing history  $\theta^s$  conditional on having been in  $\theta^t$  is  $\pi(\theta^s|\theta^t)$ .

We assume that the country orders preferences over state contingent consumption streams  $\{c(\theta^t)\}$  by a time and state additively separable function

$$(1 - \beta) \sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U(c(\theta^t)).$$

Here  $\beta \in (0, 1)$  is the discount factor of the country, while the function  $U$  is the period utility, or felicity, function which is assumed to be strictly increasing and strictly concave.

The second element of the model is a collection of  $M$  risk neutral international banks. Each bank is assumed to maximize the discounted sum of expected future profits

$$(1 - q) \sum_{t=0}^{\infty} q^t \sum_{\theta^t} \pi(\theta^t) T(\theta^t).$$

Here,  $q \in [\beta, 1)$  is the intertemporal price of consumption, while  $T(\theta^t)$  is the profit earned by a banker (or transfer to the banker) in history  $\theta^t$ ; aggregating across bankers, the sequence of total profits is given by the difference between the country's endowment and its consumption, or  $\theta_t - c(\theta^t)$ .

In the model below, we will allow the bank and country to make transfers to each other that are fully contingent on the state of the world. We will interpret these transfers as being part of an implicit contract between the country and the banks. This may co-exist with an explicit contract that is, itself, renegotiated as differing states of the world arise. The ability to offer such state contingent contracts is what distinguishes a bank from anonymous capital markets in general. That is, we are assuming that markets for fully state contingent bonds cannot be established. This may be motivated as arising due to a lack of liquidity. Such an ability to offer state contingent contracts also requires that banks be sufficiently large so as to cover all contingencies, and motivates our assumption of a finite number of banks. That is, entry is restricted.

Note that we are allowing the intermediary to borrow and lend as much as it wants at a price  $q$ , and consequently refer to the model as one of partial equilibrium. However, it should be immediate that these assumptions are consistent with a general equilibrium model in which the banker receives a sufficiently large endowment each period (as long as transfers to the bank remain bounded below) and has discount factor  $q$ , or one in which the banker has an endowment and access to a linear technology parameterized by  $q$ . Importantly, we allow the international interest rate  $R = 1/q$  to be lower than the country's discount rate; that is, we allow  $q$  to be greater than  $\beta$ .

With these preferences, and under this assumption, there are gains to be made from trading the risk in the country's endowment, and in tilting the country's consumption profile. Both will be necessary to generate borrowing (as opposed to saving) in equilibrium in our model. Without the motive implied by risk sharing, borrowing would only occur if  $q > \beta$ , in which case the country would adopt a smoothly declining consumption profile. Such a profile has the implication that, after some finite amount of time  $T$ , the country no longer receives transfers from abroad and only repays its debts. A country that can default will always do so at this point, as continued access to international markets has no other benefit like consumption smoothing, which upon iterating backwards in time implies there can be no lending in equilibrium. A similar argument shows that the infinite horizon assumption cannot be dispensed with, at least in this complete information environment. Without the motive implied by a low interest rate, incomplete risk sharing will in general lead the country to accumulate assets to self insure, leading to a model of international saving by the country, and not a model of borrowing.

As we will see below, the amount of trade that actually occurs will depend on the existence and form of the institutions governing enforcement and coordination of agents. If no trade

occurs we will say that the economy is in *autarky*, and denote by

$$V^A(\theta_0) = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U(\theta^t),$$

the autarkic utility of the country, which is state dependent. At the opposite extreme, if there are no enforcement problems and information is perfect, a *first-best* allocation is attainable. If  $\beta = q$ , this involves providing the country with a constant consumption level  $c_{FB}$  whose level depends upon the exact distribution of surplus between the parties. For general  $q > \beta$ , this will involve a declining consumption profile over time. We denote by  $V^{FB}(Z; \phi)$  the value to the country of the first best allocation given that they begin with resources  $Z$  and receive the fraction  $\phi$  of the surplus.

In the sections below we will consider an intermediate regime in which bankers are able to commit to honoring one period ahead contracts, but countries are unable to commit. As a result, a contract between a banker and a country will need to be formulated in such a way so as to be self enforcing.

## 2.1 A helpful example

Before continuing, it is useful to record the properties of a simplified version of the above framework, which we will draw upon in making some points below. Specifically, consider a specialization of the above model to logarithmic preferences, and to an endowment process that cycles deterministically between two values  $y_H > y_L$ , where  $\pi(y_H|y_L) = \pi(y_L|y_H) = 1$ . The deterministic cycle serves to drastically simplify the problem, at the expense of removing the advantage owned by banks in the provision of state contingent loan contracts: in this simple example, access to a non-contingent bond suffices as a savings instrument.

Bearing these caveats in mind, the simplicity of the example has pedagogical merit. For now, we simply record that the value the country would receive in autarky is denoted by  $V_t^A$ , or

$$V_t^A = \begin{cases} \frac{1}{1+\beta} \log y_H + \frac{\beta}{1+\beta} \log y_L & \text{if } y_t = y_H \\ \frac{1}{1+\beta} \log y_L + \frac{\beta}{1+\beta} \log y_H & \text{if } y_t = y_L \end{cases}.$$

## 3 Bilateral Lending

Before turning to a discussion of the coordination of bankers in sustaining punishments against creditors in default, it is useful to record the nature of an optimal contract between

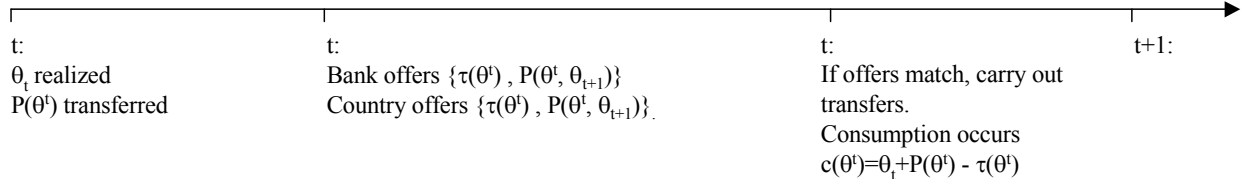


Figure 1: Within Period Timing in Bilateral Borrowing Problem

the country and one banker alone. In addition to establishing notation, this contract also defines the constrained efficient level of intertemporal trade in this economy; adding extra banks does not change the opportunities for risk sharing.

It is assumed that countries are unable to commit to honoring their contracts. A crucial question in understanding the nature of sovereign lending is the extent to which bankers are able to commit to honoring their contracts. Some authors, such as Eaton and Gersovitz [12] and Worrall [34], assume that bankers are able to fully commit to honoring an infinite sequence of state and history dependent contracts. Others, such as Kletzer and B. Wright [20], and M. Wright [35], assume that bankers are unable to commit at all to honoring their contracts. For expositional purposes, we follow authors such as Bulow and Rogoff [6] and Cole and Kehoe [8] in assuming that banks can commit to honoring one-period ahead contracts; we demonstrate below that this formulation is equivalent to one in which agents can commit to an infinite sequence of contracts.

The interaction between the country and the banker is modeled as a dynamic game, the timing of which is indicated in Figure 1. In this *bilateral borrowing game*, at each date  $t$  and after each realization of the endowment  $\theta_t$ , any payments  $P(\theta^t)$  previously committed to by the bank are carried out. A key element is the commitment ability of the bank. We assume that at any point in the period, the bank has the ability to commit itself to making a payment to the country at the start of the following period. Further, we allow the bank to condition these transfers on the size of the endowment at the time the transfers are to be made. In the game, this is modeled by allowing the bank to make a binding promise to make non-negative payments in period  $t+1$ ,  $P(\theta^t, \theta_{t+1})$ , which can be conditioned upon the state  $\theta_{t+1}$ . Note that the bank cannot, by its action at this point, bind itself not to make a transfer next period: the contemporaneous transfer to the bank  $\tau(\theta^t)$  can be negative in each period.

As the country is assumed to be relatively impatient, there is no loss in restricting  $P(\theta^t) \leq \bar{P}$  for some  $\bar{P}$  large, which serves to ensure that the value of the program is bounded. We add two other technical assumptions. First, it is assumed that there exists a  $C$  such that

$$(1 - \beta) U(C) + \beta V^{FB}(\theta_N + \bar{P}; 1) = V^A(\theta_N).$$



This is sufficient to guarantee that in every state of the world there exists a consumption level sufficiently low so that the country can be given the autarkic lifetime level of utility even if the continuation of the contract provides the maximum possible utility; it is automatically satisfied if period utility is unbounded below<sup>4</sup>. Second, note that if the world interest rate is very low, or equivalently if  $q$  is very high, there will never be trade in this economy. Specifically (and numbering  $\Theta$  in increasing order) if in the best state of the world the country does not want to save, or  $U'(\theta_N)q \geq \beta U'(\theta_1)$ , there can never be any intertemporal trade. Consequently, and so as to avoid complications as we vary the interest rate, we assume that  $\Theta$  and  $\beta$  are such that  $U'(\theta_N) < \beta U'(\theta_1)$ . This condition is necessary for long run gains from trade to be exploited, and we provide sufficient conditions that depend upon the nature of the stochastic process for the endowment, below.

As indicated in the introduction, the ability to sustain borrowing depends upon the size and distribution of the gains from trade in the contract. It will emerge below that it is the *ex post* distribution of this surplus, and not the *ex ante* distribution, that is crucial. An important consideration is the timing by which the transfers are announced and made. To begin with, we specify the timing in such a way that the distribution of surplus between the parties is left indeterminate. Specifically, at the beginning of each period, it is assumed that both the bank and country simultaneously announce  $N + 1$  numbers, corresponding to contemporaneous transfer between the bank and country  $\tau(\theta^t)$  (which may be positive or negative) and promised payments by the bank conditioned upon tomorrow's state  $P(\theta^t, \theta_{t+1})$  (which must be non-negative). If these announced "offers" match, contemporaneous transfers take place, and banks are bound to deliver their promised transfers next period. Then consumption takes place. Obviously, a country's total transfers cannot be greater than the total amount of goods available. Information is complete, so that transfers can be conditioned upon current and past realizations of the endowment as well as past payments and transfers by all agents.

Note that we are allowing for a great deal of state contingency, both in the form of the deposits, and in the form of lending and repayments. The former might as easily be thought of as an insurance contract, as a deposit contract. We motivate the degree of state contingency in lending contracts, following Grossman and van Huyck [15], in terms of excusable default. The idea is that lending contracts are implicit, and that while a formal contract might specify a set of repayment terms, banks and countries expect to renegotiate them at a later date. According to this interpretation, what is observed empirically as default and rescheduling is really just the state contingencies that we are allowing for explicitly above. This is to be contrasted with inexcusable default, in which a country does not honor the implicit contract, and which will not occur in the equilibrium of our model.

Before completing the specification of the game, it is useful to record the properties of a

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<sup>4</sup>This assumption suffices to deliver an analogue of the subjectivity assumption of Rey and Salanie [30]. See that paper for a discussion of alternative approaches to this issue.

particular feasible strategy that will be important below. As the bank is able to commit, one feasible contract is for the bank to offer deposit contracts in which in each period the country makes a non-negative deposit of size  $d(\theta^t)$  and in return the bank commits to making a series of non-negative payments  $P(\theta^t, \theta_{t+1})$  next period conditioned on each state of the world. If we assume that the bank always makes zero profits on these contracts, then these deposit contracts are of the *cash-in-advance* form supposed by Bulow and Rogoff [6]. Clearly, the country never has an incentive to default on such a contract. Further, as long as these payments satisfy

$$d(\theta^t) = q \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta^t) P(\theta^t, \theta_{t+1}),$$

the bank makes zero profits and so has an incentive to offer them *ex ante*; the fact that the bank can commit means that they do not renege on the agreement *ex post*.

Suppose that at some state date  $\theta^t$ , the country has a level of resources  $z(\theta^t)$ , made up possibly of current endowment plus any payments by the bank that were previously committed. Then from the perspective of the country, the *optimal deposit contract* solves the following programming problem: choose sequences  $\{d(\theta^s), P(\theta^s)\}$ , to maximize

$$(1 - \beta) \sum_{s=t}^{\infty} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U(z(\theta^s) - d(\theta^s)),$$

subject to

$$d(\theta^s) = q \sum_{\theta_{s+1}} \pi(\theta_{s+1}|\theta^s) P(\theta^s, \theta_{s+1}),$$

and

$$z(\theta^s) = \theta_s + P(\theta^{s-1}, \theta_s),$$

with  $d(\theta^s)$  and  $P(\theta^s, \theta_{s+1})$  non-negative, for all  $s \geq t$ , all  $\theta^s$  continuing  $\theta^t$ , with  $z(\theta^t)$  given. It is straightforward to show that this problem has a recursive representation, and that its solution can be represented by two functions,  $d(\theta, z)$  and  $P(\theta'; \theta, z)$ , which specify the profile of deposits and payments to be made at a time when the country had resources  $z$ , the current state  $\theta$ , and payments were to be conditioned on the future state  $\theta'$ . Let  $V^D(\theta, z)$  denote the value of this deposit contract to the country. The appendix shows that this function is increasing in the interest rate  $R = 1/q$ .

Although feasible, such contracts need not be optimal. To understand what contracts are optimal, we need to complete the specification of the game. A *history* of the game is made up of a sequence of past endowment realizations  $\theta^t$ , and a sequence of past payments and contemporaneous transfers up until  $t$ , denoted by  $(P^t, \tau^t)$ , while a *strategy* specifies transfers (and payments) conditional upon each possible history. A pair of strategies induces a *consumption allocation*  $c^t$  according to

$$c(\theta^t) = \theta_t + P(\theta^t) - \tau(\theta^t). \tag{1}$$

We initially focus upon subgame perfect equilibria of this game. The *autarkic strategy profile*, in which no offers are made after any history, is one subgame perfect equilibrium; the appendix shows that the autarkic strategies also deliver the lowest possible lifetime utility.

The following proposition partially characterizes the set of allocations that can be attained by subgame perfect equilibrium strategies.

**Proposition 1** *A consumption allocation  $\{c(\theta^t)\}$  can be attained in a subgame perfect equilibrium if and only if there exists a sequence of committable payments  $\{P(\theta^t)\}$  that satisfies the sequence of continuing participation constraints of the country*

$$\begin{aligned} & (1 - \beta) \sum_{s=t}^{\infty} \beta^{s-t} \sum_{\theta^s | \theta^t} \pi(\theta^s | \theta^t) U(c(\theta^s)) \\ & \geq (1 - \beta) U(\theta_t + P(\theta^t)) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1} | \theta_t) V^A(\theta_{t+1}), \end{aligned} \quad (2)$$

and the sequence of non-negative profit (after committed payments) conditions of the bank

$$(1 - q) \sum_{s=t}^{\infty} q^{s-t} \sum_{\theta^s | \theta^t} \pi(\theta^s | \theta^t) [\theta_s - c(\theta^s)] \geq -(1 - q) P(\theta^t), \quad (3)$$

for all  $t$  and all  $\theta^t$ .

**Proof.** See appendix. ■

We will typically be interested in the Pareto frontier of subgame perfect equilibrium values. The virtue of Proposition 1 is that we can characterize this frontier by first solving a programming problem in which a consumption allocation and committable payments are chosen to maximize country utility subject to (2) and (3) with  $P(\theta_0)$  given, and a constraint on the banks time-zero profits

$$(1 - q) \sum_{t=0}^{\infty} q^t \sum_{\theta^t} \pi(\theta^t) [\theta_t - c(\theta^t)] \geq \Pi,$$

for some  $\Pi$  which serves to parameterize the allocation of surplus between the parties. We will refer to this as the *bilateral borrowing programming problem*. Note that the committable payments  $P(\theta^t)$  only serve the purpose of loosening the banks non-negative profit constraints while tightening the country's continuing participation constraints. This suggests that we

can solve the problem ignoring the choice of  $P(\theta^t)$  and dropping all but the time-zero zero-profit constraint of the bank. Specifically, consider the following *relaxed programming problem*: choose the sequence  $c(\theta^t)$  to maximize country welfare

$$(1 - \beta) \sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U(c(\theta^t)),$$

subject to the continuing participation constraints of the country

$$(1 - \beta) \sum_{s=t}^{\infty} \sum_{\theta^s | \theta^t} \beta^{s-t} \pi(\theta^s | \theta^t) U(c(\theta^s)) \geq V^A(\theta^t),$$

for all  $t$  and all  $\theta^t$ , and a time zero profit constraint for the bank

$$(1 - q) \sum_{t=0}^{\infty} q^t \sum_{\theta^t} \pi(\theta^t) [\theta_t - c(\theta^t)] \geq \Pi.$$

The following proposition verifies that a solution to the relaxed problem also solves the original problem.

**Proposition 2** *A consumption allocation  $\{c(\theta^t)\}$  solves the relaxed programming problem, if and only if there exists a profile of committed payments  $\{P(\theta^t)\}$  such that  $\{c(\theta^t), P(\theta^t)\}$  solves the bilateral borrowing programming problem.*

**Proof.** See appendix. ■

Solutions to the relaxed programming problem correspond to points on the Pareto frontier of subgame perfect equilibrium values of this game. We will invoke considerations of renegotiation proofness below in order to justify focusing attention on equilibria on the Pareto frontier. The virtue of adopting the particular timing protocol discussed above is that if we can establish results for all such equilibria, we will know that they are independent of assumptions on the initial distribution of surplus. The following Proposition reassures us that identical allocations result from a game in which the bank made offers to the country, that the country may either accept or reject. As the bank enjoys a first mover advantage, the bank will appropriated all of the surplus.

**Proposition 3** *A consumption allocation  $\{c(\theta^t)\}$  solves the relaxed programming problem, if and only if there exists an initial stock of assets  $P(\theta_0)$  and a profile of committed payments  $\{P(\theta^t)\}$  such that  $\{c(\theta^t), P(\theta^t)\}$  is attained in a subgame perfect equilibrium of the game in which the bank enjoys the first mover advantage.*

**Proof.** See appendix.. ■

It is important to emphasize that the results would differ dramatically if the country was assumed to make offers to the bank. The reason is that the country would always extract the available surplus from continuing the relationship, and would never make a repayment on a loan. That is, we would never observe borrowing by the country. The bank, however, commit's itself in advance not to extract all of the surplus in the future.

Given a solution to the relaxed programming problem  $\{c(\theta^t)\}$ , we can derive the corresponding payments and transfers for the bilateral borrowing game. A question arises as to how this sequence of transfers between the bank and the country should be interpreted in terms of more traditional financial contracts such as loans and deposits. Note that as a country is indifferent between getting higher returns on it's deposits or getting a larger loan with the same repayment profile, there are many ways in which this can be done. The problem is analogous to the existence of many decentralizations of an optimal allocation.

We adopt the following convention, in which we add a distinction between deposits,  $d(\theta^t)$ , and repayments,  $r(\theta^t)$ , on past loans,  $l(\theta^t)$ . First, given values for consumption and the endowment, we can use (3) to determine the minimum level of committed payments necessary to ensure that the banks participation constraint is satisfied. Net transfers,

$$\tau(\theta^t) = - (l(\theta^t) - d(\theta^t) - r(\theta^t)),$$

can then be calculated from (1). Loans are determined in the minimal way:

$$\begin{aligned} l(\theta^t) &= \max \{l(\theta^t) - d(\theta^t) - r(\theta^t), 0\}, \\ d(\theta^t) + r(\theta^t) &= \max \{d(\theta^t) + r(\theta^t) - l(\theta^t), 0\}. \end{aligned}$$

Finally, deposits can be calculated from the zero profit condition on deposit contracts. If  $P(\theta^t, \theta_{t+1})$  is positive we will say that a deposit contract was used at  $\theta^t$ . If  $l(\theta^t)$  is positive we will say that there was lending at  $\theta^t$ . Below we present a simple example that can be computed by hand to provide further intuition on this algorithm. The example also shows that both deposits and lending can occur in equilibrium.

Decomposing the stream of transfers in this way makes clear that what distinguishes bank loans from bank deposits is that bank loans are made in expectation of future repayments that are voluntary, while a bank deposit is made in the knowledge that repayment is guaranteed. Specifically, substituting the decomposition of these transfers into the banks participation constraint gives

$$(1 - q) \sum_{s=t}^{\infty} q^{s-t} \sum_{\theta^s | \theta^t} \pi(\theta^s | \theta^t) [r(\theta^s) - l(\theta^s)] \geq 0.$$

As loans and repayments cannot both be positive, by construction, this inequality implies that loans today cannot be higher than the expected present value of repayments less new

lending expected in the future. Put another way, lending will only occur if the bank expects positive repayments in some future states. It is the threat that these repayments will be eroded by competition that has the ability to undermine lending in equilibrium. Note also that it would be equally possible to view the stream of loans as committed to in advance, in which case we would have a sequence of loan commitments. This interpretation will have some relevance below when we consider the models with multiple banks.

Versions of this relaxed programming problem have been studied by a number of authors, for example Worrall [34], and one version has received a textbook treatment in Ljungqvist and Sargent [26]. Denoting by  $\lambda$  the multiplier on the banks time zero profit constraint, and by  $\beta^t \pi(\theta^t) \mu(\theta^t)$  the multiplier on the country's continuing participation constraint in  $\theta^t$ , we get that the optimal consumption choice satisfies

$$U'(c(\theta^t)) = \left(\frac{1-q}{1-\beta}\right) \left(\frac{q}{\beta}\right)^t \frac{\lambda}{1 + \sum_{s=0}^t \mu(\theta^s)}.$$

If the continuing participation constraint does not bind, consumption declines over time by virtue of the fact that the interest rate is lower than the country discount rate. If the constraint binds, consumption is increased relative to this path. This explains our assumption that  $q > \beta$ ; if  $q = \beta$  and there is imperfect insurance (so that the participation constraints occasionally bind), the country will respond by saving until eventually it is able to perfectly self-insure. The model would be one of international saving by developing countries, and not borrowing.

Proposition 1 implies that the allocation that solves this problem can be attained as a subgame perfect equilibrium. The proof of that proposition supported allocations on the Pareto frontier by reversion to strategies that have the autarkic value. This has often, in the literature, been equated with the players being given the autarkic allocation. As there exist (in general) subgame perfect equilibria that Pareto dominate the autarkic allocation, this has led to the mistaken belief that sovereign debt contracts are not proof to renegotiation by the parties.

What does it mean to say that a strategy profile is proof to renegotiation? Intuitively, if the bank and the country can discuss their strategies they should be able to agree to move to a mutually beneficial alternative strategy profile. Of course, the bank and the country would not agree to move to a profile that was, itself, not proof to further renegotiation, where further renegotiations are themselves undominated by further renegotiations, ad infinitum. Clearly, any definition of renegotiation proofness involves an inherent recursion that can be difficult to apply in practice<sup>5</sup>. However, in the bilateral borrowing problem, the fact that there exist (under the surjectivity assumption) distinct strategy profiles that are efficient

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<sup>5</sup>This inherent recursion has also led to difficulties in defining a widely agreed upon notion of renegotiation proofness, and in turn to a proliferation of alternative concepts. In the context of two player repeated games see, for example, Bernheim and Ray [3], Farrell and Maskin [13] and Pearce [29].

and deliver to either the bank or the country the autarkic value, simplifies the problem enormously. Specifically, we can establish the following proposition, which is sufficient to imply that, for example, the equilibrium is strongly renegotiation proof in the sense of Farrell and Maskin [13].

**Proposition 4** *If a consumption allocation  $\{c(\theta^t)\}$  solves the relaxed programming problem, it can be attained by subgame perfect equilibrium strategies that generate payoffs on the Pareto frontier in every subgame.*

**Proof.** See appendix. ■

The out-of-equilibrium payments profile associated with this equilibrium has an interesting interpretation. In the event that a country defaults, there are no payments in the immediate period. In the following period, the relationship between the bank and the country is restarted, but with the entire surplus going to the bank. That is, the new payments profile is started with the country making a payment sufficient to leave it with no surplus in the continuation of the relationship. We can think of such a payment as a debt settlement, which serves to allow the country to “buy back” its reputation. How large this payment will have to be will depend on the state that occurs in that period. If the state is low, and the felicity function displays a lot of curvature, this payment might be quite small, and indeed smaller than the payment amount that was missed in the previous period. Of course, one should be wary of talking about the out-of-equilibrium “predictions” of a model, but this result implies that looking at the level of payment streams following a default can be misleading as a measure of whether or not a country was punished.

Before continuing, it is important to stress one thing. The analysis above shows that *any* allocation along the Pareto-frontier can be supported in equilibrium. This is true for both the allocation that gives autarkic utility to the country, and hence maximizes the profits of the bank, as well as for the allocation that gives zero profits to the bank, and hence maximizes country welfare. Importantly, the distribution of *ex ante* surplus does not have any impact on whether or not lending will be observed in equilibrium, although it may have an impact upon its magnitude. That is, competition between banks, as long as it is restricted to occur *ex ante*, has no effect on the ability to sustain sovereign debt.

### 3.1 Example

Returning to the simple example introduced above, the participation constraints of the country are given by

$$\sum_{s=t}^{\infty} \beta^{s-t} \ln c_s \geq V_t^A,$$

at each point in time  $t$ . The first order conditions of the relaxed programming problem specialize in this case to

$$\frac{1}{c_t} = \left(\frac{q}{\beta}\right)^t \frac{\lambda}{1 + \sum_{s=0}^t \mu_s \beta^{-s}}.$$

Inspection of this first order condition reveals that consumption will decline monotonically as long as the participation constraints do not bind, at which point the consumption allocation will become stationary, and is denoted by  $c_H$  and  $c_L$ . Under our assumption that

$$q \frac{1}{y_H} \leq \frac{1}{y_H} < \beta \frac{1}{y_L},$$

these must satisfy

$$\log c_H + \beta \log c_L \geq \log y_H + \beta \log y_L,$$

and the first order condition

$$\beta \frac{c_H}{c_L} = q,$$

which gives, evaluating the participation constraint at equality,

$$\begin{aligned} c_L &= y_H^{\frac{1}{1+\beta}} y_L^{\frac{\beta}{1+\beta}} \left(\frac{\beta}{q}\right)^{\frac{1}{1+\beta}}, \\ c_H &= y_H^{\frac{1}{1+\beta}} y_L^{\frac{\beta}{1+\beta}} \left(\frac{\beta}{q}\right)^{\frac{-\beta}{1+\beta}}. \end{aligned}$$

From this it is easy to see that the level of consumption will be lower, the more variation there is in the level of income. Additionally, the consumption profile will display more ‘tilt’, the lower is the interest rate relative to the discount factor. Essentially, when income is very variable, the threat of being excluded from consumption insurance is more severe, and hence more borrowing can be supported. In the stationary equilibrium, this implies a lower level of consumption.

The above profile specifies a path of transfers (net exports) between the country and the bank that alternates between  $y_H - c_H$  and  $y_L - c_L$ . This path of transfers can be decomposed into borrowing and deposits in a number of ways if we are prepared to allow the interest rate to vary. If, however, we restrict both deposits and borrowings to earn the market rate  $R$ , there is a unique decomposition

$$\begin{aligned} nx_H &= Rb + d, \\ nx_L &= -Rd - b, \end{aligned}$$

where, if  $d < 0$ , we will interpret it as further borrowing.

How large are the values involved? The following two numerical examples, although terribly unrealistic, are illustrative of the possibilities.



**Example 1** Let  $y_H = 3/2$ ,  $y_L = 1/2$ ,  $\beta = 2/3$ ,  $R = 7/5$  (or  $q = 5/7$ ). Then it is readily computed that the consumption levels of the agent in this stationary allocation are  $c_H = 0.9936$  and  $c_L = 0.9273$ . Net exports are therefore  $nx_H = 0.5063$  and  $nx_L = -0.4274$ . This produces profit streams for the bank of

$$\begin{aligned}\frac{\pi_H}{1-q} &= \frac{0.4603 - q \times 0.3664}{1-q^2} = 0.4105, \\ \frac{\pi_L}{1-q} &= \frac{-0.3664 + q \times 0.4603}{1-q^2} = -0.1342.\end{aligned}$$

**Example 2** Let  $y_H = 15/2$ ,  $y_L = 1/10$ ,  $\beta = 2/3$ ,  $R = 5/4$  (or  $q = 4/5$ ). Then it is readily computed that the consumption levels of the agent in this stationary allocation are  $c_H = 1.4345$  and  $c_L = 1.1954$ . Net exports are therefore  $nx_H = 6.0655$  and  $nx_L = -1.0954$ . This produces profit streams for the bank of  $\pi_H/(1-q) = 14.4142$  and  $\pi_L/(1-q) = 10.4359$ .

Note that in the first of these examples, bank profits are negative, so that deposits have been made. By contrast, the last example makes clear that it is possible for the bank to be making substantial profits in both periods so that deposits need not be observed in equilibrium. Although it is true that the bank transfers resources to the country in periods where output is low, these amounts are small relative to the amount it receives when output is high.

## 4 Sustaining Punishments with Multiple Bankers

In the previous section we showed that it is possible to support positive amounts of borrowing in this environment when there is only one banker. Moreover, we showed that borrowing can be supported by out of equilibrium strategies that are proof to renegotiation. The key was that a banker will refuse to deal with a country in default unless that country makes a payment sufficient to leave it without any positive expected future surplus. Importantly, these strategies do not require that the country in default receive the autarkic allocation, only that they receive an allocation that gives them the same value as autarky. It is this realization that leads to the resulting strategies being proof to renegotiation. However, it is not clear that such strategies will be an equilibrium if there are multiple competing bankers. The reason is that competition from these bankers may render incredible the threat not to deal with a country unless it makes a payment equal to all of its surplus. To the extent that this is true, and the country is able to form an agreement with another banker, it may not be possible to support borrowing in equilibrium.

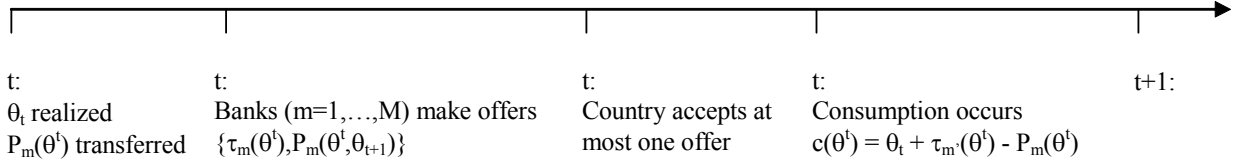


Figure 2: Within Period Timing of Multiple Bank Game

We begin our analysis by following the bulk of the literature in examining the effect of the addition of a bank on the existing bilateral relationship between the country and one bank. Typically, this is analyzed by postulating the unanticipated entry of a new bank and then demonstrating that the entrant can offer a contract that undermines the existing relationship. To get at this idea more formally, we postulate a game in which there are multiple banks, but the country is restricted to enter contracts with only one of them at any one point in time. The implicit contracts that can be struck with this bank will then have to take into account the presence of these competing banks outside the contract. Specifically, assume that in any period, loans and repayments can only move between the country and that bank. At any point in the period, the country is free to walk away from any past agreement to form an agreement with a new bank, or to default and not form a relationship with any bank. Such a relationship is started by payments with a new bank. Note that this is an important assumption: it rules out the possibility of forming a relationship with a group or syndicate of bankers, and thus limits some possibilities for cooperation among banks. We will revisit the assumption below.

The timing of this *multiple bank game* is depicted in Figure 2. Note that we adopt a timing protocol in which banks make offers to the country which then accepts at most one and rejects all of the rest. Importantly, we do not allow banks to commit to making payments to each other. At first glance, this might seem innocuous: the banks are risk neutral and hence there are no gains from trade to be made between them. However, if we allow for such payments to be fully conditioned on observed actions, we will get perfect collusion between the banks. For it would be possible for each bank to promise the others a very large payment in the event that they should not collude. In the light of anti-trust laws, ruling out such agreements seems reasonable.

The following proposition establishes an analogue to the Bulow and Rogoff [6] result: in such a model, there can be no lending in equilibrium. Note that the model does not state that there cannot be inter-temporal trade; international savings, as defined above in our decentralization, can still be supported. This also emphasizes the importance of our decentralization: under alternative definitions of borrowing by the country, we might still

observe borrowing in equilibrium.

**Proposition 5** *No consumption allocation  $\{c(\theta^t)\}$  that involves lending to the country can be supported in a subgame perfect equilibrium of the multiple bank game.*

**Proof.** See appendix. ■

Interpreted narrowly, these results only establishes that the bilateral borrowing contract cannot be sustained in the presence of another bank. This result is typically interpreted as implying that no lending can be supported in international borrowing. It is important to stress that the latter is not, in general, a necessary implication of the former. Instead, it is a result of our assumption that all lending relationships can involve at most one bank. In this case, the theorem states that competition from as little as one other bank is sufficient to undermine all lending relationships. There is a sense, then, in which this result should not be surprising: in particular, the surplus from any implicit lending contract cannot be used to support cooperation between different creditors. Nevertheless, the result is important for it establishes that any lending contract will be vulnerable to entry of a new bank that was outside the agreement. In other words, some restriction on entry is necessary to support sovereign debt. Above we motivated such a restriction by considerations of scale: in order to offer the sort of state contingent borrowing contract desired by a country, banks would have to be very large. This assumption is maintained below. This begs the question: if there is some limit on the number of banks, is it possible to design contracts that are efficient and are such that all of the banks in the market have an incentive to cooperate to punish a country in default? We turn to this question below.

## 4.1 Example

The basic point behind Bulow and Rogoff's argument can be seen quite clearly in the context of our simple example. As the profile is stationary, the present value of overseas payments (or present value of repayments, PVR) by the country is highest in the high period, in which case it equals

$$\begin{aligned} PVR_H &= Rb + d + q(-Rd - b) + q^2(Rb + d) + \dots \\ &= Rb. \end{aligned}$$

The idea of Bulow and Rogoff was that in this period, the country could, instead of repaying  $Rb$ , consume  $(R - 1)b$ , and invest  $b$  in the deposit account. This would earn  $Rb$  by next period, which would make up for lost borrowing of  $b$  and leave  $(R - 1)b$  left over for consumption. Doing this in every period where a repayment would have been made means that

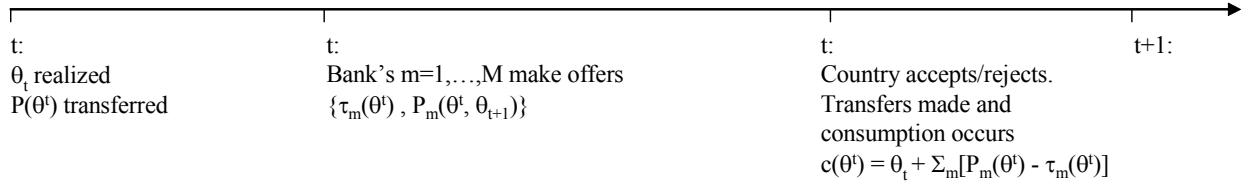


Figure 3: Within Period Timing in Syndicated Lending Game

consumption can be increased by  $(R - 1)b$  in every period. The country would be strictly better off if it defaulted. This argument, of course, assumes that the other bank would be prepared to offer deposit contracts. The arguments above establish that this is the case.

Note that, as this example is deterministic, it is sufficient only that the other bank offer a simple state non-contingent deposit contract. Equivalently, we could suppose that the country had access to a linear technology with rate of return  $R = 1/q$ . In the more general model introduced above, the sufficient condition for the Bulow-Rogoff argument to succeed would be that the deposit contracts offered bore the same state contingencies, or that the linear technology had the same set of state contingent payoffs. It is this feature that motivates our assumption that only large banks can offer these contingencies, and that entry into the market for these contracts can be reasonably modeled as restricted.

## 5 Syndicated Lending

In the previous section, we concluded with the question of whether, with a limit on the number of banks, it possible to design contracts that are efficient and are such that all of the banks in the market have an incentive to cooperate to punish a country in default? As an answer to the question, we consider a version of the above model relaxed to allow for lending and repayments between all  $M$  banks and the country. This can be thought of as syndicated lending. The timing of this *syndicated lending game* is depicted in Figure 3. As above, we assume that banks are unable to make binding promises of payments to each other.

In the game above, lending was not sustainable in equilibrium because a bank that is outside of the lending agreement had an incentive to offer a deposit contract favorable to the country whenever repayments to the lending bank were large. As, by assumption, the deposit taking bank was outside of the original agreement, it had nothing to lose by offering such a contract. The model here varies by allowing the bank a share of the profits from lending, and asks whether such a share is ever significant enough to deter that bank from deviating from the lending agreement.

In thinking about the incentive to deviate faced by any member of the syndicate, it is useful to note that there are essentially two classes of alternative contract that could be offered: those that contain lending, and those that do not. In particular, it is possible to envisage a defecting bank offering to lend at more favorable terms, or alternatively not to lend at all and simply take deposits. The key result of this section, in analogy with the related work of Kletzer and B. Wright [20], defections in which there is lending can be made unprofitable. Essentially, if the new agreement involves lending, the other member of the syndicate can respond by offering a contract that induces the country to default on these new loans. If the country defaults, the defecting bank makes strictly negative profits equal to the amount loaned. The second form of defection involves offering deposit contracts only. Unlike new loans to the country, these are initiated by a payment from the country to the bank, and even if the country were to break of trade with this bank in the future, that bank will not make losses. The other members of the syndicate are able to respond in ways that limit the defecting banks profits over time; in particular, by offering competitive deposit contracts in return, they can limit any profit earned by the defecting bank to the period of defection alone. The question of whether or not lending can be supported then reduces to a question of comparing the profits that can be achieved by charging the maximum price for accessing these deposit contracts in the future (the value to defecting) to the banks share of profits as part of the lending syndicate.

In particular, suppose we look for a subgame perfect equilibrium in which cooperation in the punishment of a bank that defects by trigger strategies in which, in the next period, banks respond to a default by offering competitive deposit contracts. As a result, the most a bank can earn by inducing a country to default with a deposit contract is the amount the country would pay, in the present period, for access to such deposits. That is, if we let

$$V^C(\theta^t + P(\theta^t)) = (1 - \beta) \sum_{s=t}^{\infty} \sum_{\theta^s | \theta^t} \beta^{s-t} \pi(\theta^s | \theta^t) U(c(\theta^s)),$$

denote the value to the country of continuing with the syndicated lending contract, the maximum a country would be willing to pay would be given by  $X(\theta^t)$  which is determined implicitly from

$$V^D(\theta_t, \theta^t + P(\theta^t) - X(\theta^t)) = V^C(\theta^t + P(\theta^t)).$$

How large is this amount? At first glance, it might appear that the bank would be prepared to pay the entire expected present value of the stream of repayments, net of loans, saved. However, this would involve the country borrowing from the bank in order to pay this amount, which the country would promptly default upon. Note that it can exceed the total amount, summing across all banks, of repayments on past loans made by the country in the present period. If the country were pay this sum, it would have the same consumption today, but more limited access to financial markets in the future, than under the original

syndicated loan contract. But, if the country is a significant debtor, the level of future consumption would also be much higher. But the amount may be large, relative to the profits a bank can expect to receive if it remains in the syndicated banking relationship. We give two examples below which illustrate the possibilities.

Clearly, then, the ability to support collusion in this way requires that the profits from the original contract,  $\Pi(\theta^t)$ , split  $M$  ways among the syndicate members, be greater than  $X(\theta^t)$ . Moreover, this must be true for all histories. This is formalized in the following proposition.

**Proposition 6** *There exists an  $M^* > 1$  such that if the number of banks  $M \leq M^*$ , an allocation that solves the bilateral programming problem can be attained from a subgame perfect profile of strategies in the syndicated lending game.*

**Proof.** See appendix. ■

Note that this is a weak result. In particular, we have not established that  $M^*$  is necessarily greater than two. That is, the theorem could, as it stands, be vacuous. Indeed, for some parameter values, it is likely that  $M^* < 2$ . However, the proposition below demonstrates that, as  $R$  approaches one we can support the efficient level of lending with any finite number of banks.

**Proposition 7** (*Quasi Folk Theorem*) *For any number of banks  $M$ , there exists an  $R > 1$  such that an allocation that solves the bilateral programming problem can be attained from a subgame perfect profile of strategies in the syndicated lending game.*

The arguments above establish conditions for the existence of lending in subgame perfect equilibrium. In that sense, the out of equilibrium strategies are credible. However, it is reasonable to ask whether or not the threat to retaliate to a deviation by inducing a deposit war is truly credible in a more restrictive sense. For to do so leaves unexploited potential gains from trade, and the deviating bank and defaulting country could presumably argue that all parties put aside the past behavior and try to exploit these potential gains from trade. That is, we are invoking considerations of coalition proofness, or renegotiation proofness with more than two players.

Unfortunately, existing concepts of renegotiation or coalition proofness are either difficult to apply in our environment (for example, *perfectly coalition proof Nash equilibrium* introduced for finite horizon games by Bernheim, Peleg and Whinston [4]), or are too strong, in the sense that solutions do not exist (for example, *strong perfection* introduced by Rubinstein [32]). Our approach will be to define a conception of coalition proofness that is strong,

in the sense of allowing a large set of blocking coalitional deviations, but not too strong, in the sense that a solution exists. We argue that our concept has been chosen so that any “reasonable” equilibrium refinement is weaker, and so will also be satisfied. In analogy with Bernheim, Peleg and Whinston’s concept, we refer to ours as perfectly strong coalition proofness.

**Definition 1** *A strategy profile is **perfectly strong coalition proof** if, in every subgame, for every coalition, and every strategy profile for that coalition that is subgame perfect in the game induced upon the coalition holding the non-coalition members strategies constant, there exists some member of that coalition that (weakly) prefers the original strategy profile.*

We argue that this represents a strong definition in the sense that it places relatively few restrictions upon deviating coalitions. In particular, coalitions may deviate to strategies that are themselves dominated by other (contemporaneous) deviations. Our restrictions require a form of dynamic consistency of unilateral deviations; that is, a coalition is not allowed to deviate to a new strategy that any of its members would unilaterally like to deviate from in the future. We do not require dynamic consistency of deviations for larger coalitions; that is, a coalition may deviate to a strategy profile that the coalition itself (or any subcoalition or more than one member) would like to deviate from in the future. Note that the concept defined is the same as that used by Kletzer and B. Wright [20] (although they refer to it as strong perfection) and M. Wright [35] in their models of sovereign lending with two-sided limited commitment.

The following proposition establishes that these threats are credible in the sense of perfectly strong coalition proofness if we allow banks to make voluntary transfers to each other. In particular, if we allow for banks to make voluntary transfers between each other, then the banking syndicate can agree to resume the old relationship upon receipt of a transfer from the deviating bank equal to all of its future profits from the syndicate.

**Proposition 8** *Allowing for voluntary transfers between banks, there exists an  $M^* > 1$  such that if the number of banks  $M \leq M^*$ , an allocation that solves the bilateral programming problem can be attained from a subgame perfect profile of strategies in the syndicated lending game that are perfectly strong coalition proof.*

**Proof.** See appendix. ■

Note that the mechanism that is enforcing lending to the sovereign is a mixture of both country and bank reputations. As long as banks have a reputation for cooperation in dealing with sovereign debtors, all the banks will cooperate in punishing a country in the event of

a default, and so threats to exclude a country in the event of a default are credible. It is bank reputations that allow a countries reputation to serve as a sufficient motivation to repay their debts.

## 5.1 Example

The optimal stationary (cyclical) deposit contract solves

$$\max_d \log(y_H - d) + \beta \log(y_L + Rd),$$

and implies a level of deposits of

$$d = \frac{\beta R y_H - y_L}{R(1 + \beta)},$$

which is positive under our assumption on the interest rate  $R = 1/q$ . This implies a level of utility of

$$V_t^D = \begin{cases} \frac{1}{1+\beta} \log\left(\frac{R y_H + y_L}{R(1+\beta)}\right) + \frac{\beta}{1+\beta} \log\left(\frac{\beta(R y_H + y_L)}{(1+\beta)}\right) & \text{if } y_t = y_H \\ \frac{1}{1+\beta} \log\left(\frac{\beta(R y_H + y_L)}{(1+\beta)}\right) + \frac{\beta}{1+\beta} \log\left(\frac{R y_H + y_L}{R(1+\beta)}\right) & \text{if } y_t = y_L \end{cases}.$$

To calculate the value of  $X_t$  needed to make the country indifferent, we need to solve a non-stationary version. It is, however, easily verified, that the country will settle down to this cyclic pattern after only two periods. In particular, denoting by  $d^*$  the level of deposits after subtracting  $X_t$ , we have that

$$d^* = \max\left\{0, \frac{\beta R (y_H - X) - y_L}{R(1 + \beta)}\right\}.$$

Together with the continuation value in the contract, this can be used to solve for  $X_t$ . Even in this simple example, the algebra quickly gets complicated. However, one result is easily established: decreasing the interest rate makes collusion more easily enforceable. This is partly for the familiar reason that the bank cares more about future profits. But in this model, there are also two other reasons. First, lower interest rates make savings a less attractive alternative to borrowing for the purpose of smoothing income fluctuations. Second, lower interest rates imply smaller profits in the high period relative to the low period.

**Proposition 9** *In the cyclical model with logarithmic preferences, for any number of banks  $M$ , there exists an  $R > 1$  such that the stationary efficient level of lending can be attained as a perfectly strong coalition proof equilibrium.*



The results of this proposition are illustrated in the following numerical example.

**Example 3** *Let  $y_H = 15/2$ ,  $y_L = 1/10$ ,  $\beta = 2/3$ ,  $R = 5/4$  (or  $q = 4/5$ ). As we showed above, net exports are  $nx_H = 6.0655$  and  $nx_L = -1.0954$  while the profits for each bank will be  $\pi_H = 14.4142$  and  $\pi_L = 10.4359$ . By defaulting in the high state, the country could bank all of the net exports. However, because the endowment stream is so variable, a country would be willing to pay 6.6296 for the privilege of accessing deposits. As a result of the response of the other bank, the bank that induced the country to default by offering them the deposit contract would expect to gain*

$$6.6296 - \frac{1}{2} \times 14.4142 = -0.5775,$$

*a net loss. That is, collusion can be supported between two banks, but no more.*

*When the interest rate is decreased to  $R = 9/8$ , the calculation changes to*

$$6.5639 - \frac{1}{2} \times 24.2766 = -5.5744,$$

*and it is almost possible to sustain three banks.*

*When the interest rate is decreased to  $R = 17/16$ , the calculation changes to*

$$6.5283 - \frac{1}{2} \times 44.1614 = -15.5524,$$

*and it is almost possible to sustain four banks.*

## 6 Other Creditor Reputations

In Section 5 above, we explored the extent to which a bank's reputation for cooperating in the banking syndicate can be sufficient to support borrowing. In such a mechanism, the profits from the relationship with the developing country itself are used to give each bank an incentive to cooperate in the relationship. To establish this, it was necessary to depart from one of the key implicit assumptions of the sovereign debt literature: that the borrowing relationship of a country be bilateral. In this section we address some alternative approaches to establishing a role for creditor reputations, and present some extensions of the basic framework.

## 6.1 Multiple Sovereign Debtors

An alternative approach preserves the bilateral nature of the borrowing relationship, and alters the canonical environment by postulating the existence of other trading relationships for the bank. That is, it removes the other key implicit assumption made in the literature, that the banks live in an economic vacuum trading (for positive profit) neither with each other, any other country, or any other agent whatsoever. In this subsection, we relax this second assumption by postulating the existence of another developing country with whom the second bank is engaged. When the gains from trade in both relationships are large enough, relative to what could be gained by inducing the country to default, lending can be sustained. Once again, a creditor reputation mechanism is at work: banks cooperate in punishing a country in default, as long as each has a reputation for cooperating in the past. It is in this sense that the mechanism comes close to justifying the folk wisdom often expressed by financial market participants, that banks have an incentive to cooperate in dealing with a country in default<sup>6</sup>.

To capture these ideas, we modify the above model by introducing a second developing country bank relationship into a two bank version of our model. As in Section 3 above, we assume that both relationships are exclusive in that each bank deals with one country. The only difference to the game in that section is that now banks and countries can make transfers to each other, conditional upon what others have done. We will focus on equilibria in which these do not occur. To economize on notation, we assume that  $\theta$  now represents a two dimensional vector of endowments in each country  $\theta_t = (\theta_{1t}, \theta_{2t})$ . Otherwise, the problems are almost completely symmetric to that studied above. The key is in ensuring that the surplus that one bank could gain from inducing the other bank's country to default can never be greater than the surplus that bank is extracting from its relationship. The only wrinkle is that a bank may also opt not to extend a loan to its own country in the same period in which it induces the other banks country to default.

Specifically, in analogy with the discussion in Section 5 above, let  $X^i(\theta^t)$  denote the maximum amount country  $i$  would be prepared to pay in the current period for access to optimal deposit contracts. If bank  $j$  induces country  $i$  to default, this is the maximum it could gain. Offsetting this is the fact that bank  $i$  can respond by offering optimal deposit contracts to country  $j$  in each period thereafter, thus eroding all of the surplus  $\Pi^j(\theta^t, \theta_{t+1})$  bank  $j$  would have earned in this relationship. Therefore, a necessary condition for collusion to be supported by trigger strategies that threaten reversion to the optimal deposit contract is that

$$q \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta_t) \Pi^j(\theta^t, \theta_{t+1}) \geq (1 - q) X^i(\theta^t), \quad (4)$$

---

<sup>6</sup>For example, the private sector Institute for International Finance has written “the principal participants not only have a mutual incentive to cooperate but also have their individual reputations at stake for credibility in any such commitments in the future” (IIF [17] p.11).

for all  $i, j$  and  $\theta^t$ . Note that the bank only loses the future profits on its current relationship.

**Proposition 10** *If (4) holds for all banks, all countries and all  $\theta^t$ , and if we allow for voluntary transfers between banks, a pair of allocations that solves the bilateral programming problem for each country can be attained from a subgame perfect profile of strategies in the multiple country game that are perfectly strong coalition proof.*

**Proof.** See appendix. ■

Once again, the condition we propose in (4) is not terribly informative, as it is not obvious that it is ever satisfied. Moreover, it is clear that in some examples it will not be satisfied; for example, if the interest rate is sufficiently high, or the output process sufficiently stable. The following example demonstrates that this is not always the case.

### 6.1.1 Example

Consider a version of the above simple deterministic example in which there are two countries that experience alternating productivity shocks. In particular, assume that whenever country one has the high endowment, country two has the low endowment.

**Example 4** *Let  $y_H = 15/2$ ,  $y_L = 1/10$ ,  $\beta = 2/3$ ,  $R = 5/4$  (or  $q = 4/5$ ). As we showed above, net exports are  $nx_H = 6.0655$  and  $nx_L = -1.0954$  while the profits for each bank will be  $\pi_H = 14.4142$  and  $\pi_L = 10.4359$ . By defaulting in the high state, the country could bank all of the net exports. However, because the endowment stream is so variable, a country would be willing to pay 6.6296 for the privilege of accessing deposits. As a result of the response of the other bank, the bank that induced the country to default by offering them the deposit contract would expect to gain*

$$6.6296 - \frac{4}{5} \times 14.4142 = -4.9018,$$

*a net loss. Hence collusion can be supported in this example.*

## 6.2 Relationships outside of sovereign lending

The modification to include an extra sovereign debtor works because it introduces the possibility that a bank that induces one country to default can be punished by having its own sovereign debtor default. In contrast to the case of syndicated lending above, in which variations in the distribution of surplus from inside the one sovereign lending relationship were used to induce cooperation, here we have introduced surplus from outside of the relationship. The exact source of this surplus is not important: we could as easily have induced cooperation by invoking the possibility of a spillover to another trading relationship of the bank. This could involve business conducted jointly by the banks, or conducted by the bank with another party altogether.

There is some evidence to suggest that such channels might have been important in history. For example, in 1864 following a violation of a bondholder embargo against Spain by a group of English and French bankers, the bondholders embarked on a campaign to disrupt the reputations of the firms involved and hinder their business with the “ordinary mercantile community”. The approach of the bondholders was endorsed in an editorial in *The Times*

it is evident that the only protection that can now be afforded to the long-suffering and defrauded bondholders of Spain must be taking care that the parties who commit themselves to such transactions shall receive all the fame they deserve. If they think well of their conduct, of course they will be glad that it should acquire notoriety. If, on the contrary, they experience misgivings, some acquaintance with the view taken of the matter by the ordinary mercantile community may, perhaps, teach them that, after all, even money may be bought too dearly. (16th September 1864, p.5)

The view that wider credit market reputations might be important also receives some support from the British Corporation of Foreign Bondholders (CFB). In its inaugural annual report in 1873, the CFB described its vision of the operation of international financial markets. According to the CFB, countries repaid their debts in order to preserve their reputations and hence credit market access. More importantly for our purposes, creditors were induced to cooperate in the punishment of countries in default out of a concern for their own reputations, and the effect that a loss of reputation would have on their own credit market access. That is, the CFB emphasized the role of creditor reputations in coordinating creditors, which in turn enables country reputations to be effective in enforcing repayment.

The very association of Bondholders brings with it elements of independent influence, the full value of which is little appreciated. It does not lend money like the great financial establishments and parties of bankers, it does not influence

prices in its dealings nor exercise any oversight over quotations like members of the Stock Exchange and Bourses, but the negative power of withholding money even from these, as well as from the aggrieving government, exercises its own effective influence when applied at a proper time. (CFB Annual Report 1873, p.60)

While we do not want to overstate the case, the above quotations are suggestive of the possibility that informal credit market interactions among banks, along with the potential for these interactions to be disrupted, can induce banks to cooperate out of a desire to preserve their own reputations.

### 6.3 Bondholders vs. Banks

In the text above we have tended to refer to creditors as banks. This was natural given our assumption that creditors are able to commit to honoring deposits. However, in the history of sovereign lending, banks have played a quantitatively minor role; bonds were much more significant than bank lending throughout the entire 19th Century and first part of the 20th Century, and in the past decade, since the debt crisis of the 1980's, bonds have begun to replace bank loans once again. Indeed, as illustrated in Figure 4, by the end of the 20th Century, bond lending had risen to a point of almost parity with bank loans.

At first glance, bond lending would appear to be different from bank lending in one important respect: bondholders tend to be more widely dispersed than banks. This would appear to have two important implications for our model. First, to the extent that bondholders are dispersed and unable to coordinate in renegotiating contracts, bond lending would appear to be able to provide much less in the way of implicit insurance than bank lending. Second, to the extent that most bondholders are small, bondholders would appear to have limited ability to offer the sorts of deposit contracts discussed above. Consequently, they would have limited ability to retaliate against any banks that attempted to steal the *ex post* rents on their lending contracts. This begs the question: Why don't banks collude to steal the rents from bondholders?

Further study of recent sovereign bond reschedulings suggests this may be a flawed interpretation. Although recent reschedulings such as those in Pakistan in 1999, and in Ukraine and Ecuador in 2000, were not free from difficulty, all three were carried out with somewhat less difficulty than might have been expected given a dispersed body of creditors. This suggests that bonds may be able to provide many of the same implicit state contingencies as bank loans. This will be further enhanced if the, so far limited, introduction of bonds with explicit state contingencies continues (see, for example, the discussion of Bulgarian and Mexican value recovery rights in Haldane [16]).

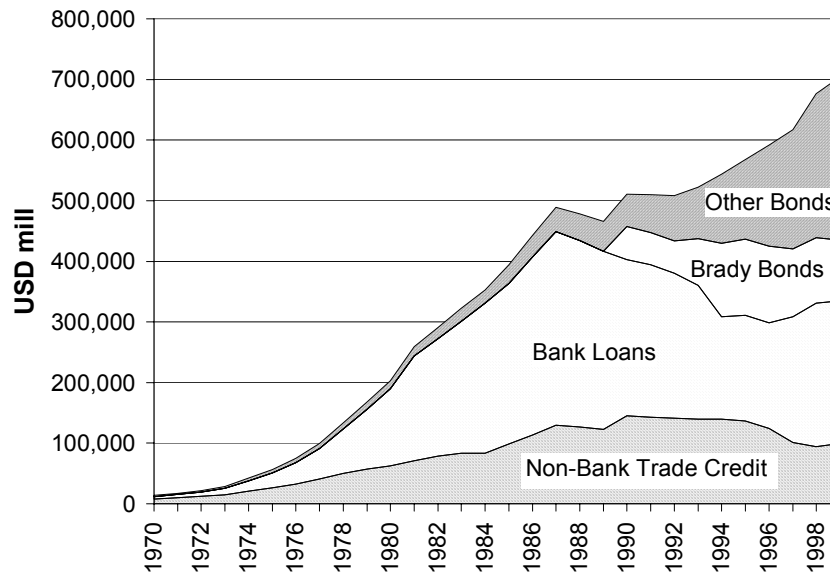


Figure 4: Emerging Market Sovereign Debt from Private Sources

Moreover, in a number of cases, including three of the four Ukrainian bonds that were rescheduled, the bonds were in fact closely held by small numbers of institutional investors. In the case of Ecuador, although the bonds were widely held, a number of significant bondholders were large institutional investors. To the extent that this is an accurate representation of bondholders more generally, the sort of retaliatory strategies discussed above would appear to be available. To the extent that this is not an accurate representation, spillovers to reputations outside of the sovereign lending relationship, such as those discussed above, would appear to be necessary to protect bondholder lending. It is notable that, in both of the above quotations, it was bondholder groups that were citing the possible effect on bank reputations in other relationships.

## 7 Concluding Remarks

Why do countries repay their debts? We have argued in this paper that *any* explanation for the existence of sovereign debt must explain how banks and other creditors can protect the *ex post* profits on their lending. Explanations based on direct sanctions or reputation spillovers grant creditors some *ex post* monopoly power by limiting the ability of a sovereign debtor to switch creditors. Explanations based on limiting the sort of contracts banks can

offer allow existing creditors to protect their rents by making credible threats to disrupt any other lending relationship that might be formed.

In this paper, we suggested a direct alternative, based on the assumption that the international banking market is not competitive. Small numbers of banks, however, turn out not to be sufficient: even with as few as two competing banks, tacit collusion between banks cannot be supported in the canonical model of sovereign lending. That is, *ex post* rents cannot be protected, and lending to sovereign debtors cannot exist in equilibrium. However, under small changes to the canonical model that allow for syndicated lending or for the presence of multiple sovereign debtors, tacit collusion can be supported. Interestingly, the mechanism through which this occurs is one in which banks collude out of a concern for maintaining their own reputations. It is creditor reputations that work to support the effectiveness of a country's reputation for repayment.

As formulated above in the bilateral banking problem, the bank is able to design highly non-linear contracts, and so acts as a perfectly price discriminating monopolist. Adding competition to this framework can only work to redistribute surplus, and to reduce efficiency to the extent that less debt can be supported in equilibrium. A necessary implication is that, ignoring concerns for equity, more competition in sovereign lending cannot be efficiency enhancing. It also leads to a natural conjecture: if banks are restricted to offer less than fully non-linear contracts, there may exist a non-monotonic relationship between the level of competition in international financial markets and aggregate welfare. At one extreme, with a monopoly bank, the monopoly distortion is large, but cooperation is assured. At the other extreme with many bank, the monopoly distortion is limited, but cooperation is impossible. Current research is investigating whether small increases in competition can have a greater effect on the monopoly distortion than on cooperation, and hence if there exists an optimal level of international financial market competition.

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## A Legal Remedies in Sovereign Defaults: A Case Study

One of the best known and most extensive examples of the attempt to use legal remedies against a country in default involves the legal actions taken against Russia by the Swiss firm known as Noga, and its owner Nessim Gaon. Nessim Gaon was born in Sudan in the 1920's and served in British army in WW2. He made his fortune exporting Sudanese snake and crocodile skins before moving on to dominate the world peanut trade. In 1950's, he moved his operations to Switzerland and moved his Compagnie Noga d'Importation et d'Exportation SA (or Noga) into property speculation and general trading. ("Vengeful David has Russian Goliath in Financial Maelstrom", Scotland on Sunday, April 29 2001).

Like many entrepreneurs, Gaon sought to profit from the reforms in Russia, and in 1991 signed a deal with the first Russian government led by Ivan Silayev. The deal involved exportation to Russia of medicines, pesticides, food and other supplies in exchange for oil. The Russian government explicitly waived its rights to sovereign immunity as part of the contract. After approximately USD1.5 billion in trade under the contract had been consummated, relations soured and the dispute began at the end of 1992 with the Russian government refusing to send any further supplies of oil (Moscow Times, July 15 2000).

Noga responded with legal action. In 1993, the first case was brought before the Trade and Industrial Chamber in Stockholm for arbitration. The Russian State Company Nafta-Moskva counterclaimed in the Arbitration Court of the Russian Trade and Industrial Chamber (BBC Summary of World Broadcasts August 6 1993). Separate legal actions were also undertaken in other countries.

In 1993, court rulings led to the freezing of Russian government bank accounts totalling USD 700 million in Luxembourg and Switzerland (Moscow Times July 15 2000). The Luxembourg accounts, containing USD279, were frozen on June 11th 1993 by the Luxembourg Arbitration Court (BBC Summary of World Broadcasts August 6 1993). The seizures led the Central Bank of Russia to establish the Financial Management Co., or FIMACO, an offshore shell company in the British Channel Islands of Jersey and Guernsey to hold its gold and foreign currency reserves (Interfax May 22 2000, Moscow Times July 15, 2000).

In 1996, these accounts were unfrozen because the Central Bank was not party to the contracts (BBC Summary of World Broadcasts, May 26 2000). The Luxembourg accounts were unfrozen in 1996, but there is some doubt as to when (and if) Switzerland accounts were ever unfrozen (Interfax May 22 2000).

In 1997, an initial judgement against Russia was made in Stockholm arbitration court for USD63 million, which was considerably less than the USD800 million in interest and damages claimed by Noga (Agence France Presse July 24th 2000). The judgement included an assessment that the initial debt was USD25 million, plus interest and damages. A New York court found a further USD25 million indebtedness against Noga at a later date (Russian Economic News June 27 2001). The current claim has risen to USD110 million (Economist

June 30 2001) in large part because of interest payments (Noviye Izvestia June 26 2001, translated in CDPSP July 25 2001).

Although the Stockholm court had ruled in favor of Noga, the enforcement of the judgment required that Russian assets be attached. Toward this end, in March 2000, Noga succeeded in having the Stockholm decision made justiciable in France. Following this ruling, in May 2000, a French court ordered the seizure of banks accounts of about 70 Russian State connected enterprises, including the Russian embassy and UNESCO delegation (Moscow Times July 15, 2000). All of these organizations had accounts in the French Branch of the Russian Central bank Eurobank (Russian Economic News October 1 2001). On 10th August 2000, a Paris judge ruled that the suit had no legal basis and ordered Noga to pay FF30,000 (more than USD4,000 in damages). The appeal had been supported by the French government (Vremya novstei Aug 11 2000 p14, translated in Current Digest of the Post-Soviet Press September 6 2000). The court cases turned on the argument of the Russian government that, under international law, Central Bank assets were immune from attachment for Russian State debts. This had been the finding of the court Luxembourg. Russian embassy property is protected under the Vienna convention (Interfax June 5th, 2000). In any case, in July 2000, the Russian embassy in Bern Switzerland, replying to threats of asset seizures by Noga's lawyers, responded that it maintained almost no money in its Swiss bank accounts and that its real estate in Switzerland was protected by treaty with the Swiss government (TASS July 21, 2000).

Concerns about seizures of other assets led a number of foreign governments to seek immunity from seizure for Russian assets. In 2000, the Royal Museum of Art and History in Belgium was forced to abandon a show of Russian Art Treasures when it could not gain legal guarantees against the seizure of the art ("Vengeful David has Russian Goliath in Financial Maelstrom", Scotland on Sunday, April 29 2001). In 2000, a French presidential decree was made to prevent the seizure of president Putin's personal aircraft at Orly Airport in Paris. (Scotland on Sunday April 29, 2001).

Despite these efforts, on 13th July 2000, the Russian tall ship Sedov, at 120m and with four masts claimed to be the worlds largest sailing ship, was impounded in the port of Brest in France. Both the ships owners, the Murmansk Marine Institute at the University of Murmansk, and the French state prosecutor argued that the University was not liable for Russian state debts. A court of appeal agreed on 24th July and ordered Noga to pay USD71,000 in damages to the ship's owners, and USD35,000 to the festival organizers. A last minute appeal by Noga against the release of the ship failed when the appeal order was not filed correctly (Agence France Presse July 24th 2000). To preempt further action on this front, foreign governments sought immunity from seizure for Russian ships. On 16th August 2000, the Harlen district court in the Netherlands ruled that the Russian sailing ships Mir, Sedov and Kruzenshtern were immune from seizure (Interfax August 17, 2000).

Threats of seizure in 2000, led Russia to halt shipments of nuclear warheads to USA for reprocessing until President Clinton signed an executive order guaranteeing immunity

of the uranium from seizure. In July 2001, President Bush extended this order. The uranium is reprocessed for use in US and Russian commercial reactors, and is a centerpiece of a Russian-USA non-proliferation agreement (Nuclear Engineering International, August 31, 2001).

In probably the most famous attempt at seizing Russian assets to date, on June 22nd, 2001, Noga attempted to seize one Su-30 fighter and one MiG-AT fighter trainer at the Le Bourget airshow in France (Manchester Guardian July 4 2001). The planes were scrambled ahead of the bailiff's after the Russian contingent were warned by the festival organizers, who also helped drag the planes to the end of the runway, gave permission for an emergency takeoff, and opened an air corridor (Noviye Izvestia June 26 2001, translated in CDPSP July 25 2001). The Russian government argued that, in any case, such planes were immune from seizure as they were the property of the Sukhoi R&D bureau, (Russian Economic News June 22, 2001), and the state owned company Komsomolsk-on-Amur Aviation Production Association (Interfax June 22 2001).

Noga's most recent attempts to seize assets were frustrated on October 1st 2001, when the Swiss Federal Court reaffirmed a lower court ruling that a CHF5 million bail bond posted on behalf of Pavel Borodin in April 2001 was immune from sequestration (TASS October 1 2000). Borodin was formerly property manager of the Russian president.

## B Technical Appendix

**Lemma 1** *In the bilateral lending game, the autarkic strategies are perfect and deliver an allocation that provides weakly lower lifetime utility and profits than any other subgame perfect equilibrium.*

**Proof.** That the autarkic strategies are perfect follows from the fact that if one agent plays the autarkic strategy, the other is indifferent amongst all other strategies. To show that it provides the lowest lifetime utility, let  $M^b(\theta, \{P(\theta'|\theta)\})$  and  $M^c(\theta, \{P(\theta'|\theta)\})$  be the infimum over the set of all profit and utility levels attainable in subgame perfect equilibrium, given current state  $\theta$  and payments  $\{P(\theta'|\theta)\}$ . We will establish the result for the country; an analogous argument follows for the banker.

Consider an arbitrary subgame perfect equilibrium which delivers  $V(\theta, \{P(\theta'|\theta)\})$  to the country given current state  $\theta$  and current payments  $\{P(\theta'|\theta)\}$ . Then

$$V(\theta, \{P(\theta'|\theta)\}) = \sum_{\theta'} \pi(\theta'|\theta) \left\{ (1-\beta) U(c(\theta')) + \beta V(\theta') \right\},$$

for some allocation  $c(\theta')$ , some committed payments  $\{P(\theta''|\theta')\}$ , and some subgame perfect equilibrium values  $V(\theta', \{P(\theta''|\theta')\})$  attainable from  $\theta'$  and  $\{P(\theta''|\theta')\}$ . As it is feasible for the country and bank to never offer a transfer, and by definition of  $M^c(\theta, \{P(\theta'|\theta)\})$ , we have

$$\begin{aligned} V(\theta, \{P(\theta'|\theta)\}) &\geq \sum_{\theta'} \pi(\theta'|\theta) \left\{ (1-\beta) U(\theta' + P(\theta'|\theta)) + \beta M^c(\theta', \{P(\theta''|\theta')\}) \right\} \\ &\geq \sum_{\theta'} \pi(\theta'|\theta) \left\{ (1-\beta) U(\theta') + \beta M^c(\theta', \{P(\theta''|\theta')\}) \right\}, \end{aligned}$$

as committed payments are non-negative. But the subgame perfect equilibrium considered was arbitrary, so that the infimum also satisfies

$$\begin{aligned} M^c(\theta, \{P(\theta'|\theta)\}) &\geq \sum_{\theta'} \pi(\theta'|\theta) \left\{ (1-\beta) U(\theta' + P(\theta'|\theta)) + \beta M^c(\theta', \{P(\theta''|\theta')\}) \right\} \\ &\geq \sum_{\theta'} \pi(\theta'|\theta) \left\{ (1-\beta) U(\theta') + \beta M^c(\theta', \{P(\theta''|\theta')\}) \right\}, \end{aligned}$$

which by recursive substitution, exploiting  $\beta \in (0, 1)$ , gives

$$M^c(\theta, \{P(\theta'|\theta)\}) \geq V^A(\theta, \{P(\theta'|\theta)\}),$$

where  $V^A$  refers to the value attained given that the autarkic strategies are followed from this stage onwards. But as autarky is a subgame perfect equilibrium, this inequality must be an equality. ■

**Remark 1** *The above proof relies on boundedness of the set of subgame perfect equilibrium values: if the infimum over the set of subgame perfect equilibrium values is  $-\infty$ , the above inequalities are well defined. An alternative, and much simpler proof would note that as the autarkic offers are always feasible regardless of the strategy played by the other player, and deliver the autarkic value, the best response to any strategy played must also deliver at least the autarkic value. As autarky is a subgame perfect equilibrium, the autarkic values are the lowest subgame perfect equilibrium values.*

**Lemma 2** *Let  $V_R^D(\theta, z)$  denote the value of following the optimal deposit strategy given an interest rate of  $R = 1/q$ . Then if  $R' > R$ , we have*

$$V_{R'}^D(\theta, z) > V_R^D(\theta, z).$$

**Proof.** The result follows from the fact that the constraint correspondence for  $R$  is a strict subset of the constraint correspondence for  $R'$  as  $d$  is restricted to be positive, and the fact that  $U$  is strictly increasing. ■

**Proof of Proposition 1.** Suppose a strategy profile delivers an allocation that satisfies these conditions. Then as autarky is a subgame perfect equilibrium, the strategy profile is a subgame perfect equilibrium supported by reversion to the autarkic strategies.

To show the converse, let  $\{c(\theta^t), P(\theta^t)\}$  be the consumption allocation and committed payments delivered by a profile of subgame perfect strategies. As it is always feasible for the country not to offer a deposit, and as autarky is the lowest possible continuation value, we must have

$$(1 - \beta) \sum_{s=t}^{\infty} \beta^{s-t} \sum_{\theta^s | \theta^t} \pi(\theta^s | \theta^t) U(c(\theta^s)) \geq (1 - \beta) U(\theta_t + P(\theta^t)) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1} | \theta_t) V^A(\theta_{t+1}).$$

Analogous arguments holds for the bank. ■

**Proof of Proposition 2.** First, note that the countries preferences are continuous in the product topology, and the constraint set for the relaxed problem is compact in the product topology. Hence a solution exists. Second, note that if a consumption allocation solves the bilateral borrowing problem for some path of committed payments, then, as committed payments are non-negative, the consumption allocation also satisfies the constraints of the relaxed problem. Further, as the constraint set for the bilateral borrowing problem is closed, a solution exists. Hence, the value of the relaxed program is at least as large as the value of the bilateral borrowing program.

Third, note that the relaxed problem is convex, so that the first order necessary conditions are also sufficient. The proof will be complete if we can show that a solution to the first

order conditions of the bilateral borrowing problem also satisfies the first order conditions of the relaxed problem, which include

$$U'(c(\theta^t)) = \left(\frac{1-q}{1-\beta}\right) \left(\frac{q}{\beta}\right)^t \frac{\lambda}{1 + \sum_{s=0}^t \mu(\theta^s)},$$

where  $\lambda$  is the multiplier on the banks time zero profit constraint, and  $\beta^t \pi(\theta^t) \mu(\theta^t)$  is the multiplier on the country's continuing participation constraint in  $\theta^t$ .

Consider the bilateral borrowing problem. The first order necessary conditions include

$$U'(c(\theta^t)) = \left(\frac{1-q}{1-\beta}\right) \left(\frac{q}{\beta}\right)^t \frac{\sum_{s=0}^t \gamma(\theta^s)}{1 + \sum_{s=0}^t \eta(\theta^s)},$$

where  $q^t \pi(\theta^t) \gamma(\theta^t)$  is the multiplier on the banks profit constraint at  $\theta^t$ , and  $\beta^t \pi(\theta^t) \eta(\theta^t)$  is the multiplier on the country's participation constraint at  $\theta^t$ , and

$$-\beta^t \eta(\theta^t) (1-\beta) U'(\theta_t + P(\theta^t)) + (1-q) q^t \gamma(\theta^t) \leq 0,$$

which hold with equality if  $P(\theta^t)$ , is greater than zero. Note that it is impossible for the bank's constraint to bind, and the consumer's not to bind. If both the first bank profit and first country participation constraints bind at  $\theta^t$ , we have  $c(\theta^t) = \theta_t + P(\theta^t)$ . Hence, if  $P(\theta^t) > 0$ , the multipliers cancel in the other first order condition. Therefore, letting

$$\phi(\theta^t) = \begin{cases} \eta(\theta^s) & \text{if } P(\theta^t) = 0 \\ 0 & \text{if } P(\theta^t) > 0 \end{cases},$$

we have

$$U'(c(\theta^t)) = \left(\frac{1-q}{1-\beta}\right) \left(\frac{q}{\beta}\right)^t \frac{\gamma(\theta_0)}{1 + \sum_{s=0}^t \phi(\theta^s)}.$$

Hence, the consumption allocation that solves the bilateral borrowing problem also satisfies the first order conditions of the relaxed problem. ■

**Proof of Proposition 3.** An allocation  $\{c(\theta^t), P(\theta^t, \theta_{t+1})\}$  can be attained in a subgame perfect equilibrium of the game where the bank moves first only if it satisfies the continuing participation constraints of the country

$$(1-\beta) \sum_{s=t}^{\infty} \sum_{\theta^s | \theta^t} \beta^{s-t} \pi(\theta^s | \theta^t) U(c(\theta^s)) \geq V^A(\theta^t + P(\theta^t)),$$

with equality for all  $t$  and all  $\theta^t$ . For if the constraint did not bind at  $\theta^t$ , the bank could do better by making the same offer of committed payments with a slightly lower consumption



today. As the bank makes offers, and as it is always feasible to offer not to trade, the bank never makes negative profits. The result then follows from the duality of the profit maximization and utility maximization problems for an appropriately chosen  $P(\theta_0)$ . ■

**Proof of Proposition 4.** Given  $\{c(\theta^t)\}$ , let  $\sigma$ , where

$$\sigma = \{P(\theta_{t+1}, \theta^t), \tau(\theta^t), d(\theta^t)\},$$

be known as the “expected transfer profile”. After a history of endowment shocks  $\theta^t = (\theta^{t-1}, \theta_t)$ , for all  $\theta^s$  that continue  $\theta^t$  denote by

$$\sigma(\theta^t, 0) = \{P(\theta_{s+1}, \theta^s; \theta^t, 0), \tau(\theta^s; \theta^t, 0), d(\theta^s; \theta^t, 0)\},$$

the profile of payments corresponding to the solution of the bilateral programming problem starting from  $\theta_t + P(\theta^t)$  with zero profits to the bank ( $\Pi = 0$ ), and by

$$\sigma(\theta^t, V^A(\theta^t)) = \{P(\theta_{s+1}, \theta^s; \theta^t, V^A(\theta^t)), \tau(\theta^s; \theta^t, V^A(\theta^t)), d(\theta^s; \theta^t, V^A(\theta^t))\},$$

the profile that gives the country the autarkic value.

Consider the following strategy profile. For the bank, as long as the country made the expected deposit in  $\theta^{t-1}$ , make the expected loan, and promise the expected payments for  $\theta_{t+1}$ , at  $\theta^t$ . If the country did not make the expected deposit, make no loan or promise payments and update the expected profile to  $\sigma(\theta^t, V^A(\theta^t))$ . For the country, as long as the bank has made the expected loan in  $\theta^{t-1}$  and offered payments in  $\theta^t$ , make the expected deposits and repayments in  $\theta^t$ . If the bank did not make the expected loan in  $\theta^{t-1}$ , or promise the expected payments in  $\theta^t$ , make no deposit or repayment and update the expected profile to  $\sigma(\theta^t, 0)$ .

It is easily verified that this is a subgame perfect equilibrium; on the equilibrium path, no agent has a unilateral incentive to deviate by virtue of the fact that the transfers and payments satisfy the constraints (2) and (3) by construction. Off the equilibrium path, there are no unilateral incentives to deviate. For example, if the bank promised an inferior set of payments, the new expected profile gives the country the maximum possible surplus. The country can only get less by offering a different deposit. The bank can do no better by not making the expected loan and promising the expected repayments.

By construction, the allocations generate payoffs on the Pareto frontier of subgame equilibrium payoffs in all subgames so that there is no joint incentive to deviate. ■

**Proof of Proposition 5.** Assume not. Then let

$$\sigma = \{P^m(\theta_{t+1}, \theta^t), \tau(\theta^t), d(\theta^t)\},$$

be the profile of transfers and payments from such a strategy for banks  $m = 1, \dots, M - 1$ , and construct from  $\sigma$  the sequence of deposits at bank  $m$  by

$$d^m(\theta^t) = q \sum_{\theta_{t+1}} \pi(\theta_{t+1} | \theta_t) P^m(\theta_{t+1}, \theta^t).$$

In what follows we suppress the bank superscript when referring to aggregates over banks one through  $M$ . We interpret

$$r(\theta^r) = d(\theta^r) - d^m(\theta^r),$$

as repayments on past borrowings.

The result will be established if we can demonstrate the existence of a profitable deviation for one of the parties. Note that in any subgame perfect equilibrium, the country accepts the current contract that gives them the highest payoff (rejecting an offer is never part of a subgame perfect equilibrium). The proof follows by utilizing the construction of Theorem 1 in Bulow and Rogoff [6] to show that there exists a subgame in which it is feasible for one of the banks not involved in lending to the country to deviate and offer a deposit contract that the country would strictly prefer to accept. As this contract is feasible and makes the country better off, the optimal deposit contract does at least as well. The deviation we consider is then one in which a bank that was not a part of the original lending agreement charges an up-front fee to the country for access to the optimal deposit contract in the future. This can be done in such a way as to give the bank positive profits, and leave the country better off. It is also invulnerable to the best response of the other banks, which is simply to offer an optimal deposit contract.

By assumption there is borrowing by the country, and so there exists a  $\theta^t$  such that  $l(\theta^t) > 0$ . The fact that the strategy profile is subgame perfect means that it satisfies (3), which implies that there exists a  $\theta^s$  that continues  $\theta^t$  such that  $r(\theta^s) > 0$ . For each  $\theta^s$ , construct

$$\begin{aligned} W(\theta^s) &= \sum_{r=s}^{\infty} q^{r-s} \sum_{\theta^r | \theta^s} \pi(\theta^r | \theta^s) \theta_r, \\ D(\theta^s) &= \sum_{r=s}^{\infty} q^{r-s} \sum_{\theta^r | \theta^s} \pi(\theta^r | \theta^s) [r(\theta^r) - \tau(\theta^t)], \end{aligned}$$

which can be interpreted as the discounted expected values of the endowment stream, and of the payment stream on debts. The present value of the endowment stream is strictly positive.

Define

$$k = \sup_{\theta^s} \frac{D(\theta^s)}{W(\theta^s)}.$$

Hence  $k \in (0, 1)$ . As  $\min \Theta > 0$ , there exists a  $\theta^s \in B$  such that

$$D(\theta^s) \geq k(W(\theta^s) - \theta_s).$$

Fix

$$d(\theta^s) = r(\theta^s) + k(W(\theta^s) - \theta_s) - D(\theta^s) \leq r(\theta^s),$$

and for all  $\theta^t$  that continue  $\theta^s$ , we can construct a zero profit deposit contract by setting

$$\begin{aligned} P(\theta^t) &= kW(\theta^t) - D(\theta^t), \\ d(\theta^t) &= P(\theta^t) + r(\theta^s) - k\theta_s. \end{aligned}$$

Note that as  $r(\theta^r) \leq \theta_r$ , we have  $k \leq 1$ , and hence by construction the  $P(\theta^t) \geq 0$ . Further,

$$\begin{aligned} q \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta_t) P(\theta^t, \theta_{t+1}) &= q \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta_t) [kW(\theta^{t+1}) - D(\theta^{t+1})] \\ &= q \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta_t) [kW(\theta^{t+1}) - D(\theta^{t+1})] \\ &= q [k(W(\theta^t) - \theta_t) - D(\theta^t) + r(\theta^t)] \\ &= d(\theta^t), \end{aligned}$$

where the third line comes from the construction of  $W(\theta^t)$  and  $D(\theta^t)$ , and the fourth by construction of  $P(\theta^t)$  and  $d(\theta^t)$ . But payments are no higher in the first period, and are strictly less in every period thereafter by

$$d(\theta^t) = P(\theta^t) + r(\theta^s) - k\theta_s < P(\theta^t) + r(\theta^s).$$

Hence consumption is no lower in the first period, and is strictly higher thereafter. ■

**Proof of Proposition 6.** Define  $M^* = \inf \{X(\theta^t) / \Pi(\theta^t)\}$ , and assume that  $M \leq M^*$ . Obviously,  $M^* > 1$ , for if the country were to pay the entire present value of its expected future repayments for the privilege of accessing deposits, it would have the same wealth and less opportunities to smooth consumption.

Given  $\{c(\theta^t)\}$ , construct the expected profile of payments and transfers  $\{P(\theta^t), d(\theta^t), l(\theta^t), r(\theta^t)\}$ . We consider a situation in which the banks are symmetric, each making and receiving one  $M$ 'th of the transfers and payments. For future reference, after a history of endowment shocks  $\theta^t = (\theta^{t-1}, \theta_t)$ , for all  $\theta^s$  that continue  $\theta^t$  denote by

$$\sigma(\theta^t, 0) = \{P(\theta_{s+1}, \theta^s; \theta^t, 0), d(\theta^s; \theta^t, 0), l(\theta^s; \theta^t, 0), r(\theta^s; \theta^t, 0)\},$$

the profile of payments corresponding to the solution of the bilateral programming problem starting from  $\theta_t$  with zero, profits to the bank ( $\Pi = 0$ ), and by

$$\sigma(\theta^t, V^A(\theta^t)) = \{P(\theta_{s+1}, \theta^s; \theta^t, V^A(\theta^t)), d(\theta^s; \theta^t, V^A(\theta^t)), l(\theta^s; \theta^t, V^A(\theta^t)), r(\theta^s; \theta^t, V^A(\theta^t))\}.$$

Once again, we consider symmetric profiles in which each bank makes on  $M$ 'th of these payments and transfers. Note that we allow for banks to offer linear multiples of the deposit contract, as it makes zero profits. Consider the following strategy profile:

**Bank Strategy** ( $m=1, \dots, M$ ):

If the country made the expected repayments and deposits in the previous period, offer the expected payments and make the expected loans today.

If the country made less than the expected transfer in the previous period, make no loans and offer no payments, and update the expected profile to  $\sigma(\theta^t, V^A(\theta^t))$ .

If both the country and another bank did not make the expected transfers and payments, offer the optimal deposit contract forever after.

**Country Strategy:**

At each date, accept contracts from all banks  $m$  in the set  $M'$ , where  $M'$  solves

$$\max_{M' \in \mathcal{P}(M)} (1 - \beta) U \left( \theta_t - \sum_{m \in M'} \tau_m(\theta^t) \right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1} | \theta_t) V^A \left( \theta_{t+1} + \sum_{m \in M'} P_m(\theta_{t+1}, \theta^t) \right),$$

where  $\mathcal{P}(M)$  is the power set of  $M$ .

It is easily verified that this strategy profile constitutes a subgame perfect equilibrium. On the equilibrium path, no bank has an incentive to deviate to offer a different contract with lending, as the other banks will retaliate by offering the optimal deposit contract, the loan will not be repaid, and the bank would make a strictly negative profit. Also, no bank has an incentive to deviate to offer a different contract with only deposits, as the other banks will retaliate by offering the optimal deposit contract which limits to profits from such a strategy to  $X(\theta^t)$  which is less than what can be earned in the syndicate by assumption that  $M \leq M^*$ . Similarly, the country has no incentive to reject an offer by construction of the allocation as satisfying the countries participation constraints.

Off the equilibrium path, consider first a subgame in which the country has rejected an offer in the previous period. No bank has an incentive to change it's offer as, by construction, the strategy profile gives the bank syndicate the maximum profit. Nor does the country have an incentive to reject these offers, as they give no less surplus than autarky.

Now consider a subgame off the equilibrium path in which a bank has offered something other than the expected contract. The country has no incentive to deviate as they can only lower their surplus from doing so. None of the banks that did not defect have any incentive to deviate: offering any other deposit contract can only make them losses, or will not be accepted; offering to make a loan will make losses as it will not be repaid. The original defecting bank cannot gain for the same reason. ■

**Proof of Proposition 7.** The proof of this proposition will follow if we can show that the maximum a country is willing to pay for access to deposits is bounded above, and bank profits are bounded above zero, as  $R$  tends to one. These results are proven in the following three Lemmata.

**Lemma 3** *The maximum amount that the country is willing to pay for access to deposit contracts,  $X(\theta^t)$ , is bounded above as  $R \rightarrow 1$ .*

**Proof.** It is straightforward to show that if  $R' > R$ , then  $V_{R'}^D > V_R^D$ . Then for any given continuation value, the country is willing to pay less for access to deposits. Further, as long as  $V^C(\theta_t + P(\theta^t)) \geq V^A(\theta_t + P(\theta^t))$ , we have  $V^D(\theta_t + P(\theta^t)) \leq V^C(\theta_t + P(\theta^t))$ . But as  $V^A$  is independent of  $R$ , the result follows. ■

**Lemma 4** *As  $R \rightarrow 1$ ,  $\Pi(\theta^t)$  converges to the long run average level of profits.*

**Proof.** The proof will follow from the results of Dutta [10] if we can establish that our problem satisfies Dutta's "value boundedness" condition. But this follows from the arguments of Dutta's Proposition 2. ■

It remains to show that long run average profits are strictly positive. This amounts to showing that it is feasible to provide the country with a welfare level at least as large as autarky at a positive profit. The following Lemma establishes a sufficient condition for this result for the case of iid endowment for the country. The proof in the case of Markov shocks is analogous with the appropriate substitutions for the probabilities. The case of deterministic cycles is proven separately below.

**Lemma 5** *If the endowment is iid, and*

$$\frac{\pi(\theta_N)}{1 - \beta + \beta\pi(\theta_N)} \beta \frac{U'(\theta_1)}{U'(\theta_N)} > 1,$$

*then long run average profits are positive.*

**Proof.** Under this assumption, we construct an allocation for the country that is feasible with respect to incentives and gives the bank positive long run profits. Specifically, consider an allocation that is autarkic except for providing a lower level of consumption in the best state and a higher level of consumption in the worst state. For a small change of this type to satisfy the countries participation constraints when output is high, it must be that

$$(1 - \beta + \beta\pi(\theta_N)) U'(\theta_N) [C_N - \theta_N] + \beta\pi(\theta_1) U'(\theta_1) [C_1 - \theta_1] \geq 0.$$

It is easily verified that this does not violate the constraint when output is low.

Suppose we construct such a deviation that leaves the participation constraint of the agent in the high state binding. The change in the long run average profits of the bank is given by

$$\pi(\theta_N) [\theta_N - C_N] + \pi(\theta_1) [\theta_1 - C_1].$$

Substituting from the above expression we get that this equals

$$\left( \frac{\pi(\theta_N)}{1 - \beta + \beta\pi(\theta_N)} \beta \frac{U'(\theta_1)}{U'(\theta_N)} - 1 \right) \pi(\theta_1) [C_1 - \theta_1],$$

which is greater than zero under our assumption. ■ ■

**Proof of Proposition 8.** Define  $M^* = \inf \{X(\theta^t) / \Pi(\theta^t)\}$ , and assume that  $M \leq M^*$ . Obviously,  $M^* > 1$ , for if the country were to pay the entire present value of its expected future repayments for the privilege of accessing deposits, it would have the same wealth and less opportunities to smooth consumption.

Given  $\{c(\theta^t)\}$ , construct the expected profile of payments and transfers  $\{P(\theta^t), d(\theta^t), l(\theta^t), r(\theta^t)\}$ . We consider a situation in which the banks are symmetric, each making and receiving one  $M$ 'th of the transfers and payments. For future reference, after a history of endowment shocks  $\theta^t = (\theta^{t-1}, \theta_t)$ , for all  $\theta^s$  that continue  $\theta^t$  denote by

$$\sigma(\theta^t, 0) = \{P(\theta_{s+1}, \theta^s; \theta^t, 0), d(\theta^s; \theta^t, 0), l(\theta^s; \theta^t, 0), r(\theta^s; \theta^t, 0)\},$$

the profile of payments corresponding to the solution of the bilateral programming problem starting from  $\theta_t$  with zero profits to the bank ( $\Pi = 0$ ), and by

$$\sigma(\theta^t, V^A(\theta^t)) = \{P(\theta_{s+1}, \theta^s; \theta^t, V^A(\theta^t)), d(\theta^s; \theta^t, V^A(\theta^t)), l(\theta^s; \theta^t, V^A(\theta^t)), r(\theta^s; \theta^t, V^A(\theta^t))\}.$$

Once again, we consider symmetric profiles in which each bank makes on  $M$ 'th of these payments and transfers. Note that we allow for banks to offer linear multiples of the deposit contract, as it makes zero profits. Consider the following strategy profile:

**Bank Strategy** ( $m=1, \dots, M$ ):

If the country made the expected repayments and deposits in the previous period, offer the expected payments and make the expected loans today.

If the country made less than the expected transfer in the previous period, make no loans and offer no payments, and update the expected profile to  $\sigma(\theta^t, V^A(\theta^t))$ .

If both the country and another bank did not make the expected transfers and payments, offer the optimal deposit contract and update expected transfers to  $\sigma(\theta^t, V^D(\theta^t))$ , which is the efficient profile giving the country the same utility as the deposit contract at  $\theta^t$ , modified in that the deviating bank is required to make a transfer to the other banks sufficient to leave it with no expected future discounted surplus.

**Country Strategy:**

If a bank has made the expected payments and loans in the previous period, make the expected repayments, and distribute among all such banks the expected deposit, in return.

If a bank  $m$  has not offered the expected payment, or did not make a loan, make no payment or transfer, and update the  $m$ 'th component of the expected payment profile to include

a loan equal in size to the expected discounted surplus to that bank of being readmitted to the expected payment profile.

On the equilibrium path, no player has a unilateral incentive to deviate by construction of the profile and the linearity of the bank participation constraints. The grand coalition cannot do better by deviating as the profiles are on the Pareto frontier of subgame perfect equilibrium values. It remains to consider other subcoalitions. Clearly, no coalition of banks alone can do better: if they offer an inferior deposit contract, any one bank can deviate, offer a slightly better contract, and extract all of the surplus. If they make a different loan, the country will not make a repayment.

Consider a coalition of the country and one or more banks. Any deviation that involves borrowing by the country and is Pareto improving is not perfect, by the argument of Proposition ?? above as other banks offer the optimal deposit contract. Consider a deviation without borrowing. As the other banks offer the optimal deposit contract thereafter, the maximum any one bank can receive is  $X(\theta^t)$ , which is less, by assumption that  $M \leq M^*$ , than what they would have received if they had not deviated.

Off the equilibrium path, suppose a bank did not make a loan. No bank can gain by making an additional loan, and the country cannot gain by making an additional repayment, as the profile will restart with the country getting the maximum surplus from this bank. As the deviating bank can do no better by not making the extra loan next period, it will do so.

Suppose a bank did not offer the expected deposit. The country will simply redistribute its deposits amongst the other banks, and neither the country or any bank can gain by not continuing to make the expected loans and repayments.

Suppose the country did not make a repayment. No bank can gain by making a loan or offering a different deposit contract, as they will receive all the surplus from the country in the next period.

Suppose a group of banks and the country (less than the grand coalition) deviated. The banks in the coalition can do no better than zero profits. They will therefore make the new expected repayment. The remaining banks have no incentive to deviate as they get the maximum surplus in the restarted profile. ■

**Proof of Proposition 9.** In the light of Proposition 8, the proof will be complete if we can establish that there exists an  $R$  such that

$$\frac{(1 - q)[r_t - l_t] + q\Pi_{t+1}}{M} \geq (1 - q)X_t,$$

for all  $t$  and all  $M$ .

Note first that the level of profits when output is  $y_L$  is given by

$$\begin{aligned}
\Pi_L &= \frac{nx_L + q \times nx_H}{1 + q} \\
&= \frac{1}{1 + R} [R \times nx_L + nx_H] \\
&= \frac{1}{1 + R} [R \times y_L + y_H - (R \times c_L + c_H)].
\end{aligned}$$

But

$$\begin{aligned}
Rc_L + c_H &= y_H^{\frac{1}{1+\beta}} y_L^{\frac{\beta}{1+\beta}} \left[ \beta^{\frac{1}{1+\beta}} R^{\frac{2+\beta}{1+\beta}} + \beta^{\frac{-\beta}{1+\beta}} R^{\frac{-\beta}{1+\beta}} \right] \\
&= (\beta y_H)^{\frac{1}{1+\beta}} (R y_L)^{\frac{\beta}{1+\beta}} \left[ R^{\frac{2}{1+\beta}} + \frac{1}{\beta} R^{\frac{-2\beta}{1+\beta}} \right] \\
&< [y_H + R y_L] \frac{\beta}{1 + \beta} \left[ R^{\frac{2}{1+\beta}} + \frac{1}{\beta} R^{\frac{-2\beta}{1+\beta}} \right],
\end{aligned}$$

where the last line follows from the properties of geometric and arithmetic means. But this is true for all  $R$ . As

$$R^{\frac{2}{1+\beta}} + \frac{1}{\beta} R^{\frac{-2\beta}{1+\beta}},$$

is continuous in  $R$ , there exists an  $R^* > 1$  such that  $\Pi_L > 0$  for all  $R \in [1, R^*)$ .

For  $R \in [1, R^*)$ , as  $\Pi_L > 0$ , we have that  $P_L = 0$ . Hence,  $d_H = 0$  (there are no deposits, but there could be further borrowing). As a result,  $r_H - l_H \geq nx_H$ . Then it is sufficient to establish the stronger result, that there exists an  $R$  such that

$$\frac{\Pi_t}{M} \geq (1 - q) X_t,$$

for all  $t$  and all  $M$ .

Note first that by Lemma 2,  $V^D$  is decreasing pointwise in  $R$ . As the value of autarky is independent of  $R$ , this implies that  $X_H$ , the value taken on by  $X_t$  when output is  $y_H$  is decreasing in  $R$ . Hence, as  $R \rightarrow 1$ ,  $(1 - q) X_H \rightarrow 0$ . Hence, it suffices to show that as  $R \rightarrow 1$ ,  $\Pi_H > 0$ . Now

$$\begin{aligned}
\Pi_H &= \frac{nx_H + q \times nx_L}{1 + q} \\
&= \frac{1}{1 + R} [R \times nx_H + nx_L] \\
&= \frac{1}{1 + R} [R \times y_H + y_L - (R \times c_H + c_L)].
\end{aligned}$$



But

$$\begin{aligned}
Rc_H + c_L &= y_H^{\frac{1}{1+\beta}} y_L^{\frac{\beta}{1+\beta}} \left[ (\beta R)^{\frac{1}{1+\beta}} + \beta^{\frac{-\beta}{1+\beta}} R^{\frac{1}{1+\beta}} \right] \\
&= (\beta R y_H)^{\frac{1}{1+\beta}} y_L^{\frac{\beta}{1+\beta}} \frac{1+\beta}{\beta} \\
&< R y_H + y_L,
\end{aligned}$$

where the last line follows from the properties of geometric and arithmetic means, and the fact that  $\beta R y_H \geq \beta y_H > y_L$  by assumption. But this is true for all  $R$ , including  $R \in [1, R^*)$ , which proves the result. ■

**Proof of Proposition 10.** Assume that (4) is satisfied.

Given  $\{c(\theta^t)\}$ , construct the expected profile of payments and transfers

$$\sigma^m = \{P(\theta^t), d(\theta^t), \tau(\theta^t)\},$$

for each bank-country pair. For future reference, after a history of endowment shocks  $\theta^t = (\theta^{t-1}, \theta_t)$ , for all  $\theta^s$  that continue  $\theta^t$  denote by

$$\sigma^m(\theta^t, 0) = \{P(\theta_{s+1}, \theta^s; \theta^t, 0), d(\theta^s; \theta^t, 0), \tau(\theta^s; \theta^t, 0)\},$$

the profile of payments corresponding to the solution of the bilateral programming problem starting from  $\theta_t$  with zero profits to the bank ( $\Pi = 0$ ), and by

$$\sigma^m(\theta^t, V^A(\theta^t)) = \{P(\theta_{s+1}, \theta^s; \theta^t, V^A(\theta^t)), d(\theta^s; \theta^t, V^A(\theta^t)), \tau(\theta^s; \theta^t, V^A(\theta^t))\}.$$

As before, we allow both banks to offer deposits to a country, but only one offers loans and expects repayments. Consider the following strategy profile:

**Bank Strategy** ( $m=1,2$ ):

If country  $m$  made the expected repayments and deposits in the previous period, offer the expected payments and make the expected loans today.

If country  $m$  made less than the expected transfer in the previous period, make no loans and offer no payments, and update the expected profile to  $\sigma(\theta^t, V^A(\theta^t))$ .

If both country  $m$  and bank  $\sim m$  did not make the expected transfers and payments, offer the optimal deposit contract to both countries and update expected transfers to  $\sigma(\theta^t, V^D(\theta^t))$ , which is the efficient profile giving the country the same utility as the deposit contract at  $\theta^t$ , modified in that the deviating bank is required to make a transfer to the other banks sufficient to leave it with no expected future discounted surplus.

**Country Strategy** ( $m=1,2$ ):

If bank  $m$  has made the expected payments and loans in the previous period, make the expected repayments and deposits.

If bank  $m$  has not offered the expected payment, or did not make a loan, make no payment or deposit with  $m$ , and update the  $m$ 'th component of the expected payment profile to include a loan equal in size to the expected discounted surplus to that bank of being readmitted to the expected payment profile.

On the equilibrium path, no player has a unilateral incentive to deviate by construction of the profile and the linearity of the bank participation constraints. The grand coalition cannot do better by deviating as the profiles are on the Pareto frontier of subgame perfect equilibrium values. It remains to consider other subcoalitions. Clearly, no coalition of banks alone can do better: if they offer an inferior deposit contract, any one bank can deviate, offer a slightly better contract, and extract all of the surplus. If they make a different loan, the country will not make a repayment.

Consider a coalition of the country and one or more banks. Any deviation that involves borrowing by the country and is Pareto improving is not perfect, by the argument of Proposition ?? above as the other bank offer the optimal deposit contract. Consider a deviation without borrowing. As the other banks offer the optimal deposit contract thereafter, the maximum any one bank can receive is  $X(\theta^t)$ , which is less, by assumption than what the bank would have received if it had not deviated.

Off the equilibrium path, suppose a bank did not make a loan. No bank can gain by making an additional loan, and the country cannot gain by making an additional repayment, as the profile will restart with the country getting the maximum surplus from this bank. As the deviating bank can do no better by not making the extra loan next period, it will do so. The grand coalition cannot do better as the profiles generate payoffs on the Pareto frontier of subgame equilibrium values.

Suppose a bank did not offer the expected deposit. The country will simply redistribute its deposits to the other bank, and neither the country or any bank can gain by not continuing to make the expected loans and repayments.

Suppose country  $m$  did not make a repayment or deposit. Bank  $m$  cannot gain by making a loan or offering a different deposit contract, as they will receive all the surplus from the country in the next period. The other bank cannot gain by making a loan as bank  $m$  will offer the optimal deposit contract and the bank will never get a repayment. The other bank cannot gain by offering the deposit contract by the fact that 4) is satisfied.

Suppose a coalition of bank  $m$  and country  $\sim m$  deviated. The banks in the coalition can do no better than to make the expected repayment, and the country has no incentive but to go along as the new profile gives at least as much utility by construction. The payoffs are on the frontier of subgame perfect equilibrium values, and so the grand coalition cannot do any better. ■