Department of Economics
University of California
Berkeley

Spring 2019
Economics 100B
Professor Olney

PROBLEM SET #2

2 % of grade; 13 points possible; max you can earn=10 points
DUE: via gradescope, 10:00.00 a.m. on Friday February 15
See Jan. 30 email from Prof. Olney & links on Econ 100B website re submitting PS on gradescope

Problem sets must be uploaded and submitted by 10 am. No problem sets accepted after gradescope window closes. Your submitted work must be your own: Problem sets that are identical (in whole or in part) to another student's problem set will receive a zero.

Follow these guidelines. Write your GSI's name and your name on the first answer sheet. You can hand write or type your answers; just be sure they are neat and legible. No answer sheet is provided; use your own paper if you are writing by hand. When you are done, you will scan your problem set answers(see gradescope hints on course website) and convert to a pdf file. No need to scan this sheet of questions. You are responsible for submitting the right pdf file. We will grade what you submit, not circle back to you to tell you that you gave us the wrong file. You must then tell gradescope on which page we will find your answers. If you don't tell gradescope which page your answer is on, we won't grade what you submit. Gradescope is accessible via our bcourses site.

1. (2 points total, 1 point each)

A. Starting from the national income identity (Y = C + I + G + GX - IM), derive the expression sY = I.

B. Starting from $\frac{Y}{L}=\left(\frac{K}{L}\right)^{\alpha}E^{(1-\alpha)}$, derive the expression $\frac{Y}{L}=\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}E$

2. (1 point total, ½ point each)

Assume the production function is the usual Cobb-Douglas production function, $\frac{Y}{L}=\left(\frac{K}{L}\right)^{\alpha}E^{1-\alpha}$.

A. What is the value of output per worker per year when K = $90,000,000 million, L = 150 million workers, E = $75,000 per year, and α = 0.33? Put a box around your numerical answer.

B. Suppose instead the value of α = 0.4. Now what is the value of output per worker per year? Put a box around your numerical answer.

3. (2 points; 1 point each)

Starting from the Cobb-Douglas production function $\frac{Y}{L}=\left(\frac{K}{L}\right)^{\alpha}E^{1-\alpha}$ answer these two questions.

A. In which case, when α = 0.33 or when α = 0.4, are the returns to investment greater?

B. Explain, in words that would make sense to someone who doesn't yet understand, what it means for the returns to investment to be greater. (Remember this pedagogical trick: when someone doesn't understand an abstract concept, it often helps to start with an example.)

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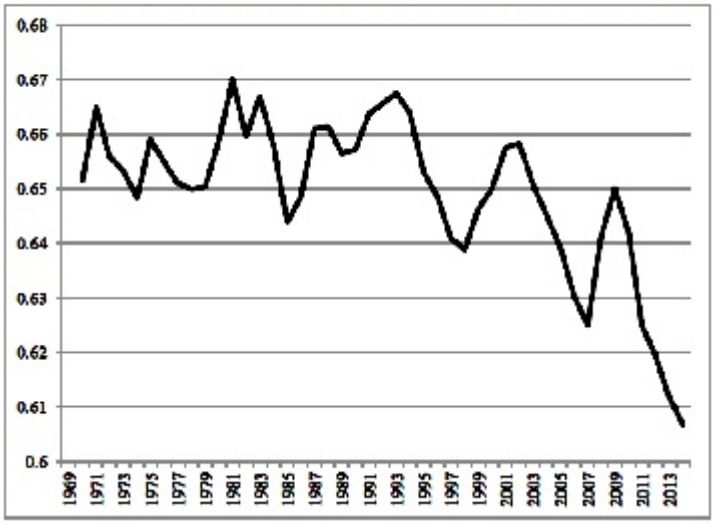
4. (2 points; 1 point each)

A. The value of α measures capital's share of total income, Y_K/Y. The value of 1-α measures labor's share of income, Y_L/Y. Starting from the Cobb-Douglas production function in this format $Y = A*K^{\alpha}L^{(1-\alpha)}$, where A stands for $E^{1-\alpha}$, and assuming perfectly competitive markets in which the factors of production K and L are paid their marginal product, and assuming that capital and labor are the only two inputs (so $Y_L + Y_K = Y$), derive the expression that shows labor's share of income equals 1-α.

B. In an NBER working paper by Robert Lawrence (https://www.nber.org/papers/w21296), Lawrence shows in Figure 1 (repeated here) that labor's share of income in the U.S. has declined from about 66% to about 60% over the last two decades. Comparing today with the early 1990s, does this fact tell you that α is larger or smaller today than it was in the early 1990s? Does it tell you that returns to investment are larger or smaller today than they were in the early 1990s?

Figure 1: Share of labor compensation in US national income 1969 to 2013



5. (3 points total; ½ point each)

A. The definition of the balanced growth equilibrium is that *what* is balanced with *what*? What is the equation that expresses the balanced growth equilibrium condition?

B. From the equation expressing the balanced growth equilibrium condition, derive the implication: K/Y is constant in BGE.

C. From the equation expressing the balanced growth equilibrium condition and using a Cobb-Douglas production function, derive the implication: in BGE, the growth rate of the standard of living (Y/L) equals the growth rate of efficiency.

D. Suppose n = 0.01, g = 0.02, δ = 0.04, s = 0.21, α = ⅓ and E = 8,000. What is the value of the capital-output ratio when the economy is in balanced growth equilibrium? What is the balanced-growth-equilibrium value of output per worker when E=8,000?

E. When E = 8,000 and K/L = 4,000, is this economy in balanced growth equilibrium? Give two different mathematical conditions, both of which support your answer.

F. Provide expressions that show, for the economy described in part d, the BGE values of Y/L and K/L after 10 years.

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6. (2 points total, ½ point each)

(From the textbook, page 117) Suppose that Mexico in 2000 was on its balanced-growth path. Output per worker in Mexico in the year 2000 was about $10,000 per year. Labor-force growth was 2.5 percent per year. The depreciation rate was 3 percent per year. The rate of growth of the efficiency of labor was 2.5 percent per year. The saving rate was 16 percent of GDP. The diminishing-returns-to-investment parameter α is 0.5

A. What is Mexico's equilibrium capital-output ratio? What was the value of efficiency in 2000?

B. What is your forecast of output per worker in Mexico for 2040?

C. Suppose instead the labor-force growth rate fell to 1 percent per year in 2000 and remained at 1 percent. What is your new forecast of output per worker in Mexico for 2040?

D. In words, not equations, explain why your forecasted level of the standard of living changed as a result of the lower labor-force growth rate.

7. (1 point total)

Start from the Cobb-Douglas production function $\frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha} E^{\beta}$ where the parameters α and β are constant, $0 < \alpha < 1$ and $\alpha + \beta = 1.2$. Derive the expression for the growth rate of output per worker in balanced growth equilibrium. Show all your steps. In this case, will the growth rate of output per worker be equal to, less than, or greater than the growth rate of efficiency in balanced growth equilibrium?