Suggested Solutions for Problem Set #1

1. *You needed to upload the handwritten honor pledge. Without that, you get zero points on the entire problem set.*

2. *GDP = C + I + G + GX - IM. Explain why we subtract IM rather than just ignore them.*

We subtract IM because C, I, and G all contain purchases of both domestically-produced and foreign-produced goods and services. But GDP is production of just domestically-produced goods and services. Therefore aggregate demand must also be total demand for just domestically-produced goods and services. (remember: AD = C + I + G + GX – IM) Therefore we need to subtract out the demand for foreign-produced goods and services. We do that by subtracting IM, which is the sum of consumption of foreign-produced goods and services, business investment in foreign-produced capital goods, and government agency purchases of foreign-produced goods and services.

3. *Starting from the standard Cobb-Douglas production function, derive the expression* $\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} E$

$\frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha} E^{1-\alpha}$	Cobb-Douglas form of production function
$\frac{Y}{L} = \left(\left(\frac{K}{L}\right)\left(\frac{Y}{Y}\right)\right)^{\alpha} \cdot E^{1-\alpha}$	Multiply K/L by Y/Y = 1
$\frac{Y}{L} = \left(\left(\frac{K}{Y}\right)\left(\frac{Y}{L}\right)\right)^{\alpha} \cdot E^{1-\alpha}$	Rearrange to get terms that have meaning, K/Y & Y/L
$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\alpha}\left(\frac{Y}{L}\right)^{\alpha} E^{1-\alpha}$	Exponent rule
$\frac{\frac{Y}{L}}{\left(\frac{Y}{L}\right)^{\alpha}} = \left(\frac{K}{Y}\right)^{\alpha} E^{1-\alpha}$	Gather like terms
$\left(\frac{Y}{L}\right)^{1-\alpha} = \left(\frac{K}{Y}\right)^{\alpha} E^{1-\alpha}$	Exponent rule
$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} E$	Raise both sides to the power $\frac{1}{1-\alpha}$; simplify using exponent rules

4A. From the equation expressing the balanced growth equilibrium condition, derive the implication: K/Y is constant in BGE.

Two approaches, both equally valid. If Y/K is constant, then K/Y is constant.

g(Y/L) = g(K/L)	g(Y/L) = g(K/L)	definition of BGE
g(Y/L) – g(K/L) = 0	0 = g(K/L) – g(Y/L)	algebra
g[(Y/L) / (K/L)] = 0	g[(K/L) / (Y/L)] = 0	growth rules (see PS 1, #2)
g(Y/K) = 0	g(K/Y) = 0	algebra, L's cancel
Y/K constant	K/Y constant	meaning of zero growth

4B. From the equation expressing the balanced growth equilibrium condition and using a Cobb-Douglas production function, derive the implication: in BGE, the growth rate of the standard of living (Y/L) equals the growth rate of efficiency.

Two approaches, both equally valid.

$g\left(\frac{K}{L}\right) = g\left(\frac{Y}{L}\right)$	$g\left(\frac{K}{L}\right) = g\left(\frac{Y}{L}\right)$	BGE condition
$g\left(\frac{K}{L}\right) = g\left(\left(\frac{K}{L}\right)^{\alpha} E^{1-\alpha}\right)$	$g\left(\frac{K}{L}\right) = g\left(\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \cdot E\right)$	Substitute in production function
$g\left(\frac{K}{L}\right) = \alpha \cdot g\left(\frac{K}{L}\right) + (1-\alpha) \cdot g(E)$	$g\left(\frac{K}{L}\right) = \left(\frac{\alpha}{1-\alpha}\right) \cdot g\left(\frac{K}{Y}\right) + g(E)$	Growth rules
$(1-\alpha) \cdot g\left(\frac{K}{L}\right) = (1-\alpha) \cdot g(E)$	$g\left(\frac{K}{L}\right) = 0 + g(E)$	Algebra (col. 1); invoking K/Y constant in BGE (col. 2)
$g\left(\frac{K}{L}\right) = g(E)$		Algebra (col. 1)
$g\left(\frac{Y}{L}\right) = g\left(\frac{K}{L}\right) = g(E)$	$g\left(\frac{Y}{L}\right) = g\left(\frac{K}{L}\right) = g(E)$	BGE condition

5.

Suppose s = 36 % = 0.36, n = ½ % = 0.005, g = 2 % = 0.02, δ = 1.5 % = 0.015, α = ⅓ and Eo =$20,000 per year.

a. In BGE, what are the values of K/Y, Y/L, and K/L when E = Eo?

To find the value of the capital-output ratio when the economy is in balanced growth equilibrium, we use the equation for the value of K/Y in balanced growth equilibrium condition: $\frac{K}{Y} = \frac{s}{n+g+\delta}$

$$\frac{K}{Y} = \frac{s}{n+g+\delta} = \frac{0.36}{0.005+0.02+0.015} = \frac{.36}{.04} = \frac{36}{4} = 9$$

In balanced growth equilibrium, the value of the capital-output ratio K/Y = 9

To determine the value of output per worker Y/L when the economy is in balanced growth equilibrium, we use this form of the production function: $\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} E$. Plug in the values of K/Y, α, and E to get:

$$\frac{Y}{L} = (9)^{\frac{1/3}{2/3}} \cdot 20{,}000 = 9^{\frac{1}{2}} \cdot 20{,}000 = 3 \cdot 20{,}000 = \$60{,}000 \; per\ worker\ per\ year$$

The easiest way to determine the value of K/L is to multiply K/Y and Y/L:

$$\frac{K}{L} = \left(\frac{K}{Y}\right)\left(\frac{Y}{L}\right) = 9 * 60{,}000 = \$540{,}000 \; per\ worker\ at\ time\ t$$

Note that we have assumed that g = 0.02, which means that efficiency *E* will increase from one year to the next. So in the next year, efficiency (E) will equal 20,000*(1.02) = $20,400. And so next year, the balanced-growth-equilibrium output per worker will be $61,200 per worker. In the year after that, BGE Y/L will equal $62,424 per worker. So the value of BGE annual output per worker of $60,000 is not "what Y/L will be forever, and ever" but "what Y/L will be in BGE when the annual value of E equals $20,000."

b. Provide expressions that show the BGE values of Y/L and K/L after 10 years. Use your expressions to compute the BGE values of Y/L and K/L after 10 years.

We know that when the economy is in BGE, $g(E) = g\left(\frac{Y}{L}\right) = g\left(\frac{K}{L}\right)$. From the prompt we know that g(E) = 0.02. Therefore in BGE, Y/L and K/L will also grow by 2 percent per year.

Therefore, $\left(\frac{Y}{L}\right)_{t+10} = \left(\frac{Y}{L}\right)_t \cdot (1 + growth\ rate)^{10} = \left(\frac{Y}{L}\right)_t \cdot (1.02)^{10}$

And $\left(\frac{K}{L}\right)_{t+10} = \left(\frac{K}{L}\right)_t \cdot (1 + growth\ rate)^{10} = \left(\frac{K}{L}\right)_t \cdot (1.02)^{10}$.

The values are therefore: $\left(\frac{Y}{L}\right)_{t+10} = 60{,}000(1.02)^{10} = \$73{,}140\ per\ worker\ per\ year$

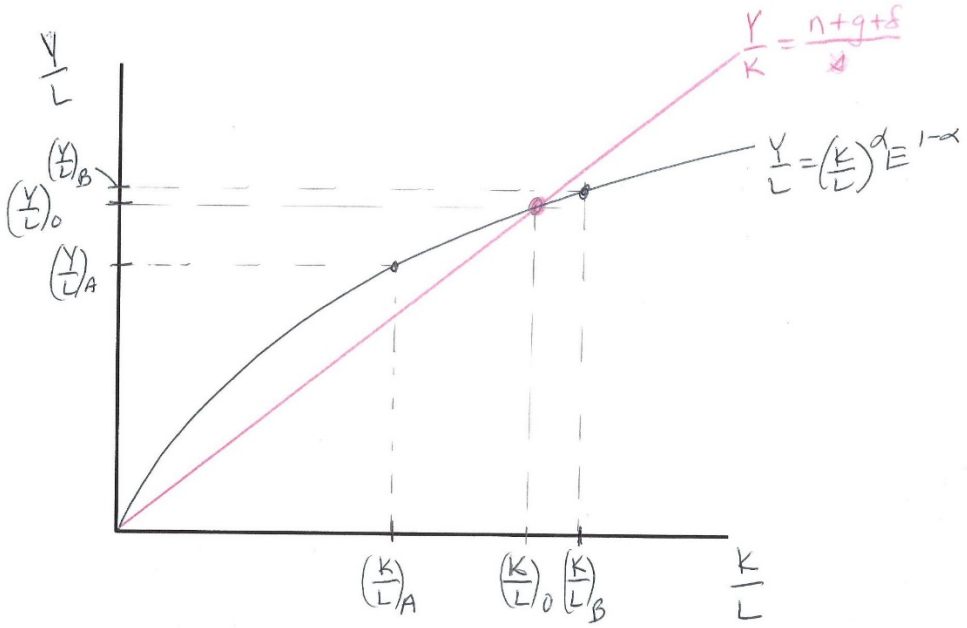
And $\left(\frac{K}{L}\right)_{t+10} = 540{,}000(1.02)^{10} = \$658{,}257\ per\ worker\ in\ year\ 10$

6. Consider two economies – Economy A and Economy B – that are initially in BGE. Both economies have the same production function. Initially both economies have the same levels of output per worker. Then war between Economy A and Economy B begins. The battles of the war take place only in Economy A, destroying about 25 percent of its capital stock. In both economies, about 1 percent of the labor force is killed in the way. Assume that there is no change to the saving rate as a result of the war Assume efficiency (E) is constant in both economies.

a. After the war ends, is Economy A in BGE? Is Economy B in BGE? Defend your answers.

Neither economy is in BGE after the economy ends. Both economies had been at their balanced growth equilibrium before the war began (stated in the prompt). In both economies, the post-war level of K/L is different than its pre-war level: K/L has increased in Economy B (no change in K, 1% drop in L) and decreased in Economy A (25% drop in K, 1% drop in L). So neither economy remains at its initial equilibrium which (per the prompt) was its BGE.

b. Draw a graph that shows the determination of the pre-war and post-war K/L and Y/L for Economy A and for Economy B.



c. Suppose that natural population growth is endogenous: in both economies, natural population growth is inversely related to output per worker. Explain why the war will therefore cause the economies of these two countries to diverge.

A change in the natural population growth rate will change the value of n, the growth rate of the labor force. That will change the value of K/Y, the BGE value of the capital-output ratio.

Immediately after the war, Y/L has risen in economy B. This will make its natural population growth rate decline, lowering the value of n in economy B. A lower value of n leads to a decrease in Y/K (and increase in K/Y). On the graph, that pivots the BGE line down/flatter. The new postwar BGE values of K/L & Y/L for economy B will be higher than their prewar values.

On the other hand, for economy A, K/L and Y/L have fallen after the war. The decline in the standard of living raises natural population growth, which will increase the rate of growth of the labor force, increasing the value of n in Economy A. A higher value of n will increase Y/K (and decrease K/Y). On the graph, that pivots the BGE line up / steeper. The new postwar BGE values of K/L and Y/L for economy A will be lower than their prewar values.

In less mathy terms: the war time destruction of physical capital in Economy A lowered the standard of living in A. Because standards of living had fallen, couples had more children (interested in the linkages between fertility and economic growth? Take Econ C175, Demography.) But over time, larger families means more people entering the work force, causing the labor force to grow at a faster rate than it had in the pre-war years. In equilibrium, then, workers in Economy A end up with less capital per worker, making the workers unable to produce as much output, lowering their output per worker, which is a measure of the standard of living.

d. Redraw the graph from part B. Illustrate your answers from part C by adding to your redrawn graph.

There are 2 options for drawing the 2nd graph. One approach is to make $\left(\frac{K}{L}\right)_A$, $\left(\frac{Y}{L}\right)_A$ and $\left(\frac{K}{L}\right)_B$, $\left(\frac{Y}{L}\right)_B$ the new BGE points, drawing the new BGE lines through those points. That's consistent with the prompt and perfectly acceptable, though it would be quite the coincidence to have the immediate effects of the war line up precisely with the changes in the BGE value of Y/K. In that case your graph would look like the graph at the top of the next page.

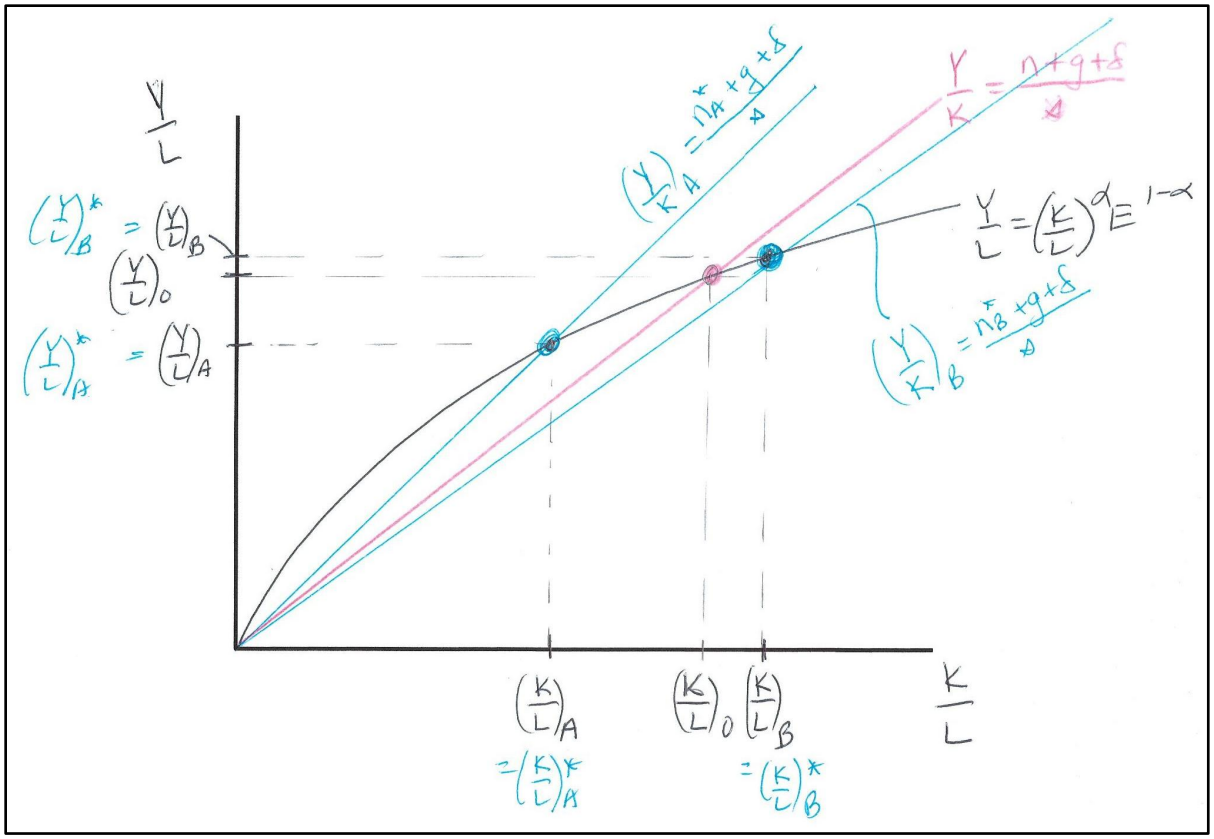
The other option is to not make the new BGE line and the immediate postwar position coincidental. Pivot the BGE line up for Economy A (whether it ends up to the left of the immediate postwar point doesn't matter; what matters is that it is to the left of the prewar point). Pivot the BGE line down for Economy B (whether it ends up to the right of the immediate postwar point doesn't matter; what matters is that it is to the right of the prewar point). In that case your graph would look like the 2nd graph on the next page.

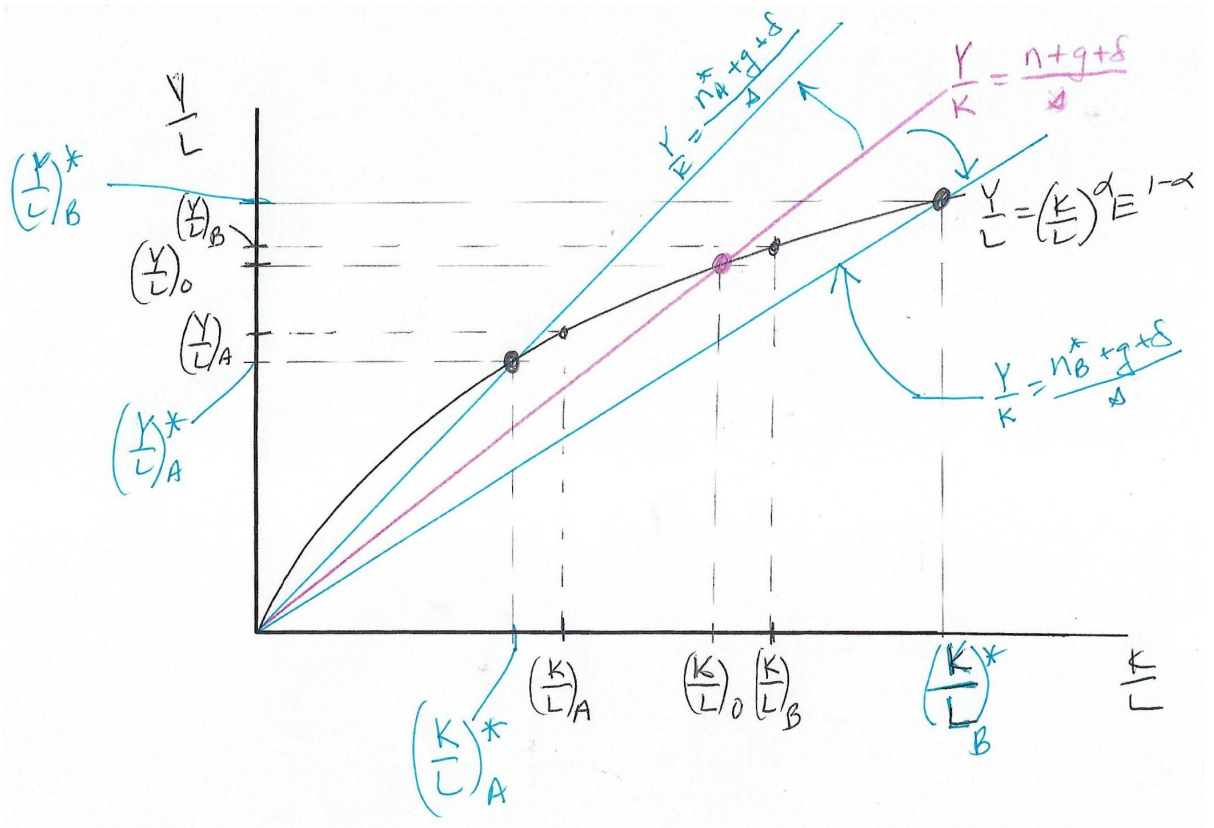
Either way, you now have 2 BGE lines. The BGE line for economy A is steeper (to the left) of the BGE line for economy B, as a result of n increasing for A and decreasing for B. This will cause the BGE values of K/L and Y/L to diverge. The destruction caused by the war will make economy A (where the war was fought) permanently poorer than economy B (which fought the war, but not on its own soil).

This is an important part of understanding the impact of war on an economy.

If we had assumed that labor force growth was exogenous, then after the war both A and B would ultimately return to their pre-war standards of living. The fact the war was found on the soil of A wouldn't have a permanent impact on A.

But if instead we assume that labor force growth is endogenous, and inversely related to the standard of living, then we come to a different conclusion about the impact of war on an economy. With the assumption of endogenous labor force growth, a war fought between two countries but <u>in</u> (on the soil of) just one of those countries will have long run effects on their standard of living, leading the country that was ravaged by war permanently poorer than the country it was at war with.


Y/L
K/L
Y/K = (n+g+δ)/α
Y/L = (K/L)^α E^(1-α)
(Y/K)_A = (n_A^* +g+δ)/α
(Y/K)_B = (n_B^* +g+δ)/α
(Y/L)_B^* = (Y/L)_B
(Y/L)_0
(Y/L)_A^* = (Y/L)_A
(K/L)_A = (K/L)_A^*
(K/L)_0
(K/L)_B = (K/L)_B^*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Y/K = (n_B^* +g+δ)/α
(Y/L)_B^*
(Y/L)_B
(Y/L)_0
(Y/L)_A
(Y/L)_A^*
(K/L)_A^*
(K/L)_A
(K/L)_0
(K/L)_B
(K/L)_B^*

Question 7: The Essay (3 points total)

Updated 1/29/2021 4 pm (updates in red)

Consider this argument: "I don't care whether government spending is to build and maintain new and efficient transportation and communication networks, to hire more teachers, or to increase payments to unemployed people. Increased government spending lowers the standard of living in the long run."

Follow steps 1, 2*, 3, and 4 of "The Olney Method" (the 5-step method Prof. Olney presented on 1/21 for critiquing arguments) and write a 1-page essay in which you critique the argument above. Your essay should reflect your understanding of how to critique an argument and your understanding of the long-run growth model.

**You need not identify all assumptions in the argument, but do need to at a minimum identify the assumption that you change in step 3.*

There are lots of approaches to take here, so we can't provide you with "this is what you should have written."

You needed to follow the specs: 400 words and 1 page max, double spaced, 10-11-12 pt font, 1" margins on all sideds, your name, date, and word count in top right corner. If you didn't follow the specs, you were penalized a point so the most you got was 2 points

You needed to also submit the essay on bcourses so that bcourses could do the plagiarism check. If you didn't submit on bcourses, you got a 0 for the essay.

Beyond that you were demonstrating two things: [1] that you understand how to critique an argument, and [2] that you understand the implications of the growth model.

An excellent or very good job on both got you full credit for the content
An excellent or very good job on one and a not good job on the other cost you 30 – 50% of the credit for the content
A not good job on both cost you 60-80% of the credit for the content.
A clueless job cost you 100% of the credit