# Firms and Labor Market Inequality: A Review

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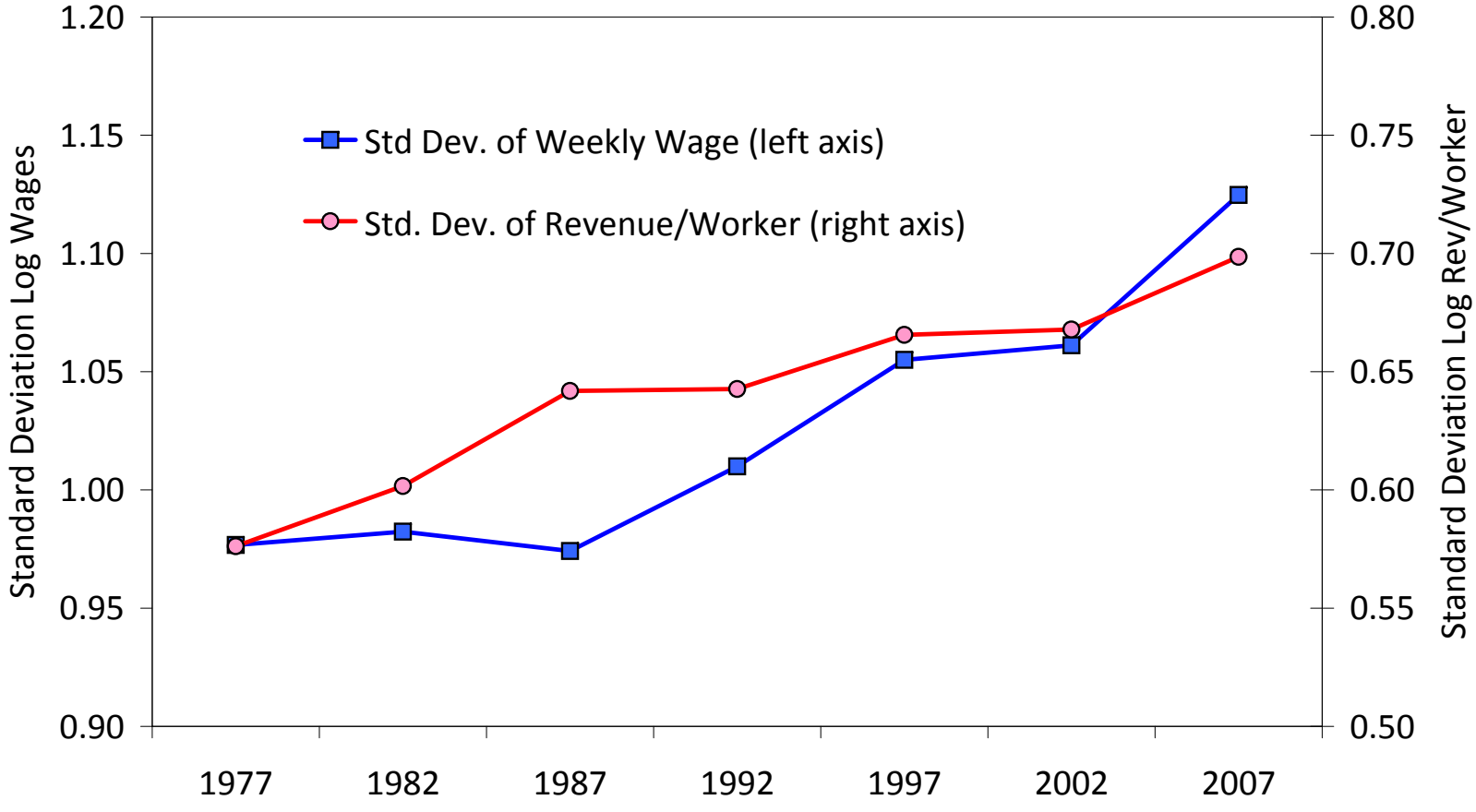
# Motivation

- Well known that firms exhibit substantial heterogeneity in productivity
  - Syverson (2004): 90-10 TFP ratios in US manufacturing $\approx 2$ (within 4-digit industry!)
  - Hseih and Klenow (2009): 90-10 ratios in India/China $\approx 4$.
- How does that heterogeneity influence wage inequality?
- Facts:
  1. Firms/plants exhibit substantial dispersion in average wages (Slichter, 1950; Davis and Haltiwanger, 1991; Groshen, 1991)
  2. Between firm/plant dispersion increasing (CHK, 2013; Song et al., 2015)
  3. Trends in productivity dispersion track trends in wage inequality (Faggio et al., 2010; Barth et al., 2015)

Figure 1: Trends in Between-Establishment Dispersion in Wages and Productivity




Source: Barth et al (2015)

# Sorting or Firm Wage Premia?

- Neoclassical models: firm heterogeneity influences who you hire, not what you pay them!
  - Industry wage premium wars (Krueger and Summers, 1989; Katz and Summer, 1988; Murphy and Topel, 1987)
- Can we get to the bottom of this w/ longitudinal E-E data?
- Two literatures address sorting concerns:
  - "Rent sharing" lit: effect of shock to firm on wages of stayers
  - "Firm movers" lit: effect of switching between firms on wages

# Today

- Review empirical literatures on rent sharing and firm movers
- Rent-sharing elasticities:
  - "Micro" estimates clustered in range 0.05-0.15
  - But room for more work w/ quasi-experimental design
- Firm movers studies:
  - Wage effects of firm moves surprisingly well characterized by additive model
  - Firm wage effects explain $\sim$20% of wage variance
  - Firm effs strongly related to productivity measures
  - But still some outstanding technical problems

# Today

- Develop a model of imperfect labor market competition capable of rationalizing findings:
  - Full information about jobs
  - Firms differentiated by TFP and workplace amenities
  - Heterogeneous preferences over amenities lead to upward sloping supply to firm

- Main insights:
  - AKM-style decomposition into worker and firm heterogeneity within broad skill groups
  - Plausible rent-sharing elasticities
  - Rising heterogeneity across firms $\Rightarrow$rising wage inequality
  - New predictions regarding relationship between skill group specific wage premia and employment shares
  - Also: "SBTC shocks"

# Rent Sharing Literature

- Basic idea:

$$\Delta \ln w_{it} = \alpha + \beta \Delta \ln Rent_{J(i,t)t} + \varepsilon_{it}$$

- Problems:
  - How to measure "rents"?
  - Want firm- (not industry-) level variation
  - Measurement error and transitory / permanent distinction
  - Mechanical negative relationship between some measures of $Rent_{jt}$ (e.g. profits) and wages
  - Stayers non-randomly selected?

# Measuring Rents

- Interested in how firm's "ability to pay" affects wages, so treat (revenue-based) TFP as "ideal" forcing variable
- Standard CRTS model:

$$\pi_j = VA_j - w_j N_j - r_j K_j,$$

$$VA_j \equiv R_j - M_j = P_j T_j f(N_j, K_j) = P_j T_j N_j g(k_j)$$

- Suppose $k_j = k^*$. Then:

$$\ln\left(\frac{VA_j}{N_j}\right) = \ln TFP_j + \ln g\left(k^*\right)$$

- Suppose also that $\frac{M_j}{R_j} = m^*$. Then:

$$\ln\left(\frac{R_j}{N_j}\right) = \ln TFP_j + \ln g(k^*) - \ln(1 - m^*).$$

# Measuring Rents

- de Menil (1971) firm and worker split "quasi-rent":

$$Q_j = VA_j - w_j^a N_j - r_j K_j$$

- Average quasi-rent:

$$\frac{Q_j}{N_j} = TFP_j g\left(k^*\right) - w_j^a - r_j k^*$$

- Can show:

$$\frac{\partial \ln(\frac{Q_j}{N_j})}{\partial \ln TFP_j} = \frac{\partial \ln(\frac{\pi_j}{N_j})}{\partial \ln TFP_j} = \frac{VA_j/N_j}{Q_j/N_j} \approx 2$$

- Bottom line: sales / VA elasticities $\approx 2\times$ quasi-rent / profit elasticities

## Summary of Estimated Rent Sharing Elasticities - Preferred Specifications, Adjusting to TFP Basis

| | | | | |
|---|---|---|---|---|
| Industry-Level Profit Measure | 1. Christofides and Oswald (1992) | 0.140 | (0.035) | Mean=0.16 |
| | 2. Blanchflower, Oswald, Sanfey (1996) | 0.060 | (0.024) | |
| | 3. Estevao and Tevlin (2003) | 0.290 | (0.100) | |
| Firm-Level Profit but Mean Wage | 4. Abowd and Lemieux (1993) | 0.220 | (0.081) | Mean=0.15 |
| | 5. Van Reenen (1996) | 0.290 | (0.089) | |
| | 6. Hildreth and Oswald (1997) | 0.040 | (0.010) | |
| | 7. Hildreth (1998) | 0.030 | (0.010) | |
| | 8. Barth et al (2014) | 0.160 | (0.002) | |
| Firm-Level Profit and Indiv. Wage | 9. Margolis and Salvanes (2001), France | 0.062 | (0.041) | Mean=0.08 |
| | 9. Margolis and Salvanes (2001), Norway | 0.024 | (0.006) | |
| | 10. Arai (2003) | 0.020 | (0.004) | |
| | 11. Guiso, Pistaferri, Schivardi (2005) | 0.069 | (0.025) | |
| | 12. Fakhfakf and FitzRoy (2004) | 0.120 | (0.045) | |
| | 13. Caju, Rycx, Tojerow (2009) | 0.080 | (0.010) | |
| | 14. Martins (2009) | 0.039 | (0.021) | |
| | 15. Guertzgen (2009) | 0.048 | (0.002) | |
| | 16. Cardoso and Portela (2009) | 0.092 | (0.045) | |
| | 17. Arai and Hayman (2009) | 0.068 | (0.002) | |
| | 18. Card, Divincienti, Maida (2014) | 0.073 | (0.031) | |
| | 19. Carlsson, Messina, and Skans (2014) | 0.149 | (0.057) | |
| | 20. Card, Cardoso, Kline (2014), Between Firm | 0.156 | (0.006) | |
| | 20. Card, Cardoso, Kline (2014), Stayers | 0.049 | (0.007) | |
| | 21. Bagger et al. (2014), Mfg | 0.090 | (0.020) | |

Table 1: Summary of Estimated Rent-Sharing Elasticities

| Study | Design Features | Measure of Profitability | Elasticity |
|---|---|---|---|
| ***A. Industry-Level Profit Measures*** | | | |
| 1. Christofides and Oswald (1992) | Canadian union contracts; 120 narrowly defined manufacturing industries | Industry profits/worker (wage changes) | 0.07 |
| 2. Blanchflower, Oswald, Sanfey (1996) | US individual wage data (CPS), grouped to industry×year cells; manufacturing only | Industry profits/worker (within-industry changes) | 0.01-0.06 |
| 3. Estevao and Tevlin (2003) | US manufacturing industry data; adjusted for labor quality; **instrument** for value-added = demand shocks in downstream sectors | Value added per worker (first differences) | 0.29 |
| | | Profit per worker (first differences) | 0.14 |
| ***B. Firm-Level Profit Measures, Average Firm-level Wages*** | | | |
| 4. Abowd and Lemieux (1993) | Canadian union contracts merged to corporate accounts; **instruments** for revenues = industry selling prices, import and export prices | Quasi-rent/worker (wage change model) | 0.22 |
| 5. Van Reenen (1996) | Large British manufacturing firms merged with corporate accounts; **instruments** for rents = innovations, imports, R&D, industry concentration | Quasi-rent/worker (wage change model) | 0.29 |
| 6. Hildreth and Oswald (1997) | British firms (EXSTAT); firm-specific profits (from financial statements); **instruments** = lagged values of wages and profits | Profit per worker | 0.02 |
| 7. Hildreth (1998) | British manufacturing establishments; establishment-specific value added; **instruments** for rents = innovation measure | Quasi-rent/worker | 0.03 |
| 8. Barth et al (2014) | US establishments in LBD. Establishment-specific revenues; **instrument** for revenues/worker = revenues/worker in same industry, other regions | Sales/worker (within-establishment changes) | OLS = 0.32<br>IV = 0.16 |

Note: Table continues.

Table 1 (continued): Summary of Estimated Rent-Sharing Elasticities

| Study | Design Features | Measure of Profitability | Elasticity |
|---|---|---|---|
| ***C. Individual Wages and Firm-Level Profit Measures*** | | | |
| 9. Margolis and Salvanes (2001) | Worker and firm data for France and Norway; full time male workers in manufacturing; profit from financial filings; **instruments** = sales/worker and subsidies/worker | Profit per worker | France: 0.03<br>Norway: 0.01 |
| 10. Arai (2003) | Swedish worker panel matched to employer (10-year stayers design); profits from financial statements | Change in 5-year average profit per worker | 0.01-0.02 |
| 11. Guiso, Pistaferri, Schivardi (2005) | Italian worker panel matched to larger firms; value added from financial statements; model-based decomposition of value-added shocks | Permanent shock to log value added per worker<br>Transitory shock to log value added per worker | 0.07<br>0.00 |
| 12. Fakhfakf and FitzRoy (2004) | Larger French manufacturing establishments; value added from establishment survey | Mean log value-added/worker over past 3 years | 0.12 |
| 13. Caju, Rycx, Tojerow (2009) | Belgian establishment panel; value added and labor cost from financial statements | Value added minus labor costs per worker | 0.03-0.04 |
| 14. Martins (2009) | Larger Portuguese manufacturing firms; revenue and capital costs from financial statements; **instruments**=export share of sales × exchange rate changes | Revenue-capital costs/worker (differenced) | 0.03-0.05 |
| 15. Guertzgen (2009) | German establishment/worker panel (LIAB) value added from establishment survey. **instruments** for change in quasi-rent = lags of value added and wages | Quasi-rent/worker (no adjustment for capital)<br>Change in quasi-rent/worker (stayers design), instrumented | 0.03-0.04<br>0.01-0.06 |

Note: Table continues.

Table 1 (continued): Summary of Estimated Rent-Sharing Elasticities

| Study | Design Features | Measure of Profitability | Elasticity |
|---|---|---|---|
| ***C. Individual Wages and Firm-Level Profit Measures*** | | | |
| 16. Cardoso and Portela (2009) | Portuguese worker panel; sales from firm reports; model-based decomposition of sales shocks | Permanent shock to log sales | 0.09 |
| | | Transitory shock to log sales | 0.00 |
| 17. Arai and Hayman (2009) | Swedish worker/firm panel (1996-2000); profits from financial statements; stayers; **instrument**=change in foreign sales | Change in profit per worker | 0.05 |
| 18. Card, Divincienti, Maida (2014) | Italian worker panel matched to firms; value added and capital from financial statements; **instrument** for value added = sales/worker at firms in other regions | Value added per worker (within job match) | 0.06-0.08 |
| 19. Carlsson, Messina, and Skans (2014) | Swedish worker panel matched to firms; mining and manufacturing only; firm-specific output and selling price indexes; **instruments** for productivity = indexes of firm-specific and sectoral TFPQ; | Firm-specfic output/worker (within-job-match) | 0.05 |
| | | Sectoral average output/worker (within-job-match) | 0.15 |
| 20. Card, Cardoso, and Kline (2014) | Portuguese worker panel matched to firms; value added and capital from financial statements; wage measure=estimated firm effect from AKM model | Mean Value added per worker | Males: 0.16<br>Females: 0.14 |
| | | Mean Value added per worker (changes for stayers) | Males: 0.05<br>Females: 0.04 |
| 21. Bagger, Christensen, and Mortensen (2014) | Danish worker panel matched to firms; output from firm survey; non-parametric regressions within sector of wages on labor productivity | Output per worker | Manuf: 0.09<br>Trade: 0.13<br>Transp/Comm: 0.05<br>Finance/Real Est: 0.07 |

# Summary

- Bigger elasticities for:
  - Aggregate wages (composition effects)
  - Aggregate shocks (equilibrium effects)
  - Permanent shocks (insurance)
  - Instrumented specifications (measurement error / mechanical)
- Would be nice to have more studies w/ E-E microdata exploiting *observable firm-level* shocks ala Van Reenen (1996)
- Converting to VA/Sales units, most elasticities $\in [0.05, 0.15]$
  - Too small to explain all covariance in trends between productivity and wage dispersion
  - But sorting can *amplify* effects of dispersion in wage premia

# Firm Movers

- Abowd, Kramarz, and Margolis (AKM, 1999):

$$\ln w_{it} = \alpha_i + \psi_{J(i,t)} + X'_{it}\beta + \varepsilon_{it}$$

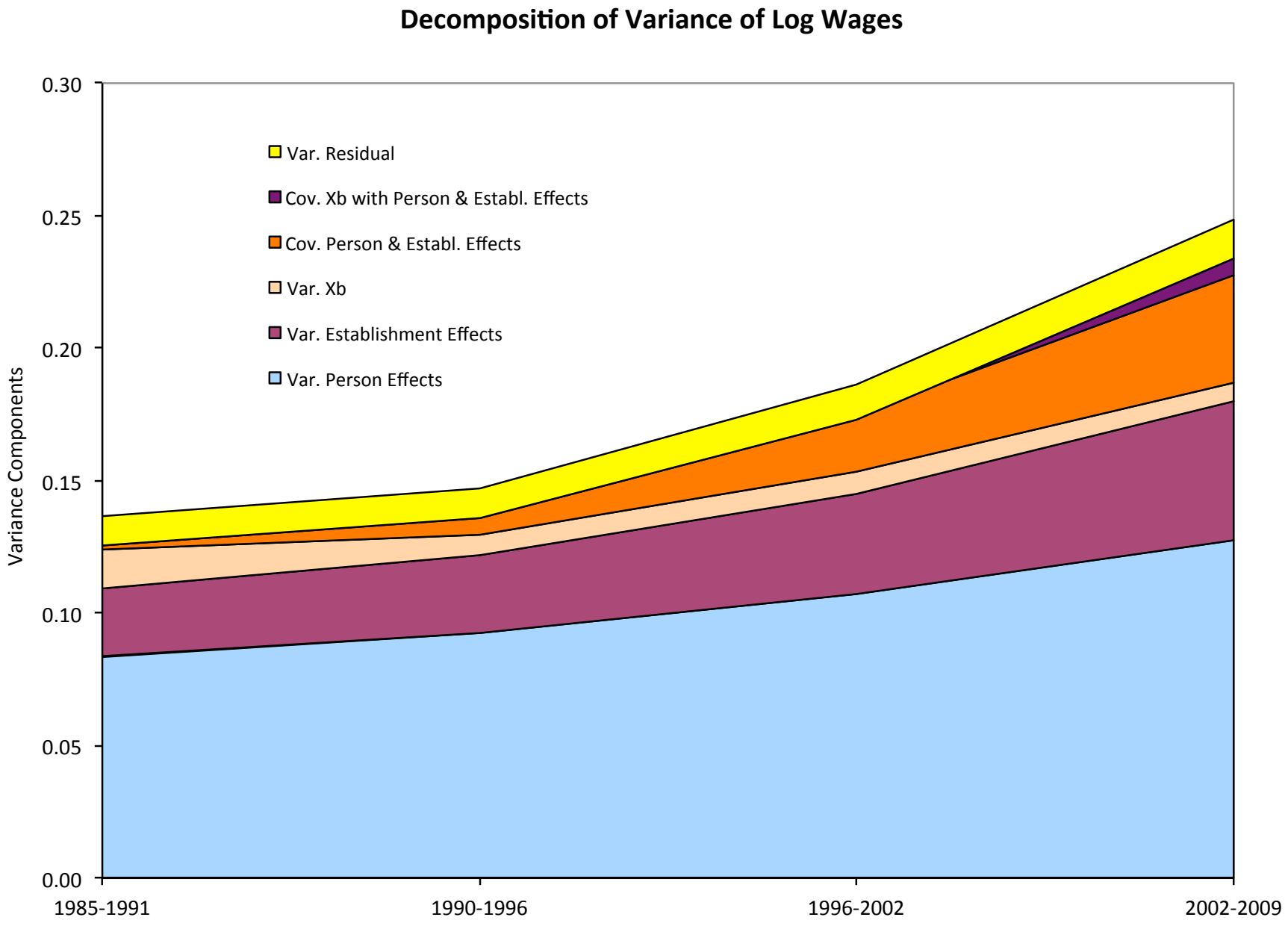
- Average wage effect of moving from firm $j$ to firm $k$ given by $\psi_k - \psi_j$

- Decomposing inequality:

$$\begin{aligned} Var\left(\ln w_{it}\right) = & \underbrace{Var\left(\alpha_i\right)}_{\text{workers}} + \underbrace{Var\left(\psi_{J(i,t)}\right)}_{\text{firms}} + Var\left(X'_{it}\beta\right) + Var\left(\varepsilon_{it}\right) \\ & + 2\underbrace{Cov\left(\alpha_i, \psi_{J(i,t)}\right)}_{\text{sorting}} + 2Cov\left(\alpha_i, X'_{it}\beta\right) + 2Cov\left(\psi_{J(i,t)}, X'_{it}\beta\right) \end{aligned}$$

- Recent studies: $\frac{Var\left(\psi_{J(i,t)}\right)}{Var(\ln w_{it})} \in [0.15, 0.25]$

# Variance Decomposition (CHK, 2013)



# Are AKM-style estimates credible?

- Additive Separability: proportional markup/down for all workers

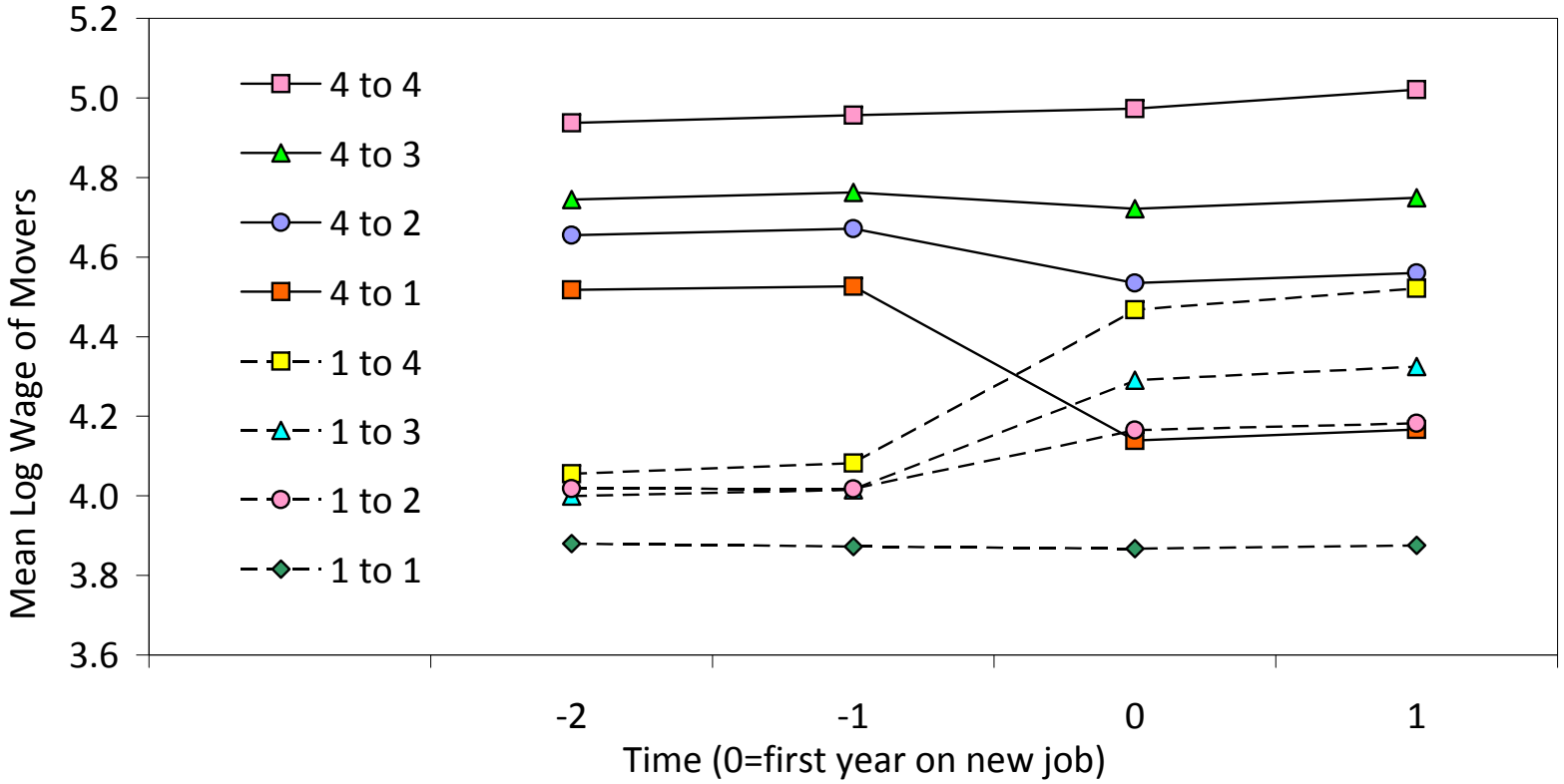
- Exogeneous mobility: no selection on time-varying errors or match component

$$P\left(J\left(i,t\right)=j|\alpha_i,\boldsymbol{\psi},\varepsilon_{i1},...,\varepsilon_{iT}\right)=P\left(J\left(i,t\right)=j|\alpha_i,\boldsymbol{\psi}\right)$$

- Statistical issues: fixed effect estimates inconsistent in short panels

# Event-studies (CHK, 2013)

Figure 2: Mean Wages of West German Male Job Changers, Classified by Quartile of Co-worker Wages at Origin and Destination (2002-09)




Notes: figure shows mean wages of male workers observed in 2002-2009 who change jobs in 2004-2007 and held the preceding job for 2 or more years, and the new job for 2 or more years. Jobs are classified into quartiles based on mean wage of co-workers.

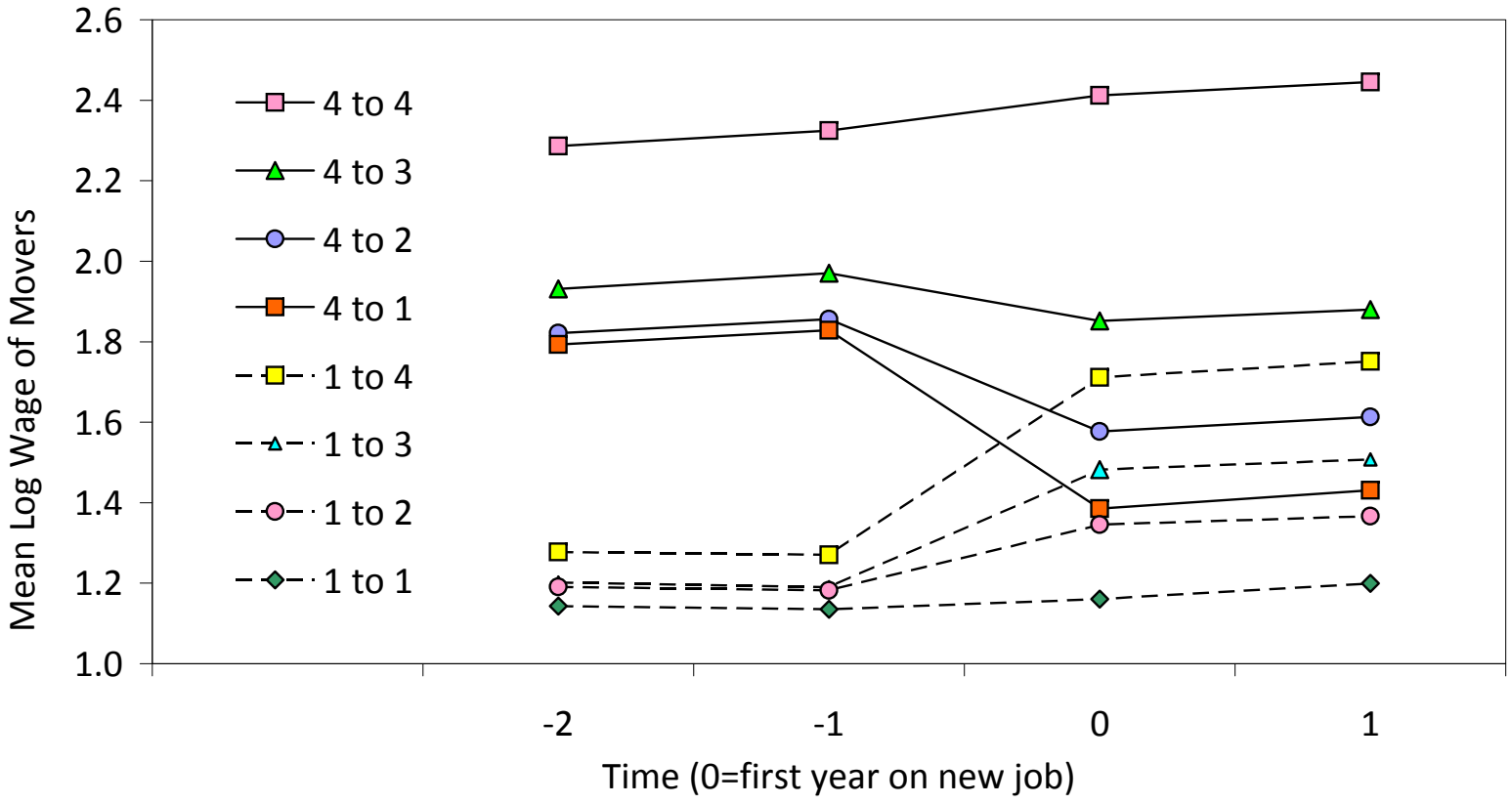
# Mobility is directed (CHK, 2012)

Appendix Table 3: Mean Log Wages Before and After Job Change, for Movers with Two or More Years of Wage Data Before and After Job Change

| Origin/destination quartile* | Number of Changes: (1) | Mean Log Wages of Movers: 2 years before (2) | 1 year before (3) | 1 year after (4) | 2 years after (5) | 4 Year Change: Raw (6) | Adjusted** (7) |
|---|---|---|---|---|---|---|---|
| Interval 1: 1985-1991 | | | | | | | |
| 1 to 1 | 333,648 | 4.003 | 4.025 | 4.085 | 4.113 | 0.110 | 0.000 |
| 1 to 2 | 206,251 | 4.063 | 4.085 | 4.207 | 4.248 | 0.185 | 0.075 |
| 1 to 3 | 136,119 | 4.064 | 4.087 | 4.271 | 4.323 | 0.260 | 0.150 |
| 1 to 4 | 82,193 | 4.102 | 4.132 | 4.380 | 4.444 | 0.342 | 0.232 |
| 2 to 1 | 125,376 | 4.160 | 4.178 | 4.144 | 4.175 | 0.015 | -0.072 |
| 2 to 2 | 204,787 | 4.229 | 4.251 | 4.286 | 4.316 | 0.087 | 0.000 |
| 2 to 3 | 158,360 | 4.258 | 4.278 | 4.359 | 4.395 | 0.137 | 0.051 |
| 2 to 4 | 86,038 | 4.298 | 4.324 | 4.474 | 4.529 | 0.231 | 0.144 |
| 3 to 1 | 59,334 | 4.245 | 4.261 | 4.163 | 4.194 | -0.051 | -0.153 |
| 3 to 2 | 91,474 | 4.315 | 4.337 | 4.333 | 4.371 | 0.056 | -0.046 |
| 3 to 3 | 173,160 | 4.384 | 4.409 | 4.452 | 4.486 | 0.102 | 0.000 |
| 3 to 4 | 136,569 | 4.460 | 4.487 | 4.594 | 4.635 | 0.175 | 0.073 |
| 4 to 1 | 30,110 | 4.373 | 4.396 | 4.252 | 4.284 | -0.089 | -0.220 |
| 4 to 2 | 41,079 | 4.459 | 4.488 | 4.447 | 4.487 | 0.028 | -0.103 |
| 4 to 3 | 91,177 | 4.552 | 4.584 | 4.596 | 4.633 | 0.080 | -0.051 |
| 4 to 4 | 290,921 | 4.678 | 4.710 | 4.777 | 4.809 | 0.131 | 0.000 |

# Event-studies (CCK, 2015)

Figure 3a: Mean Log Wages of Portuguese Male Job Changers, Classified by Quartile of Co-Worker Wages at Origin and Destination




Notes: figure shows mean wages of male workers at mixed-gender firms who changed jobs in 2004-2007 and held the preceding job for 2 or more years, and the new job for 2 or more years. Job is classified into quartiles based on mean log wage of co-workers of both genders.

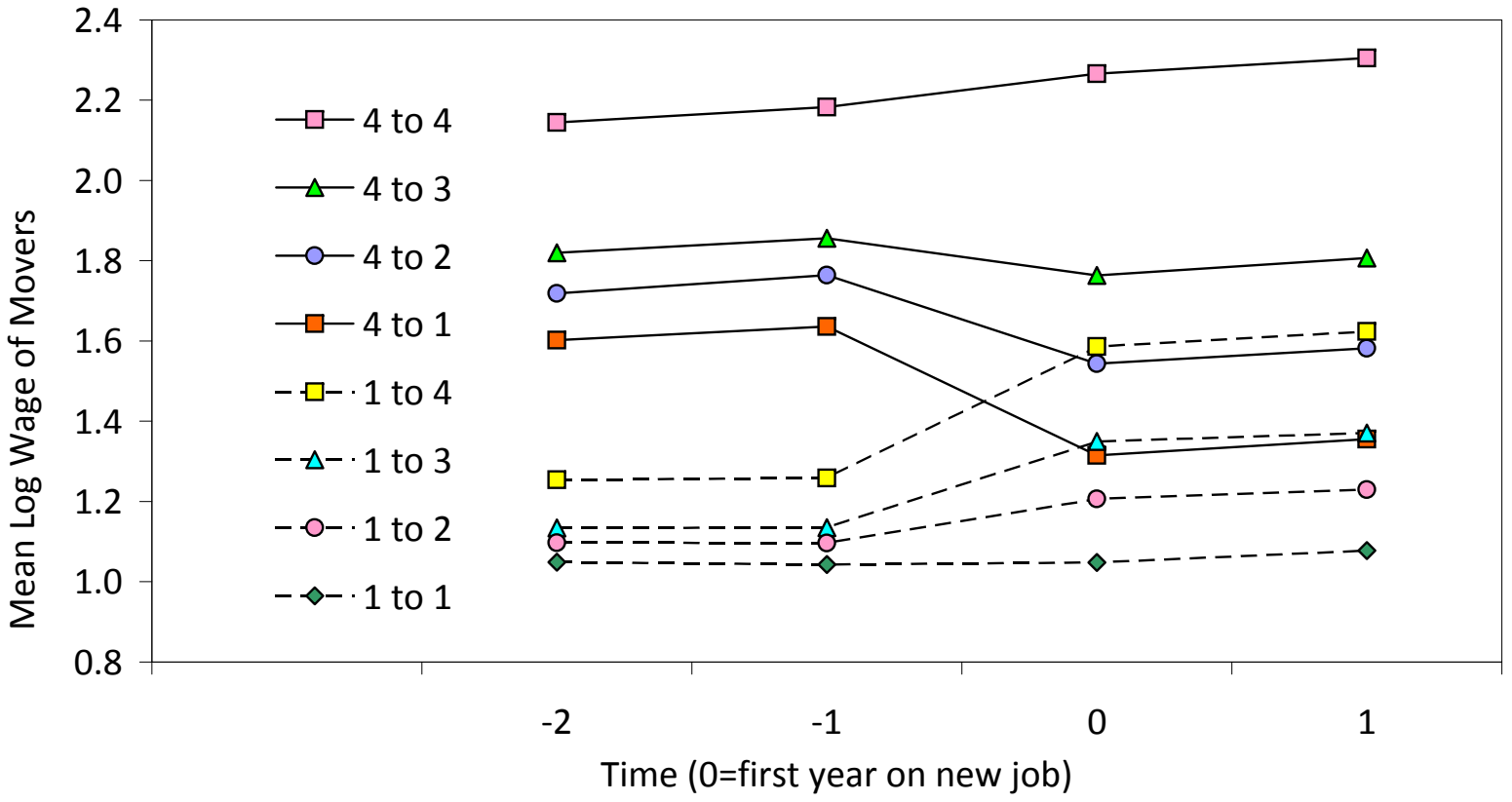
# Mobility Directed in Portugal too (CCK, 2015)

Appendix Table B2: Wages of Job Changes for Movers with 2+ Years of Data Before/After Job Change

| Origin/ destination quartile | Number Changes (1) | Pct. Of Changes (2) | Mean Log Real Wages of Movers: 2 years before (3) | 1 year before (4) | 1 year after (5) | 2 years after (6) | 3 Year Change (%) Raw (7) | Adjusted* (8) | (Std Err) (9) |
|---|---|---|---|---|---|---|---|---|---|
| Males | | | | | | | | | |
| 1 to 1 | 13,787 | 43.2 | 1.14 | 1.14 | 1.16 | 1.20 | 5.6 | 0.5 | (0.5) |
| 1 to 2 | 9,139 | 28.7 | 1.19 | 1.18 | 1.35 | 1.37 | 17.6 | 11.6 | (0.6) |
| 1 to 3 | 6,283 | 19.7 | 1.20 | 1.19 | 1.48 | 1.51 | 30.6 | 23.9 | (0.7) |
| 1 to 4 | 2,682 | 8.4 | 1.28 | 1.27 | 1.71 | 1.75 | 47.3 | 39.0 | (1.2) |
| 2 to 1 | 7,293 | 21.2 | 1.34 | 1.35 | 1.22 | 1.27 | -6.5 | -12.0 | (0.6) |
| 2 to 2 | 12,326 | 35.8 | 1.37 | 1.38 | 1.40 | 1.42 | 5.0 | -0.8 | (0.6) |
| 2 to 3 | 10,356 | 30.0 | 1.41 | 1.42 | 1.54 | 1.57 | 15.9 | 9.3 | (0.5) |
| 2 to 4 | 4,496 | 13.0 | 1.49 | 1.49 | 1.81 | 1.84 | 35.3 | 27.0 | (0.9) |
| 3 to 1 | 4,356 | 11.9 | 1.49 | 1.52 | 1.24 | 1.30 | -19.4 | -25.6 | (0.7) |
| 3 to 2 | 8,835 | 24.2 | 1.54 | 1.55 | 1.45 | 1.48 | -5.8 | -12.2 | (0.6) |
| 3 to 3 | 15,107 | 41.3 | 1.61 | 1.63 | 1.65 | 1.67 | 6.4 | -0.3 | (0.5) |
| 3 to 4 | 8,246 | 22.6 | 1.73 | 1.75 | 1.94 | 1.97 | 24.7 | 16.0 | (0.7) |
| 4 to 1 | 1,634 | 5.4 | 1.79 | 1.83 | 1.39 | 1.43 | -36.2 | -43.3 | (1.6) |
| 4 to 2 | 3,245 | 10.7 | 1.82 | 1.86 | 1.58 | 1.61 | -20.9 | -28.1 | (1.2) |
| 4 to 3 | 6,589 | 21.7 | 1.93 | 1.97 | 1.85 | 1.88 | -5.2 | -13.1 | (0.9) |
| 4 to 4 | 18,830 | 62.1 | 2.29 | 2.32 | 2.41 | 2.45 | 15.9 | 6.1 | (0.9) |

# Event-studies (CCK, 2015)

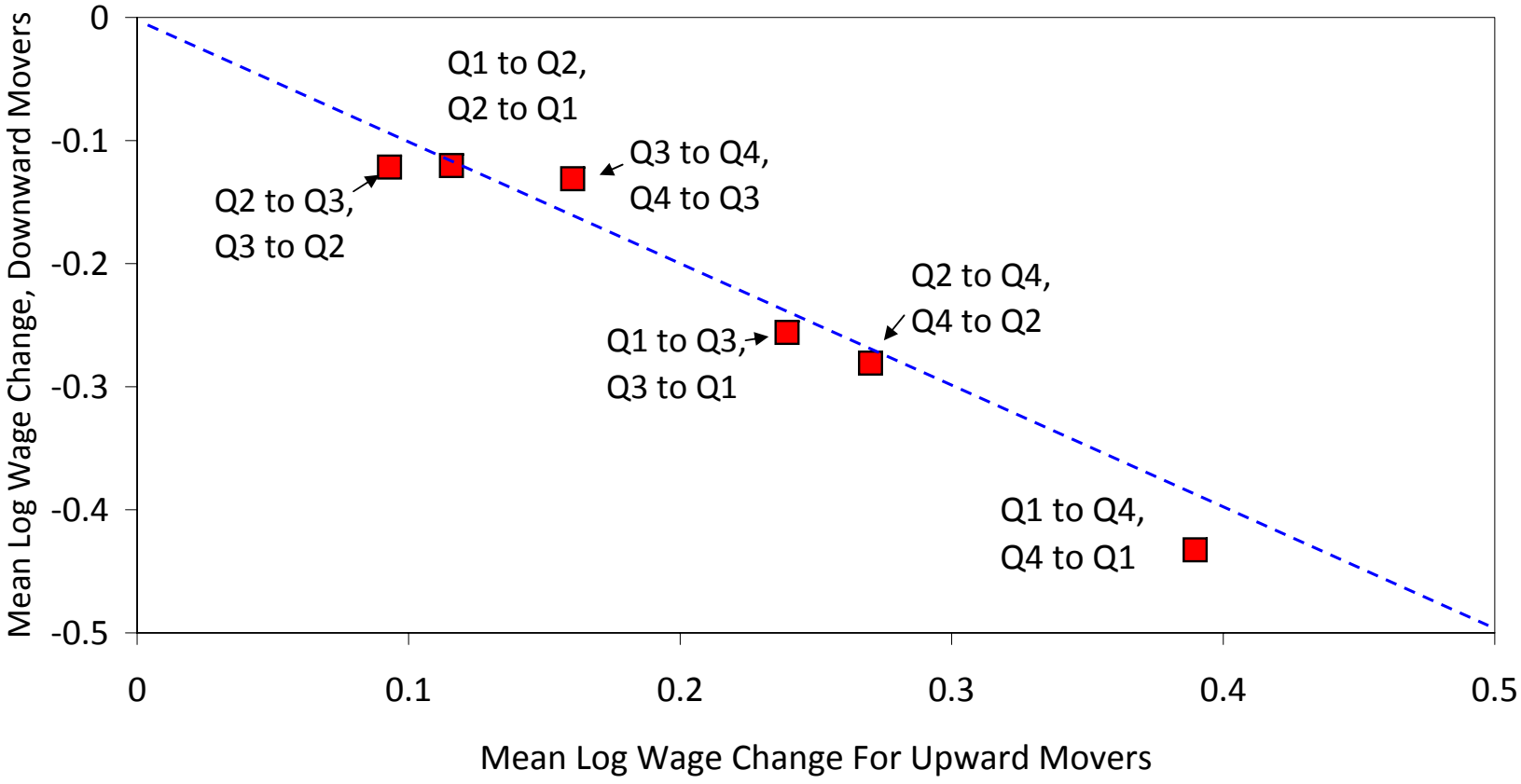
Figure 3b: Mean Wages of Portuguese Female Job Changers, Classified by Quartile of Co-Worker Wages at Origin and Destination




Notes: figure shows mean wages of female workers at mixed gender firms who changed jobs in 2004-2007 and held the preceding job for 2 or more years, and the new job for 2 or more years. Jobs are classified into quartiles based on mean log wage of co-workers of both genders.

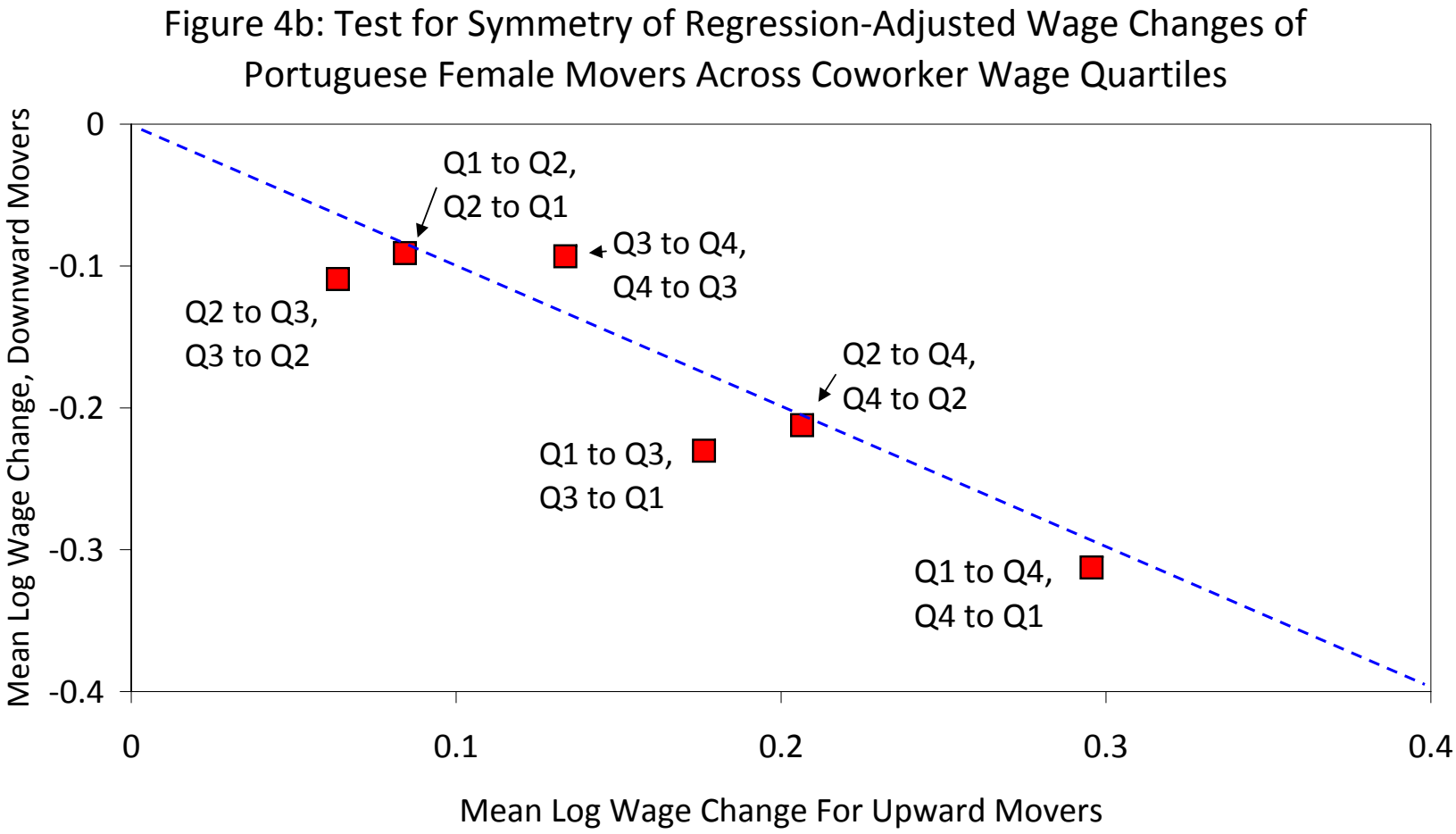
# Symmetry Plot (CCK, 2015)

Figure 4a: Test for Symmetry of Regression-Adjusted Wage Changes of Portuguese Male Movers Across Coworker Wage Quartiles




Note: Figure plots regression adjusted mean wage changes over 4 year interval for job changers who move across coworker wage quartile groups indicated. Dashed line represents symmetric changes for upward and downward movers.

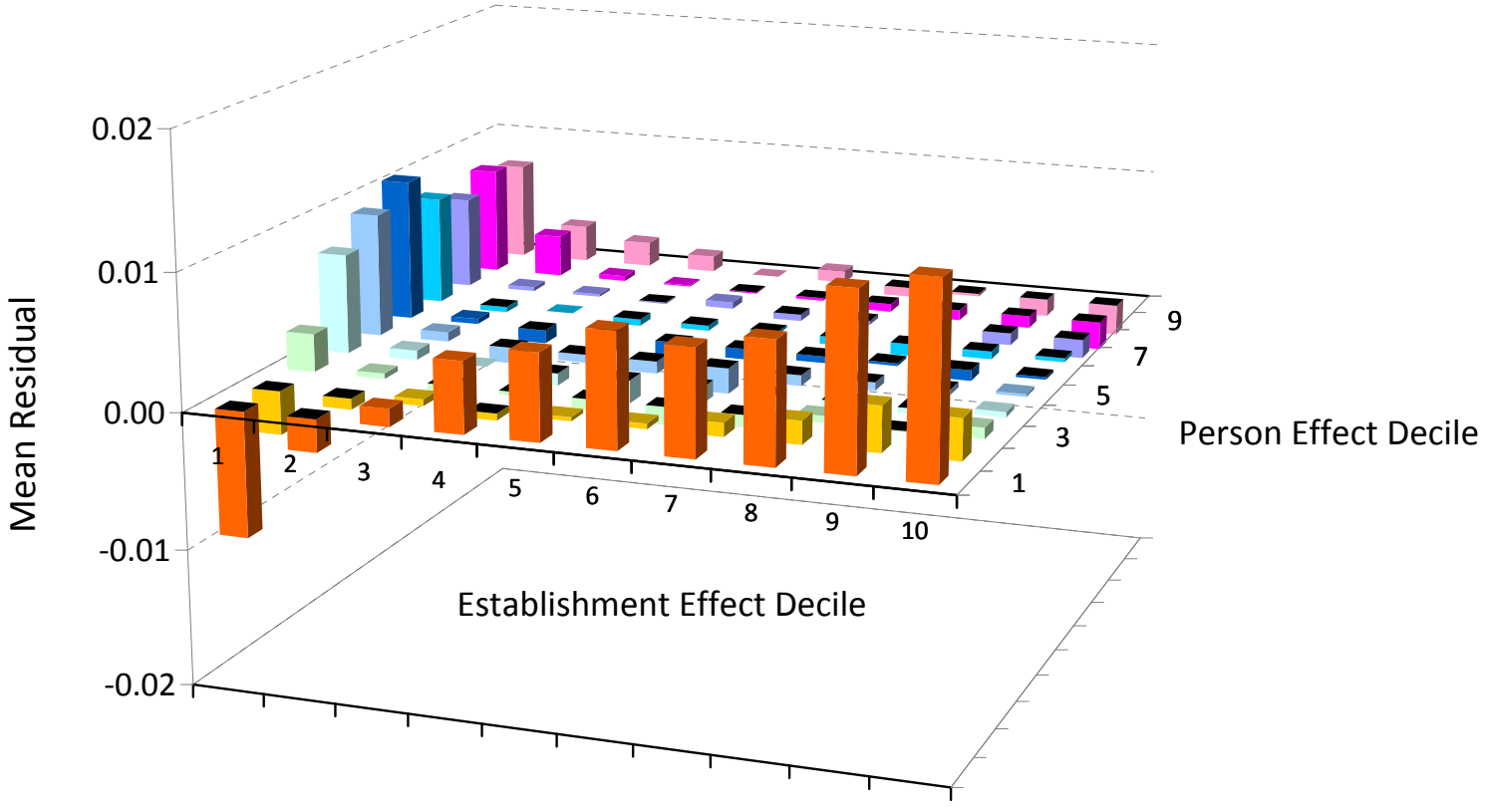
# Symmetry Plot (CCK, 2015)




Note: Figure plots regression adjusted mean wage changes over 4 year interval for job changers who move across coworker wage quartile groups indicated. Dashed line represents symmetric changes for upward and downward movers.

# Separability Plot (CHK, 2013)

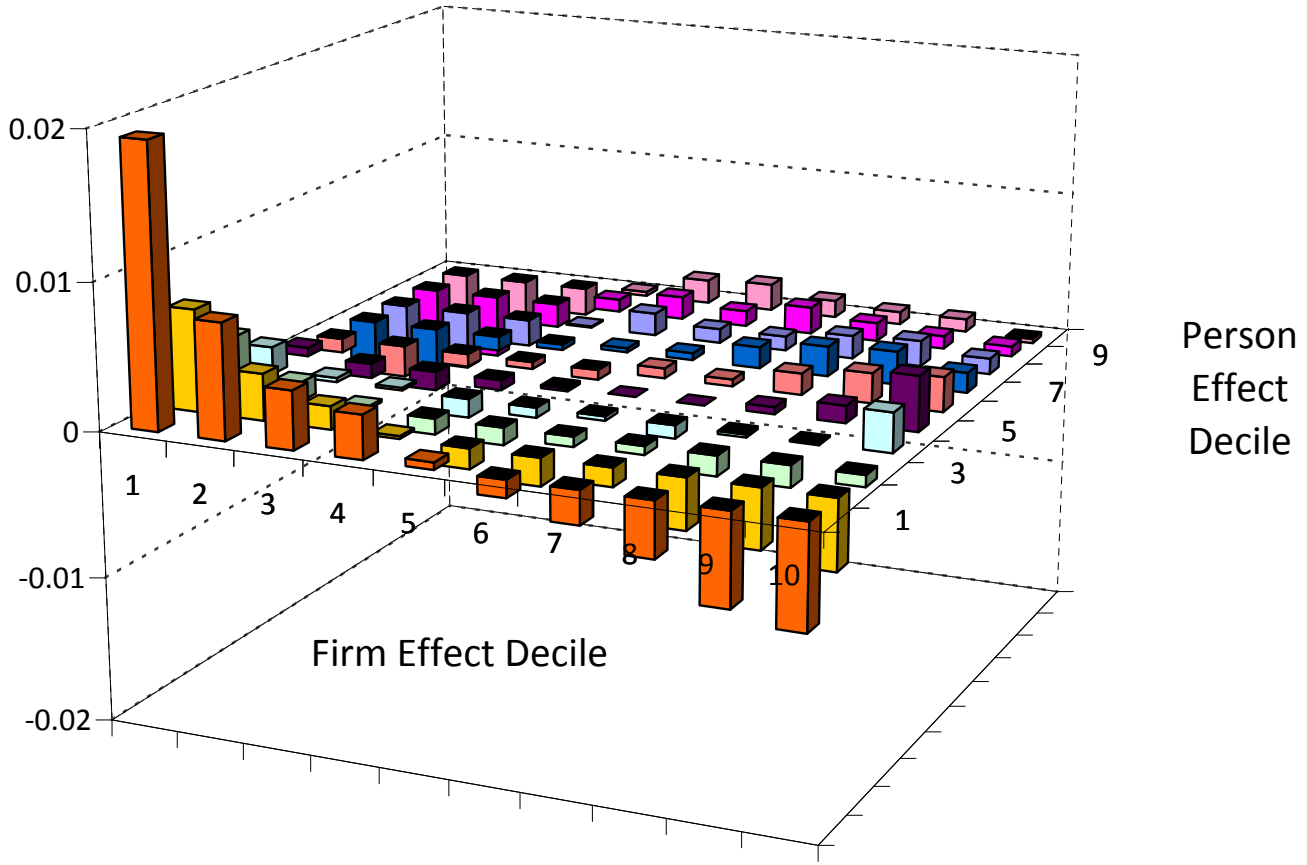
Figure 5: Mean Residuals by Person/Establishment Deciles, German Male Workers 2002-09




Notes: figure shows mean residuals from estimated AKM model with cells defined by decile of estimated establishment effect, interacted with decile of estimated person effect.

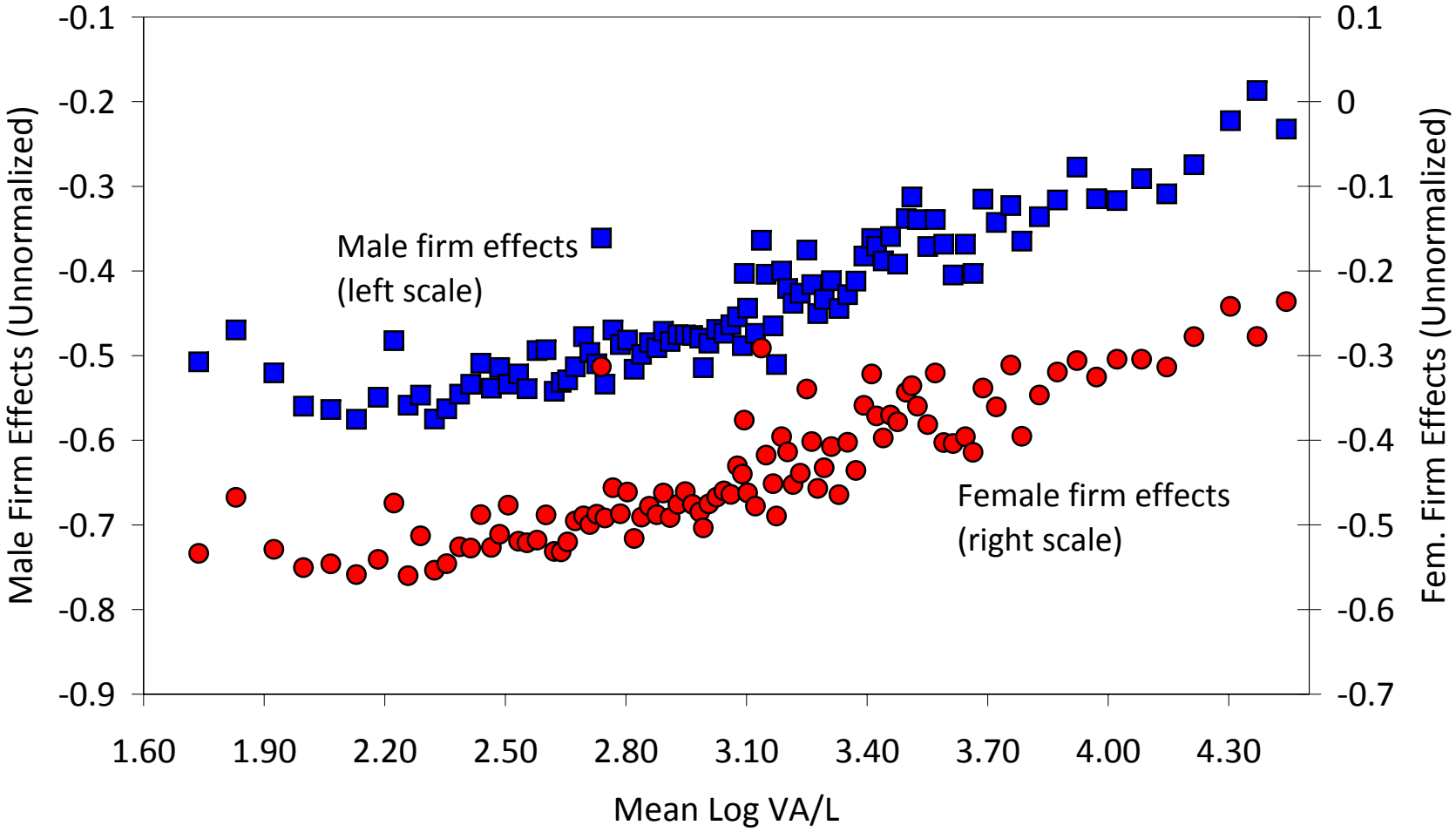
# Separability Plot (CCK, 2015)

Figure 6a: Mean Residuals by Person/Firm Deciles, Portuguese Male Workers




Note: figure shows mean residuals from estimated AKM model with cells defined by decile of estimated firm effects interacted with decile of estimated person effect.

Figure 7: Relationship of Estimated Firm Fixed Effects with Log Value Added/Worker




Note: points shown represent mean estimated firm-specific wage premiums from AKM models for men and women, averaged across firms in 100 percentile bins of mean log value added per worker.

# Finite sample biases

- Under the AKM assumptions, FE estimates are unbiased:

$$E\left[\hat{\psi}_j\right] = \psi_j \ \forall j, E\left[\hat{\alpha}_i\right] = \alpha_i \ \forall i$$

- But noise leads to upward bias in variance of estimated effects and downward bias in covariance
  - Bias concave in # movers per firm (Andrews et al, 2008)

- Bias corrections proposed by Andrews et al (2008) and Gaure (2015) rely on strong homoscedasticity / non-correlation assumptions about time-varying errors $\{\varepsilon_{it}\}$
  - But promising work on random effects alternatives (Abowd et al, 2012; Bonhomme et al, 2015)
  - Structural models: joint model of mobility and wages (e.g., Postel-Vinay and Robin, 2002)

- In practice, bias is likely to be fairly substantial if working w/ samples instead of population files

# Andrews et al (2010): Germany, 1998-2007

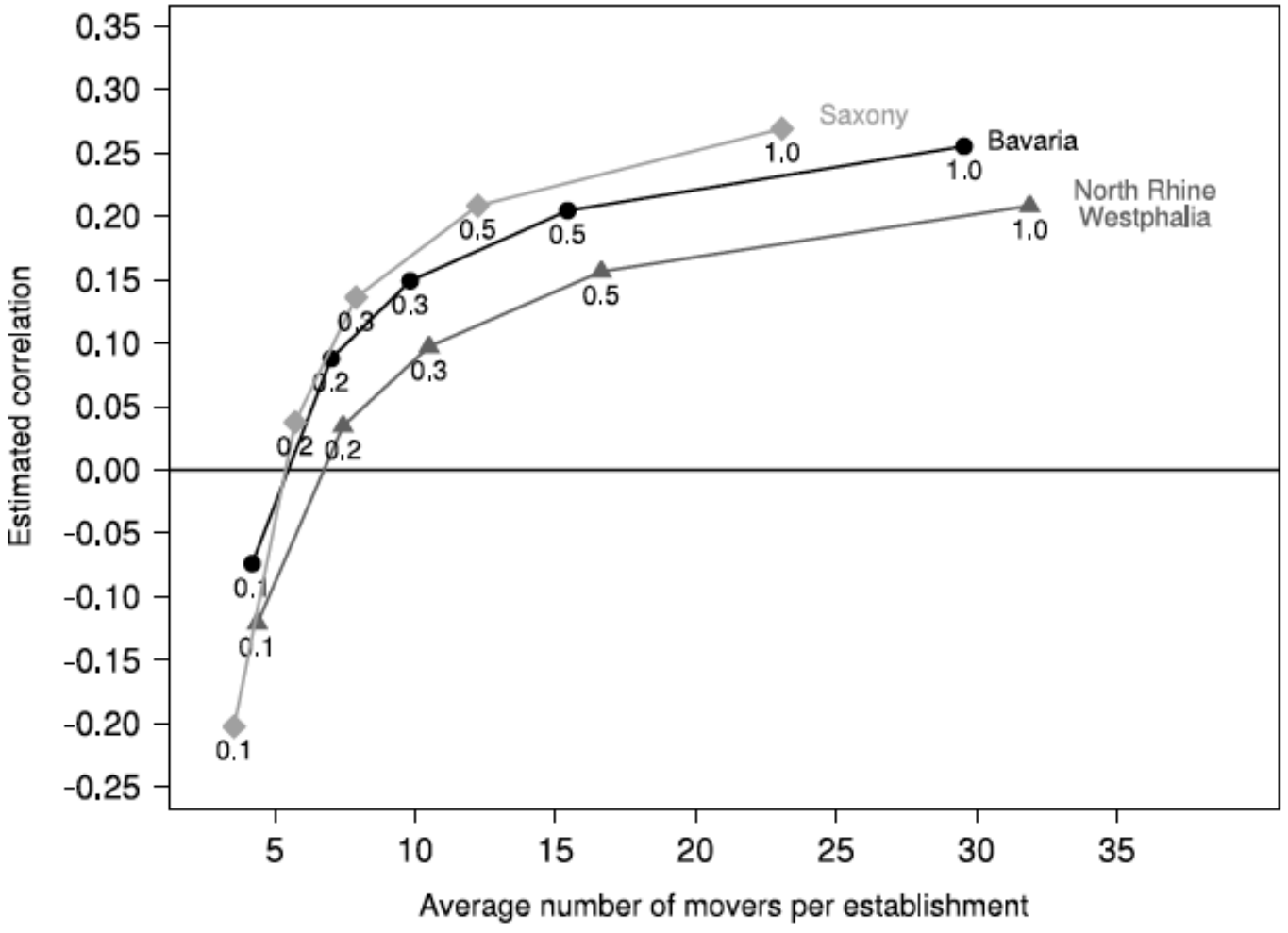



**Fig. 1.** Increasing the number of movers per establishment in a fixed sample of establishments increases Corr($\hat{\theta}_i$, $\hat{\psi}_j$).

# A Model of Imperfect Competition

- Want to think more carefully about how firm heterogeneity can generate wage premia

- Standard approach: Search and Matching (S&M)
  - S&M gives plausible account of unemployment / labor flows
  - But not really a theory of wages *per se*
  - In fact, wages often indeterminate (Edgeworth, 1932)

- Today: follow IO literature in thinking about imperfect competition due to "workplace differentiation"
  - Differentiation gives firms some power to set wages
  - Study link between productivity and wage dispersion
  - Interpretation of AKM-style firm effects
  - And some new testable predictions...

# Setup

- Two types of workers: $L$ and $H$
- $J$ firms, each with non-wage attributes
- Indirect utility for an individual $i$ of type $S \in \{L, H\}$ of working at firm $j$ is:

$$v_{iSj} = \beta_S \ln w_{Sj} + a_{Sj} + \epsilon_{iSj}$$

  - $\{a_{Lj}, a_{Hj}\}$ capture mean valuations of work environment
  - $\{\varepsilon_{iLj}, \varepsilon_{iHj}\} \sim EV1(.)$ capture heterogeneity in valuations

# Market Structure

- Logit choice probabilities:

$$p_{Sj} \equiv P(v_{iSj} \geq v_{iSk} \, \forall k \neq j) = \frac{\exp(\beta_S \ln w_{Sj} + a_{Sj})}{\sum_{k=1}^{J} \exp(\beta_S \ln w_{Sk} + a_{Sk})}$$

- Take $J \to \infty$ (large market):

$$p_{Sj} \approx \lambda_S \exp(\beta_S \ln w_{Sj} + a_{Sj}),$$

where $(\lambda_H, \lambda_L)$ are constants

# Firm-specific supply

- Iso-elastic type-specific supply curves:

$$\begin{aligned} \ln L_j(w_{Lj}) &= \ln(L\lambda_L) + \beta_L \ln w_{Lj} + a_{Lj} \\ \ln H_j(w_{Hj}) &= \ln(H\lambda_H) + \beta_H \ln w_{Hj} + a_{Hj}, \end{aligned}$$

- As $(\beta_L, \beta_H) \to \infty$, market becomes competitive
- $\{a_{Lj}, a_{Hj}\}$ break mechanical link between firm size and wages

# Firm Optimization

- Firm $j$'s production function:

$$Y_j = T_j f(L_j, H_j)$$

- Firm's problem is to choose wages to minimize cost:

$$\min_{w_L, w_H} w_L L_j(w_L) + w_H H_j(w_H) \text{ s.t. } T_j f(L_j(w_L), H_j(w_H)) \geq Y$$

- Note: firm knows shape of LS fn's but not identity of workers who comprise them (no 1st-degree price discrim)

# Wages

- Choosing $(w_{jH}, w_{jL})$ yields standard "mark down" formula:

$$\begin{aligned} w_{Lj} &= \frac{\beta_L}{1+\beta_L} T_j f_L \mu_j \\ w_{Hj} &= \frac{\beta_H}{1+\beta_H} T_j f_H \mu_j \end{aligned}$$

  where $\mu_j \equiv MC \overset{opt}{=} MR$

- Example: $\beta_L = \beta_H = 9 \Rightarrow$ workers paid 90% of MRP

# Special Case: Linear Production, Fixed Output Price

- Production function:

$$Y_j = T_j((1-\theta)L_j + \theta H_j)$$

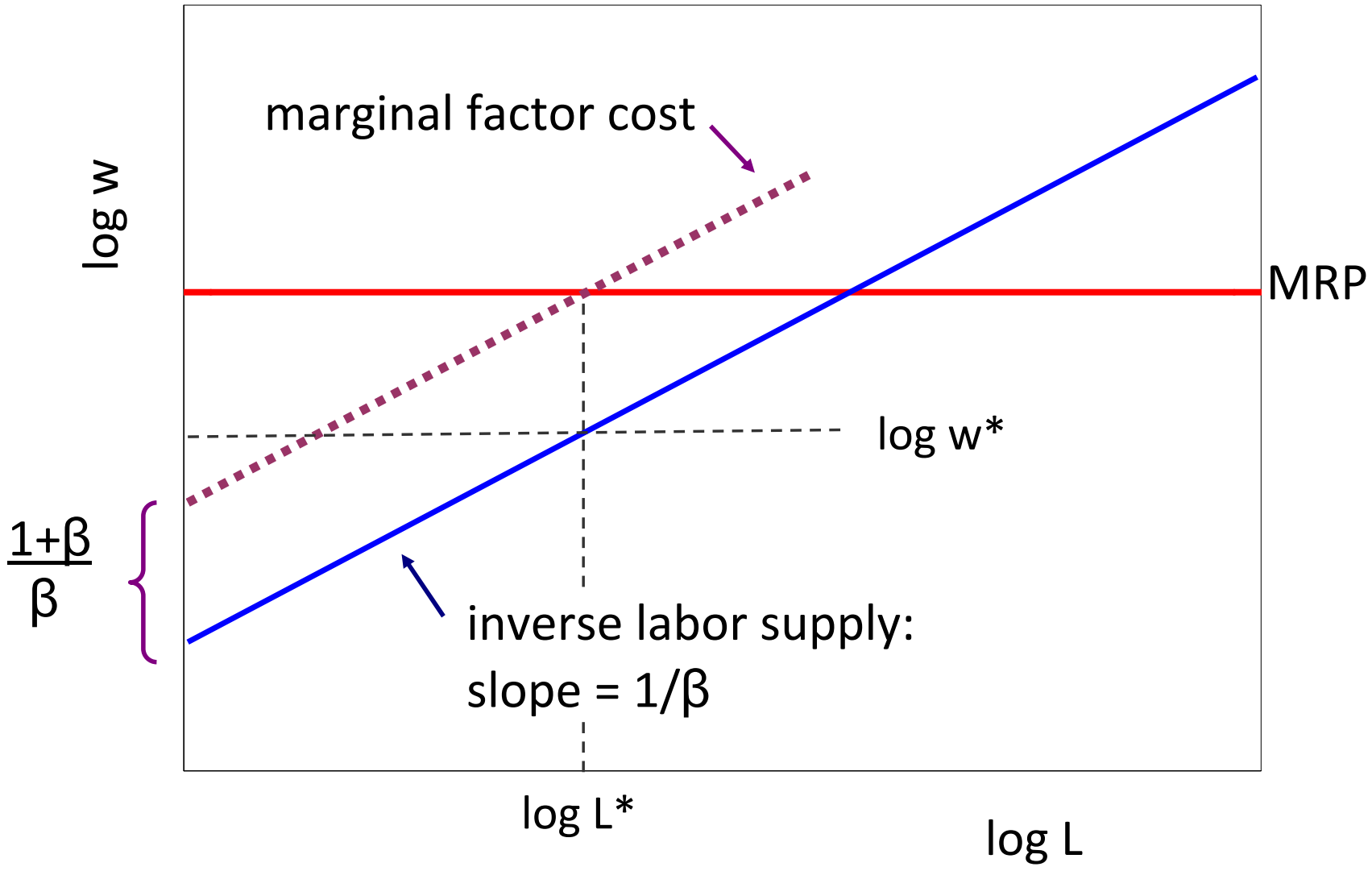
- Equilibrium Wages:

$$\begin{aligned} w_{Lj} &= \frac{\beta_L}{1+\beta_L}(1-\theta)T_j P_j^0 \\ w_{Hj} &= \frac{\beta_H}{1+\beta_H}\theta T_j P_j^0 \end{aligned}$$

- Notes:
  - "Rent-sharing" elasticity $= 1$
  - No "sharing" going on: rents captured by inframarginal workers due to asym. info
  - No compensating diffs ($a_{Sj}$'s don't influence the LS elasticity)

# Equilibrium Wages and Employment

Figure 8: Equilibrium Wages and Employment



# AKM Interpretation

$$\ln w_{jH} = \underbrace{\ln \frac{\beta_H}{1+\beta_H} + \ln 1 - \theta}_{\text{Person Eff}} + \underbrace{\ln T_j P_j^0}_{\text{Firm Eff}}$$

$$\ln w_{jL} = \underbrace{\ln \frac{\beta_L}{1+\beta_L} + \ln \theta}_{\text{Person Eff}} + \underbrace{\ln T_j P_j^0}_{\text{Firm Eff}}$$

- Stable "person effect" across firms driven by LS elasticity and technology
- Stable "firm effect" driven by productivity
- Inequality trends:
  - Variance of firm effects driven by $\sigma^2_T$,$\sigma^2_{P^0}$
  - Possible group differences due to diffs in $\beta's$ (Robinson, 1933)

# Relative Wages / Employment

- Relative wages invariant to TFP (stable person eff):

$$\ln \frac{w_{Hj}}{w_{Lj}} = \ln \frac{\beta_H}{1+\beta_H} - \ln \frac{\beta_L}{1+\beta_L} + \ln \frac{\theta}{1-\theta}$$

- But relative employment related to TFP if $\beta_H \neq \beta_L$:

$$\ln \frac{H_j}{L_j} = C + \ln \frac{a_{Hj}}{a_{Lj}} + \beta_H \ln \theta - \beta_L \ln \frac{1}{1-\theta} + (\beta_H - \beta_L) \ln T_j P_j^0$$

- Notes:
  - Firm size / sorting driven by both productivity and non-wage amenities
  - $\beta_H > \beta_L \Rightarrow$ more productive firms have higher skill share

# Adding product market power

- Downward sloping demand: $P_j = P_j^0 Y_j^{-1/\varepsilon}$, $\varepsilon > 1$
- Marginal revenue: $MR_j = \left(1 - \frac{1}{\varepsilon}\right) P_j^0 Y_j^{-1/\varepsilon}$
- Wages become:

$$\begin{aligned} w_{Lj} &= \frac{\beta_L}{1+\beta_L}(1-\theta)\, T_j^{1-1/\varepsilon} P_j^0 f(L_j(w_{Lj}), H_j(w_{Hj}))^{-1/\varepsilon} \\ w_{Hj} &= \frac{\beta_H}{1+\beta_H}\theta\, T_j^{1-1/\varepsilon} P_j^0 f(L_j(w_{Lj}), H_j(w_{Hj}))^{-1/\varepsilon} \end{aligned}$$

- Note that now "$TFPR$" $= T_j^{1-1/\varepsilon} P_j^0$
- AKM-style decomp still holds because relative wages $\frac{w_{Hj}}{w_{Lj}}$ invariant to TFP, now firm effect is (to 1st order):

$$\psi_j \approx \frac{\varepsilon}{\varepsilon + \overline{\beta_j}} \ln \frac{P_j^0}{\overline{P}} + \frac{\varepsilon - 1}{\varepsilon + \overline{\beta_j}} \ln \frac{T_j}{\overline{T}}$$

# "Rent-sharing" elasticities

- Letting $\overline{\beta_j} = \beta_L \kappa_j + \beta_H (1 - \kappa_j)$, $\kappa_j = \frac{(1-\theta)L_j}{(1-\theta)L_j + \theta H_j}$, we have:

$$\frac{\partial \ln w_{Lj}}{\partial \ln P_j^0} = \frac{\partial \ln w_{Hj}}{\partial \ln P_j^0} = \frac{\varepsilon}{\varepsilon + \overline{\beta_j}}$$

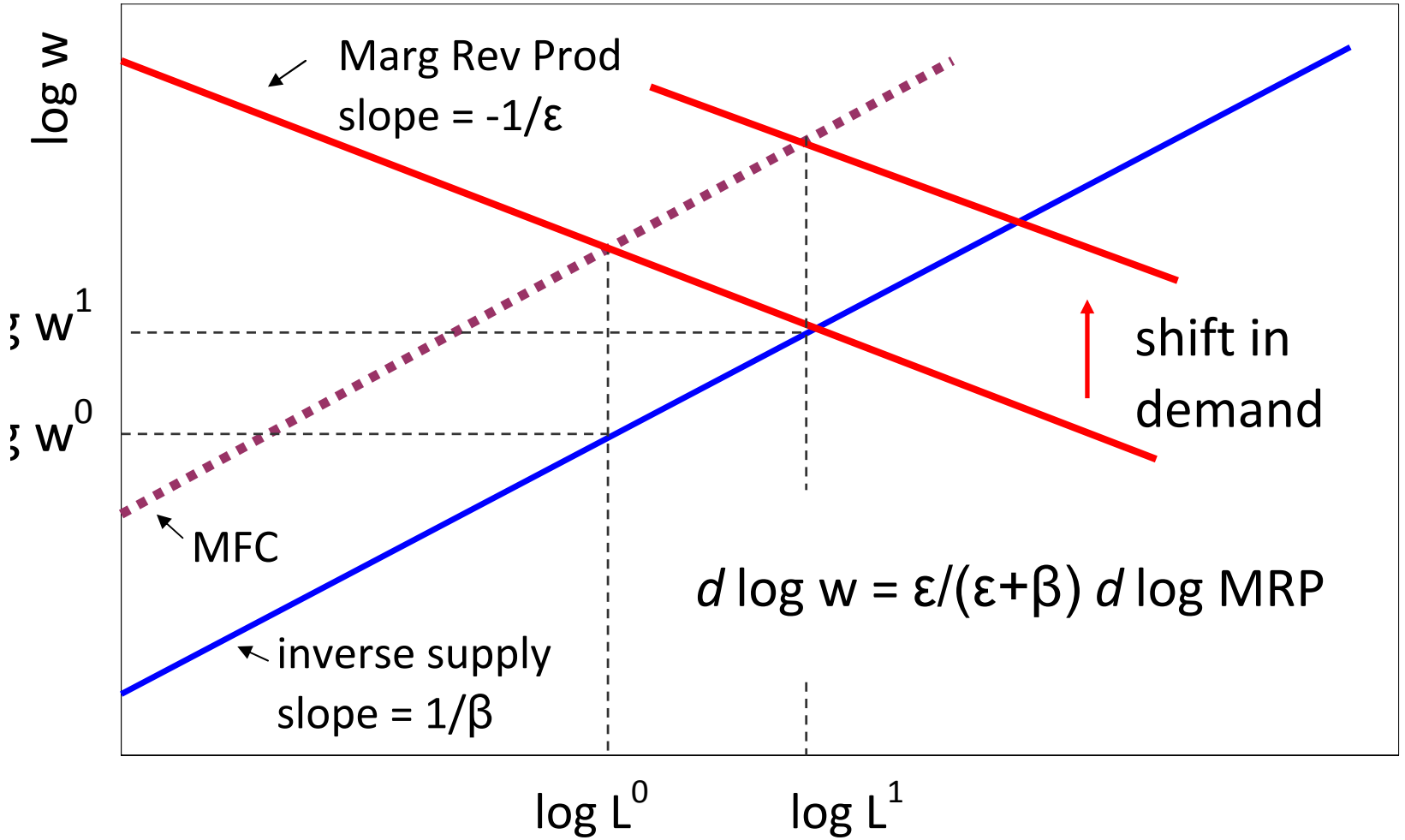
$$\frac{\partial \ln w_{Lj}}{\partial \ln T_j} = \frac{\partial \ln w_{Hj}}{\partial \ln T_j} = \frac{\varepsilon - 1}{\varepsilon + \overline{\beta_j}}$$

- Special cases:
  - As $\varepsilon \to \infty$, $\frac{\partial \ln w_{Lj}}{\partial \ln T_j} \to 1$ (constant MRP)
  - As $\varepsilon \to 1$, $\frac{\partial \ln w_{Lj}}{\partial \ln T_j} \to 0$ (inelastic demand)
  - As $\overline{\beta_j} \to \infty$, $\frac{\partial \ln w_{Lj}}{\partial \ln T_j} \to 0$ (competitive labor market)
  - As $\overline{\beta_j} \to 0$, $\frac{\partial \ln w_{Lj}}{\partial \ln T_j} \to \frac{\varepsilon - 1}{\varepsilon}$ (fixed labor supply)

- Suppose $\overline{\beta_j} = 9$, $\varepsilon = 1.5$. Then $\frac{\partial \ln w_{Lj}}{\partial \ln P_j^0} = .14$, $\frac{\partial \ln w_{Lj}}{\partial \ln T_j} = .047$
- "Rent-sharing" elasticity will be weighted average of these two based upon variance-covariance of shocks to $\left(P_j^0, T_j\right)$

# A Shift in Demand

Figure 9: Effect of Demand Variation with Decreasing Marginal Revenue Product



# Adding imperfect substitution

- Suppose we relax linear production technology to allow CES production:

$$Y_j = Tf(L, H) = T_j[(1-\theta)L_j^{\rho} + \theta H_j^{\rho}]^{1/\rho}$$

where $\rho \in (-\infty, 1]$. The elasticity of substitution is $\sigma = (1-\rho)^{-1} \in [1, \infty)$

- Wages can be written:

$$\begin{aligned}
\left(1 + \frac{1}{\sigma}\beta_L\right) \ln w_{Lj} &= \ln\left(\frac{\beta_L}{1+\beta_L}\right) + \ln(1-\theta) - \frac{1}{\sigma}a'_{Lj} + \Gamma_j \\
\left(1 + \frac{1}{\sigma}\beta_H\right) \ln w_{Hj} &= \ln\left(\frac{\beta_H}{1+\beta_H}\right) + \ln\theta - \frac{1}{\sigma}a'_{Hj} + \Gamma_j
\end{aligned}$$

- AKM-decomp holds when $\beta_L \approx \beta_H$. Otherwise it only holds locally within skill groups.

# A new prediction

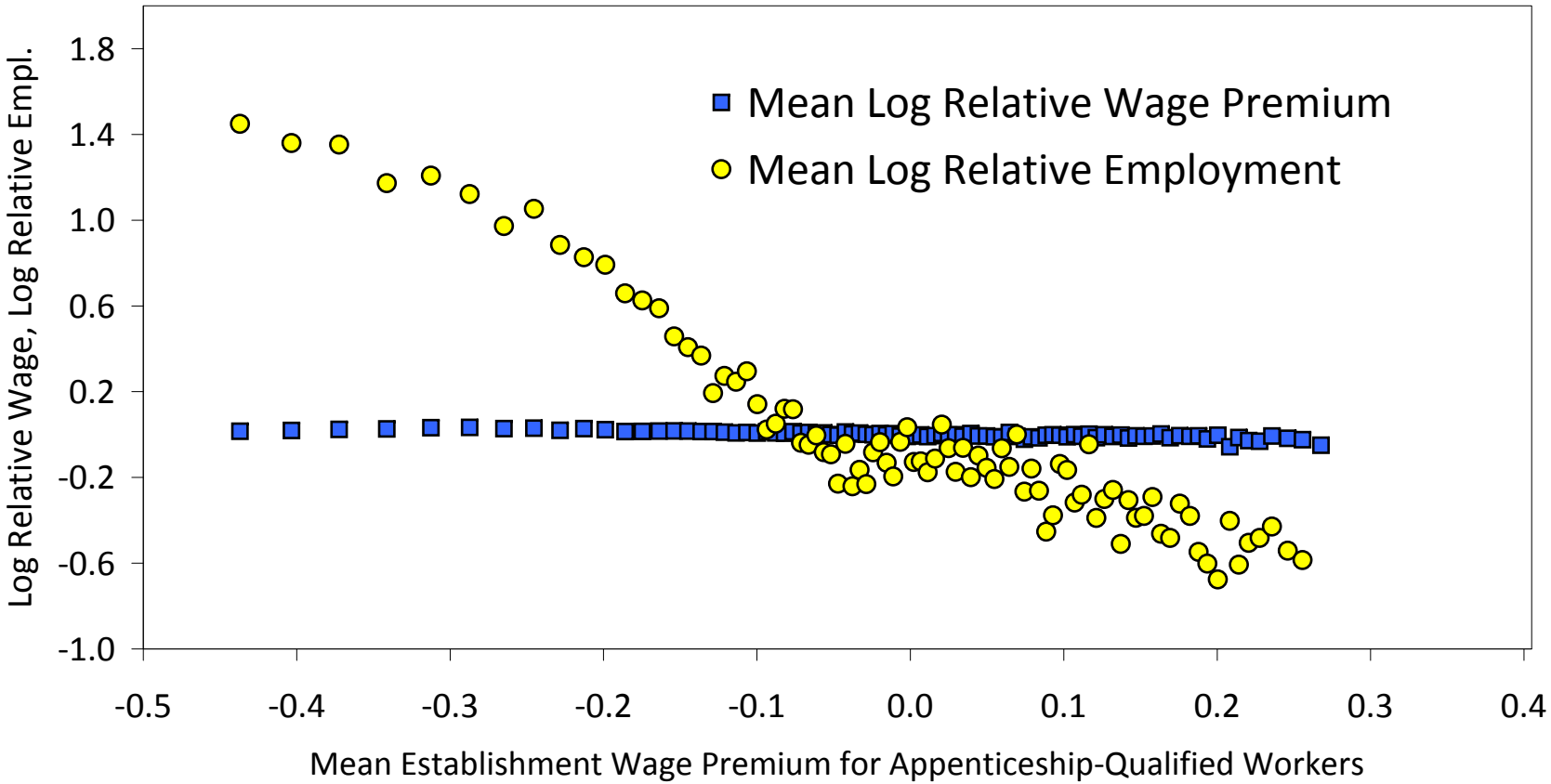
- Usual inverse relationship between relative quantities and wages now holds at firm-level:

$$\frac{\partial \ln(H_j/L_j)}{\partial \ln P_j^0} = -\sigma \frac{\partial \ln(w_{Hj}/w_{Lj})}{\partial \ln P_j^0}$$

- Firm-level evidence on what is usually considered a macro phenomenon?

# Relative Quantities and Prices: Low vs. Medium

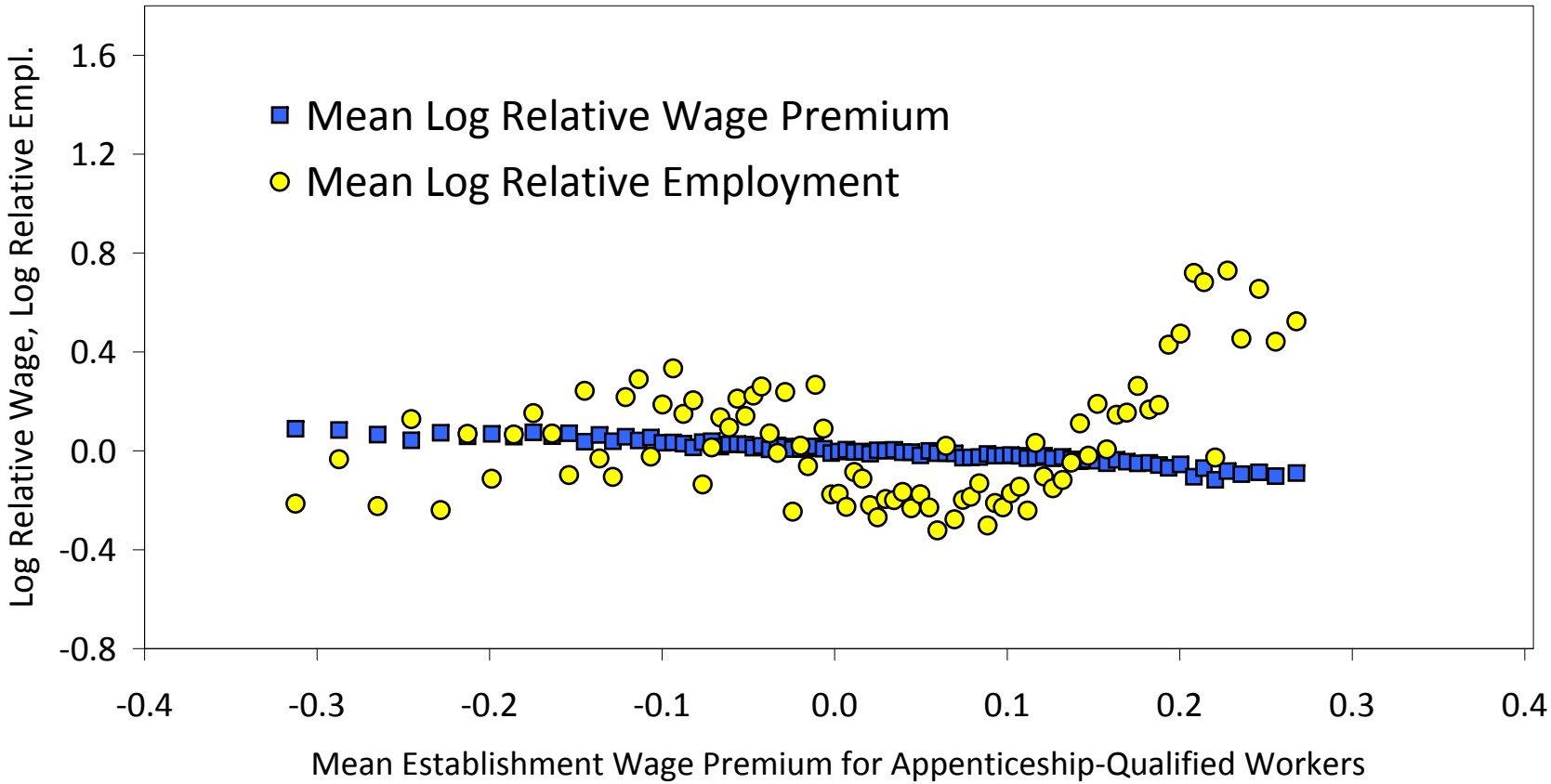
Figure 11: Relative Wages and Relative Employment of Low-Education Workers vs. Wage Premium for Apprenticeship-Qualified Workers




Note: figure shows 5th to 95th percentile groups only. Mean log relative wage premium is mean wage premium for low-education workers minus wage premium for apprenticeship qualified workers. Mean log relative employment is mean log employment of low-education workers minus log employment of apprenticeship-qualified workers. Based on establishment wage premiums and employment shares among West German male full time workers, 2002-2009.

# Relative Quantities and Prices: High vs. Medium

Figure 12: Relative Wages and Relative Employment of High-Education Workers vs. Wage Premium for Apprenticeship-Qualified Workers




Note: figure shows 10th to 95th percentile groups only. Mean log relative wage premium is mean wage premium for high-education workers minus wage premium for apprenticeship qualified workers. Mean log relative employment is mean log employment of high-education workers minus log employment of apprenticeship-qualified workers. Based on establishment wage premiums and employment shares among West German male full time workers, 2002-2009.

# "SBTC" shocks (simple version)

- Technology diffuses unevenly across firms (Griliches, 1957; Doms, Dunne, and Troske, 1997; Dunne et al., 2004)
- No reason to assume all variation is Hicks neutral: Let $\theta$ vary in addition to TFP!

- When $(\sigma, \varepsilon) \to \infty$, we get skill-group specific firm effects:

$$\begin{aligned} \psi_j^L &= \ln(1-\theta_j) + \ln T_j P_j^0 \\ \psi_j^H &= \ln \theta_j + \ln T_j P_j^0 \end{aligned}$$

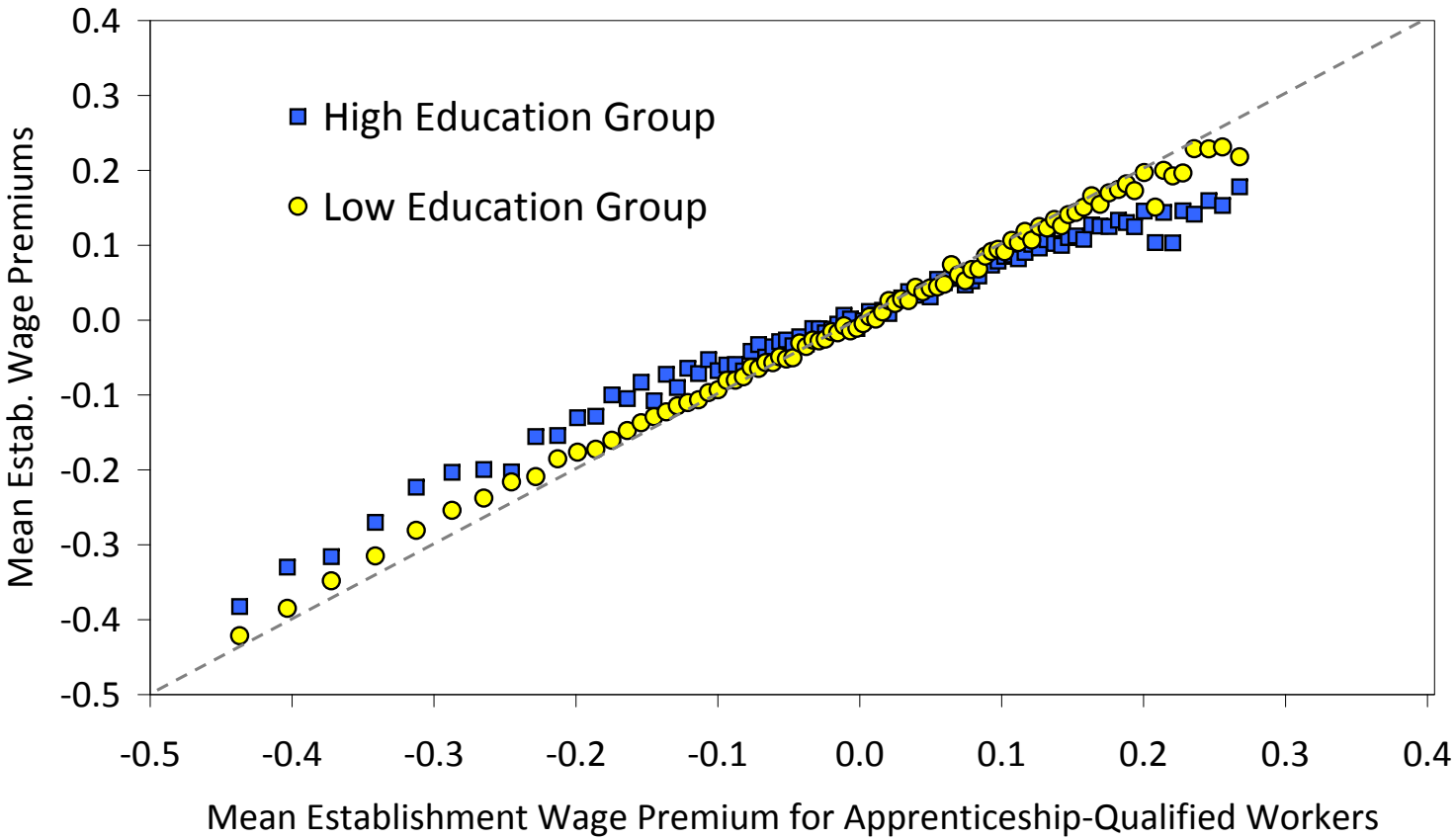
- Regression of type-L FE on type-H FE:

$$\frac{Cov[\psi_j^L, \psi_j^H]}{Var[\psi_j^H]} < 1$$

- Alternate explanation for imperfect correlation of firm effs across groups

# Firm effects by Skill Group

Figure 10: Establishment Wage Premiums for High and Low Education Groups vs. Premium for Appenticeship-Qualified Workers




Note: figure shows 5th to 95th percentile groups only. Based on estimated establishment effects for West German male full time workers, 2002-2009. Establishment effects are normalized to have mean of 0 for each education

# "SBTC" shocks (full version)

- Define $\xi_j \equiv \frac{\partial \ln f}{\partial \ln \theta_j}$ as "TFP-like" component of SBTC shock
- Distinguish from "pure" shock to relative productivity $\frac{\theta_j}{1-\theta_j}$
- Link between relative wages and quantities now ambiguous:

$$\frac{\partial \ln(w_H/w_L)}{\partial \ln \theta_j} = \frac{\frac{1}{1-\theta_j}\sigma(1+\frac{\overline{\beta}}{\varepsilon}) + (\beta_L - \beta_H)\xi_j(1-\frac{1}{\varepsilon})}{\sigma + \beta_L + \beta_H + \left(\frac{\sigma}{\varepsilon} - 1\right)\overline{\beta_j} + \frac{1}{\varepsilon}\beta_L\beta_H}$$

$$\frac{\partial \ln(H_j/L_j)}{\partial \ln \theta_j} = \frac{\frac{1}{1-\theta_j}\sigma(\widetilde{\beta}_j + \frac{1}{\varepsilon}\beta_L\beta_H) - (\beta_L - \beta_H)\xi_j(1-\frac{1}{\varepsilon})}{\sigma + \beta_L + \beta_H + \left(\frac{\sigma}{\varepsilon} - 1\right)\overline{\beta_j} + \frac{1}{\varepsilon}\beta_L\beta_H}$$

- TFP-like variation induces negative correlation, while "pure" TFP-constant component induces positive correlation
- Expect *under-estimate* of $\sigma$ from regression of relative wages on relative quantities

# Final Thoughts

- Both rent-sharing and firm-mover literatures find that firms important for wages
- Static monopsony model can explain AKM style wage structure and "rent sharing" effects
- Even a little market power ($\beta = 9$) gives interesting results

# Going forward..

- Think about static monopsony as steady state of dynamic model w/ frictions?
- Allow for finite $J$ and study strategic interactions between firms?
- How to handle aggregation with imperfect competition and heterogeneity?
- Supply shocks to individual firms? (converse of rent sharing literature)
- Effects of labor market institutions? (min wage, contracts, regulations)