Problem Set #5

ECONOMICS 141
FALL 2008

Due December 8

1. **Theoretical Question:** Suppose you are concerned that a simple regression model might have both heteroskedastic and serially-correlated errors; that is,

$$Y_t = \alpha + \beta X_t + \varepsilon_t,$$

where the errors ε_t are generated by the heteroskedastic process,

$$\begin{aligned} \varepsilon_t &= |\alpha + \beta X_t| \cdot u_t, \\ Var(\varepsilon_t) &= (\alpha + \beta X_t)^2 Var(u_t), \end{aligned}$$

where the error term u is autocorrelated,

$$u_t = \rho u_{t-1} + v_t,$$

with v_t an i.i.d. sequence with mean zero and variance σ_v^2. Describe the sequence of regressions and data transformations you would use to correct for both heteroskedasticity and serial correlation to get efficient estimates of α and β and correct standard error estimates. [Hint: the idea is to do the usual corrections in the right order, and to state why that order is right.]

2. **Sample Short Answer Questions:** Give a *brief* answer, explanation, and/or mathematical derivation to **all** of the questions below.

A. Evaluate the following statement: "It's easy to get instrumental variables if some of the right-hand side variables are correlated with the error terms. Any exogenous variables which clearly do not belong in that equation (and therefore are uncorrelated with the error terms) can be used as valid instruments."

B. Suppose you believed that the relationship between stock prices and dividend payments satisfies the assumptions of the classical linear regression model (with normal errors) for observations drawn across different firms and/or drawn from a given firm across time. However, the only data available to you are group averages of stock prices and dividend payments for a group of 500 industrial firms, collected over 43 years. Is the classical least squares estimation method appropriate to fit a linear model using these group-averaged variables, or should a WLS procedure be used? Briefly justify your answer.

C. Suppose that, in a regression problem with time series data, the Durbin-Watson statistic is 1.8. How large would the sample size T have to be for this result to lead to a rejection of the null hypothesis of no serial correlation (against a **one**-sided alternative of positive serial correlation) at an approximate 5% significance level? (You may round any relevant critical value(s) to a couple digits to simplify calculations.)

D. Pindyck & Rubinfeld, Exercise 6.3, page 175.

E. Pindyck & Rubinfeld, Exercise 6.4, page 175.

F. Pindyck & Rubinfeld, Exercise 7.1, page 200.

G. Pindyck & Rubinfeld, Exercise 7.4, page 201.

3. **Empirical Exercise:** On the course website is the file "Anipa.txt", which has annual (1929 to 1991) data from the National Income and Product Accounts. Each line of each of these files has quantities of six variables: the observation date ($DATE$); the U.S. Gross Domestic Product (GDP) for that date in constant (1987) dollars; the corresponding U.S. Personal Consumption Expenditures (PCE) in constant dollars; the Implicit Price Deflator (IPD) for GDP (which was used to convert the data from current to constant dollars), real federal government expenditures ($GOVT$); and the nominal money supply ($M1$).

Defining the time trend variable $T = DATE - 1928$ and its square $T2 = T * T$, separately regress GDP, PCE, and IPD on a constant, T, and $T2$, and use a t-test to test for significance of the quadratic term in each regression. Then, for each regression, use the Durbin-Watson statistic to test for serial correlation. Finally, for each regression where the null hypothesis of no serial correlation is rejected, use the TSP "AR1" command (or the like) to estimate the regression equation using generalized least squares. Do your conclusions about the significance of the quadratic terms change when you account for serial correlation? [Please write up just your results - don't turn in lots of computer output.]

4. **Numerical Question:** Consider the estimation of two scalar coefficients, β_1 and β_2, in the linear equation

$$y = x_1\beta_1 + x_2\beta_2 + \varepsilon,$$

where y, x_1, and x_2 are observable N-dimensional random vectors. In addition, two N-dimensional vectors of instrumental variables, z_1 and z_2, are available. In a sample of size $N = 227$, the following matrix of cross-products of the variables is observed:

$$\begin{bmatrix} y'y & y'x_1 & y'x_2 & y'z_1 & y'z_2 \\ x_1'y & x_1'x_1 & x_1'x_2 & x_1'z_1 & x_1'z_2 \\ x_2'y & x_2'x_1 & x_2'x_2 & x_2'z_1 & x_2'z_2 \\ z_1'y & z_1'x_1 & z_1'x_2 & z_1'z_1 & z_1'z_2 \\ z_2'y & z_2'x_1 & z_2'x_2 & z_2'z_1 & z_2'z_2 \end{bmatrix} = \begin{bmatrix} 22 & -11 & 10 & 8 & 8 \\ -11 & 21 & 10 & -8 & -8 \\ 10 & 10 & 20 & -2 & 0 \\ 8 & -8 & -2 & 6 & 4 \\ 8 & -8 & 0 & 4 & 6 \end{bmatrix}$$

For these data, calculate the classical LS estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ of the unknown regression coefficients, and compute the instrumental variables estimators $\tilde{\beta}_1$ and $\tilde{\beta}_2$ using z_1 and z_2 as instruments for x_1 and x_2.