Generalized Social Marginal Welfare Weights for Optimal Tax Theory

Emmanuel Saez  
(Berkeley) 

Stefanie Stantcheva  
(Harvard) 

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Standard Welfarist Approach: Critiques and Puzzles

- Maximize concave function or weighted sum of individual utilities.
  \[ \max_{T(\cdot)} SWF = \max_{T(\cdot)} \int \omega_i \cdot u_i \]

- Special case: utilitarianism, \( \omega_i = 1 \).

- Cannot capture elements important in tax practice:
  - Source of income: earned versus luck.
  - Counterfactuals: what individuals \textit{would} have done absent tax system.
  - Horizontal Equity concerns that go against “tagging.”

- Utilitarianism critique: 100% redistribution optimal with concave \( u(\cdot) \) and no behavioral responses

- Methodological and conceptual critique: Policy makers use reform-approach rather than posit and maximize objective.
A Novel Approach to Model Social Preferences

- **Tax reform approach**: weighs gains and losses from tax changes.

  $$\delta T(z) \text{ desirable iff: } - \int_i g_i \cdot \delta T(z_i) > 0 \text{ with } g_i \equiv G'(u_i) \frac{\partial u_i}{\partial c}$$

- **Optimality**: no budget neutral reform can increase welfare.

- **Weights directly come from social welfare function, are restrictive.**
A Novel Approach to Model Social Preferences

- **Tax reform approach:** weighs gains and losses from tax changes.

  \[
  \text{Change in welfare: } - \int_i g_i \cdot \delta T(z_i) \text{ with } g_i \equiv g(c_i, z_i; x_i^s, x_i^b).
  \]

- Replace restrictive social welfare weight by **generalized social marginal welfare weights**.
  - \( g_i \) measures social value of $1 \) transfer for person \( i \).
  - Specified to directly capture fairness criteria.
  - Not necessarily derived from SWF.
Generalized social welfare weights approach

\[ u_i = u(c_i - v(z_i; x_i^u, x_i^b)) \quad g_i = g(c_i, z_i; x_i^s, x_i^b) \]
Resolve Puzzles and Unify Alternative Approaches

- **Resolve puzzles:** Can depend on luck vs. deserved income, can capture counterfactuals ("Free Loaders"), can model horizontal equity concerns.

- **Unify main alternatives to utilitarianism:** Rawlsianism, Libertarianism, Equality of Opportunity, Poverty Alleviation, Fair Income Taxation.

- **Pareto efficiency** guaranteed (locally) by non-negative weights.

- As long as weights depend on taxes paid (in addition to consumption): non-trivial theory of taxation even absent behavioral responses.

- **Positive tax theory:** Can estimate weights from revealed social choices.
Related Literature


Outline

1. Outline of the Approach
2. Resolving Puzzles of the Standard Approach
3. Link With Alternative Justice Principles
4. Empirical Testing and Estimation Using Survey Data
5. Conclusion
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General Model

- Mass 1 of individuals indexed by $i$.

- Utility from consumption $c_i$ and income $z_i$ (no income effects):
  \[
  u_i = u(c_i - v(z_i; x_i^u, x_i^b))
  \]
  where $x_i^u$ and $x_i^b$ are vectors of characteristics

- $u(.)$ increasing, $v$ decreasing in $z_i$.

- Typical income tax: $T(z)$, hence $c_i = z_i - T(z_i)$.
  - More general tax systems, with conditioning variables possible, depending on what is observable and politically feasible.
Small Tax Reform Approach

Consider a small tax reform $\delta T(z)$
[formally $\delta T(z) = \text{small reform in direction } \Delta T(z)$: $\delta T(z) = \varepsilon \cdot \Delta T(z)$ with $\varepsilon \to 0$]

- Small reform $\delta T(z)$ affects individual $i$ utility by $\delta u_i$ and earnings by $\delta z_i$
- By envelope theorem: $\delta u_i = -\frac{\partial u_i}{\partial c} \cdot \delta T(z_i)$
- $\Rightarrow$ Mechanical $-\delta T(z_i)$ measures money-metric welfare impact on $i$
- Change in tax paid by individual $i$ is $\delta T(z_i) + T'(z_i)\delta z_i$.

Definition

A reform $\delta T(z)$ is budget neutral if and only if $\int_i[\delta T(z_i) + T'(z_i)\delta z_i] = 0$. 
Generalized social welfare weights approach

Definition

The generalized social marginal welfare weight on individual $i$ is:

$$ g_i = g(c_i, z_i; x_{s_i}^i, x_{b_i}^i) $$

$g$ is a function, $x_{s_i}^i$ is a vector of characteristics which only affect the social welfare weight, while $x_{b_i}^i$ is a vector of characteristics which also affect utility.

- Recall utility is: $u_i = u(c_i - \nu(z_i; x_{u_i}^i, x_{b_i}^i))$
- Characteristics $x_s^i$, $x_u^i$, $x_b^i$ may be unobservable to the government.
  - $x_b^i$: fair to redistribute, enters utility – e.g. ability to earn
  - $x_s^i$: fair to redistribute, not in utility – e.g. family background
  - $x_u^i$: unfair to redistribute, enters utility – e.g. taste for work
Optimality Criterion with Generalized Weights

Definition

**Tax reform desirability criterion.** Small budget neutral tax reform $\delta T(z)$ desirable iff $\int_i g_i \cdot \delta T(z_i) < 0$, with $g_i$ the generalized social marginal welfare weight on $i$ evaluated at $(z_i - T(z_i), z_i, x_i^s, x_i^b)$.

- Reform only requires knowing $g_i$ and responses $\delta z_i$ around current $T(z)$

Definition

**Optimal tax criterion.** $T(z)$ optimal iff, for any small budget neutral reform $\delta T(z)$, $\int_i g_i \cdot \delta T(z_i) = 0$, with $g_i$ the generalized social marginal welfare weight on $i$ evaluated at $(z_i - T(z_i), z_i, x_i^s, x_i^b)$.

- No budget neutral reform can locally improve welfare as evaluated using generalized weights (local approach by definition)
Aggregating Standard Weights at Each Income Level

Taxes depend on $z$ only: express everything in terms of observable $z$.

$H(z)$: CDF of earnings, $h(z)$: PDF of earnings [both depend on $T(.)$]

Definition

$\tilde{G}(z)$ is the (relative) average social marginal welfare weight for individuals earning at least $z$:

$$\tilde{G}(z) \equiv \frac{\int\left\{i: z_i \geq z\right\} g_i}{\text{Prob}(z_i \geq z) \cdot \int_i g_i}$$

$g(z)$ is the average social marginal welfare weight at $z$ defined so that

$$\int_{z}^{\infty} g(z') dH(z') = \tilde{G}(z) \left[1 - H(z)\right]$$
Nonlinear Tax Formula Expressed with Welfare Weights

Proposition
The optimal marginal tax at $z$:

$$T'(z) = \frac{1 - \tilde{G}(z)}{1 - \tilde{G}(z) + \alpha(z) \cdot e(z)}$$

$e(z)$: average elasticity of $z$ w.r.t $1 - T'$ at $z_i = z$

$\alpha(z)$: local Pareto parameter $zh(z)/[1 - H(z)]$.

Proof follows the same “small reform” approach of Saez (2001): increase $T'$ in a small band $[z, z + dz]$ and work out effect on budget and weighted welfare.
Proof

- Reform $\delta T(z)$ increases marginal tax by $\delta \tau$ in small band $[z, z + dz]$.
- Mechanical revenue effect: extra taxes $dz\delta \tau$ from each taxpayer above $z$: $dz\delta \tau [1 - H(z)]$ is collected.
- Behavioral response: those in $[z, dz]$, reduce income by $\delta z = -ez\delta \tau/(1 - T'(z))$ where $e$ is the elasticity of earnings $z$ w.r.t $1 - T'$. Total tax loss $-dz\delta \tau \cdot h(z)e(z)zT'(z)/(1 - T'(z))$ with $e(z)$ the average elasticity in the small band.
- Net revenue collected by the reform and rebated lump sum is: $dR = dz\delta \tau \cdot \left[1 - H(z) - h(z) \cdot e(z) \cdot z \cdot \frac{T'(z)}{1 - T'(z)}\right]$.
- Welfare effect of reform: $-\int_i g_i \delta T(z_i)$ with $\delta T(z_i) = -dR$ for $z_i \leq z$ and $\delta T(z_i) = \delta \tau dz - dR$ for $z_i > z$. Net effect on welfare is $dR \cdot \int_i g_i - \delta \tau dz \int_{\{i: z_i \geq z\}} g_i$.
- Setting net welfare effect to zero, using $(1 - H(z))\tilde{G}(z) = \int_{\{i: z_i \geq z\}} g_i / \int_i g_i$ and $\alpha(z) = zh(z)/(1 - H(z))$, we obtain the tax formula.
Linear Tax Formula Expressed with Welfare Weights

The optimal linear tax rate, such that $c_i = z_i \cdot (1 - \tau) + \tau \cdot \int z_i$ can also be expressed as a function of an income weighted average marginal welfare weight (Piketty and Saez, 2013).

Proposition

The optimal linear income tax is:

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + e} \quad \text{with} \quad \bar{g} \equiv \frac{\int g_i \cdot z_i}{\int g_i \cdot \int z_i}$$

$e$: elasticity of $\int z_i$ w.r.t $(1 - \tau)$. 

Applying Standard Formulas with Generalized Weights

- Individual weights need to be “aggregated” up to characteristics that tax system can conditioned on.
  - E.g.: If \( T(z, x^b) \) possible, aggregate weights at each \((z, x^b) \rightarrow \bar{g}(z, x^b)\).
  - If standard \( T(z) \), aggregate at each \( z \): \( \bar{G}(z) \) and \( \bar{g}(z) \).

- Then apply standard formulas. Nests standard approach.

- If \( g_i \geq 0 \) for all \( i \), (local) Pareto efficiency guaranteed.

- Can we back out weights? Optimum \( \Leftrightarrow \max SWF = \int_i \omega_i \cdot u_i \) with Pareto weights \( \omega_i = g_i / u_{ci} \geq 0 \) where \( g_i \) and \( u_{ci} \) are evaluated at the optimum allocation
  - Impossible to posit correct weights \( \omega_i \) without first solving for optimum
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1. Optimal Tax Theory with Fixed Incomes

Modelling fixed incomes in our general model.

- Focus on redistributive issues.
- \( z = z_i \) is fixed for each individual (fully inelastic labor supply).
- Concave uniform utility \( u_i = u(c_i) \)

Standard utilitarian approach.

- Optimum: \( c = z - T(z) \) is constant across \( z \), full redistribution.
- Is it acceptable to confiscate incomes fully?
- Very sensitive to utility specification
- Heterogeneity in consumption utility? \( u_i = u(x_i^c \cdot c_i) \)
1. Tax Theory with Fixed Incomes: Generalized Weights

Definition
Let \( g_i = g(c_i, z_i) = \tilde{g}(c_i, z_i - c_i) \) with \( \tilde{g}_c \leq 0, \tilde{g}_{z - c} \geq 0 \).

i) Utilitarian weights: \( g_i = g(c_i, z_i) = \tilde{g}(c_i) \) for all \( z_i \), with \( \tilde{g}(\cdot) \) decreasing.

ii) Libertarian weights: \( g_i = g(c_i, z_i) = \tilde{g}(z_i - c_i) \) with \( \tilde{g}(\cdot) \) increasing.

- Weights depend negatively on \( c \) – “ability to pay” notion.
- Depend positively on tax paid – taxpayers contribute socially more.
- Optimal tax system: weights need to be equalized across all incomes \( z \):
  \[
  \tilde{g}(z - T(z), T(z)) \text{ constant with } z
  \]
1. Tax Theory with Fixed Incomes: Optimum

Proposition

The optimal tax schedule with no behavioral responses is:

\[ T'(z) = \frac{1}{1 - \frac{\tilde{g}_{z-c}}{\tilde{g}_c}} \quad \text{and} \quad 0 \leq T'(z) \leq 1. \] (1)

Corollary

Standard utilitarian case, \( T'(z) \equiv 1 \). Libertarian case, \( T'(z) \equiv 0 \).

- Empirical survey shows respondents indeed put weight on both disposable income and taxes paid.
- Between the two polar cases, \( g(c, z) = \tilde{g}(c - \alpha(z - c)) = \tilde{g}(z - (1 + \alpha)T(z)) \) with \( \tilde{g} \) decreasing.
- Can be empirically calibrated and implied optimal tax derived.
2. Luck versus Deserved Income: Setting

- Fairer to tax luck income than earned income and to insure against luck shocks.

- Provides micro-foundation for weights increasing in taxes, decreasing in consumption.

- $y^d$: deserved income due to effort

- $y^l$: luck income, not due to effort, with average $Ey^l$.

- $z = y^d + y^l$: total income.

- Society believes earned income fully deserved, luck income not deserved. Captured by binary set of weights:

  \[ g_i = 1(c_i \leq y^d_i + Ey^l) \]

  \[ g_i = 1 \] if taxed more than excess luck income (relative to average).
2. No behavioral responses: Observable Luck Income

- If luck income observable, can condition taxes on it: $T_i = T(z_i, y_i^l)$.

- Aggregate weights for each $(z, y^l)$ pair:
  $$\bar{g}(z, y^l) = 1(z - T(z, y^l) \leq z - y^l + Ey^l).$$

- Optimum: everybody's luck income must be $Ey^l$ with
  $$T(z, y^l) = y^l - Ey^l + T(z)$$ and $T(z) = 0$.

- Example: Health care costs.
2. No behavioral responses: Unobservable Luck Income

- Can no longer condition taxes on luck income: \( T_i = T(z_i) \).

- Aggregating weights:
  \[
  \tilde{g}(c, z - c) = \text{Prob}(c_i \leq z_i - y_i^l + Ey^l | c_i = c, z_i = z).
  \]

- Under reasonable assumptions, provides micro-foundation for weights \( \tilde{g}(c, z - c) \) decreasing in \( c \), increasing in \( z - c \).

- If bigger \( z - c \) at \( c \) constant, means bigger \( z \). Then, \( y^l \) increases but typically by less than \( z \), hence person more deserving, and hence \( \tilde{g}(c, z - c) \uparrow \).

- Optimum should equalize \( \tilde{g}(z - T(z), z) \) across all \( z \).

- Non-trivial theory of optimal taxation, even without behavioral responses.
3. Transfers and Free Loaders: Setting

- Behavioral responses closely tied to social weights: biggest complaint against redistribution is “free loaders.”
- Generalized welfare weights can capture “counterfactuals.”
- Consider linear tax model where $\tau$ funds demogrant transfer.
  
  \[ u_i = u(c_i - v(z_i; \theta)) = u(c_{z_i} - \theta_i \cdot z_i) \text{ with } z_i \in \{0, 1\}. \]
- Individuals can choose to not work, $z = 0$, $c_i = c_0$.
- If they work, earn $z = 1$, consume $c_1 = (1 - \tau) + c_0$.
- Cost of work $\theta$, with cdf $P(\theta)$, is private information.
- Individual: work iff $\theta \leq c_1 - c_0 = (1 - \tau)$.
- Fraction working: $P(1 - \tau)$.
- $e$: elasticity of aggregate earnings $P(1 - \tau)$ w.r.t $(1 - \tau)$. 

3. Transfers and Free Loaders: Optimal Taxation

Apply linear tax formula:

\[ \tau = \frac{(1 - \bar{g})}{(1 - \bar{g} + e)} \]

- In this model, \( \bar{g} = \frac{\int g_i z_i}{\int g_i \cdot \int z_i} = \bar{g}_1 / [P \cdot \bar{g}_1 + (1 - P) \cdot \bar{g}_0] \) with: \( \bar{g}_1 \) the average \( g_i \) on workers, and \( \bar{g}_0 \) the average \( g_i \) on non-workers.

Standard Approach:

- \( g_i = u'(c_0) \) for all non-workers so that \( \bar{g}_0 = u'(c_0) \).

- Hence, approach does not allow to distinguish between the deserving poor and free loaders.

- We can only look at actual situation: work or not, not “why” one does not work.

- Contrasts with public debate and historical evolution of welfare.
3. Transfers and Free Loaders: Generalized Welfare Weights

- Distinguish people according to what would have done absent transfer.

  **Workers:** Fraction $P(1 - \tau)$. Set $g_i = u'(c_1 - \theta_i)$.

  **Deserving poor:** would not work even absent any transfer: $\theta > 1$. Fraction $1 - P(1)$. Set $g_i = u'(c_0)$.

  **Free Loaders:** do not work because of transfer: $1 \geq \theta > (1 - \tau)$. Fraction $P(1) - P(1 - \tau)$. Set $g_i = 0$.

- Cost of work enters weights – fair to compensate for (i.e., not laziness).

- Average weight on non-workers
  \[ \bar{g}_0 = u'(c_0) \cdot (1 - P(1)) / (1 - P(1 - \tau)) < u'(c_0) \] lower than in utilitarian case.

- Reduces optimal tax rate not just through $e$ but also through $\bar{g}_0$. 
3. Transfers and Free Loaders: Remarks and Applications

- Ex post, possible to find suitable Pareto weights $\omega(\theta)$ that rationalize same tax.
  - $\omega(\theta) = 1$ for $\theta \leq (1 - \tau^*)$ (workers)
  - $\omega(\theta) = 1$ for $\theta \geq 1$ (deserving poor)
  - $\omega(\theta) = 0$ for $(1 - \tau^*) < \theta < 1$ (free loaders).

- But: these weights depend on optimum tax rate $\tau^*$.

- Other applications:
  - **Desirability of in-work benefits** if weight on non-workers becomes low enough relative to workers.
  - **Transfers over the business cycle**: composition of those out of work depends on ease of finding job.
4. Horizontal Equity: Puzzle and the Standard Approach

- Standard theory strongly recommends use of “tags” – yet not used much.

- Illustrate in Ramsey problem, where need to raise revenue $E$.

- 2 groups of measure 1, differ according to inelastic attribute $m \in \{1, 2\}$ and income elasticities $e_1 < e_2$.

- Standard approach: apply Ramsey tax rule, generates horizontal inequity:

  $$\tau_m = \frac{1 - \bar{g}_m}{1 - \bar{g}_m + e_m} \quad \text{with} \quad \bar{g}_m = \frac{\int_{i \in m} u_{ci} \cdot z_i}{p \cdot \int_{i \in m} z_i},$$

  $p > 0$: multiplier on budget constrained, adjusts to raise revenue $E$.

- Typically $\tau_1 > \tau_2$ because $e_1 < e_2$

- Horizontal equity: aversion to treating differently people with same income.

- Social marginal welfare weights concentrated on those suffering from horizontal inequity.
  
  ▶ Horizontal inequity carry higher priority than vertical inequity.

- If no horizontal inequity, a reform that creates horizontal inequity needs to be penalized: **weights need to depend on direction of reform.**

- If \( i \in m \) then \( i \notin n \) and define weight \( g_i = g(\tau_m, \tau_n, \delta\tau_m, \delta\tau_n) \)

  - i) \( g(\tau_m, \tau_n, \delta\tau_m, \delta\tau_n) = 1 \) and \( g(\tau_n, \tau_m, \delta\tau_n, \delta\tau_m) = 0 \) if \( \tau_m > \tau_n \).
  
  - ii) \( g(\tau, \tau, \delta\tau_m, \delta\tau_n) = 1 \) and \( g(\tau, \tau, \delta\tau_n, \delta\tau_m) = 0 \) if \( \tau_m = \tau_n = \tau \) and \( \delta\tau_m > \delta\tau_n \).
  
  - iii) \( g(\tau, \tau, \delta\tau_m, \delta\tau_n) = g(\tau, \tau, \delta\tau_n, \delta\tau_m) = 1 \) if \( \tau_m = \tau_n = \tau \) and \( \delta\tau_m = \delta\tau_n \).
4. Horizontal Equity: Optimum with Generalized Weights

Regularity assumptions.
- There is a uniform tax rate $\tau_1 = \tau_2 = \tau^*$ that can raise $E$.
- Laffer curves $\tau_1 \rightarrow \tau_1 \cdot \int_{i \in 1} z_i$, $\tau_2 \rightarrow \tau_2 \cdot \int_{i \in 2} z_i$, and $\tau \rightarrow \tau \cdot (\int_{i \in 1} z_i + \int_{i \in 2} z_i)$ are single peaked.

Proposition

Let $\tau^*$ be the smallest uniform rate that raises $E$: $\tau^* (\int_{i \in 1} z_i + \int_{i \in 2} z_i) = E$.

i) If $1 / (1 + e_2) \geq \tau^*$ the only optimum has horizontal equity with $\tau_1 = \tau_2 = \tau^*$.

ii) If $1 / (1 + e_2) < \tau^*$ the only optimum has horizontal inequity with $\tau_2 = 1 / (1 + e_2) < \tau^*$ (revenue maximizing rate) and $\tau_1 < \tau^*$ the smallest tax rate s.t. $\tau_1 \cdot \int_{i \in 1} z_i + \tau_2 \cdot \int_{i \in 2} z_i = E$ (Pareto dominates $\tau_1 = \tau_2 = \tau^*$)
4. Horizontal Equity with Generalized Weights

- Horizontal inequity can be part of an optimum only if helps group discriminated against.

- Tagging must be Pareto improving to be desirable, limits scope for use.

- New Rawlsian criterion: “Permissible to discriminate against a group based on tags, only if discrimination improves this group’s welfare.”
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1. Libertarianism and Rawlsianism

Libertarianism:

- Principle: “Individual fully entitled to his pre-tax income.”
- Morally defensible if no difference in productivity, but different preferences for work.
- \[ g_i = g(c_i, z_i) = g(c_i - z_i), \] increasing (\( x_i^s \) and \( x_i^b \) empty).
- Optimal formula yields: \( T'(z_i) \equiv 0 \).

Rawlsianism:

- Principle: “Care only about the most disadvantaged.”
- \[ g_i = g(u_i - \min_j u_j) = 1(u_i - \min_j u_j = 0), \] with \( x_i^s = u_i - \min_j u_j \) and \( x_i^b \) is empty.
- If least advantaged people have zero earnings independently of taxes, \( \bar{G}(z) = 0 \) for all \( z > 0 \).
- Optimal formula yields: \( T'(z) = 1/[1 + \alpha(z) \cdot e(z)] \) (maximize demogrant \( -T(0) \)).
2. Equality of Opportunity: Setting

- Standard utility \( u(c - v(z/w_i)) \) with \( w_i \) ability to earn
- \( w_i \) is result of i) family background \( B_i \in \{0, 1\} \) (which individuals not responsible for) and ii) merit (which individuals are responsible for) = rank \( r_i \) conditional on background.
- Advantaged background gives earning ability \( w \) advantage: \( w(r_i|B_i = 1) > w(r_i|B_i = 0) \)
- Society is willing to redistribute across backgrounds, but not across incomes conditional on background.
- \( \Rightarrow \) Conditional on earnings, those coming from \( B_i = 0 \) are more meritorious [because they rank higher in merit]
- \( \bar{c}(r) \equiv (\int_{i:r_i=r} c_i) / \text{Prob}(i : r_i = r) \): average consumption at rank \( r \).
- \( g_i = g(c_i; \bar{c}(r_i)) = 1(c_i \leq \bar{c}(r_i)) \)
2. Equality of Opportunity: Results

- Suppose government cannot condition taxes on background.

- $\tilde{G}(z)$: **Representation index**: % from disadvantaged background earning $\geq z$ relative to % from disadvantaged background in population.

- Implied Social Welfare function as in Roemer et al. (2003).

- $\tilde{G}(z)$ decreasing since harder for those from disadvantaged background to reach upper incomes.

- If at top incomes, representation is zero, revenue maximizing top tax rate.

- Justification for social welfare weights decreasing with income not due to decreasing marginal utility (utilitarianism).
### 2. Equality of Opportunity vs. Utilitarian Tax Rates

<table>
<thead>
<tr>
<th>Income percentile</th>
<th>Equality of Opportunity</th>
<th>Utilitarian (log-utility)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction from low background (=parents below median) above each percentile</td>
<td>Implied social welfare weight G(z) above each percentile</td>
</tr>
<tr>
<td>z= 25th percentile</td>
<td>44.3%</td>
<td>0.886</td>
</tr>
<tr>
<td>z= 50th percentile</td>
<td>37.3%</td>
<td>0.746</td>
</tr>
<tr>
<td>z= 75th percentile</td>
<td>30.3%</td>
<td>0.606</td>
</tr>
<tr>
<td>z= 90th percentile</td>
<td>23.6%</td>
<td>0.472</td>
</tr>
<tr>
<td>z= 99th percentile</td>
<td>17.0%</td>
<td>0.340</td>
</tr>
<tr>
<td>z= 99.9th percentile</td>
<td>16.5%</td>
<td>0.330</td>
</tr>
</tbody>
</table>

3. Poverty Alleviation: Setting

- Poverty gets substantial attention in public debate.
- Poverty alleviation objectives can lead to Pareto dominated outcomes:
  - Intuition: disregard people’s disutility from work.
- Generalized welfare weights can avoid pitfall of Pareto inefficiency.
- \( \bar{c} \): poverty threshold. "Poor": \( c < \bar{c} \).
- \( u_i = u(c_i - v(z_i/w_i)) \).
- \( \bar{z} \): (endogenous) pre-tax poverty threshold: \( \bar{c} = \bar{z} - T(\bar{z}) \).
- Poverty gap alleviation: care about shortfall in consumption.
  - \( g_i = g(c_i, z_i; \bar{c}) = 1 > 0 \) if \( c_i < \bar{c} \) and \( g_i = g(c_i, z_i; \bar{c}) = 0 \) if \( c_i \geq \bar{c} \).
  - \( \bar{g}(z) = 0 \) for \( z \geq \bar{z} \) and \( \bar{g}(z) = 1/H(\bar{z}) \) for \( z < \bar{z} \).
  - \( \bar{G}(z) = 0 \) for \( z \geq \bar{z} \) and \( 1 - \bar{G}(z) = \frac{1/H(\bar{z}) - 1}{1/H(z) - 1} \) for \( z < \bar{z} \).
3. Optimal Tax Schedule that Minimizes Poverty Gap

Proposition

\[ T'(z) = \frac{1}{1 + \alpha(z) \cdot e(z)} \]

if \( z > \bar{z} \)

\[ T'(z) = \frac{1}{1 + \alpha(z) \cdot e(z) \cdot \frac{1/H(z) - 1}{1/H(\bar{z}) - 1}} \]

if \( z \leq \bar{z} \)

\[ c = z - T(z) \]

(a) Direct poverty gap minimization

(b) Generalized weights approach
4. Fair Income Taxation: Principle

- Agents differ in preference for work (laziness) and skill.


- Want to favor hard working low skilled but cannot tell them apart from the lazy high skilled.

- Show how their $w_{\text{min}}$-equivalent leximin criterion translates into social marginal welfare weights.

- We purely reverse engineer here to show usefulness of formula and generalized weights.
4. Fair Income Taxation: Setting and Optimal tax rates

- \( u_i = c_i - \nu(z_i/w_i, \theta_i) \), \( w_i \): skill, \( \theta_i \): preference for work.

- Labor supply: \( l_i = z_i/w_i \in [0, 1] \) (full time work \( l = 1 \)).

- Criterion: full weight on those with \( w = w_{\text{min}} \) getting smallest net transfer from government.

- Fleurbaey-Maniquet optimal tax system: \( T'(z) = 0 \) for \( z \in [0, w_{\text{min}}] \),
  \( T'(z) = 1/(1 + \alpha(z) \cdot e(z)) > 0 \) for \( z > w_{\text{min}} \).

- Implies \( \bar{G}(z) = 1 \) for \( 0 \leq z \leq w_{\text{min}} \).

- Hence, \( \int_{z}^{\infty} [1 - g(z')] dH(z') = 0 \).

- Differentiating w.r.t \( z \): \( \bar{g}(z) = 1 \) for \( 0 \leq z \leq w_{\text{min}} \).

- For \( z > w_{\text{min}} \), \( \bar{G}(z) = 0, \bar{g}(z) = 0 \).

- Let \( T_{\text{max}} \equiv \max_{i: w_i = w_{\text{min}}} (z_i - c_i) \).

- \( g(c_i, z_i; w_i, w_{\text{min}}, T_{\text{max}}) = \tilde{g}(z_i - c_i; w_i, w_{\text{min}}, T_{\text{max}}) \) with:
  - \( \tilde{g}(z_i - c_i; w_i, w_{\text{min}}, T_{\text{max}}) = 0 \) for \( w_i > w_{\text{min}} \), for any \((z_i - c_i)\) (no weight on those above \( w_{\text{min}} \)).
  - \( \tilde{g}(.; w_i = w_{\text{min}}, w_{\text{min}}, T_{\text{max}}) \) is an (endogenous) Dirac distribution concentrated on \( z - c = T_{\text{max}} \).

- Forces government to provide same transfer to all with \( w_{\text{min}} \).

- If at every \( z < w_{\text{min}} \) can find \( w_{\text{min}} \) agents, forces equal transfer at all \( z < w_{\text{min}} \).

- Zero transfer above \( w_{\text{min}} \) since no \( w_{\text{min}} \) agents found there.
Outline

1. Outline of the Approach
2. Resolving Puzzles of the Standard Approach
3. Link With Alternative Justice Principles
4. Empirical Testing and Estimation Using Survey Data
5. Conclusion
Online Survey: Goals and Setup

Two goals of empirical application:

1. Discover notions of fairness people use to judge tax and transfer systems.
   - Focus on themes addressed in theoretical part.

2. Quantitatively calibrate simple weights

Online Platform:

- Amazon mTurk (Kuziemko, Norton, Saez, Stantcheva, 2015).
- 1100 respondents with background information.
Evidence against utilitarianism

- Respondents asked to compare families w/ different combinations of $z$, $z - T(z)$, $T(z)$.
- Who is most deserving of a $1000 tax break?
- **Both disposable income and taxes paid matter** for deservedness
  - Family earning $40K, paying $10K in taxes judged more deserving than family earning $50K, paying $10K in taxes
  - Family earning $50K, paying $15K in taxes judged more deserving than family earning $40K, paying $5K in taxes
- **Frugal vs. Consumption-loving** person with same net income

<table>
<thead>
<tr>
<th>Consumption-lover more deserving</th>
<th>Frugal more deserving</th>
<th>Taste for consumption irrelevant</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>22%</td>
<td>74%</td>
</tr>
</tbody>
</table>
Which of the following two individuals do you think is most deserving of a $1,000 tax break?

Individual A earns $50,000 per year, pays $10,000 in taxes and hence nets out $40,000. She greatly enjoys spending money, going out to expensive restaurants, or traveling to fancy destinations. She always feels that she has too little money to spend.

Individual B earns the same amount, $50,000 per year, also pays $10,000 in taxes and hence also nets out $40,000. However, she is a very frugal person who feels that her current income is sufficient to satisfy her needs.

- Individual A is most deserving of the $1,000 tax break
- Individual B is most deserving of the $1,000 tax break
- Both individuals are exactly equally deserving of the tax $1,000 break

Source: survey in Saez and Stantcheva (2013)
Does society care about effort to earn income?

- Hard-working vs. Easy-going person with same net income

  “A earns $30,000 per year, by working in two different jobs, 60 hours per week at $10/hour. She pays $6,000 in taxes and nets out $24,000. She is very hard-working but she does not have high-paying jobs so that her wage is low.”

  “B also earns the same amount, $30,000 per year, by working part-time for 20 hours per week at $30/hour. She also pays $6,000 in taxes and hence nets out $24,000. She has a good wage rate per hour, but she prefers working less and earning less to enjoy other, non-work activities.”

<table>
<thead>
<tr>
<th>Hardworking</th>
<th>Easy-going</th>
<th>Hours of work irrelevant conditional on total earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>more deserving</td>
<td>more deserving</td>
<td>54%</td>
</tr>
</tbody>
</table>
Do people care about “Free Loaders” and Behavioral Responses to Taxation?

Starting from same benefit level, which person most deserving of more benefits?

<table>
<thead>
<tr>
<th></th>
<th>Disabled unable to work</th>
<th>Unemployed looking for work</th>
<th>Unemployed not looking for work</th>
<th>On welfare not looking for work</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average rank (1-4)</strong></td>
<td>1.4</td>
<td>1.6</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>% assigned 1st rank</td>
<td>57.5%</td>
<td>37.3%</td>
<td>2.7%</td>
<td>2.5%</td>
</tr>
<tr>
<td>% assigned last rank</td>
<td>2.3%</td>
<td>2.9%</td>
<td>25%</td>
<td>70.8%</td>
</tr>
</tbody>
</table>
Calibrating Social Welfare Weights

- Calibrate $\tilde{g}(c, T) = \tilde{g}(c - \alpha T)$
- 35 fictitious families, w/ different net incomes and taxes
- Respondents rank them pair-wise (5 random pairs each)

Which of these two families is most deserving of the $1,000 tax break?

- Family earns $100,000 per year, pays $50,000 in taxes, and hence nets out $50,000
- Family earns $25,000 per year, pays $1,250 in taxes, and hence nets out $23,750

Which of these two families is most deserving of the $1,000 tax break?

- Family earns $50,000 per year, pays $2,500 in taxes, and hence nets out $47,500
- Family earns $500,000 per year, pays $170,000 in taxes, and hence nets out $330,000
Eliciting Social Preferences

Is A or B more deserving of a $1,000 tax break?
Eliciting Social Preferences

Is A or B more deserving of a $1,000 tax break?
Eliciting Social Preferences

\( S_{ijt} = 1 \) if \( i \) ranked 1st in display \( t \) for respondent \( j \), \( \delta T_{ijt} \) is difference in taxes, \( \delta c_{ijt} \) difference in net income for families in pair shown.

\[
S_{ijt} = \beta_0 + \beta_T \delta T_{ijt} + \beta_c \delta c_{ijt}
\]

\[
\alpha = \frac{\delta c}{\delta T} \bigg|_S = -\frac{\beta_T}{\beta_c} = -\text{slope}
\]

\( g(c, T) = g(c - \alpha T) \)

indifference curves
### Table 5: Calibrating Social Welfare Weights

<table>
<thead>
<tr>
<th>Sample</th>
<th>Probability of being deemed more deserving in pairwise comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Excludes cases with income of $1m</td>
</tr>
<tr>
<td></td>
<td>Full</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>d(Tax)</strong></td>
<td>0.0017***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
</tr>
<tr>
<td><strong>d(Net Income)</strong></td>
<td>-0.0046***</td>
</tr>
<tr>
<td></td>
<td>(0.00012)</td>
</tr>
<tr>
<td><strong>Number of observations</strong></td>
<td>11,450</td>
</tr>
<tr>
<td><strong>Implied α</strong></td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
</tr>
<tr>
<td><strong>Implied marginal tax rate</strong></td>
<td>73%</td>
</tr>
</tbody>
</table>
Outline

1. Outline of the Approach
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Conclusion

- Generalized marginal social welfare weights are fruitful way to extend standard welfarist theory of optimal taxation.
  - Allow to dissociate individual characteristics from social criteria.
  - Which characteristics are fair to compensate for?

- Helps resolve puzzles of traditional welfarist approach.

- Unifies existing alternatives to welfarism.

- Weights can **prioritize social justice principles** in lexicographic form:
  1. Injustices created by tax system itself (horizontal equity)
  2. Compensation principle (health, family background)
  3. Luck component in earnings ability
  4. Utilitarian concept of decreasing marginal utility of consumption.