Journal of Public Economics 4 (1975) 335–342.

A MANY-PERSON RAMSEY TAX RULE

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Received January 1975, revised version received February 1975

Define the social marginal utility of an individual's income as the gain to society of a unit of consumption by the individual plus the value of his marginal propensity to pay taxes out of income. This concept rather than the social marginal utility of consumption (equal to the first term above) seems helpful in understanding optimal tax first order conditions. For example, with many consumers (and a poll tax as well as excise taxes) the change in aggregate compensated quantity demanded is proportional to the covariance between individual quantities demanded and social marginal utilities of income.

1. Introduction

In setting out the first-order conditions for optimal excise taxes in a one person (or many identical individuals) economy, it has become standard to use the Ramsey (1927) formulation[1] that the optimal taxes induce (approximately) equal percentage reductions in (compensated) demands for all commodities (with the approximation being valid for small amounts of tax revenue). Mirrlees (1975) has given an alternative interpretation of these same conditions – that at the optimum, a small proportional increase in all tax rates results in a proportional decrease in all (compensated) demands. Consideration of the first-order conditions for optimal excise taxes in a general many-person economy has not yet yielded similarly simple interpretations when cast into a similar quantity change form. In considering the two-class economy, (i.e., two types of consumers), Mirrlees (1975) has modified the standard problem by considering simultaneously excise taxes and a poll tax. For this problem he gets a generalized Ramsey formulation that the induced changes in aggregate demand be proportional to demand differences between typical members of the two classes. This paper will examine the Ramsey rule for a many-person economy with excise taxes and a poll tax. Instead of using the social marginal utilities of consumption (i.e., increase in social welfare from increased consumption of the numeraire good by different individuals), the interpretation will use the

*Financial support by the National Science Foundation is gratefully acknowledged.

[1]The directly derived first-order conditions have the form that at the optimum the impact of any tax increase on social welfare is proportional to the marginal tax revenue collected, or alternatively to the cost of producing the induced changes in demand.

social marginal utilities of income (i.e., gain in social welfare from provision of additional income in numeraire units, which is the sum of gains from individual consumption and from the marginal propensity to pay taxes out of income). The many-person Ramsey rule is that the (approximate) percentage change in (compensated) demands depends on the social marginal utilities of income, being positive (negative) for goods demanded on average by individuals with above (below) average social marginal utilities of income.[2] Denoting the social marginal utility of man h by γ^h, and his consumption of good k by x_k^h, the many-person Ramsey rule is

$$\frac{\Delta X_k}{X_k} = \frac{\sum(\gamma^h - \lambda)x_k^h}{\lambda X_k}, \tag{1}$$

where λ is the average of γ^h (and also equals the Lagrangian on the government budget constraint) and X_k is aggregate demand for good k.

This modification of familiar first-order conditions might appear to be simply replacing a complicated expression by an arbitrary definition, γ^h, which thereby automatically simplified the expression. However, by briefly considering three problems already analyzed in the literature, we shall see that the use of the social marginal utility of income seems to give more natural interpretations than use of the social marginal utility of consumption. We shall see that in the two-class model of Mirrlees the individuals in the class with lower social marginal utility of income pay more in excise taxes. In the many-consumer economy this generalizes to a negative covariance between social marginal utilities of income and excise taxes paid. The same statement does not appear to hold generally with consumption replacing income. Without using this terminology, Atkinson and Stern (1974) have noted that in the one-consumer economy the relative size of social marginal utility of consumption and of the Lagrangian on the government budget constraint appears to depend on the choice of numeraire. As they noted, the sign of the social marginal utility of income less the government Lagrangian, however, is the opposite of that of tax revenue, independent of choice of numeraire. In addition we will consider the rules for optimal public good expenditures, expressed in terms analogous to the social marginal utility of income.

2. Many-person Ramsey rule

Since optimal tax derivations are now so familiar we will proceed directly. For convenience in later use we shall set up the model with public goods.

q	vector of consumer prices,
p	vector of producer prices,

[2]This result has also been developed by Atkinson and Stiglitz (1974).

$t = q-p$	vector of taxes,
I	lump-sum income (the same for all consumers),
e	level of public good expenditures,
$v^h(q, I, e)$	indirect utility function for consumer h,
$\alpha^h = \dfrac{\partial v^h}{\partial I}$	marginal utility of income (consumption),
$W(v^1, \ldots, v^H)$	social welfare function depending on utilities of the H consumers,
$\beta^h = \dfrac{\partial W}{\partial v^h} \alpha^h$	social marginal utility of consumption,
$x^h(q, I, e)$	vector of consumer h demands,
$X = \sum_h x^h$	aggregate demand,
$F(X, e)$	production constraint.

We can now set up the welfare function maximization as

$$\text{Max } W(v^1(q, I, e), \ldots, v^H(q, I, e)), \tag{2}$$

subject to

$$F(X(q, I, e), e) = 0.$$

Forming a Lagrangian expression with multiplier λ we are in a position to generate first-order conditions. Assuming I and e are given and zero, we can calculate the first-order conditions for q,

$$\sum_h \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial q_k} = \lambda \sum_i F_i \frac{\partial X_i}{\partial q_k}. \tag{3}$$

Choosing good one as numeraire and selecting units appropriately we shall write $p_1 = F_1 = 1 = q_1$. Using the properties of the indirect utility function we can write this in the familiar form (e.g., see, Diamond and Mirrlees (1971))

$$-\sum_h \beta^h x_k^h = \lambda \sum_h \sum_i p_i \frac{\partial x_i^h}{\partial q_k}. \tag{4}$$

Replacing p_i by $q_i - t_i$, noting that $\sum_i q_i(\partial x_i^h/\partial q_k) = -x_k^h$ (from the individual's budget constraint), and using the Slutsky equation, we have

$$\sum_h \beta^h x_k^h = \lambda \sum_h \left(x_k^h + \sum_i t_i \left(s_{ik}^h - x_k^h \frac{\partial x_i^h}{\partial I} \right) \right), \tag{5}$$

where s_{ik}^h is the derivative of the compensated demand curve.

Defining the social marginal utility of income, γ^h, as the gain to society from additional income given to consumer h, we see that γ^h is made up of two parts. One part is the social evaluation of the increased utility of h made possible by higher income. This equals β^h. The second part is the social evaluation of the additional tax revenue collected, $\sum t_i\,(\partial x_i^h/\partial I)$, as a consequence of his having more income. We shall elaborate on this definition in section 4. Thus

$$\gamma^h = \beta^h + \lambda \sum_i t_i\,(\partial x_i^h/\partial I). \tag{6}$$

Using this definition we can write the first-order conditions (5) as

$$\sum_h (\gamma^h - \lambda) x_k^h = \lambda \sum_i \sum_h t_i s_{ik}^h. \tag{7}$$

From the symmetry of the Slutsky matrix, so that $s_{ik}^h = s_{ki}^h$, this has the form of eq. (1), where

$$\Delta X_k = \sum_h \sum_i s_{ki}^h t_i \tag{8}$$

is the change in compensated aggregate demand for good k as a result of a marginal proportional increase in all tax rates. Eq. (7) holds as a consequence of the optimal excise taxes. The interpretation of (7) becomes more interesting if we also have an optimal poll tax. From (2) the first-order condition coming from differentiation with respect to I is

$$\sum_h \beta^h = \lambda \sum_i F_i\,(\partial X_i/\partial I). \tag{9}$$

Following the same sequence of steps as before we can write this as

$$\sum_h \beta^h = \lambda \sum_i \sum_h p_i \frac{\partial x_i^h}{\partial I} = \lambda \sum_h \sum_i (q_i - t_i) \frac{\partial x_i^h}{\partial I} = \lambda \sum_h \left(1 - \sum_i t_i \frac{\partial x_i^h}{\partial I}\right). \tag{10}$$

Thus we have the result that λ is equal to the average of γ^h in the economy

$$\sum_h \gamma^h = \lambda H. \tag{11}$$

With λ equal to the average of the γ^h, we can interpret (1) as a covariance formula, since we can subtract $\sum (\gamma^h - \lambda)\bar{x}_k$ from the left hand side, where $\bar{x}_k$ is the average of x_k^h. Thus, for each good, the change in aggregate compensated quantity demanded is proportional to the covariance between individual quantities demanded and social marginal utilities of income. The percentage change in demand equals the covariance divided by the product of the two means.

3. Two-class economy

We can move directly from (7) and (11) to the results of Mirrlees. Assume there are m consumers of type 1 and n consumers of type 2. Then, from (11),

$$(m+n)\lambda = m\gamma^1 + n\gamma^2. \tag{12}$$

Thus using (12), eq. (7) becomes

$$\begin{aligned}\lambda \sum_i \sum_h t_i s_{ik}^h &= m(\gamma^1-\lambda)x_k^1 + n(\gamma^2-\lambda)x_k^2 \\ &= n(\lambda-\gamma^2)x_k^1 + n(\gamma^2-\lambda)x_k^2 \\ &= n(\gamma^2-\lambda)(x_k^2-x_k^1).\end{aligned} \tag{13}$$

Thus the induced changes in compensated aggregate demand are proportional to the differences in demand between the two types. Multiplying (13) by t_k and summing over k we have

$$n(\gamma^2-\lambda)\left(\sum_k t_k x_k^2 - \sum_k t_k x_k^1\right) = \lambda \sum_h \sum_i \sum_k t_i s_{ik}^h t_k \leq 0. \tag{14}$$

The sign follows from the negative semidefiniteness of the Slutsky matrix. Since the signs of $\gamma^2-\lambda$ and $\gamma^2-\gamma^1$ are the same, we see that an individual with greater social marginal utility of income pays less in excise taxes under the optimal excise and poll tax regime.

Applying the same procedure to the general economy, from (7) we have the result that with optimal excise taxes

$$\sum_h \left((\gamma^h-\lambda)\sum_k t_k x_k^h\right) \leq 0. \tag{15}$$

If we add an optimal poll tax we can again go to a covariance formulation. With $\sum(\gamma^h-\lambda)$ equal to zero we can multiply it by the average over h of $\sum_k t_k x_k^h$ and subtract it from (15).

Denoting average values by a bar we thus have the result that with optimal excise and poll taxes,

$$\sum_h \left\{(\gamma^h-\bar{\gamma})\left(\sum_k t_k x_k^h - \overline{\sum_k t_k x_k}\right)\right\} \leq 0. \tag{16}$$

That is, with optimal excise and poll taxes there is a negative covariance between social marginal utility of income and excise taxes paid.

4. One-consumer economy

Consider an outside agency planning to give aid to a one-consumer economy with optimal excise taxes. The agency might give the aid to the consumer directly or to the government, and the aid might be given in any commodity. One would expect that it is better to give the aid to the government if revenue is being raised by distorting taxes, whatever the good being considered. (And to give it to the consumer if the government is disposing of a surplus by distorting subsidies.) This is precisely the answer given by (15), evaluating the social worth of aid to the consumer and government respectively by γ and λ. From the definition of γ, it is clear we are evaluating aid assuming it is provided while markets are still open. Thus the consumer engages in trade with the income provided him, generating a change in tax revenue, as well as a direct utility rise for the consumer.

Suppose, alternatively, that aid is provided 'after markets are shut'. That is, no changes in trades are allowed after aid is provided. For arbitrarily small amounts of aid, the fact that the consumer was at a utility maximizing consumption plan implies that his direct gain in utility from the aid is unaffected by the prohibition of further trading. Thus the value to society of aid provided to the consumer in this way is β. The question of the comparative advantage of giving this aid to the consumer rather than the government is a comparison of β with λ.[3] However, the government's rate of substitution between different commodities is equal to the ratio of producer prices, p, while the consumer's rate of substitution is equal to the ratio of consumer prices q. Thus a change in units in which aid is given (corresponding to a change in choice of numeraire) has the potential of altering the answer to the question of the choice of recipient which most increases social welfare.

5. Public good expenditures

The first-order condition for public expenditures, like any equation, can be arranged with different terms on either side of the equation. We shall consider a rearrangement which parallels the structure considered above. For some of the interpretations, it will not be necessary to assume that all taxes are optimally set since the equations derived will also hold when those taxes not being optimally set are held constant at given levels. Let us define δ^h to be the value to society of providing the public good to consumer h. It is made up of two parts, the social evaluation of his utility increase, $(\partial W/\partial v^h)(\partial v^h/\partial e)$, and the value of any change in taxes paid, $\lambda \sum_i t_i (\partial x_i^h/\partial e)$,

[3]Atkinson and Stern (1974) discuss this issue in terms of α and λ. These are obviously the same where, in the one-consumer economy, the social welfare function and the utility function are the same.

$$\delta^h = \frac{\partial W}{\partial v^h}\frac{\partial v^h}{\partial e} + \lambda \sum_i t_i \frac{\partial x_i^h}{\partial e} \tag{17}$$

$$= \beta^h \frac{\partial v^h/\partial e}{\partial v^h/\partial I} + \lambda \sum_i t_i \frac{\partial x_i^h}{\partial e}.$$

Returning to the problem of social welfare maximization, (2), differentiation with respect to the public good expenditure gives

$$\sum_h \frac{\partial W}{\partial v^h}\frac{\partial v^h}{\partial e} = \lambda \sum_i F_i \frac{\partial X_i}{\partial e} + \lambda F_e \tag{18}$$

$$= \lambda \sum_h \sum_i (q_i - t_i) \frac{\partial x_i^h}{\partial e} + \lambda F_e.$$

Since $\sum_i q_i\,(\partial x_i^h/\partial e)$ is zero by the consumers budget constraint, we can write the first-order condition for public expenditures as

$$\sum_h \delta^h = \lambda F_e. \tag{19}$$

Thus, for the optimum, the sum over individuals of the value to society of providing each of them with the public good is equated to the resource cost of public provision of the public good, measured in units of social welfare. Put this way, this is an obvious first-order condition. The complication is to measure correctly the value to society of provision of the public good to the individual. Dividing (19) by λ we can, alternatively, express the first-order condition in units of numeraire. The right-hand side is the resource cost of public good provision. Since the public good expenditure comes out of the government budget it makes sense that the left-hand side is the sum over individuals of the marginal rate of substitution between the social cost of expenditure from the public budget and the social gain from the individual's enjoyment of the public good. This first-order condition is valid whatever mix of excise and poll taxes and other public expenditures is varied optimally, the remaining government choice variables being held constant.

To get an expression more closely resembling that in the lump-sum tax world, we can assume that the poll tax is among the variables being optimally set. Then λ is the average of social marginal utilities of income in the economy, and the first-order condition for public goods equates the marginal rate of transformation in production to the sum over consumers of social marginal rates of substitution between public good consumption by the consumer and income averaged over the population.

We are still a long way from having an intuition for resource allocation questions in economies with distorting taxes which parallels the level of intuition in first-best economies. Perhaps by using the social marginal utility of income rather than the seemingly more natural social marginal utility of consumption we can develop such a level more rapidly.

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