Preference Heterogeneity and Optimal Commodity Taxation

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Abstract

We analytically and quantitatively examine a prominent justification for capital income taxation: goods preferred by the high-skilled ought to be taxed. We study an environment where commodity taxes are allowed to be nonlinear functions of income and consumption and find that optimal commodity taxes on these goods may be regressive. We first derive an expression for optimal commodity taxation, allowing us to study the forces for and against regressivity in that more general setting. We then parameterize the model to evidence on the relationship between skills and preferences and examine the quantitative case for regressive taxes on future consumption (saving). The relationship between skill and time preference delivers quantitatively small, regressive capital income taxes and does not justify substantial capital income taxation, whether regressive or linear. We also apply the model to a second category of expenditure, owner-occupied housing, and find a stronger case for a sizeable, regressive tax on this good.

Introduction

One justification for positive capital income taxation is that the goods preferred by high-ability individuals ought to be taxed because the consumption of these goods provides a signal of individuals’ otherwise unobservable ability. If individuals’ abilities are positively related to preferences for saving, this argument implies that capital income should be taxed. Two prominent expositions of this justification are Saez (2002) and Banks and Diamond (2009). Saez shows that a small linear tax on a commodity preferred by individuals with higher skills generates a smaller efficiency loss than does an increase in the optimal nonlinear income tax that raises the same revenue from each individual. He applies this logic to capital income taxation and concludes "...the discount rate \( \delta \) is probably negatively correlated with skills. This suggests that interest income ought to be taxed even in the presence of a non-linear optimal earnings tax." Banks and Diamond...
(2009) is the chapter on direct taxation in the Mirrlees Review. Commissioned by the Institute for Fiscal Studies, the Review is the successor to the influential Meade Report of 1978 and is the authoritative summary of the current state of tax theory as it relates to policy. Their chapter concludes:

"With the plausible assumption that those with higher earnings abilities discount the future less (and thus save more out of any given income), then taxation of saving helps with the equity-efficiency tradeoff by being a source of indirect evidence about who has higher earnings abilities and thus contributes to more efficient redistributive taxation."

In this paper, we analytically and quantitatively study this justification for taxing goods preferred by those with high ability, in particular future consumption (i.e., saving) and housing, when commodity taxes are allowed to be nonlinear functions of both income and consumption.³

We first derive analytical expressions that indicate the shape of optimal commodity taxation. We start in a two-type, two-commodity economy and demonstrate that the high ability type faces no distortion to its chosen commodity basket while the low type faces a positive marginal tax on the good preferred by the high type. In other words, taxes are regressive in this case. We then derive the condition describing optimal commodity taxes in an economy with a continuum of types. The commodity tax on the agent with the highest skill is again equal to zero and is positive for other types. As is common in Mirrleesian models (e.g., Saez 2001) we then analytically study the forces for and against regressivity. The intuition for why regressive commodity taxation may be optimal starts with the realization that the goal of optimal tax policy (in the Mirrleesian framework) is to redistribute from high-ability workers without discouraging their work effort. With this as the goal, the optimal use of commodity taxation is to increase the attractiveness of earning a high income. Commodity taxes that are regressive (i.e., that fall with income) on those goods most valued by high-ability individuals will encourage them to earn more, allowing the tax authority to levy higher income taxes on them and redistribute more resources to the low-skilled.⁴

The second objective of the paper is to examine the quantitative case for regressive taxes on two important commodities: future consumption (saving) and housing. For saving, we find that the optimal nonlinear (and also linear) capital income tax is less than five percent in our main simulations. Thus, our results do not justify substantial capital income taxation despite the positive relationship between ability and patience that we find in the data. For housing, we find that a quantitatively significant regressive tax may be optimal.

To quantitatively characterize the optimal taxation of saving, we use existing evidence from the National Longitudinal Survey of Youth (NLSY) to show a positive correlation between ability⁵ and relative preference for future consumption. Using these data to estimate a mean value for time preference by ability quantile, we find that optimal capital income tax rates are regressive but quantitatively small relative to existing rates. For the baseline quantitative example the maximal capital income tax rate in the nonlinear case is less than 4.5%, and the optimal linear capital income tax rate is 2.5%. Moreover, welfare gains from these taxes on them and redistribute more resources to the low-skilled.⁴

³Though most research on this issue has focused on the linear tax problem, Mirrlees (1976) is clear that his results apply to nonlinear marginal commodity tax rates. A few later authors also noted the potential for optimal nonlinear rates: e.g., Kaplow (2008). Banks and Diamond (2009) look for but find no work on the nonlinear problem. They write: "In the context of this issue, how large the tax on capital income should be and how the marginal capital income tax rates should vary with earnings levels has not been explored in the literature that has been examined."

⁴The standard argument against nonlinear commodity taxation is arbitrage or retrading (see Hammond 1987, Golosov and Tsyvinski 2006). That may be an appropriate restriction for many goods, but important categories of personal expenditure can feasibly be taxed nonlinearly or as a function of income.

⁵We measure ability by the survey respondent’s score on the cognitive ability portion of the Armed Forces Qualification Test (AFQT). While it is impossible to measure ability perfectly, the AFQT score is commonly used, such as in the study of the returns to education.
optimal capital income taxes are negligible. These results provide little support for the claim that preference heterogeneity may justify substantial capital income taxation, whether in a linear or regressive form.

The quantitative case for regressive housing taxes is stronger. Again using data from the NLSY, we show that individuals with higher ability own houses of greater market value relative to their income history, conditional on accrued lifetime income, gender, and age. Using an estimate of the mean preference for housing consumption by ability quintile, we calculate the optimal policy treatment of housing. In our baseline case, owner-occupied housing consumption should be regressed, with the maximum distortion reaching 19 percent for the lowest-ability workers and 0 percent for the highest ability workers. The welfare gain from this optimal policy is orders of magnitude greater than that from capital income taxes and equals 6 billion dollars in terms of consumption variation for the United States, approximately 6% of the tax revenue lost to the mortgage interest deduction each year. The pattern of optimal distortions we simulate resembles the slope of the effective mortgage interest deduction in the United States as estimated in Poterba and Sinai (2009), but the optimality of a tax on housing rather than a subsidy implies that the existing mortgage interest deduction is far from optimal in our model.

Finally, this paper studies the importance of preference normalization in our optimal taxation model. We normalize preferences over commodities in two ways. These normalizations are similar to two assumptions made by Saez (2002) in his analysis of optimal commodity taxes with preference heterogeneity. First, we normalize preferences to eliminate any incentive for the planner to redistribute across agents based simply on their preferences over goods. Specifically, the marginal social value to a Utilitarian planner of allocating resources to an undistorted individual is independent of that individual’s preferences over consumption goods. This normalization makes it more likely that the optimal capital income tax is positive than if we assumed more patient individuals generated higher social welfare for the planner. We also normalize preferences in a second way. We model preferences over commodities, including future and current consumption, as having no direct effect on the labor supply decisions of individuals. Because the challenge of optimal tax policy is to encourage the high-skilled to work despite redistributive taxation, this normalization has a direct influence on the resulting form of optimal policy. This second normalization contrasts with the approach in recent work on optimal taxation of capital with heterogeneous discount rates by Diamond and Spinnewijn (2009), who model preferences such that more patient individuals are more willing to work.

The paper proceeds as follows. Section 1 provides an illustrative example of our theoretical results in an economy with two skill types and heterogeneity in preferences over two goods. Section 2 specifies a general model of optimal taxation with heterogeneity in ability and preferences and derives conditions on the optimal policy. In Sections 3 and 4, we parameterize the model with data on heterogeneous preferences for consumption over time and calculate the optimal taxes for these data. We then turn to housing, for which we calculate and decompose the welfare benefits of the optimal policy, and we compare the results of our baseline simulation to the existing mortgage interest deduction in U.S. tax policy. In Section 5, we test the robustness of our results to a wide range of parameterizations. Section 6 discusses the importance of preference normalization in these models. An Appendix contains technical details referred to in the text.

If we took into account variation around mean preference values within ability levels, the optimal taxes and welfare gains are likely to be even smaller.
1 A simple example

In this section we provide a simple example that captures the main intuition behind the more general model.\textsuperscript{7} We show that, in this setting, the optimal commodity tax is regressive for goods preferred by the skilled. In particular, the tax is positive on the low-skilled individual’s purchase of goods that are preferred by the high-skilled, while the high-skilled individual faces no distortion.

Consider an economy populated by a continuum of measure 1 of two types of individuals $i = \{1, 2\}$, where the size of each group is equal to $1/2$. These individuals differ in wage $w^i$, where $w^2 > w^1$. The wage is private information to the agent. Suppose there are two commodities, $c_1$ and $c_2$. The utility function for an individual with wage $w^i$ is given by:

$$u\left(c_1^i, c_2^i, \frac{y^i}{w^i}\right).$$

The planner’s problem is to specify consumption and income allocations for each individual to maximize a Utilitarian social welfare function.

**Problem 1 Planner’s problem in two-type example**

$$\max_{\{c_1, c_2, y^i\}} \sum_{i=1,2} u\left(c_1^i, c_2^i, \frac{y^i}{w^i}\right)$$

subject to

$$u\left(c_1^2, c_2^2, \frac{y^2}{w^2}\right) \geq u\left(c_1^1, c_2^1, \frac{y^1}{w^1}\right),$$

$$\sum_i y^i - c_1^i - c_2^i \leq 0.$$  \hspace{1cm} (3)

Constraint (2) is an incentive compatibility constraint stating that an individual of type $i = 2$ prefers the consumption and income bundle intended for it by the planner $\{c_1^2, c_2^2, y^2\}$ to a bundle $\{c_1^1, c_2^1, y^1\}$ allocated to an individual of type $i = 1$.\textsuperscript{8} Constraint (3) is feasibility, where we assume that the marginal rate of transformation of commodities is equal to 1.

Let $u_n$ be the partial derivative of $u(c_1, c_2, l)$ with respect to the $n^{th}$ argument. Note that these partial derivatives may depend on the wage rate.\textsuperscript{9} Let $\mu$ be the multiplier on constraint (2). Using the first order conditions for consumption in the above problem, we obtain the following expressions for and individual of type $i = 2$:

$$\frac{u_1\left(c_1^2, c_2^2, \frac{y^2}{w^2}\right)}{u_2\left(c_1^2, c_2^2, \frac{y^2}{w^2}\right)} = 1.$$  \hspace{1cm} (4)

\textsuperscript{7}Similar examples are found in Diamond (2007) and Diamond and Spinnewijn (2008). However, as discussed in Section 6, we normalize preferences in important ways that these other examples do not. This normalization has direct effects on the optimal policies we derive.

\textsuperscript{8}Writing this constraint we assumed that only an individual of type $i = 2$ can misrepresent his type. This is easy to ensure if the ratio $w^2/w^1$ is high enough.

\textsuperscript{9}For example, using the utility function from the general model stated later, (18), $u_1\left(c_1^1, c_2^1, l^1\right) = \frac{a^1(w^1)}{1+a^1(w^1)} \frac{1}{c_1^1}$. 

4
and for the individual of type \( i = 1 \):

\[
\frac{u_1(c_1^1, c_2^1, \frac{w_i}{y_i})}{u_2(c_1^2, c_2^1, \frac{w_i}{y_i})} = \frac{1 - \frac{u_2(c_1^1, c_2^1, \frac{w_i}{y_i})}{u_2(c_1^2, c_2^1, \frac{w_i}{y_i})}}{1 - \frac{u_1(c_1^2, c_2^2, \frac{w_i}{y_i})}{u_1(c_1^1, c_2^1, \frac{w_i}{y_i})}}. \tag{5}
\]

Equation (4) shows that the consumption choices of the high-skill individual are undistorted. The marginal rate of substitution \( u_1(c_1^1, c_2^1, \frac{w_i}{y_i}) \) is equal to the marginal rate of transformation. Equation (5) shows that if the multiplier \( \mu \) on the incentive compatibility constraint is not equal to zero, then the consumption choices of the low-skill individual are distorted. In particular, if an individual’s ratio \( \frac{u_1}{u_2} \) is less than 1, the policy has caused him to consume more of good 1 relative to good 2 than he would have chosen in autarky.

Now, suppose we impose a condition requiring that if all individuals are given the same consumption and income allocation, the marginal utility of good 2 relative to good 1 is higher for the high-ability individual \( j \) (type 2) than for the low-ability individual \( i \) (type 2).

**Assumption 1** If \( w_j > w_i \):

\[
\frac{u_2(c_1, c_2, \frac{w_j}{y_j})}{u_1(c_1, c_2, \frac{w_i}{y_i})} > \frac{u_2(c_1, c_2, \frac{w_i}{y_i})}{u_1(c_1, c_2, \frac{w_i}{y_i})} \tag{6}
\]

for any \((c_1, c_2, y)\).

We now can summarize the argument in a proposition characterizing the distortions in the optimal allocation.

**Proposition 2** Suppose that \( \{c_1^i, c_2^i, y_i\}_{i=1,2} \) is an optimal allocation solving (1). Then the optimal choice of consumption for the high-skill individual is not distorted. Suppose that Assumption 1 holds. Then the optimal choice of consumption of good 1 versus consumption of good 2 for the low-skill agent is distorted downwards:

\[
\frac{u_1(c_1^1, c_2^2, \frac{w_i}{y_i})}{u_2(c_1^1, c_2^2, \frac{w_i}{y_i})} < 1.
\]

This Proposition states that if good 2 is particularly enjoyed by high-skilled workers, the planner should impose a distortion (a positive relative tax) on the consumption of good 2 by the low-skilled workers (but not on consumption of that good by high-skilled workers). The intuition for this result is as follows. The planner wants to discourage a high-skill individual from deviating and claiming that he is a low type. A high-skill agent will find deviating less attractive if doing so will cause him to face a positive tax on the good that he values highly. The cost of such a positive tax is a distortion in the consumption choices by the low-skill agent. Assumption 1 ensures that the costs of such distortion are smaller than the gain from relaxing the incentive compatibility constraint.

It is important to be clear that this result depends on preferences varying by skill level, not income. In particular, it does not apply to goods with an income elasticity of demand greater than 1 but for which preferences are unrelated to skill. For those goods, the inequality in (6) would be an equality because each type would have the same ratio of marginal utilities given the same consumption and income bundle. Instead, the case for regressive taxes requires the high-skilled to prefer good 2 even when at the same income level as the low-skilled.
2 Model

In this section, we set up a model with a continuum of ability types, as in the classic Mirrlees (1971) framework. We derive a formula for optimal relative commodity taxes that are allowed to be nonlinear in consumption and to depend on income. To capture preference heterogeneity, we assume that preferences across consumption goods are a function of ability. This simplifies the planner’s problem by retaining a single dimension of heterogeneity: two or more dimensions introduce multiple screening problems for which a tractable analytical approach has not been developed.\(^{10}\)

There is a continuum of measure one of individual agents. We index agents by \(i \in [0, 1]\). Individuals differ in their abilities, which we measure with their wages, denoted by \(w^i\) and distributed according to the density function \(f(w)\) over the interval \([w_{\text{min}}, w_{\text{max}}]\). The ability is private information to the agent. The utility function of an individual depends on \(\alpha(w^i)\), so that the preference parameter for an individual depends directly on his or her wage.

Each individual maximizes the utility function:

\[
U(w^i) = u(c_1^i, c_2^i, l^i, \alpha(w^i)).
\] (7)

Note that utility is a function of the consumption of good 1, \(c_1\), and the consumption of good 2, \(c_2\), as well as of labor effort \(l\), and the preference parameter \(\alpha(w^i)\). Superscripts \(i\) on consumption and labor denote the values of these variables for the individual of wage \(w^i\).

A social planner maximizes a utilitarian social welfare function. The planner offers incentive compatible triplets of \(\{c_1^i, c_2^i, y^i\}\).

Problem 3

\[
\max_{\{c_1^i, c_2^i, y^i\}} \int_{w_{\text{min}}}^{w_{\text{max}}} \left( c_1^i, c_2^i, \frac{y^i}{w^i} \right) f(w^i) \, dw^i
\] (8)

subject to

\[
\int_{w_{\text{min}}}^{w_{\text{max}}} (y^i - c_1^i - c_2^i) \leq 0.
\] (9)

and

\[
u \left( c_1^i, c_2^i, \frac{y^i}{w^i} \right) \geq u \left( c_1^j, c_2^j, \frac{y^j}{w^j} \right),\] (10)

for all \(i, j\).

Constraint (10) is the incentive compatibility constraint stating that an individual of type \(i\) prefers the consumption and income allocation intended for it by the planner to an allocation intended for an individual of type \(j\).

Solving the planner’s problem in equations (8) through (10) can yield insights into the wedges that optimal policy drives into private optimization.

It is standard to rewrite the planner’s problem with explicit tax functions. In this alternative formalization of the problem, each individual maximizes the utility function (7) subject to the individual’s after-tax budget constraint,

\[
l^i w^i - T(w^i l^i) - \left( (c_1^i + t_1^i) (w^i l^i, c_1^i) \right) - \left( (c_2^i + t_2^i) (w^i l^i, c_2^i) \right) \geq 0.
\] (11)

\(^{10}\)See Kleven, Kreiner, and Saez (2009), Tarkiainen and Tuomala (2007), and Judd and Su (2008) for discussions of the approach to optimal taxation with multi-dimensional heterogeneity.
The budget constraint requires careful examination. The nonlinear income tax \( T \left( w^i l^i \right) \) is a continuous, differentiable function of income \( y^i = w^i l^i \). The two other tax functions, \( t^1 \left( w^i l^i, c^i_1 \right) \) and \( t^2 \left( w^i l^i, c^i_2 \right) \), are commodity tax functions that we also assume to be continuous and differentiable. Importantly, note that we explicitly allow for the taxation of each commodity to be nonlinear in consumption of that good and to depend on income.\(^{11}\) The budget constraint (11) has the multiplier \( \mu \).

To characterize optimal taxes with this formalization of the planner’s problem, we follow the formal techniques of the Mirrleesian literature. In particular, we consider the following social planner’s problem:

**Problem 4 Planner’s Problem**

\[
\max_{ \{c^i_1, c^i_2, l^i \} } \int_{w_{\min}}^{w_{\max}} U \left( w^i \right) f \left( w^i \right) dw^i \tag{12}
\]

subject to feasibility

\[
\int_{w_{\min}}^{w_{\max}} \left( w^i l^i - c^i_1 - c^i_2 \right) f \left( w^i \right) dw^i \tag{13}
\]

and incentive compatibility, which is that each individual maximizes (7) subject to (11) given tax policies \( T \left( w^i l^i \right), t^1 \left( w^i l^i, c^i_1 \right), \) and \( t^2 \left( w^i l^i, c^i_2 \right) \).

In words, the social planner chooses a tax system to maximize Utilitarian social welfare subject to a budget constraint that assumes no government spending for simplicity. The government must also take into account that each individual will choose labor supply to maximize his or her utility subject to the specified tax system.

### 2.1 The optimal commodity choice wedge

We now derive a formula for the optimal commodity wedge, i.e., the wedge distorting commodity choices.\(^{12}\) We formulate the Hamiltonian from the planner’s problem above. The Hamiltonian includes the following differential constraint:

\[
\frac{\partial U^i}{\partial w^i} = u_{w^i} \left( c^i_1, c^i_2, l^i, \alpha \left( w^i \right) \right) + \mu \left( l^i \left( 1 - T^i \left( w^i l^i \right) - t^i_y \left( w^i l^i, c^i_1 \right) - t^i_y \left( w^i l^i, c^i_2 \right) \right) \right), \tag{14}
\]

derived using the envelope condition on the individual’s utility maximization problem. To remove the tax functions from this expression, we use the individual’s first order condition with respect to labor \( l^i \):

\[
u_{l^i} \left( c^i_1, c^i_2, l^i, \alpha \left( w^i \right) \right) = -\mu w^i \left( 1 - T^i \left( w^i l^i \right) - t^i_y \left( w^i l^i, c^i_1 \right) - t^i_y \left( w^i l^i, c^i_2 \right) \right). \tag{15}\]

Substituting (15) into (14) yields:

\[
\frac{\partial U^i}{\partial w^i} = u_{w^i} \left( c^i_1, c^i_2, l^i, \alpha \left( w^i \right) \right) - \frac{l^i w^i}{w^i} \frac{\partial U^i}{\partial w^i} \left( c^i_1, c^i_2, l^i, \alpha \left( w^i \right) \right).
\]

The Hamiltonian is then:

\[
H \left( w^i \right) = \left( U \left( w^i \right) + \lambda \left( w^i l^i - c^i_1 - c^i_2 \right) \right) \frac{\pi^i}{d w^i} + \phi \left( u_{w^i} \left( \cdot \right) - \frac{l^i w^i}{w^i} \left( \cdot \right) \right).
\]

\(^{11}\)These tax instruments are redundant, in that a single tax function of the consumption of one good and income would be sufficient to characterize the full policy. Separating taxes into these functions aids interpretation.

\(^{12}\)For a textbook treatment, see Salanie (2003), chapter 5.2.
where subscripts denote partial derivatives and \((\cdot)\) denotes the set of arguments of the utility function, \((c^1, c^2, l^i, \alpha(w^i))\). The first term of the Hamiltonian is the utility of the individual with wage \(w^i\). The second is government’s budget constraint multiplied by a shadow price \(\lambda\). The third term is the evolution of the state variable \(U(w^i)\) with respect to \(w^i\), as derived above, and is multiplied by the costate variable \(\phi\).

To solve for the optimal policy, choose \(l\) and \(c^1\) as the control variables, with \(c^2\) an implicit function defined by the budget constraint. The first order condition with respect to \(c^1\) is:
\[
\lambda \left( -1 - \frac{dc^1}{dc^1} \right) \pi^i + \phi \left( u_{w^i c^1}(\cdot) + u_{w^i c^2}(\cdot) \frac{dc^2}{dc^1} - \frac{l^i u_{l^i c^1}(\cdot)}{w^i} - \frac{l^i u_{l^i c^2}(\cdot)}{w^i} \frac{dc^2}{dc^1} \right) = 0,
\]
or, rearranging
\[
\frac{dc^2}{dc^1} = -\frac{\pi^i}{\lambda} - \phi \left( u_{w^i c^1}(\cdot) - \frac{l^i u_{l^i c^1}(\cdot)}{w^i} \right).
\]

Individuals maximizing (7) subject to (11) will allocate their after-tax income so that the following relationships hold:
\[
\frac{dc^2}{dc^1} = \frac{u_{c^1}}{u_{c^2}} = \frac{-1 + t1_{c^1}(w^i, l^i, c^1)}{1 + t2_{c^2}(w^i, l^i, c^2)}
\]
so we write:
\[
\frac{1 + t1_{c^1}(w^i, l^i, c^1)}{1 + t2_{c^2}(w^i, l^i, c^2)} = \frac{\pi^i}{\lambda} \frac{u_{w^i c^1}(\cdot)}{\phi(w^i)} \frac{l^i u_{l^i c^1}(\cdot)}{w^i}.
\]
To fully characterize the optimal distortion to commodity purchases given by (17), we solve for \(\lambda\) and \(\phi(w^i)\) in a specific example.

### 2.1.1 A specific example

We assume the individual utility function is
\[
U^i = u(c^1, c^2, l^i, \alpha(w^i)) = \frac{\alpha(w^i)}{1 + \alpha(w^i)} \ln c^1_i + \frac{1}{1 + \alpha(w^i)} \ln c^2_i - \frac{1}{\sigma} (l^i)^\sigma.
\]

It is important to note that this utility function normalizes preferences over consumption goods in the two ways mentioned in the Introduction. The first normalization, following the techniques of Weinzierl (2009), ensures that the marginal social value to a Utilitarian planner of allocating resources to an undistorted individual is independent of that individual’s preference parameter \(\alpha(w^i)\). This prevents preference heterogeneity, which is inherently ordinal, from artificially driving redistribution by making the cardinal utility of consumption higher for an individual depending on his or her preferences. The second normalization separates heterogeneity in commodity preferences from the consumption-leisure choice of individuals. Specifically, it ensures that two individuals of the same ability \(w^i\) will choose the same labor effort when undistorted.\footnote{Logarithmic utility of consumption makes it possible to achieve these two normalizations simultaneously. For a more general case, the Appendix to this paper contains the details of both normalizations.}

The next proposition derives an expression for the optimal commodity taxes.

Proposition 5  Given the individual utility function (18), the solution to the Planner’s Problem satisfies:

\[
\frac{1 + t_{c_1}^1 (w^1 l, c_1^1)}{1 + t_{c_2}^1 (w^1 l, c_2^1)} = \frac{f (w^i) + u_{w^1 c_1^1} (1 - F_i (w^i)) \left( \frac{-1 + F_i (w^i)}{1 - F_i (w^i)} \int_{w^i}^{w^i_{max}} f (w^j) dw^j \right) - \left( \frac{1}{1 - F_i (w^i)} \int_{w^i_{min}}^{w^i_{max}} f (w^j) dw^j \right)}{f (w^i) + u_{w^2 c_2^1} (1 - F_i (w^i))} \left( \frac{-1 + F_i (w^i)}{1 - F_i (w^i)} \int_{w^i}^{w^i_{max}} f (w^j) dw^j \right) - \left( \frac{1}{1 - F_i (w^i)} \int_{w^i_{min}}^{w^i_{max}} f (w^j) dw^j \right) \right) \]

(19)

Proof. In the Appendix, we derive the following expressions for \( \lambda \) and \( \phi (w^i) \):

\[
\lambda = \frac{1}{\int_{w^i_{min}}^{w^i_{max}} f (w^j) dw^j} \]

\[
\phi (w^i) = (1 - F_i (w^i)) \left( 1 - \frac{\left( \frac{1}{1 - F_i (w^i)} \int_{w^i_{min}}^{w^i_{max}} f (w^j) dw^j \right)}{\left( \frac{1}{1 - F_i (w^i)} \int_{w^i_{min}}^{w^i_{max}} f (w^j) dw^j \right)} \right). \]

Using these results in expression (17), we obtain (19).

As with the conditions for optimal marginal income tax rates from, e.g., Saez (2001), concave utility of consumption prevents result (19) from being fully closed-form, instead relying on optimal utility and consumption levels. Nevertheless, we can establish some important lessons from it.

First, on the top-type, \((1 - F_i (w^i))\) is zero, and the result reduces to

\[
\frac{1 + t_{c_1}^1 (w^1 l, c_1^1)}{1 + t_{c_2}^1 (w^1 l, c_2^1)} = 1.
\]

so the commodity distortion is zero on the highest ability worker.

Second, the distortion is also zero on the lowest ability worker, as the terms in large parentheses in the numerator and denominator are zero.

In addition, examination of terms in (19) gives detail about the determinants of the optimal distortion.

The parenthetical term common to the numerator and denominator is the difference in the average cost of raising utility for the population with wages above \(w^i\) and for the entire population. It is positive, since if it were negative the planner could raise social welfare by incentive-compatible and feasible transfers of \(c_2\) from the overall population to the high-skilled. As such, this difference measures the loss in welfare that results from having to satisfy the incentives of the high-skilled rather than being able to spread resources across all workers. When this loss is large, the optimal distortion to consumption at wage \(w^i\) is larger because that distortion discourages higher-skilled workers from working less.

The relationship between \(u_{w^1 c_1^1}\) and \(u_{w^2 c_2^1}\) determines whether policy discourages consumption of good 1 or good 2 for intermediate ability levels. With utility function (18) this relationship is determined by the sign of \(\alpha' (w^i)\). If \(\alpha' (w^i) < 0\), then high-ability workers relatively prefer good 2, and \(u_{w^1 c_1^1} < 0\) while \(u_{w^2 c_2^1} > 0\). Then, the ratio on the right-hand side of (19) is less than one, and the optimal distortion discourages marginal consumption of good 2. That is, the good preferred by the more able workers ought to be marginally taxed.

The term \(f (w^i)\) provides a measure of the share of the population distorted by a given commodity
tax. When this share is high, the optimal consumption distortion is smaller, as the planner wants to avoid distortions on large sub-populations. Mathematically, \( f(w^i) \) enters both the numerator and the denominator, pushing the tax ratio toward unity.

The term \((1 - F(w^i))\) is the share of individuals with higher wages who are encouraged to exert more effort due to the distortion at \( w^i \). The larger this term, the more valuable is the distortion to the planner, all else the same. Mathematically, \((1 - F(w^i))\) multiplies the terms in the numerator and denominator that push the tax ratio away from unity. We know that \((1 - F(w^i))\) falls as the wage rises, so this lowers the optimal distortion as we move up the ability distribution.

Finally, suppose there exists an ability level \( \tilde{w} \) such that the distribution of all abilities above that level follows a Pareto form, as in Saez (2001). Then for all such \( w^i > \tilde{w} \), \( \frac{w^i f(w^i)}{(1 - F(w^i))} \) is constant. Rearrange the expression (19) to obtain

\[
\frac{1 + t_{c_1}^i \left( w^i f(w^i), c_2^i \right)}{1 + t_{c_2}^i \left( w^i f(w^i), c_2^i \right)} = \frac{w^i f(w^i) + u_{w^i} c_1^i w^i \left( \frac{1}{1 - F(w^i)} \int_{w^j = w^i}^{w^j = w_{\max}} \frac{1}{u^j} f(w^j) \, dw^j - \frac{1}{1 - F(\hat{w}_{\min})} \int_{w^j = \hat{w}_{\min}}^{w^j = w_{\max}} \frac{1}{u^j} f(w^j) \, dw^j \right) + u_{w^i} c_2^i w^i \left( \frac{1}{1 - F(w^i)} \int_{w^j = w^i}^{w^j = w_{\max}} \frac{1}{u^j} f(w^j) \, dw^j - \frac{1}{1 - F(\hat{w}_{\min})} \int_{w^j = \hat{w}_{\min}}^{w^j = w_{\max}} \frac{1}{u^j} f(w^j) \, dw^j \right)}{1 + t_{c_2}^i \left( w^i f(w^i), c_2^i \right) + u_{w^i} c_1^i w^i \left( \frac{1}{1 - F(w^i)} \int_{w^j = w^i}^{w^j = w_{\max}} \frac{1}{u^j} f(w^j) \, dw^j - \frac{1}{1 - F(\hat{w}_{\min})} \int_{w^j = \hat{w}_{\min}}^{w^j = w_{\max}} \frac{1}{u^j} f(w^j) \, dw^j \right)}.
\]

From above, we know that the parenthetical terms are positive; they are also increasing in \( w^i \) following the same argument. Therefore, assuming \( u_{w^i} c_1^i < 0 \) and \( u_{w^i} c_2^i > 0 \), whether the optimal tax on good 2 is regressive or progressive in the upper tail of the income distribution depends on how quickly \( u_{w^i} c_1^i \) and \( u_{w^i} c_2^i \) converge to zero. If they do not converge quickly enough, the tax on good 2 is progressive in the tail.

Though these interpretations aid in understanding result (19), we may want to reformulate that result in terms of observable quantities in the spirit of Saez (2001). The Appendix derives the following version of result (19):

\[
\frac{1 + t_{c_1}^i}{1 + t_{c_2}^i} = \frac{w^i f(w^i) + \varepsilon c_1^i w^i \left( 1 - F(w^i) \right) \left( \frac{1}{1 - F(w^i)} \int_{w^j = w^i}^{w^j = w_{\max}} \frac{1}{u^j} f(w^j) \, dw^j - \frac{1}{1 - F(\hat{w}_{\min})} \int_{w^j = \hat{w}_{\min}}^{w^j = \hat{w}_{\max}} \frac{1}{u^j} f(w^j) \, dw^j \right)}{\varepsilon c_2^i w^i \left( 1 - F(w^i) \right) \left( \frac{1}{1 - F(w^i)} \int_{w^j = w^i}^{w^j = w_{\max}} \frac{1}{u^j} f(w^j) \, dw^j - \frac{1}{1 - F(\hat{w}_{\min})} \int_{w^j = \hat{w}_{\min}}^{w^j = \hat{w}_{\max}} \frac{1}{u^j} f(w^j) \, dw^j \right)},
\]

where \( \varepsilon c_{\text{cum}} \) denotes the Frisch elasticity (holding marginal utility constant) of consumption of good \( m \) with respect to the wage, \( \hat{y}^j \) is the disposable income individual \( i \) would choose to earn in an economy with income taxes only (i.e., before the introduction of optimal commodity taxes, the planner can observe the distribution of \( \hat{y}^j \)). This alternative representation of the main result on optimal commodity taxes can be more readily applied with observable data.

If we restrict attention to commodity taxes that are a linear function of the consumption of the good, a modification of result (19) confirms the results of the previous literature (e.g., Saez 2002, Salanie 2003) that goods preferred by the highly able ought to be taxed.

### 3 Example 1: Capital Taxes

The results of Sections 1 and 2 suggest that optimal commodity taxes may be regressive on goods preferred by the high-skilled, but the analytical expression (19) made it clear that the shape of optimal commodity taxes will depend on many details of the economy. In the next two sections we study the shape of optimal commodity taxation numerically for two important categories of expenditure, future consumption (savings) and housing. First, we simulate the optimal tax treatment of capital income using empirical evidence on
the relationship between ability and time preference, or intertemporal discounting.

We use data from the National Longitudinal Survey of Youth (NLSY), a nationally representative sample of individuals born between 1957 and 1964 and first interviewed in 1979. This sample has been interviewed annually or biannually since. The key advantage of the NLSY for our purposes is that it contains data on individuals’ net worth and income over time as well as, most importantly, a standard, direct measure of ability. In 1980, the NLSY administered the Armed Forces Qualification Test (AFQT) to 94 percent of its participants. This test measured individuals’ aptitudes in a wide range of areas, including some mechanical skills relevant to military service.

We use an aggregation of scores in some of the areas covered by the AFQT as the indicator of ability. This aggregation, the AFQT89, is calculated by the Center for Human Resource Research at Ohio State University, as follows:

Creation of this revised percentile score, called AFQT89, involves (1) computing a verbal composite score by summing word knowledge and paragraph comprehension raw scores; (2) converting subtest raw scores for verbal, math knowledge, and arithmetic reasoning; (3) multiplying the verbal standard score by two; (4) summing the standard scores for verbal, math knowledge, and arithmetic reasoning; and (5) converting the summed standard score to a percentile.

Our measure of preferences will be the discount factor implied by using NLSY data on income and net worth in a simple model of individual optimization. Suppose individuals live for three periods. In the first two periods, roughly corresponding to ages 20 through 42 and 43 through 65, they work, consume, and perhaps borrow or save. In the third period, they are retired and live for 23 years (for simplicity, as this makes all three periods of similar length). The individual solves the following utility maximization problem:

$$\max_{c_1, c_2, c_3} \left[ \ln(c_1) + \delta \ln(c_2) + \delta^2 \ln(c_3) - v(y_1, y_2) \right]$$

subject to

$$\left((y_1 - c_1) R^2 + (y_2 - c_2)\right) R - c_3 = 0.$$ 

where \(c_t\) and \(y_t\) are consumption and income in period \(t\), \(\delta\) is the discount factor across 23-year periods (i.e., if the one-year-ahead discount factor is \(\beta\), then \(\delta = \beta^{23}\)), \(R = (1.05)^{23}\) is the average return to saving over a 23-year period, and \(v(\cdot)\) is an unspecified function for the disutility of earning income.

We make the assumption that an individual’s total value of income prior to age 43 is identical to the income it will earn from age 43 until retirement. In the notation of the model, we assume \(y_1 = y_2\) for all individuals. The first-order conditions of the individual’s problem yield the following expression for \(\delta\):

$$1 + \delta + \delta^2 = \frac{y_1}{c_1} \left(1 + \frac{R}{R^2} \right).$$

or

$$\delta = \frac{1}{2} \left( \left( -3 + 4 \frac{y_1}{c_1} \frac{1 + R}{R} \right)^{\frac{1}{2}} - 1 \right).$$

As expected, the higher is income relative to consumption, the greater the estimated \(\delta\) for an individual. We drop 37 individuals whose estimated \(\delta\) is negative or exceeds two, leaving 7,008 observations.

To estimate \(\delta\), we need values for \(y_1\) and \(c_1\) for each individual. For \(y_1\), we use the NLSY’s observations on income over time for each individual to calculate the "future value" of income earned prior to and including
2004.\textsuperscript{14} Formally, \( y_t = \sum_{t=1979}^{2004} R_t^{(2004-t)} y_t \). Using the full time series of income rather than simply the most recent observation of income is important for two reasons. First, it gives a better measure of the individual’s likely lifetime or permanent income. Second, to calculate \( c_1 \), we assume that any income not accumulated as net worth by 2004 was consumed. Formally, we denote the NLSY variable "family net worth" \( NW \) and calculate \( c_1 = y_{10} - NW \).

In the following table, we show the mean and standard deviations of \( \delta \) by AFQT quintile:

<table>
<thead>
<tr>
<th>AFQT89 quintile</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>0.34</td>
<td>0.37</td>
<td>0.39</td>
<td>0.42</td>
<td>0.47</td>
</tr>
<tr>
<td>st.dev. of ( \delta )</td>
<td>0.16</td>
<td>0.18</td>
<td>0.18</td>
<td>0.21</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The variation in \( \delta \) within AFQT quintiles is large relative to the variation across wage levels. Of course, the data are likely to be very noisy, and our inference of \( \delta \) is based on a highly simplified model. Nevertheless, our findings are consistent with the patterns found in Lawrance (1991), who estimates discount factors by income (not ability) group, and with the findings of Benjamin, Brown, and Shapiro (2006), who find a "positive relationship between AFQT score and the propensity to have positive net assets" in the NLSY.

Consistent with these patterns, Table 1 shows the results of a regression of \( \delta \) on age, age squared, gender, a 4th-degree polynomial in \( y_1 \) (to control for income effects), and AFQT score for the same sample.\textsuperscript{15} Though the explanatory power of this set of independent variables is low, the coefficient on AFQT score is positive and significant at the 1 percent level. The magnitude of the coefficient, 0.00066, implies that a twenty-point increase in AFQT raises \( \delta \) by .013, roughly in keeping with the pattern by quintile shown above.

Our baseline case for these simulations will use the utility function previously given in expression (18):

\[
\begin{align*}
  u(c^t, l^t, w^t) &= \frac{\alpha(w^t)}{1 + \alpha(w^t)} \ln c^t + \frac{1}{1 + \alpha(w^t)} \ln c^t - \frac{1}{\sigma} (l^t)^\sigma \\
\end{align*}
\]

with \( \sigma = 3 \), for a constant-consumption elasticity of labor supply with respect to the wage of \( \frac{1}{2} \). In the context of capital income taxation, we interpret \( c_1 \) and \( c_2 \) as consumption in two different time periods. To perform the optimal policy simulations, we convert \( \delta \) into an annualized discount rate \( \rho^t \) for each AFQT quintile using the following identities: \( \delta^{\frac{1}{\sigma}} = \beta^t \), \( \alpha(w^t) = \frac{1}{\beta^t} \), and \( \alpha(w^t) = \exp(\rho^t) \). The NLSY also has data on wage and salary earnings and hours worked. We use these to impute a wage for each individual. We use data from 1993, the middle of the observed data range, to estimate wages by AFQT89 quintile.\textsuperscript{16} The results are:

<table>
<thead>
<tr>
<th>AFQT89 quintile</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^t )</td>
<td>0.954</td>
<td>0.958</td>
<td>0.960</td>
<td>0.963</td>
<td>0.967</td>
</tr>
<tr>
<td>( \rho^t ) (discount rate)</td>
<td>0.047</td>
<td>0.043</td>
<td>0.040</td>
<td>0.038</td>
<td>0.033</td>
</tr>
<tr>
<td>( \alpha(w^t) )</td>
<td>1.049</td>
<td>1.044</td>
<td>1.041</td>
<td>1.039</td>
<td>1.034</td>
</tr>
<tr>
<td>Mean wage (ability)</td>
<td>$12.27</td>
<td>16.21</td>
<td>19.20</td>
<td>21.62</td>
<td>25.73</td>
</tr>
</tbody>
</table>

\textsuperscript{14}We do not observe income in all years for each individual. To obtain an income figure comparable to ending net worth for each individual, we calculate the future value of the observed incomes for each individual. Then, we scale that future value by the maximum number of years observable over the number of years observed for each individual.

\textsuperscript{15}We also have run simulations controlling for the slope of income during the 1979-2004 period and over the past ten years for each individual. These controls reduce the coefficient on AFQT to 0.0043, but it remains significant at the 1% level.

\textsuperscript{16}Specifically, we compute the wage from the total wage and salary income divided by the total hours worked in 1992, as reported in 1993. We calculate mean wages by AFQT quintile limiting the sample to workers who reported more than 1,000 hours worked. Using all workers does not change the pattern, but all wage levels rise because some workers with low reported hours have high imputed hourly wages.
Using these data, we simulate optimal policy. We will study the expression:

$$\tau = 1 - \frac{\frac{u_{ci}}{u_{cj}} - 1}{r}$$  \hspace{1cm} (21)$$

where \( r \) is the annual rate of return to savings.\(^{17} \) The variable \( \tau \) measures the relative distortion toward good 1 and away from good 2 at a given income level. Under the capital income tax interpretation, \( \tau \) is the implicit tax on the interest income earned on good 2, i.e., capital. If this expression is positive, the tax policy is discouraging future consumption relative to current consumption. More informally, it is taxing the return saving, so we will refer to it as the implied capital income tax.

Figure 2 plots \( \rho \) and the optimal capital income tax from expression (21) for each AFQT89 quintile against the wage. The optimal nonlinear capital income tax rates are regressive but small relative to rates seen in modern developed economies, with a maximum of 4.5 percent on the lowest-ability worker. The best linear capital income tax rate is positive but still smaller, at 2.5 percent. The welfare gain from optimal capital income taxation is nearly zero in this simulation, which is not surprising given the relatively small variation in preferences apparent from the NLSY data.

In sum, evidence yields too weak a relationship between ability and time preferences to justify, in our model, substantial capital income taxation, whether linear or regressive.

4 Example 2: Housing

While our analysis above did not find a justification for quantitatively substantial regressive (or even linear) capital income taxation, nonlinear taxation may still be important for other categories of consumption. In this section, we consider one example: owner-occupied housing. Building on the results of the previous sections, if individuals of greater ability have a greater preference for the consumption of housing services, regressive subsidization of housing may be optimal.\(^{18} \)

To derive a measure of the preference for housing, suppose individuals maximize the following utility function

$$\max_{c_1, c_2, c_3} \left[ \ln (c) + \eta \ln (h) - v(y) \right]$$

where \( c \) is consumption, \( h \) is spending on housing, and \( y \) is income, subject to the budget constraint

$$y - c - h = 0.$$ 

The first order conditions yield:

$$\frac{1}{\eta} h = c$$

which, in the budget constraint, implies

$$\frac{h}{y} = \frac{\eta}{1 + \eta}.$$ 

\(^{17} \)In the simulations, we assume that \( 1 + r = \frac{1}{J \alpha(w) f(w)} \). The implicit tax \( \tau \) is on net capital income, i.e., the implicit after-tax return to saving is \( (1 + r (1 - \tau)) \).

\(^{18} \)Why would high-ability workers spend more on housing, holding income constant? A possible explanation is that these workers value a high quality public school system. In the United States, public schools differ in quality across local jurisdictions, and the prices of homes rise with the quality of the schools to which they give access (see, e.g., Black 1999). On the positive relationship between ability and the returns to schooling, which could generate the preference pattern suggested here, see Belzil and Hansen (2002).
for the value of expenditure on housing as a share of total income.

To examine whether $\eta$ varies with ability, we return to the NLSY data from above. We use the reported 2004 market value of the respondent’s primary residence as the measure of $h$.\(^{19}\) For income $y$, we use the same cumulative income measure as in the capital income tax simulations from the previous section. There are 7,280 observations for $\frac{h}{y}$. The median value of $h$ is $85,000, the mean is $152,058, and the maximum is $1.816 million.\(^{20}\) The following table shows, by AFQT quintile for these observations, the ratio $\frac{h}{y}$, its standard deviation, the value of the taste parameter $\alpha (w^i) = \left( \frac{1}{y} - 1 \right)$, and the average wage for individuals.

<table>
<thead>
<tr>
<th>AFQT89 quintile</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{h}{y}$</td>
<td>0.080</td>
<td>0.116</td>
<td>0.131</td>
<td>0.157</td>
<td>0.170</td>
</tr>
<tr>
<td>st.dev. of $\frac{h}{y}$</td>
<td>0.150</td>
<td>0.169</td>
<td>0.161</td>
<td>0.221</td>
<td>0.188</td>
</tr>
<tr>
<td>$\alpha (w^i)$</td>
<td>10.5</td>
<td>6.6</td>
<td>5.6</td>
<td>4.4</td>
<td>3.9</td>
</tr>
<tr>
<td>Mean wage (ability)</td>
<td>$12.27$</td>
<td>16.21</td>
<td>19.20</td>
<td>21.62</td>
<td>25.73</td>
</tr>
</tbody>
</table>

These are the same AFQT quintiles, with the same corresponding mean wages, as were used in the capital income tax simulation of Section 3. Note that the variation in $\frac{h}{y}$ is large within quintile. We will ignore this variation and use the point estimates of $\frac{h}{y}$ to simulate optimal tax policy toward housing consumption. The data imply the highest-skilled individuals place approximately twice the weight on housing relative to other goods compared to the lowest-skilled individuals.

Regression results provide support for the relationship between ability and preferences seen in this table. Table 2 shows the results of a regression of $\frac{h}{y}$ on age, age squared, gender, a 4th-degree polynomial in $y_1$ (to control for income effects), and AFQT score.\(^{21}\) The coefficient on AFQT score is positive and highly significant, with a t-statistic of 8.38. In magnitude, the coefficient of 0.0007 implies that a twenty-point increase in AFQT, i.e., approximately a one-quintile increase, would generate an increase in $\frac{h}{y}$ of approximately 0.014. This magnitude is consistent with the pattern of $\frac{h}{y}$ across AFQT quintiles shown above.

Using the distribution of ability and preferences for housing given above, we simulate optimal taxation of housing. We will summarize policy with the expression

$$\frac{u_h - u_c}{u_h} = \frac{\eta}{\bar{y}} - \frac{1}{\bar{y}},$$

which measures the relative distortion toward non-housing consumption and away from housing consumption at each income level.\(^{22}\) As in the capital income tax simulations, we assume that the normalized utility function in the social welfare function takes the form (18) with $\sigma = 3$.

Figure 3 plots $\frac{h}{y}$, expression (22), and the best linear housing tax against wages. The positive relationship between the mean values of $\frac{h}{y}$ and AFQT scores generates a sizeable and regressive optimal tax policy toward housing consumption. Figure 3 shows that the optimal tax on housing consumption, relative to other consumption, starts at 20 percent at the bottom of the income distribution and falls, with one blip up in the

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\(^{19}\)We do not have data on imputed rent or rent paid by non-owners. The relevant policy for this section is therefore a distortion to non-rental housing.

\(^{20}\)If we restrict the sample to the 4,716 observations for the ratio $\frac{h}{y}$ is positive, the median value is $160,000 and the mean value is $234,531.

\(^{21}\)We also have run simulations controlling for the slope of income during the 1979-2004 period and over the past ten years for each individual. These controls reduce the coefficient on AFQT to 0.0005, but it remains significant at the 1% level.

\(^{22}\)Note that this distortion applies to the level of housing consumption, not to a "rate of return" on housing as it did with capital income taxation.
middle of the distribution, as income rises.

The welfare gain from this regressive housing distortion is orders of magnitude larger than the gain estimated from the optimal capital income taxes, though it remains moderate in absolute terms. The estimated gain is 0.05 percent of total income, or just over $6 billion in the current U.S. economy. About half of this gain could be captured by using the optimal linear tax on housing shown in Figure 3.

We can decompose the welfare gain from the optimal nonlinear housing tax into four components, as shown in Table 3. The first component is "efficiency," or the increase in overall production in the economy due to lower distortions to labor effort. The second is the reallocation of consumption across individuals in general, and toward less able individuals in particular, that is made possible by the housing subsidy. The third is the reallocation of consumption across goods for each individual. Finally, the fourth component is the reallocation of required income across individuals, in particular toward those with high ability and, therefore, low marginal disutility of earned income.

Table 3 shows that the largest contributor to the welfare gain is the redistribution of consumption across types, which yielded a gain equal to 175% of the overall increase. Partially offsetting this was, as would be expected, a large decrease in welfare due to reallocation of consumption from housing to non-housing consumption due to the distortion. This loss was equal to -121% of the overall increase in welfare. Smaller positive components of the change were greater overall efficiency due to lower distortions (21% of the gain), and a redistribution of required income to those of higher ability (25% of the gain).

One motivation for our study of the optimal treatment of housing is the existence of a regressive housing subsidy in the United States, specifically the mortgage interest deduction. Poterba and Sinai provide average tax savings from the mortgage interest deduction and average market values of the homes of households in five income brackets. For instance, for those with over $250,000 in annual income, the annual tax saving from the deduction is $5,459, and the mean home value is $1.072 million. For each income bracket, we calculate the implied subsidy to housing by assuming a real discount rate and a term length for the mortgage. We assume each household has a 5 percent real discount rate and has financed its house with a standard 30-year mortgage. Then, we calculate the present value of tax deductions as a percent of the market value of the home for each income bracket. We label this series the "existing housing subsidy," and we show it in Figure 3. We also plot the average distortion to housing implied by our model in each income bracket. Figure 3 shows that the existing mortgage interest deduction in the United States resembles the shape and size of the optimal results we have derived. Despite those similarities, it differs in a dramatic manner from the optimal policy: i.e., it is a subsidy rather than a tax. This makes the existing mortgage interest deduction far from optimal.\footnote{To see an example of why the existing deduction is not optimal, note that in the optimal policy the highest-ability workers go undistorted. In the existing policy, they are distorted toward housing. Such a distortion does not help the planner increase the extent of redistribution.}

5 Extrapolations and Robustness

The data used above for simulating optimal capital and housing tax policy were limited to a narrow range of wages and preferences. To supplement these simulations and to consider robustness, we extrapolate the patterns estimated from the NLSY data to a wider, realistic distribution of wages. We also vary the values of $\gamma$ and $\sigma$, key parameters in the model.

We use a wage distribution that runs from $4 to $100 with 25 equally-spaced discrete values. Based on Saez (2001), we assume that the distribution of the population across these wages is lognormal up to $62.50.
and Pareto with a parameter value of two for higher wages. We calibrate the lognormal distribution with
the 2007 wage distribution for full-time workers in the United States as reported in the Current Population
Survey.

To extend the preference patterns across this wider wage distribution, we assume the preferences follow
a power distribution.\footnote{Power distributions fit the empirical data better than polynomial, linear, or exponential alternatives.} To determine the parameters of the power distribution for time preference as a
function of the wage, we first estimate the following regression

$$\ln \delta = \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{gender} + \beta_4 \ln (\text{income}) + \beta_5 \ln (\text{AFQT}),$$

where \text{income} is the cumulative income measure described in previous sections. This regression yields a
highly significant estimate for $\beta_5$ of 0.026 (standard error of 0.004). We then fix the value of $\delta$ for the wage
corresponding to the middle AFQT quintile. Finally, we calculate a simple regression of AFQT score on the
wage in the NLSY data. These steps allow us to use the following expression to calculate $\delta$ at each wage
in our distribution:

$$\delta = 0.356 \times (-67.46 + 6.14 \times \text{wage})^{0.026}.$$  

This extrapolation is shown in Figure 4. A similar procedure applied to the housing preference data leads
to

$$\frac{h}{y} = 0.102 \times (-67.46 + 6.14 \times \text{wage})^{0.064}.$$  

For each value of the wage we use these expressions and the fact that $\alpha (w^i) = \frac{1}{\gamma}$ for capital and $\alpha (w^i) = \frac{1}{\gamma} - 2$ for housing to calculate the values of $\alpha (w^i)$ for the utility functions in our simulations.

Aside from extrapolating to a wider wage distribution, we examine the robustness of our results to a range
of parameter values for the utility function. One complication in considering alternative parameterizations
is that the simple utility function assumed in (18) is no longer appropriate. Recall that we normalized
all individuals’ utility functions according to two criteria. With logarithmic utility of consumption, these
restrictions are captured by the normalization in expression (18), but with a more general utility function,
a more complex normalization is required. In the Appendix, such a normalization is derived. Here, we use
the general expression for utility:

$$U = \frac{1}{\varphi^i} \left( \frac{\alpha (w^i)}{1 + \alpha (w^i)} \right)^{\frac{\gamma}{(c^i_1)^{1-\gamma} - 1} \left( \frac{1}{1 + \alpha (w^i)} \right)^{\frac{\gamma}{(c^i_2)^{1-\gamma} - 1} - \frac{1}{\sigma} (t)^{\gamma}} \right)$$

where $\varphi^i$ is a normalization factor that depends only on the preference parameters $\alpha (w^i)$ and the parameters
$\gamma$ and $\sigma$. We solve the same planner’s problem as above, but for a range of values for $\gamma$ and $\sigma$.

Figure 6 shows the optimal capital income taxes for each combination of parameter values, and Table
4 shows the implied optimal capital income taxes, the implied best linear capital income tax rate, and the
welfare gains from nonlinear capital income taxation for each combination. For the parameterizations with
moderate values for $\gamma$ and $\sigma$, the optimal nonlinear capital income tax rates are less than 5 percent and flat
over much of the distribution, becoming regressive at higher incomes. In all cases, optimal capital income
taxes are zero for the highest-wage, highest-patience type. Larger capital income tax rates can be obtained
only by raising the labor supply elasticity (which makes the income tax more distortionary) or the concavity
of utility from consumption (which makes redistribution more valuable to the planner). Related to these
characteristics of the optimal nonlinear capital income taxes, the best linear capital income tax rates shown
in Table 4 are positive but generally low, ranging from below 1% to just below 7%. Again, the larger best linear rates come from increasing the elasticity of labor supply or the concavity of utility from consumption.

The welfare gains from capital income taxation in this setting are negligible, with the highest gain reaching only 0.00005 percent of total output in the case with the highest elasticity of labor supply ($\sigma = 1.5$). The incremental reform from the best linear capital income tax to the optimal nonlinear tax consistently makes up a small fraction of the gain obtained from reforming a system with no capital income taxation at all.

Figure 7 shows the optimal taxes on housing for the same range of parameter values, and Table 5 gives the same information as did Table 4, but for housing. For most parameter combinations, the optimal distortions to housing are sizeable. They start at approximately 14%, are relatively flat up to around $100,000 in income, then turn up for a narrow range of wages before a steady fall to zero at a high income. As with capital income taxes, raising the labor supply elasticity or concavity of consumption utility increases the size of the optimal distortions. The best linear distortions are also sizeable, ranging from 5% to 20%.

The welfare gains from optimal policy are not large but are orders of magnitude larger than those found for capital income taxation. The gains reach 0.17% of total consumption for the calibration with low elasticity of labor supply.

6 Role of preference normalization

In this section, we explore the role of preference normalization in the study of optimal commodity taxation. We normalize preferences in two ways: to neutralize the role of preferences over goods in how much the planner values individuals; and to neutralize the effect of preferences over goods on the labor supply choices of individuals.

First, we normalize so as to eliminate any incentive for the planner to redistribute across agents based simply on their preferences over goods. Consider the following two representations of the same preferences over consumption goods:

$$U = \frac{\alpha (w^i)}{1 + \alpha (w^i)} \ln c^i_1 + \frac{1}{1 + \alpha (w^i)} \ln c^i_2 + \frac{1}{\sigma} (t^i)^{\sigma}$$

(24)

and

$$U = \ln c^i_1 + \frac{1}{\alpha (w^i)} \ln c^i_2 - \frac{1}{\sigma} (t^i)^{\sigma},$$

(25)

Expression (24) normalizes preferences as in our main analysis, whereas (25) does not. Specifically, starting an individuals’ undistorted optimal allocations, a planner using (24) has no desire to redistribute across preference types (conditional on the wage) because the marginal social value of resources is equalized across preference types. In contrast, a planner using (25) obtains a larger increase in social welfare from allocating a marginal unit of resources to the individual with lower $\alpha (w^i)$. Therefore, optimal tax policy will favor individuals with lower $\alpha (w^i)$. For example, in the context of capital income taxation when ability is positively correlated with patience (i.e., $\alpha' (w^i) < 0$), using (25) rather than (24) will cause the planner to favor those with lower $\alpha (w^i)$. Since these individuals value saving, the planner will be discouraged from taxing capital. Importantly, if we were to multiply expression (25) through by $\alpha (w^i)$, the impact of preferences would reverse even though we would be using an observationally equivalent representation of them. In that case, lower $\alpha (w^i)$ types would yield smaller increases in social welfare to the planner, so the planner would be predisposed toward capital income taxation.

Second, we use a representation of preferences that implies no relationship between preferences across.
goods and the willingness to work. To do so we use the utility function (a generalization of (24) for γ not necessarily equal to one):

\[ U = \frac{1}{\varphi^i} \left( \left( \frac{\alpha (w^i)}{1 + \alpha (w^i)} \right)^{\gamma (c^i_1)^{(1-\gamma)}} - 1 \right) + \left( \frac{1}{1 + \alpha (w^i)} \right)^{\gamma (c^i_2)^{(1-\gamma)}} - 1 - \frac{1}{\sigma (i^i)^{\sigma}} \]

To see that chosen income is independent of preferences, note that individual maximization yields:

\[ \varphi^i \lambda = (w^i)^{\frac{2 - 2\gamma}{\sigma + 2 + \gamma}} , \]

and thus

\[ y^i = (w^i)^{\frac{\sigma (\sigma - 1 +\gamma) - 2\gamma}{\sigma (\sigma - 1 +\gamma) - (\sigma - 1 + 1)}} . \]

Without this normalization, we would be forced to assume that preferences over goods were systematically related to preferences between leisure and consumption.

These normalizations are similar to two assumptions Saez (2002) states in his analysis of this topic. His Assumption 1 is that the planner’s marginal social welfare weights on individuals are independent of their tastes for goods, conditional on their incomes. Our first normalization pursues the same neutrality of marginal social welfare weights, though we use the laissez-faire allocations rather than the optimal allocations as the starting point for the normalization. This normalization captures the idea that the government does not want to redistribute resources across individuals simply because they will spend them on different consumption baskets. Our second normalization parallels Saez’s Assumption 2, which states that, conditional their income, individuals’ labor supply responses to tax changes are unaffected by their preferences. Though our normalization focuses on isolating from preferences the chosen level of labor supply, rather than its response to tax changes, the idea of the two approaches is similar. Intuitively, this normalization means that individuals choose how much to work without regard to how they plan to spend their disposable income. Saez notes that both of his Assumptions seem like reasonable ones in the context of capital income taxation. We take a similar perspective, believing that our normalizations provide a natural, and neutral, starting point for modeling preference heterogeneity and its effects on optimal commodity taxation.

Diamond and Spinnewijn (2009) use a representation of preferences similar to (25) to study optimal capital income taxation. As the discussion above implies, the use of this utility function introduces two factors in determining the optimal tax policy that are absent from our setup. First, if the planner’s objective function simply sums utilities of the form in (25), a positive capital income tax is less likely to be part of the optimum policy, as \( \alpha' (w^i) < 0 \) means that high-wage individuals are more valuable to their planner. For example, recall that our baseline simulation of optimal capital income taxes in Section 3 yielded a constrained optimal linear capital income tax of 0.08%. Using (25) instead yields a constrained optimal linear capital income tax of only 0.04%. Diamond and Spinnewijn consider a planner’s objective function that weighs different individuals differently. For their main analysis they assume that the planner puts a small enough weight on the high-skilled so as to want to redistribute away from them. Second, their use of (25) makes capital income taxation less distortionary, as the individuals who value saving also respond less elastically to tax changes. As Diamond and Spinnewijn note, if they were to assume an observationally equivalent representation of preferences in which more patient individuals would choose to work less, their results on optimal capital income taxation are reversed. Only limited empirical evidence is available to determine which is the appropriate assumption.
7 Conclusion

Among others, Mirrlees (1976) and Saez (2002) have argued that goods preferred by the high-skilled ought to be taxed as part of an optimal tax policy that seeks to redistribute from the (unobservably) high-skilled. This argument has been used, in particular, to justify positive capital income taxation. We show that, contrary to these previous results, optimal commodity taxation when preferences vary with ability may be regressive in income on those goods preferred by those who are more able. We obtain this result by allowing taxes on goods to be nonlinear functions of income and the consumption of the good, which is plausible for many important categories of consumption such as education, health, housing, and future consumption.

We parameterize the model with data on preferences for current relative to future consumption and for housing. We find that the relationship between ability and time discounting is unlikely to justify substantial capital income taxation, whether regressive or linear. There appears to be a stronger quantitative case for regressive taxes on housing.

References


### Table 1. Results of OLS regression of discount factor (delta) on ability and controls

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>age</td>
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<td>1.60E-02</td>
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<td>afqt**</td>
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<td>9.36E-05</td>
<td>7.05</td>
</tr>
</tbody>
</table>

Note: * indicates significance at the 5% level or lower; ** at 1%

| Observations | 7,008 |
| F-statistic  | 92.44 |
| R-squared    | 0.095 |

### Table 2. Results of OLS regression of h/y on ability and controls

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-statistic</th>
</tr>
</thead>
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<tr>
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<td>afqt**</td>
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<td>8.38</td>
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</tbody>
</table>

Note: * indicates significance at the 15% level or lower; ** at 1%

| Observations | 7,280 |
| F-statistic  | 46.07 |
| R-squared    | 0.047 |

### Table 3: Welfare gain decomposition of housing subsidy

<table>
<thead>
<tr>
<th>Share of gain due to:</th>
<th>Increase in efficiency</th>
<th>Redistribution of total consumption</th>
<th>Individual allocation of consumption</th>
<th>Redistribution of required income</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>21%</td>
<td>175%</td>
<td>-121%</td>
<td>25%</td>
<td>100%</td>
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### Table 4. Robustness Checks: results varying curvature of utility and elasticity of labor supply

<table>
<thead>
<tr>
<th>gamma</th>
<th>sigma</th>
<th>Capital tax rates at select wages</th>
<th>Optimal flat capital tax rate</th>
<th>Welfare gain over no capital tax, (as % of output)</th>
<th>Welfare gain over flat capital tax, (as % of output)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
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<td>$4=Min 16.7% $12 7.8% $22 5.1% $30 3.8% $50=Max 0%</td>
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<td>0.0259%</td>
<td>0.0048%</td>
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<tr>
<td>2.0</td>
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<td>0.0079%</td>
<td>0.0004%</td>
</tr>
<tr>
<td>2.0</td>
<td>4.5</td>
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<td>0.0034%</td>
<td>0.0001%</td>
</tr>
<tr>
<td>1.0</td>
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<td>0.0046%</td>
<td>0.0004%</td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>$4=Min 4.5% $12 4.1% $22 3.6% $30 3.0% $50=Max 0%</td>
<td>3.55%</td>
<td>0.0079%</td>
<td>0.0004%</td>
</tr>
<tr>
<td>4.0</td>
<td>3.0</td>
<td>$4=Min 9.3% $12 9.0% $22 7.7% $30 6.4% $50=Max 0%</td>
<td>7.83%</td>
<td>0.0179%</td>
<td>0.0009%</td>
</tr>
</tbody>
</table>

**Memo: cdf at wage**
- 0.02
- 0.37
- 0.76
- 0.90
- 1.00

**Memo: Discount rate**
- 0.25
- 0.14
- 0.08
- 0.05
- 0.00

### Table 5. Robustness Checks for Housing

<table>
<thead>
<tr>
<th>gamma</th>
<th>sigma</th>
<th>Housing tax rates in select ranges (simple average)</th>
<th>Best linear housing tax rate</th>
<th>Welfare gain over no housing tax, (as % of output)</th>
<th>Welfare gain over flat housing tax, (as % of output)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$4-$10 51% $10-22 20% $22-$42 13% $42-$62 13% $62-$82 13% $82-100 4% $100=Max 0%</td>
<td>$4-$10 51% $10-22 20% $22-$42 13% $42-$62 13% $62-$82 13% $82-100 4% $100=Max 0%</td>
<td>$4-$10 51% $10-22 20% $22-$42 13% $42-$62 13% $62-$82 13% $82-100 4% $100=Max 0%</td>
<td>$4-$10 51% $10-22 20% $22-$42 13% $42-$62 13% $62-$82 13% $82-100 4% $100=Max 0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lognormal-Pareto wage distribution</td>
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<tr>
<td>2.0</td>
<td>1.5</td>
<td>$4-$10 51% $10-22 20% $22-$42 13% $42-$62 13% $62-$82 13% $82-100 4% $100=Max 0%</td>
<td>19.69%</td>
<td>0.17%</td>
<td>0.0462%</td>
</tr>
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<td>2.0</td>
<td>3.0</td>
<td>$4-$10 13% $10-22 11% $22-$42 10% $42-$62 11% $62-$82 11% $82-100 4% $100=Max 0%</td>
<td>10.71%</td>
<td>0.04%</td>
<td>0.0005%</td>
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<td>2.0</td>
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<td>$4-$10 2% $10-22 6% $22-$42 8% $42-$62 9% $62-$82 10% $82-100 3% $100=Max 0%</td>
<td>6.52%</td>
<td>0.01%</td>
<td>0.0009%</td>
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<td>10.71%</td>
<td>0.04%</td>
<td>0.0005%</td>
</tr>
<tr>
<td>4.0</td>
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<td>$4-$10 20% $10-22 24% $22-$42 21% $42-$62 24% $62-$82 24% $82-100 7% $100=Max 0%</td>
<td>22.97%</td>
<td>0.09%</td>
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<td>Lognormal wage distribution</td>
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<tr>
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<td>$4-$10 13% $10-22 11% $22-$42 9% $42-$62 8% $62-$82 7% $82-100 3% $100=Max 0%</td>
<td>10.35%</td>
<td>0.0342%</td>
<td>0.0006%</td>
</tr>
</tbody>
</table>

**Memo: cdf at wage**
- 0.04
- 0.21
- 0.41
- 0.72
- 0.95
- 0.96
- 0.99

**Memo: alpha**
- 8.84
- 7.28
- 6.50
- 5.63
- 4.64
- 4.64
- 3.59
Figure 1: Implied capital income taxes using NLSY

Optimal capital income tax

Best linear capital income tax

ρ^i

Wage ($1992)

Discount rate
Figure 2: Implied housing consumption taxes

(h/y) = home value over income

Optimal housing consumption tax

Best linear housing tax

Implied relative housing tax vs. Wage ($1992)
Figure 3: U.S. Mortgage Interest Deduction and Simulated Optimal Housing Distortion

Simulated optimal distortion (gamma=1, sigma=3)

Existing housing subsidy*

Poterba and Sinai (2009) income bracket

* Calculations based on Poterba and Sinai (2008). Data shown are the present value of mortgage interest deduction savings as a percent of home value.
Figure 4: Extrapolating Alpha from NLSY Data for Capital

- Original data
- Alpha, using regression results
- Power distribution fitted to regression-based alpha
Figure 5: Extrapolating Alpha from NLSY Data for Housing

- Original data
- Alpha, using regression results
- Power distribution fitted to regression-based alpha
Figure 6: Optimal capital income distortions

Capital income distortion (rate)

Annual Labor Income ($2005)

- gamma=1, sigma=3
- gamma=4, sigma=3
- gamma=2, sigma=1.5
- gamma=2, sigma=4.5
- gamma=2, sigma=3
- LogNorm, gamma=2, sigma=3
Figure 7: Optimal housing distortions vs. income

- red: gamma=1, sigma=3
- pink: gamma=4, sigma=3
- blue: gamma=2, sigma=1.5
- green: gamma=2, sigma=4.5
- black: gamma=2, sigma=3
- LogNorm: gamma=2, sigma=3