1. Introduction

The effects of taxation on the volume and composition of private saving has traditionally been considered one of the central questions in public finance. This is hardly surprising. From a policy point of view one can point to a series of arguments for the importance of the problem. If alternative tax systems can lead to different rates of private saving, then the choice between them should take into account the short-run effects on employment and inflation, the medium-term effects on the rate of growth, and the long-term effect on the capital intensity of the economy. These are basically issues of the efficiency of resource allocation, but distributional policy is also involved. A tax policy designed to encourage saving may transfer income from “workers” to “capitalists” and from the present to future generations. Evidently, there are all sorts of tradeoffs to consider in policy design.

Although it is clear that much of the interest in this particular question is derived from a concern with policy problems, it is important to emphasize the conceptual distinction between positive and normative issues. Thus the question of whether an expenditure tax will lead to a higher or lower level of private saving than an equivalent income tax is a positive one. Whether the answer is one or the other does not in itself have any implications for tax policy. It is only when we introduce criteria for social welfare or efficiency that we can begin to consider the normative question of the desirability of an expenditure tax.

In principle, savings decisions can be made by consumers, firms and governments. The tradition in the literature has been to concentrate on consumer decisions and to take the personal saving rate as being the main determinant of the overall rate of saving. This approach is reflected in the emphasis given to consumer decision making in the present paper; however, there is also a need to consider the role of private corporations and government. At the level of private firms the tradition has been to see their saving and investment decisions as reflections of the preferences and market opportunities of the owners, so that

*I am grateful to Alan Auerbach, Mervyn King, Hans-Werner Sinn and Jon Vislie for helpful comments.
these remain the basic explanatory factors. But behind this view lie a number of assumptions about market structure, incentives, etc., which need both theoretical justification and empirical verification. As far as the government is concerned it does of course intervene directly in private saving decisions through public pension plans and social security provision, and the interaction between private and public saving decisions has recently become a very active area of research.

The tradition in the public finance literature has been to study the effects of taxation on savings as a whole more or less in isolation from the problem of the tax effects on portfolio composition, particularly with regard to its risk characteristics. This tradition is followed in the present survey, which first considers a series of models in which savings take the form of holdings of a single financial asset. Within this framework we then consider successively the tax effects on the saving decisions of an individual consumer (including their connections with corporate saving and social security), the question of the incidence of taxation, and the problem of the optimum tax treatment of savings. Tax effects on portfolio composition and risk taking are then analyzed with reference to the same set of questions; on some of these the literature is not very extensive, so that this part of the survey occupies less space than that which is concerned with pure savings models.

2. Taxation and saving: Models of individual choice

2.1. The two-period model: Perfect markets

The simplest context in which one can analyze the intertemporal consumption decisions of a single individual, is the two-period model which was first introduced by Irving Fisher (1930). The simplest version of this model takes labour income to be exogenous and concentrates on the allocation of consumption between the two periods via the saving decision. This has proved to be a fruitful model for many purposes and has been extensively used in public finance; see, e.g., Hansen (1955) and Musgrave (1959).

We imagine a consumer whose preferences are defined over the amounts of consumption enjoyed in the two periods of his life. His preference ordering can be represented by the utility function

\[ U = U(C_1, C_2), \]  

which is assumed to be increasing, strictly quasi-concave and differentiable. Incomes in the two periods are given as \( y_1 \) and \( y_2 \). The consumer can borrow or lend in a perfect capital market at a rate of interest equal to \( r \). The budget constraint for the first period is

\[ C_1 + S = y_1, \]  

(2.2)
where $S$ is saving, which can be either positive or negative. In the second period, consumption is limited by income in that period plus the amount of saving with interest added, i.e.,

$$C_2 = y_2 + S(1 + r).$$  \hfill (2.3)

Combining (2.2) and (2.3) we have that

$$C_1 + \frac{C_2}{1 + r} = y_1 + \frac{y_2}{1 + r},$$  \hfill (2.4)

which simply states that the present value of consumption must be equal to the present value of income.

To find the maximum of (2.1) subject to (2.4) we form the Lagrangian

$$\mathcal{L} = U(C_1, C_2) - \lambda \left( C_1 + \frac{C_2}{1 + r} - y_1 - \frac{y_2}{1 + r} \right).$$

Setting the partial derivatives equal to zero we obtain (with subscripts denoting partial derivatives)

$$U_1 - \lambda = 0,$$ \hfill (2.5)

$$U_2 - \lambda \frac{1}{1 + r} = 0,$$ \hfill (2.6)

which can be combined to give

$$\frac{U_2}{U_1} = \frac{1}{1 + r} \quad \text{or} \quad \frac{U_1}{U_2} - 1 = r.$$ \hfill (2.7)

The first version says that the marginal rate of substitution should be equal to the price of future in terms of present consumption, which is the discount factor. The second version is the famous "rule" that the marginal rate of time preference should be equal to the rate of interest.

So far we have not introduced taxation into the model. It is in fact quite useful to study the properties of the model as it stands; the implications of alternative tax systems can then be inferred fairly directly.

From equations (2.4)–(2.6) we can derive demand functions for consumption in the two periods; of particular interest is the demand for first-period consumption, which can be written as

$$C_1 = C_1(r, y_1, y_2) \quad \text{or} \quad C_1 = C_1(r, y),$$ \hfill (2.8)
where \( y = y_1 + (1 + r)^{-1} y_2 \). The latter formulation reflects the fact that consumption depends on income only via its present value; a shift of income between periods such that \( y \) were unchanged would leave consumption unaffected.

As usual in demand theory there are no \emph{a priori} restrictions on the income effects; consumption could be normal or inferior. However, in view of the aggregate interpretation of consumption in this model it is natural to assume that it is a normal good.

The effect of a change in the rate of interest can be characterized by means of the Slutsky equation. This is easily derived as follows. Taking the differential of the demand function with \( y_1 \) constant we have that

\[
dC_1 = \frac{\partial C_1}{\partial r} dr + \frac{\partial C_1}{\partial y_2} dy_2.
\]

For a \emph{compensated} change in the rate of interest it must be the case that

\[
dU = U_1 dC_1 + U_2 dC_2 = 0,
\]
or, substituting from the first-order conditions (2.5)–(2.6),

\[
\lambda \left( dC_1 + \frac{1}{1 + r} dC_2 \right) = 0.
\]

From the budget constraint we have that

\[
dC_1 + \frac{1}{1 + r} dC_2 = - \frac{y_2 - C_2}{(1 + r)^2} dr + \frac{1}{1 + r} dy_2.
\]

Constant utility therefore requires that

\[
\frac{dy_2}{1 + r} dr = -(y_1 - C_1) dr,
\]
where the last equality follows from (2.4). Substituting for \( dy_2 \) in (2.9) and dividing through by \( dr \), we obtain

\[
\left( \frac{dC_1}{dr} \right)_{\lambda = \text{const.}} = \frac{\partial C_1}{\partial r} - (y_1 - C_1) \frac{\partial C_1}{\partial y_2},
\]
or, rearranging terms,

\[
\frac{\partial C_1}{\partial r} = (y_1 - C_1) \frac{\partial C_1}{\partial y_2} + \left( \frac{\partial C_1}{\partial r} \right)_U.
\]
The last term is the substitution effect which can be shown to be negative using the second-order maximum conditions. The first term is the income effect which is positive for a lender ($C_1 < y_1$) and negative for a borrower ($C_1 > y_1$). Thus, for a borrower it is clear that an increase in the interest rate implies reduced consumption, while for a lender the outcome depends on the relative magnitudes of the income and substitution effects. These results are demonstrated graphically in Figure 2.1a for the case of a borrower and in Figure 2.1b for the case of a lender. In both diagrams the original budget constraint is the line $AA$ and the consumer's optimum is at $a$. With an increase in the rate of interest the budget line swings around the income point $(y_1, y_2)$ to the new position $CC$ with the corresponding optimum at $c$. If a lump sum payment were made to bring the consumer back to his original indifference curve, his optimum would be at $b$. The substitution effect on present consumption is therefore the horizontal distance between $a$ and $b$, while the income effect corresponds to the horizontal distance between $b$ and $c$.

What about the effect on saving itself? From the first-period budget constraint (2.2) it must be the case that $\frac{dS}{dr} = -\frac{dC_1}{dr}$, so that our interpretation of the comparative statics results can simply be applied to saving by changing the algebraic sign of the effects. However, Feldstein (1978) has pointed out that if

![Figure 2.1a. Substitution and income effects of a change in the interest rate: the borrower.](image-url)
saving instead is defined by means of the second-period budget constraint (2.3), an unexpected ambiguity arises. It is easy to show that the compensated interest effect on future consumption is positive, i.e., \((\partial C_2/\partial r)_U > 0\). But since the rate of interest enters into the expression which links saving to future consumption, the compensated effect on saving is ambiguous, while the conventional view is certainly that it is positive. As argued in Sandmo (1981a), rather than proving the conventional view to be wrong, Feldstein has presented a case for paying more attention to the definition of saving than has typically been the case. In the analysis above it was assumed that the compensated demand functions for consumption and saving were established through variations in second-period income; it is easy to see that Feldstein’s definition of saving implies that compensation is made in terms of first-period income. In the latter case the income point on the compensated budget line \(BB\) in Figure 2.1 lies to the right of \(y_2\), and it is then obvious that the compensated effect on saving becomes ambiguous. However, ambiguity can be avoided by stating all comparative static results in terms of consumption, which is the basic choice variable over which preferences are defined. In other words, “the effects of the rate of interest on saving” should be interpreted as simply a short-hand expression for “the effects of the rate of interest on the intertemporal allocation of consumption”.

Figure 2.1b. Substitution and income effects of a change in the interest rate: the lender.
We finally come to the effects of taxation. Four types of taxes will be considered: lump sum tax, income tax, expenditure tax and indirect taxation. For simplicity, in the last three cases we limit the discussion to proportional taxes, i.e., taxes for which the rates do not vary with the size of the tax base.

A lump sum tax is one whose magnitude cannot be influenced by any decision taken by the consumer. If \( a \) is the amount of the tax payment, it would enter the budget constraint (2.4) simply as a deduction from the right-hand side,

\[
C_1 + \frac{C_2}{1 + r} = y_1 + \frac{y_2}{1 + r} - a. \tag{2.11}
\]

The lump sum payment could be interpreted as a tax which is paid in the first period only, or as the present value of such taxes paid in the future period only or in both. In any case the lump sum tax has pure income effects on consumption; it does not affect the relative price between present and future consumption. Although it is of limited practical importance, the lump sum tax is of interest as a useful benchmark case when we come to discuss the efficiency of alternative tax systems with respect to savings decisions.

We now consider a tax on all income, i.e., both on exogenous labour income \( (y_1 \text{ and } y_2) \) and on income from capital \( (rS) \). If the tax rate is assumed to be constant over time and equal to \( t \), the budget constraint becomes

\[
C_1 + \frac{C_2}{1 + r(1 - t)} = y_1(1 - t) + \frac{y_2(1 - t)}{1 + r(1 - t)}. \tag{2.12}
\]

An income tax thus works like a combination of a lump sum tax and a special tax on interest income. Since it reduces the rate of interest, it clearly has a substitution effect in favour of present and against future consumption. If the tax rate were allowed to vary between the two periods, the conclusions would be unaffected; interest income would be taxed at the rate \( t_2 \), and even though other income would be taxed at rates \( t_1 \) and \( t_2 \), respectively, the difference in rates would not in itself have any incentive effects for the consumer's adjustment of his consumption decisions.

Indirect taxation at the rate \( s \) would raise the price of consumption from 1 to \( 1 + s \). Consequently, the budget constraint in this case would have to be written as

\[
(1 + s)C_1 + \frac{(1 + s)C_2}{1 + r} = y_1 + \frac{y_2}{1 + r}, \tag{2.13a}
\]

or, equivalently,

\[
C_1 + \frac{C_2}{1 + r} = \frac{y_1}{1 + s} + \frac{y_2}{(1 + s)(1 + r)}. \tag{2.13b}
\]
From (2.13b) it is clear that the case of a general indirect tax at a rate which is constant over time is equivalent to a tax on labour income alone, leaving the relative price of present and future consumption unaffected. In terms of the present model, indirect taxation is accordingly also equivalent to a lump sum tax being levied on all consumers in proportion to their labour income. An expenditure tax at the rate \( s \) would of course have to be modelled in exactly the same way, and the same conclusion holds.

This line of reasoning is sensitive, however, to the assumption that the tax rate is constant over time. If \( s_i \) \((i = 1, 2)\) is the tax rate in period \( i \), it is immediate from (2.13a) that the tax system will indeed influence the relative price of present and future consumption; if, e.g., \( s_2 > s_1 \), the effect, as compared with a lump sum tax, is similar to a reduction in the rate of interest. This point becomes important if one considers, e.g., the gradual substitution of an expenditure tax for an income tax. Although the former system is neutral with respect to the rate of interest facing the consumer, during a reform process where the tax rate was gradually increased to its permanent level, the system would have a distortion similar to that implied by the general income tax. The same complication would arise if marginal tax rates were increasing under an expenditure tax system. The basic point is that neutrality with respect to the intertemporal consumption decision can only be achieved by a system which leaves the price of future in terms of today's consumption unaffected by the tax rates.

2.2. The two-period model: Imperfect markets

The perfect markets assumption is an idealization which has eventually to be judged against its empirical usefulness. Two features of the model stand out as being particularly strong abstractions from real world conditions. The first is the assumption that the borrowing and lending rates are the same, the second that there is no credit rationing. In considering the implications of relaxing these assumptions one should keep in mind that inequality of borrowing and lending rates and the existence of some form of credit rationing do not necessarily constitute "imperfections" in a real sense; the former clearly arises because of the transactions costs of credit institutions, while the latter can be justified by the asymmetric information possessed by agents in the credit markets. For an interesting early discussion of the notion of perfect capital markets, see Stigler (1967), and for a modern analysis, Stiglitz and Weiss (1981).

Let the lending and borrowing rates be \( r_L \) and \( r_B \) with \( r_L < r_B \). The second-period budget constraint (2.3) now becomes

\[
C_2 = y_2 + S(1 + r_L) \quad \text{if} \quad S > 0,
\]

\[
= y_2 + S(1 + r_B) \quad \text{if} \quad S < 0.
\]

(2.14)
Ch. 5: The Effects of Taxation on Savings and Risk Taking

The graphical picture of this situation is shown in Figure 2.2. If, in addition, there is a quantity constraint on the amount that can be borrowed, we must have

\[ S > S, \quad (2.15) \]

where \( S \) is some negative number which could possibly depend upon \( y_1 \). When both (2.14) and (2.15) hold we have the situation shown in Figure 2.3. It is clear that the effects of the imperfections are to introduce kinks in the budget constraints, and it is likely that the kink points will in fact be the optimum choices for many consumers. The implication of this for the effects of taxation is that taxes which would otherwise be non-neutral with respect to the rate of interest, will in fact not have any substitution effects for the consumers who have chosen their optima at the kink points. To the extent that this is true it diminishes the importance of the substitution effects for the overall results of tax policy.\(^1\)

\(^1\)It should be observed that a kink of the type depicted in Figure 2.2 could also emerge – or become more pronounced – by a tax system in which interest income is taxed but where no deduction is allowed for interest payments.
2.3. Multi-period models and the bequest motive

In principle the extension of the analysis to the multi-period case is straightforward. If the consumer's horizon extends over $T$ periods, his problem in the perfect markets case without taxation can be formulated as that of maximizing

$$U = U(C_1, \ldots, C_T),$$

subject to

$$\sum_{t=1}^{T} (1 + r)^{t-1}C_t = \sum_{t=1}^{T} (1 + r)^{t-1}y_t. \tag{2.17}$$

When taxation is introduced, one can as before study its effects as equivalent combinations of income and interest rate changes. From the point of view of descriptive analysis, one is of course particularly interested in the demand for present consumption $(C_1)$, since this is the decision which is actually binding for the consumer; the optimum choices of $C_2, \ldots, C_T$ constitute an optimal plan which can be revised as the future gradually becomes the present.

Figure 2.3. Different borrowing and lending rates with a quantity constraint on borrowing.
There is one respect in which the multi-period model significantly modifies a result of the two-period analysis. In the two-period model, the substitution effect of an interest rate change on future consumption is positive; this is the own-substitution effect. Because there are only two goods, these are necessarily substitutes in the Hicksian sense, so that the compensated interest effect on present consumption is necessarily negative; this is essentially the result demonstrated in Figure 2.1 above. However, with more than two periods, consumption in any one future period can be either a complement or a substitute with respect to first-period consumption. Since a change in the rate of interest changes the relative prices of future consumption in all periods, it cannot be ruled out that the compensated interest effect on present consumption is positive. It is worth noting that in the most popular version of (2.16), where \( U \) is additive and strictly concave, all goods are necessarily substitutes, and this result is ruled out. However, from a theoretical point of view, there is nothing paradoxical in the possibility of present and future consumption being complements rather than substitutes.

So far we have concentrated on versions of the pure consumption model which explain saving as arising from the adjustment of the life cycle pattern of consumption to the exogenously given time profile of income. This leaves out the bequest motive as a determinant of saving. Formally this is easy to incorporate: one could, e.g., interpret consumption in the final period \( C_T \) as bequests and allow for the possibility that this could be taxed at a special bequest tax. Given that bequests is a normal good, both the income and substitution effects of such a tax would lead to a lower level of bequests. Whether it would also lead to increased consumption and less saving in the initial period is a question which involves cross-substitution effects in the same way which was discussed above, and little can be said on an \textit{a priori} basis. For a fuller discussion of the bequest motive, inheritance taxation and its long-run effects on income distribution, the reader is referred to Stiglitz (1978).

2.4. Uncertainty

The precautionary motive for saving comes from the fact that future income, the rate of return on saving, etc. are not known with certainty in the present. Consequently, saving may be influenced by the extent to which it can act as a protection against unfavourable realizations of current expectations. This leads immediately to a consideration of problems of portfolio choice, the demand for insurance, etc., which will be postponed to the section on taxation and risk taking. At the present stage we should note, however, that the re-interpretation of the parameters in terms of expected values is not an adequate way to handle uncertainty. When the rate of interest and/or future income are uncertain, taxation changes not only the expected value, but also the variance and higher
moments of the probability distribution. Taking account of uncertainty therefore leads not only to a new set of questions which can be asked; it may also lead to revisions of the qualitative implications of certainty models. As an example of this we may consider the substitution effect on present consumption of a tax on interest income. On the basis of the two-period certainty model we would conclude that the effect is positive, since it is equivalent to a fall in the rate of interest. But, if the rate of interest is uncertain, an increase in the tax rate not only reduces its expected value but also its variance. As shown in Sandmo (1970) the substitution effect of reduced riskiness is likely to be negative, so that the overall tax substitution effect is ambiguous.

2.5. The role of labour supply

The pure consumption model with exogenously given income has played an important role in the literature and has formed the theoretical foundation for much empirical work. However, when the model is used to analyse tax policy it is important to be aware of the problems that cannot be adequately handled by the model. Of the implicit assumptions contained in the model probably the most serious of all is that labour income is exogenous. This implies, e.g., that the choice between direct and indirect taxation can be considered without reference to the effect on labour supply. This is clearly unsatisfactory and leads us to consider a model which gives an integrated treatment of saving and labour supply decisions.

We again postulate a two-period model in which the consumer now works in the first period and is retired in the second. His utility function is

\[ U = U(C_1, C_2, L), \]  
(2.18)

where \( L \) is leisure enjoyed in the first period. The time available in the first period is equal to \( T \), so that we have \( L + H = T \), where \( H \) is working time. According to the specific use that one wants to make of the model, \( H \) could be interpreted as the number of hours worked per day or per week, days per year, etc. If \( a \) is lump sum income, possibly adjusted for lump sum taxation, the budget constraint in the absence of distortionary taxation is

\[ C_1 + \frac{C_2}{1 + r} = w(T - L) + a. \]  
(2.19)

The first-order maximum conditions can be written as

\[ \frac{U_2}{U_1} = \frac{1}{1 + r}, \]  
(2.20)

\[ \frac{U_k}{U_l} = w. \]  
(2.21)
We now have a model with three commodities and two prices, and we wish to consider its comparative statics properties. As far as income effects are concerned, it is again the case that these are typically assumed to be positive; such an assumption is of course an implicit restriction on the indifference map. As far as substitution effects are concerned, the direct or "own"-effects are negative, implying that

\[
\left( \frac{\partial C_2}{\partial r} \right)_U > 0, \quad \left( \frac{\partial L}{\partial w} \right)_U < 0. \tag{2.22}
\]

For a constant level of utility the interest rate effects must clearly satisfy

\[
U_1 \left( \frac{\partial C_1}{\partial r} \right)_U + U_2 \left( \frac{\partial C_2}{\partial r} \right)_U + U_L \left( \frac{\partial L}{\partial r} \right)_U = 0,
\]

or, substituting from (2.20) and (2.21),

\[
\left( \frac{\partial C_1}{\partial r} \right)_U + \frac{1}{1 + r} \left( \frac{\partial C_2}{\partial r} \right)_U + w \left( \frac{\partial L}{\partial r} \right)_U = 0. \tag{2.23}
\]

From (2.22) and (2.23) it is clear that we cannot deduce the sign of the substitution effect of the rate of interest on present consumption. In the pure consumption model this was necessarily negative, since leisure was assumed to be fixed. But since we cannot in general exclude the case where leisure and future consumption are Hicksian substitutes, present and future consumption could well be complements. In other words, the compensated demand function for present consumption could depend positively on the rate of interest. In some of the literature one gets the impression that a negative relationship between present consumption and the rate of interest is a very robust implication of economic theory. But as the discussion here and in previous sections has shown, this is not the case. A negative relationship can be established only if restrictive assumptions are made about the consumer's choice set or his utility function or both. This should be kept in mind when we turn to the tax policy implications of the analysis.

Similar remarks apply of course to models of labour supply which implicitly take the saving decision to be exogenous or at least separable from the labour–leisure choice. The narrow framework may lead both to misspecification of theoretical and empirical descriptive models and to misleading conclusions in normative models of taxation.

Table (2.1) summarizes the comparative statics properties of the integrated model and compares the implications with those of the two "pure" models. In each part of the table the first row of signs follows from the assumption that
Table 2.1
Comparative statics properties of alternative models.

1. Pure consumption model

\[ U = U(C_1, C_2), \quad C_1 + C_2/(1 + r) = y_1 + y_2/(1 + r) + a \]

<table>
<thead>
<tr>
<th>Parameter changing</th>
<th>Decision variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>+</td>
</tr>
<tr>
<td>( r ) (comp.)</td>
<td>-</td>
</tr>
</tbody>
</table>

2. Pure labour supply model

\[ U = U(C, L), \quad C = w(T - L) + a \]

<table>
<thead>
<tr>
<th>Parameter changing</th>
<th>Decision variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>+</td>
</tr>
<tr>
<td>( w ) (comp.)</td>
<td>+</td>
</tr>
</tbody>
</table>

3. Integrated model

\[ U = U(C_1, C_2, L), \quad C_1 + C_2/(1 + r) = w(T - L) + a \]

<table>
<thead>
<tr>
<th>Parameter changing</th>
<th>Decision variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>+</td>
</tr>
<tr>
<td>( r ) (comp.)</td>
<td>?</td>
</tr>
<tr>
<td>( w ) (comp.)</td>
<td>?</td>
</tr>
</tbody>
</table>

Consumption and leisure are normal goods. The signs related to the compensated effects of changes in the wage and interest rates are those which follow from the hypothesis of utility maximization. In the integrated model two of the latter sign restrictions are survivors from the pure models: Future consumption is positively related to the rate of interest, and leisure depends negatively on the wage rate. Non-survivors are the positive relationship between present consumption and the rate of interest as well as that between the wage rate and consumption. The general theoretical point is that models with only two goods are very special in that these have to be substitutes; some of the implications of two-good models will therefore not hold when the models are extended to three or more goods.

Having examined the properties of the model in the absence of taxation, it is now a straightforward task to examine the effects of alternative systems. A lump sum tax increase is equivalent to a decrease of the parameter \( a \). As for the other equivalence results, they are more complex than in the pure consumption model.
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Take first the income tax at a constant rate $t$, which, however, does not apply to the lump sum element $a$. This could be interpreted to mean either that $a$ is simply tax exempt, or it could be seen as implying a linear but non-proportional tax schedule which is progressive or regressive according to whether $a \geq 0$. The budget constraint is

$$C_1 + \frac{C_2}{1 + r (1 - t)} = w (1 - t) (T - L) + a. \quad (2.24)$$

Clearly, a change in the tax rate $t$ will have both income and substitution effects, the former being analogous to the effects of a decrease in $a$. As far as the substitution effects are concerned, the tax rate changes both the net interest and wage rates, and the total effects of these on present and future consumption cannot be assessed without empirical evidence about the magnitudes of the various effects involved. Thus, whereas in the pure consumption model it was fairly straightforward to conclude that the income tax “discriminated against saving”—in the sense that the substitution effects were in favour of present consumption—this is no longer obvious in the present model. Whereas in the previous model it was clear that a lump sum tax which left the consumer at the same level of utility as the income tax would induce less present consumption, in the present model the overall pattern of substitution effects is too complex for any such conclusion to be drawn.

With the linear expenditure tax or a general indirect tax at rate $s$, we have the budget constraint

$$(1 + s) C_1 + \frac{(1 + s) C_2}{1 + r} = w (T - L) + a, \quad (2.25a)$$

or, dividing by $(1 + s)$,

$$C_1 + \frac{C_2}{1 + r} = \frac{w}{1 + s} (T - L) + \frac{a}{1 + s}. \quad (2.25b)$$

From (2.25b) it is clear that the imposition of either of these forms of taxation have effects equivalent to a simultaneous reduction of the wage rate and lump sum income, leaving the net rate of interest unchanged. Thus, it is no longer true that these forms of taxation are essentially lump sum in nature, since they do lower the price of leisure in terms of consumption. It might be tempting to compare these taxes with the income tax and conclude that the latter is more discriminatory with respect to saving, since it lowers the net rate of interest. But the argument is clearly unfounded. First, it is not clear what the basis of the comparison ought to be. Suppose, however, that it is constant utility. It is true
that the net interest rate is higher under, e.g., indirect taxation, but constant utility must then imply a lower net wage rate. Thus, what is involved is a comparison of two cross-substitution effects on present consumption, and the outcome of this cannot be decided on \textit{a priori} grounds.

The most immediate interpretation of labour supply in this type of model is in terms of hours supplied in a given occupation, and most empirical studies are in fact based on this interpretation; the reader is referred to the survey of this area in Chapter 4. An alternative interpretation is in terms of occupational choice, where the budget constraint can be seen as a market opportunity locus with each point on it representing an occupation offering a fixed income-leisure package. This interpretation suggests that the cross-substitution effects emphasized above may be especially relevant when one thinks of saving as being partly determined by the pattern of occupational choice. E.g., if the tax system were to discriminate in favour of occupations with much leisure relative to income, and if such occupations were also believed to be characterized by relatively stable patterns of annual income over time, one would also believe that the amount of life cycle saving would tend to diminish. In the public finance context, these problems have so far received little attention.

The present discussion has been focussed on the derivation of empirical hypotheses concerning the effects of alternative tax systems. However, it is clear that the issues raised easily lead into normative questions. Is neutrality of the tax system with respect to the interest rate a property which it is particularly desirable to achieve? What determines the relative rates of tax on income from labour and capital in an “optimal” tax system? These and related questions are postponed to Section 7.

3. Taxation and saving: Aggregation and empirical estimation

It is well known from the general theory of consumer behaviour that the conditions for perfect aggregation of demand relationships are extremely restrictive.\footnote{See Deaton and Muellbauer (1980, ch. 6) for a good modern treatment of the theory of aggregation.} Perfect aggregation requires that the aggregate consumption function, i.e., the market demand function for present consumption, is such that it could have resulted from the maximization of a single utility function subject to a single budget constraint. A condition ensuring that this is the case is that all consumers have identical homothetic utility functions. This condition implies that any redistribution of income between consumers leaves aggregate consumption unaffected. Although the condition is in itself strong, it is important to keep in mind that it is derived on the assumption that consumers face the same prices. This...
may not be an unreasonable assumption for ordinary non-durable household goods, but for intertemporal decisions involving the expectations of future prices and interest rates it becomes far from trivial. Where labour supply is involved it clearly becomes untenable. Moreover, taxation itself is important for the possibility of aggregation. E.g., if the income tax has an increasing marginal tax rate, high-income individuals will be faced with a lower rate of interest than low-income individuals. Moreover, if different individuals face different kinds of constraints in the labour and credit markets, we have another set of difficulties preventing the aggregation of individual into market relationships.

In spite of all these difficulties, the estimation of aggregate consumption functions has been one of the most active areas of econometric research. It is also one of the areas where most attention is paid to the theoretical foundations. The transition from the theory of the individual consumer to the estimation of aggregate relationships, however, is usually made on the basis of a number of aggregation problems being assumed away. One would have to be a purist to call this procedure an illegitimate one. But it is still necessary to bear in mind that, e.g., attempts at the aggregate level to estimate the income and substitution effects of interest rate changes may be subject to serious errors of aggregation.

The influential early work of Wright (1969), while neglecting the aggregation problem, provided evidence of a significant negative substitution effect of the rate of interest on present consumption, the compensated elasticity being in the neighbourhood of \(-0.03\). This would mean that an increase in the rate of interest by one percentage point from 4 to 5 percent — brought about by, e.g., a reduction in the rates of tax on interest income — would decrease present consumption by approximately 0.75 percent. Wright emphasized the significance of this result both for the traditional Keynesian view of the effectiveness of monetary policy and for the assessment of the deadweight loss associated with capital income taxation.

Blinder (1975) in a later study, which explicitly took the distribution of income into account, found a substitution effect of an order of magnitude of only one tenth of that found by Wright. However, Blinder unlike Wright did not explicitly take account of the effects of taxation on the net rate of interest. Blinder also found that contrary to the usual Keynesian belief, an equalisation of incomes tends to lower aggregate consumption, at least as judged from the evidence of post-war American data. These results may serve as a reminder that the equivalence between changes in tax rates on the one hand and of income and interest rate changes on the other must be interpreted with a great deal of caution. In particular, the distribution of the income effects across the population may be very different for changes in the rate of interest and for changes in taxation.

More recent work by Boskin (1978) and Boskin and Lau (1978) seems to indicate that the interest elasticity of consumption could easily be considerably

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3 Wright claims (1969, p. 284) that prior to his own work only two published attempts had been made to estimate the interest elasticity of consumption or saving.
higher than was previously thought. The authors’ preferred value of the savings elasticity is 0.4, which would correspond to a consumption elasticity of $-0.30$, which is ten times the magnitude of Wright’s estimate; indeed, since this estimate is of the uncompensated elasticity, the increase in the estimate of the compensated elasticity should be even higher. Boskin (1978), like Wright and Blinder, estimated a version of the pure consumption model. The later work of Boskin and Lau is particularly notable for the extension of the theoretical framework to an integrated model of consumption and labour supply, where they found significant cross-effects of interest and wage changes on labour supply and consumption, respectively.

Much of the evidence for tax substitution effects on consumption and saving is indirect, in the sense that tax effects are inferred from the estimated interest rate effects using the equivalence results of the theoretical model, while ideally one would like to see direct tests of the tax effects. Another difficulty with using evidence related to the rate of interest is the measurement problem involved in the use of this variable. While earlier studies used nominal before-tax rates of interest, the increases in both the rate of inflation and marginal tax rates have made it important to use some measure of real after-tax rate of return; see the discussion of this in Feldstein (1982). This is done, e.g., by Boskin (1978), while Blinder (1975) does not correct for the effects of taxation; this discrepancy could conceivably account for the difference in the estimated magnitude of the interest elasticity.

Another measurement problem is the choice of the appropriate nominal interest rate. Since in reality we observe different borrowing and lending rates, a varying degree of access to capital markets and increasing marginal tax rates, it is clear that the relevant interest variable should differ between socioeconomic groups of consumers. However, in practice one has had to compromise by choosing some “representative” interest rate, and it therefore becomes to some extent a judgement of the individual researcher to choose such a rate. While Blinder chooses a weighted average of rates paid on time deposits by financial institutions, Boskin relies on the estimates of Christensen and Jorgenson (1973) of rates of return to the household sector, computed as income from assets divided by asset value. Both of these studies, on the other hand, use data for rates of return on an annual basis. It is not clear that annual rates are the relevant ones if the underlying model is that of life cycle saving, for which some long-run measure would seem more appropriate. When the results are used to consider the relative merits of income and expenditure taxation, they are in fact used to predict the effect of a long-run shift in the real after-tax rate of return.

It is not easy to give a summary characterization of the empirical work which has been done in this area. It certainly seems to indicate that the relative price effects could be rather substantial and that the consequence of taxation for saving incentives clearly go beyond those of a pure reduction of real disposable income.
These conclusions have important implications both for the positive assessment of the effects of taxation and for the normative issues of tax design. However, it should be kept in mind that there are as yet relatively few studies in this area, and that these differ both in terms of model specification and in the nature of the data used. With the gradual accumulation of empirical results one will hopefully achieve a firmer basis both for prediction and for policy advice.

4. Corporate savings

In accounting for total private saving it might seem obvious that close attention should be paid to saying by corporations in the form of retained earnings. In theory, there are at least two views of the relationship between corporate and personal saving. The first view is that dividends but not retained earnings should be included in the disposable income which enters the consumption function; Feldstein (1973) refers to this as the Keynesian view, although not necessarily that of Keynes himself. The second point of view is to start from the assumption that the value of corporate assets is reflected in individual wealth; therefore, whether corporate earnings are retained or distributed makes no difference for personal consumption behaviour. This approach goes back to Irving Fisher (1930) and is continued in modern life cycle theories. According to this view, consumers always see through "the corporate veil"; on the other hand the flow of capital income has no effect on personal saving.

The empirical results of Feldstein (1973) on U.S. data and of Feldstein and Fane (1973) on British data give qualified support to the Fisher view. Consumers do see through the corporate veil by adjusting personal saving so as to offset changes in corporate saving. On the other hand the flow of capital income, whether dividends or retained earnings, has an effect on personal consumption and saving. It should be noted that later work by Bhatia (1979) on a different data set for the U.S. concluded that retained earnings did not have any independent effect on consumer spending.

5. Social security, pensions and saving

Both social security and public and private pension schemes can be regarded as a kind of compulsory saving. In return for social security taxes and pension contributions in the present the individual is promised payments to provide for his old age. Suppose now that a social security scheme is introduced into an environment where consumers have already drawn up optimal consumption-saving plans covering their entire lifetimes. Since retirement consumption is now to be provided for from public funds, an optimal response on the part of
individuals is to reduce their personal saving. How will this affect total saving in the economy? The answer obviously depends on the use which the government makes of the contributions which it collects. If these are used for public consumption and transfers, the overall rate of saving will fall. If, on the other hand, the contributions are left to accumulate in a fund, the rate of saving will be upheld and may even increase. In view of the very large amounts involved, the growth of social security and pension schemes could easily have a powerful effect on the rate of saving.\footnote{The effects of social security and pension schemes on saving are, of course, only one aspect of the growth of these types of saving. The effects on retirement behaviour have been studied by Diamond and Hausman (1984) and others. A survey of the social security reform debate in the United States has been provided by Thompson (1983).}

To see the theoretical issues more precisely, let us go back to our basic two-period consumption model of Section 2. Let $b$ be social security benefits received in period 2 (the retirement period), and let $c$ be the contribution paid in period 1. If we begin by assuming that the contribution is paid in a lump sum manner, the consumer's budget constraint is

$$C_1 + \frac{C_2}{1 + r} = y_1 + \frac{y_2}{1 + r} - c + \frac{b}{1 + r}.$$  \hspace{1cm} (5.1)

Let us assume that $b$ is calculated as a multiple of some reference contribution $\bar{c}$ which may but need not be equal to $c$. If the multiplicative factor is $(1 + g)$ we can write $b = (1 + g)\bar{c}$ and (5.1) as

$$C_1 + \frac{C_2}{1 + r} = y_1 + \frac{y_2}{1 + r} - c(1 - \frac{1 + g}{1 + r} \frac{\bar{c}}{c}).$$  \hspace{1cm} (5.2)

From this formulation several conclusions follow immediately:

1. If $c = \bar{c}$ and $g = r$, the social security scheme has no effect on present or future private consumption. This implies that personal saving will fall by an amount equal to the amount of the contribution. This is the case where there is no redistribution involved in the scheme, and where the rate of return available to the individual saver equals that available to the government.

2. If $c = \bar{c}$ and $g \neq r$, there is an income effect on consumption in both periods which is positive or negative according to whether $g > r$. If marginal propensities to consume are positive and less than one, saving will change in the same direction as consumption. The case $g > r$ may be of particular interest as that in which the government is able to offer a higher rate of return on social security contributions than savers can get in the private capital market.

3. If $g = r$ and $c \neq \bar{c}$, there are income effects of the same kind dependent on whether $c > \bar{c}$. In many countries benefits increase less than proportionately with contributions, so that there might be positive income effects for low-income
groups, while the effects could be negative for individuals in high-income brackets. There might then be a redistributive effect on aggregate personal saving, the sign of which cannot be determined on a priori grounds.

The assumptions on which this model is based are extremely restrictive; of these, a fixed retirement period, complete certainty and the absence of a bequest motive and of distortionary taxes are potentially important as leading to further deviations from conclusion (1).

(4) It has been argued by, e.g., Feldstein (1976a) that retirement age is a decision variable which should not be taken as fixed. The introduction of social security induces individuals to retire earlier and to save more during their working years to provide for a longer period of retirement. This effect comes in addition to the “replacement effect” outlined under (1), and makes the total effect indeterminate.

(5) The implicit yield on social security contributions might be more certain than the rate of return in the capital market, e.g., because the public sector has a risk diversification advantage over private individuals. This would probably lead to a further reduction in private saving, over and above the displacement effect. On the other hand, the yield on social security contributions is based on political guarantees which may not be considered particularly trustworthy; in that case the risk effect could go the other way.

(6) Suppose that the present generation cares not only about its own utility but also about that of its descendants; more precisely, each generation has a welfare function over its own utility and that of its heirs. Because the same is true of the next generation, each generation acts as an individual with an infinite life. Barro (1974) has then argued as follows. Suppose that an unfunded pension scheme were introduced with the immediate effect of reducing private saving. This in itself would lead to a lower capital stock in the future and hence impose a burden on future generations. However, the present generation would realize this and therefore increase their bequests. This would exactly offset the replacement effect and so leave personal saving unaffected.

(7) The formulation (5.2) assumes that social security payments are financed by lump sum taxes. In reality the financing typically takes place through distortionary income taxes which may have further effects on saving via changes in after-tax interest and wage rates; see the discussion in Section 2.

Empirical evidence in this area has been accumulating rather rapidly in recent years. Feldstein (1974c) found that for the United States during the period 1929–71, social security reduced private saving by about 40 percent; however, the picture is much less clear when attention is restricted to the post-war period 1947–71. Feldstein (1977) has also used international cross-section data to study the relationship between private saving and social security. This study confirms the time series evidence of a negative relationship. Feldstein interprets this as evidence that the replacement effect outweighs the induced retirement effect.
The conclusions in Feldstein’s pioneering work do not receive unequivocal support from later studies. For the case of private pensions, which have important similarities with social security, Munnell (1976) found a negative effect on saving, while Kotlikoff (1979) found mixed support for the Feldstein position. Both of these papers use cross-section data for men aged 45–59 in the United States. A recent paper by Kurz (1981), using a much more representative sample of households, concludes that there is no effect of social security wealth on the private rate of savings. On the other hand, he finds that private pension plans do have effects on saving, but these are rather complex; there are, e.g., significant differences between the responses of men and women to the availability of pensions.

Barro and MacDonald (1979) have re-examined the international evidence on a basis of mixed cross-section and time series data for sixteen Western countries over the period 1951–60. The conclusions, in contrast to the Feldstein (1977) study, are rather indecisive. The data do not support the conclusion of a pronounced displacement effect, but neither is there evidence of a positive effect on saving.

Detailed studies of data from other countries are also becoming available. Dicks-Mireaux and King (1984) have studied cross-section data for Canadian households for 1977. They conclude that there is a small but significant displacement effect. Using time series data for the United Kingdom, Browning (1982) finds that public pensions tend to increase consumption, but that the effect is very small and hardly significant. The effects of occupational pension schemes in the U.K. have been studied by Green (1981) and Hemming and Harvey (1983).

From a theoretical point of view the influence of social security and pensions on consumer saving is a complex one. Few single hypotheses emerge once we move beyond the framework of the simplest life cycle model. It is therefore not surprising that the econometric evidence should also be conflicting.

6. The incidence of taxation

Tax incidence will be dealt with separately and in more detail in a separate chapter. Here we shall limit the survey to a brief consideration of the main issues, and a few references to the literature on incidence which deals especially with the taxation of savings.

5Williamson and Jones (1983) have argued that the conflicting evidence from empirical studies is due to insufficiently detailed theoretical specification of the econometric models.

6A later study by Feldstein (1980), using new international cross-section data developed by the U.S. Social Security Administration, confirms his earlier results of a substantial negative impact of social security on private saving. An appendix to this paper by C.Y. Horioka discusses the reasons for the differences between the Feldstein and Barro–MacDonald results.
A simple partial equilibrium analysis of the incidence of an income tax on the rate of interest would focus on the interest rate as an equilibrator of saving and investment. If there were no tax distortions on the investment side, the equilibrium condition would be simply

\[ S(r(1-t)) = I(r). \]  

(6.1)

If either saving or investment were totally inelastic, this would of course require that both sides of the market face a net rate of interest which is independent of the tax. Thus, if investment is inelastic, a change in the tax rate must imply a reverse change in the rate of interest to keep \( r(1-t) \) constant; in this case the "cost" of taxation is borne by investors. If saving is inelastic, it is savers who pay the cost through a reduction in their after-tax rate of interest. In general, the effect of the tax rate on the equilibrium rate of interest would depend on the relative magnitudes of the two elasticities.

However, this analysis is unsatisfactory for at least two reasons. First, it is a partial and not a general equilibrium formulation. Second, it is a static theory, while it seems obvious that the equilibrium effects of taxes on saving and investment require a dynamic equilibrium formulation. The first objection could be overcome by incorporating the analysis in a model of tax incidence of the Harberger (1962) type. However, this suffers from the weakness of assuming fixed factor supplies and is not suited for the purpose. Clearly what is required is a model of economic growth which takes account of the long-run consequences for the capital stock of the tax effects on the equilibrium level of saving and investment.

Such an extension of the model was first achieved by Diamond (1970). His point of departure is the integrated consumption–labour model which was discussed above but with labour being supplied inelastically. Population grows exponentially with generations overlapping in the manner of the famous Samuelson (1958) model. There are constant returns to scale in production and competitive behaviour in all markets. Diamond then studies the incidence effects of a tax on interest income as compared to a situation with only lump sum taxation. He finds that the differential incidence of an interest income tax raises the before-tax rate of interest and lowers the wage rate. The effects of this on the functional distribution of income depend of course on the elasticity of substitution.

A similar framework of analysis was used by Feldstein (1974a, 1974b). The latter of these articles comes close to Diamond’s analysis in assuming an exogenous labour supply. On the savings side the explicit optimization framework of the overlapping generations model is dropped in favour of a two-class model with different savings propensities. Feldstein shows that a tax on capital income in this

\(^7\)For a detailed discussion of Harberger type models, see Atkinson and Stiglitz (1980, ch. 6).
model is in general borne both by labour and capital, the division of the burden depending on the difference in propensities to save between the two groups and on the sensitivity of these propensities to changes in the rate of interest. One might think that the results would be crucially dependent on the exogeneity of labour supply; however, Feldstein (1974a) showed that in the case of a tax on labour income, the asymptotic incidence of the tax in long-run steady state equilibrium was independent of the elasticity of labour supply. In the short run, the results are different and come closer to those of comparative statics analysis. These results thus demonstrate very clearly how careful one must be in the analysis of tax incidence where saving decisions are involved.\(^8\)

The models which we have been discussing all treat the savings decision as being concerned with a single financial asset with a well defined rate of interest. This is certainly a legitimate simplification for many purposes, but it also abstracts from problems which in practice may be serious ones. Taxable income from capital is usually defined in nominal terms, and when inflation is present this may lead to an effective tax rate which is much higher than the nominal rate. This is clearly a problem even in one-asset models, but with many assets the tax rules typically lead to a system whereby different types of capital income are taxed at very different effective rates of tax. A particularly interesting discussion of the problems raised by this in the U.K. context is in Kay and King (1978). Tax effects on the composition of saving are taken up in more detail in Sections 8–12 below.

7. The optimum tax treatment of savings

The optimum tax treatment of saving—like the optimum tax treatment of anything else—must involve a tradeoff between distributional objectives on the one hand and economic efficiency on the other. To the extent that saving can somehow be treated just like another commodity in the context of general equilibrium analysis, the results of optimum tax theory could be applied fairly directly to this problem.\(^9\) However, there are a number of special issues related to the taxation of saving which are not easily captured in a general framework. One of these is the old argument about the “double taxation of savings”. It has been claimed by many writers that a general income tax with the same rates applying to income from labour and capital would involve a discrimination against saving because the income from which asset purchases are made has already been subjected to taxation. Another argument, which can be associated with the names of Böhm-Bawerk and Pigou, is that economic agents are myopic and have a

\(^8\) Other contributions to tax incidence in a growing economy include Friedlaender and Vandendorpe (1978), Kotlikoff and Summers (1979) and Summers (1981).

\(^9\) For surveys of the theory of optimum commodity taxation, see Chapter 2 or Sandmo (1976).
tendency to save too little and thus not provide a sufficiently large capital stock for the future. These and related issues have recently been examined in a series of papers written in an optimum taxation framework; e.g., Ordover (1976), Ordover and Phelps (1979), Atkinson and Sandmo (1980) and King (1980). The analysis which we shall present here is a simplified version of the Atkinson–Sandmo analysis; it is also similar to that of King.

The analytical model is one of steady state growth in which population is increasing at the rate \( n \). Each consumer lives for two periods, working in the first and being retired in the second as in the integrated model described earlier. The government has an exogenously given revenue requirement which has to be financed through distortionary taxes on income from labour and capital; lump sum taxation is ruled out. Within each generation all individuals are identical, and the problem of the representative consumer is to maximize his utility \( U(C_1, C_2, L) \) as in (2.18), subject to the budget constraint

\[
C_1 + p_2 C_2 = w(1 - t_w)(T - L) + a. \tag{7.1}
\]

Here \( p_2 = (1 + r(1 - t_r))^{-1} \) is the price of future consumption. Compared to our earlier formulation (2.24), allowance has now been made for the possibility of taxing income from capital and labour at different rates. A lump sum term \( a \) has been included although, as already explained, this is in fact constrained to be zero; however, it serves a useful analytical purpose for the derivation of the Slutsky equations.

The consumer’s optimum can now be characterized by the first-order conditions

\[
U_1 - \lambda = 0, \tag{7.2}
\]

\[
U_2 - \lambda p_2 = 0, \tag{7.3}
\]

\[
U_L - \lambda w(1 - t_w) = 0. \tag{7.4}
\]

From (7.1)–(7.4) we can solve for the demand functions, and substituting these back into the utility function, we have the indirect utility function

\[
V = V(p_2, w(1 - t_w), a), \tag{7.5}
\]

which has the partial derivatives

\[
\frac{\partial V}{\partial a} = \lambda, \tag{7.6a}
\]

\[
\frac{\partial V}{\partial t_r} = \frac{\partial V}{\partial p_2} \cdot \frac{\partial p_2}{\partial t_r} = -\lambda C_2 r p_2^2, \tag{7.6b}
\]

\[
\frac{\partial V}{\partial t_w} = -\lambda w(T - L). \tag{7.6c}
\]
The government’s problem can now be posed as that of choosing \( t_r \) and \( t_w \) so as to maximize \( V^{10} \), subject to the government’s budget constraint

\[
t_w w(T - L) + t_r \frac{r p_2 C_2}{1 + n} = R,
\]

where \( R \) is given. The second term on the left is the government’s revenue from capital income taxation. The savings of the older generation living now is \( p_2 C_2 \) and the income from this is accordingly \( r p_2 C_2 \). In order to make the tax revenue from this comparable with the revenue from labour income taxation we must divide by \((1 + n)\) to take account of the smaller number of people in the older generation.

We solve this problem by taking the derivatives with respect to \( t_r \) and \( t_w \) of the Lagrange function

\[
\mathcal{L} = V(P_2, w(1 - t_w), a) + \mu \left( t_w w(T - L) + t_r \frac{r p_2 C_2}{1 + n} - R \right),
\]

and equating them to zero. This yields

\[
-\lambda C_2 r p_2^2 + \mu \left( -t_w w \frac{\partial L}{\partial t_r} + t_r \frac{r p_2}{1 + n} \frac{\partial C_2}{\partial t_r} + \frac{r p_2 C_2}{1 + n} + t_r \frac{r^2 p_2 C_2}{1 + n} \right) = 0,
\]

\[
-\lambda w(T - L) + \mu \left( w(T - L) - t_w w \frac{\partial L}{\partial t_w} + t_r \frac{r p_2}{1 + n} \frac{\partial C_2}{\partial t_w} \right) = 0.
\]

Dividing (7.9) by (7.10), we obtain

\[
\frac{C_2 r p_2^2}{w(T - L)} = \frac{-t_w w \frac{\partial L}{\partial t_r} + t_r \frac{r p_2}{1 + n} \frac{\partial C_2}{\partial t_r} + \frac{r p_2 C_2}{1 + n} (1 + t_r r p_2)}{w(T - L) - t_w w \frac{\partial L}{\partial t_w} + t_r \frac{r p_2}{1 + n} \frac{\partial C_2}{\partial t_w}}.
\]

\(^{10}\) I.e., the government maximizes the utility of a representative generation. For a discussion of the dynamic optimization problem, see Atkinson and Sandmo (1980). We also assume here that \( w \) and \( r \) are given from the production side; the full analysis is presented in Atkinson and Sandmo and in King (1980).
The expression can be considerably simplified if we substitute from the Slutsky equations. The income effects then cancel out and we are left with

\[
\frac{n - r}{1 + n} C_2(T - L) + \frac{t_w r p_2}{1 + n} C_2 S_{2L} - t_w r p_2 C_2 S_{2L} - \frac{t_r p_2}{1 + n} C_2 S_{2L} = t_w w(T - L) S_{L2} + \frac{t_r p_2}{1 + n} (T - L) S_{22}.
\]

It is convenient to rewrite this in terms of the compensated effects on labour supply rather than on leisure. Defining the former as \( S_{HH} = -S_{LL} \) and \( S_{H2} = -S_{L2} \), and dividing the equation by \( C_2(T - L) \), we finally obtain

\[
\frac{t_r}{1 + n} (\sigma_{22} + \sigma_{H2}) = \frac{t_w}{1 - t_w} (\sigma_{HH} - \sigma_{H2}) + \frac{r - n}{1 + n},
\]

where the \( \sigma \)'s are the compensated elasticities. This equation characterizes the relative levels of the tax rates on income from labour and capital, with the absolute levels being determined by the government's revenue requirement.

We note first that the characterization depends on compensated and not on gross elasticities. The absence of income effects is explained by the fact that the income effects are analogous to the effects of lump sum taxation and therefore irrelevant for an assessment of the relative merits of different kinds of distortionary taxation.

Assume for simplicity of interpretation that \( r = n \) and that the cross-elasticities are zero. Then we have that

\[
\frac{t_r}{1 + r} \frac{t_w}{1 - t_w} = \frac{\sigma_{HH}}{-\sigma_{22}}.
\]

If labour supply is completely inelastic (along the compensated supply curve), the optimal tax on interest income is zero, while the tax on labour income is

\[
\frac{\frac{\partial L}{\partial t_r}}{\frac{\partial L}{\partial t_w}} = \left[ (T - L) \frac{\partial L}{\partial a} + S_{LL} \right] (-w), \quad \frac{\partial C_2}{\partial t_r} = \left[ (T - L) \frac{\partial C_2}{\partial a} + S_{2L} \right] (-w),
\]

\[
\frac{\partial L}{\partial t_r} = \left[ -C_2 \frac{\partial L}{\partial a} + S_{L2} \right] r p_2, \quad \frac{\partial C_2}{\partial t_r} = \left[ -C_2 \frac{\partial C_2}{\partial a} + S_{22} \right] r p_2.
\]

In each case the bracketed expression is the Slutsky equation with respect to the net price, and the last factor is the derivative of the net price with respect to the tax rate.

\[\text{footnote}{11\text{These are}}\]

\[\text{footnote}{12\text{For a more detailed derivation of this equation, see King (1980).}}\]
equivalent to a lump sum tax and could be set arbitrarily high. If, on the other hand, the demand for future consumption is inelastic, the argument is reversed, and interest income is the ideal tax base from an efficiency point of view. In general, the relative rates of tax depend on the relative magnitudes of the two elasticities, and there is no particular reason to believe that the optimal rates should be the same for the two sources of income. This interpretation carries over, with appropriate modifications, to the case of non-zero cross-elasticities. The importance of these is explained by the fact that, for any given rate of tax on labour income, the case for an interest income tax is strengthened if it leads to changes in demand and supply which counteract those associated with the wage tax.

The final term in (7.11) has the sign of the difference between the rate of interest and the rate of growth; it is accordingly zero when the economy is on the golden rule growth path.\(^3\) If we take the case where the coefficients of the tax rates are positive, i.e., where the direct substitution effects dominate, to be the normal one, we see immediately that on an inefficient growth path with \( r < n \), the case for an interest income tax would be weakened.\(^4\) Since the rate of interest equals the marginal productivity of capital, this case is where the capital intensity of the economy is too high and where, consequently, one would expect that saving ought to be discouraged. This apparent paradox is resolved by noting that a lower tax on interest income means that the tax on wage income must be increased in order to keep tax revenue constant. This reduces labour supply (in a compensated sense) and hence the income from which saving is generated. With the assumptions which have been made about the elasticities, the overall effect of the switch in taxation is precisely to discourage saving, as one's economic intuition would have it.

This is as close as we come in this model to capturing the arguments of Böhm-Bawerk and Pigou. There is a kind of pecuniary externality from population growth if the growth rate is not equal to the rate of return. This social myopia does not, however, capture the idea of individual myopia which was central to their arguments. If individuals underestimate their "true" willingness to pay for future consumption, there is a merit good argument for the subsidization or public provision of saving. There can be no doubt that in practice this type of paternalistic argument has been seen by many as being of decisive importance for the public provision of social security and pension schemes.

The conclusions of optimum taxation models are sensitive to the assumptions made concerning the range of instruments available to the government. In the present case the importance of the externality corrective last term in (7.11)

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\(^3\) For a discussion of this concept and its optimality properties, see Dixit (1976).

\(^4\) This line of reasoning presupposes that the compensated elasticities, which are in general functions of \( r \) and, therefore, of the capital intensity of the economy, remain constant over the range in which the comparisons are made.
provides a good illustration of this point. As shown by Atkinson and Sandmo (1980), if the government can transfer income between generations in a lump sum fashion, or if it can pursue an independent debt policy, it can always attain the golden rule growth path with \( r = n \). In that case, the results of the analysis are in complete accordance with standard optimum tax theory.

What about the argument of the double taxation of saving? First of all, it should be clear that what finally counts for the evaluation of efficiency is not the number of taxes to which a commodity is subjected, but the final effective rate. Apart from this we see that on purely theoretical grounds there is no strong reason to suspect that efficiency considerations should imply a tax rate on capital income either below or above that on wages. The case has to rest on an empirical assessment of the demand and supply elasticities involved.

Many studies of the welfare loss of capital income taxation start from the pure consumption model in which labour income is given exogeneously while saving is interest elastic. It is obvious that this implies that taxation of interest income involves a deadweight loss and that efficiency calls for a zero rate. King (1980) has considered the empirical evidence very carefully, drawing in particular on the results reported by Boskin and Lau (1978), who use the overlapping generations model with variable labour supply. He finds that the evidence tends to support a negative rate of tax on capital income. The implications of the theory and of the empirical evidence for the choice of the tax base is examined in more detail in another chapter of the Handbook and will not be pursued here. Although it is clear that the choice of tax base has to rest on many more considerations than the efficiency arguments which have been examined here, we have at least indicated the importance of both theoretical arguments and empirical evidence for a rational approach to the tax treatment of savings.

8. Taxation and risk taking: Portfolio choice

In looking at the saving decision as if it takes the form of purchases of a single homogeneous asset we have obviously made a drastic simplification. Although such simplifications are clearly necessary in order to focus attention on the strategically important issues, they may also be misleading. Thus, it may well be that the tax effects on the total volume of saving are less important than their effects on the composition of saving.

Of course, tax effects on portfolio composition may be of many kinds, not all of which may be expected to have any systematic bias in favour of assets with particular characteristics. The classic argument for a systematic effect of taxation on portfolio choice runs in terms of risk-taking behaviour. The popular view has traditionally been that the taxation of income from assets discriminates against risk taking through its lowering of the expected rates of return. However, at least
since the seminal article by Domar and Musgrave (1944) it has been common among economists to emphasize a different point of view. In addition to taking a share in the expected return, the government also shares in the risk. If there are perfect loss offset provisions, so that losses can be written off against other taxable income, the government will in fact carry the same share of a possible loss as it takes in a gain. If individuals ascribed a sufficiently large weight to the loss sharing property of the tax, the direction of the tax discrimination could possibly go in the opposite direction.

The modern version of this argument, using the expected utility framework of von Neumann and Morgenstern, is due to Mossin (1968) and Stiglitz (1969), and it is useful to start by examining the simplest possible model. This is one in which individuals have preferences for the probability distribution of their wealth \(Y\) at the end of the investment period, and they evaluate this according to a strictly concave utility function \(U(Y)\). Initial wealth can be invested in two assets, money \(m\) bearing a certain return of zero and a risky asset \(a\) having a stochastic rate of return of \(x\). The return is taxed at the rate \(t\). The budget constraint is

\[
m + a = A, \tag{8.1}
\]

where \(A\) is initial wealth, and final wealth is

\[
Y = a[1 + x(1 - t)] + m = A + ax(1 - t), \tag{8.2}
\]

where the last equality is obtained after substitution from (8.1). Expected utility, assuming that \(x\) is continuously distributed on the interval \([-1, \infty)\), is

\[
E[U] = \int_{-1}^{\infty} U(A + ax(1 - t))f(x)\,dx. \tag{8.3}
\]

The first-order condition for an interior solution can be written as

\[
E[U'(Y)x(1 - t)] = 0. \tag{8.4}
\]

The second-order maximum condition is satisfied by the assumption of concavity. It is easy to show \(^{15}\) that the optimal holding of the risky asset is positive if and only if \(E[x] > 0\). We shall assume that this is the case.

Differentiating (8.4) with respect to \(t\) we can write

\[
E\left[U''(Y) \left( \frac{\partial a}{\partial t} x(1 - t) - ax \right) x(1 - t) - U'(Y)x \right] = 0.
\]

\(^{15}\)See, e.g., the analysis in Arrow (1970).
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The last term vanishes because of (8.4) and since $t$ itself is non-stochastic.\footnote{For an analysis of the case where $t$ is stochastic due to "political risk", see Ekern (1971). Of course the marginal tax rate could also be stochastic if it depended on $Y$.} It is then easy to see that we must have

$$\frac{\partial a}{\partial t} = \frac{a}{1-t} \quad \text{or} \quad \frac{\partial a}{\partial t} \frac{\partial t}{\partial a} = \frac{t}{1-t}. \quad (8.5)$$

This result is striking in its simplicity. The investor's response to taxation can be predicted without knowledge of his risk preferences. The only information required is his holding of the risky asset and his rate of tax.

The result must necessarily have a simple interpretation. By adjusting his portfolio according to the "rule" (8.5), the investor is in fact able to keep the probability distribution of final wealth constant.\footnote{From (8.2) we can compute the mean and variance of final wealth as $E[Y] = A + \mu(1-t)$ and $\text{var}[Y] = \sigma^2(1-t)^2$, where $\mu$ and $\sigma^2$ are the mean and variance of the rate of return. The reader may easily convince himself that portfolio adjustment according to (8.5) keeps the mean and variance of final wealth constant, and that the same is true of higher moments of the probability distribution.} Since this was the distribution chosen as the optimal one in the first place, it is not surprising that the investor should continue to choose it, given that it is still available. Thus, the investor responds to the higher tax rate by increasing his holding of the risky asset. The risk subsidization involved outweighs the taxation of expected return.

This is of course a very simple model. Its most obvious shortcoming is that it takes no account of other types of decisions made by the individual, so that, e.g., the total size of the savings portfolio is unaffected by the tax rate. We shall return to this problem later. However, even in the context of the pure portfolio framework the model is based on a number of restrictive assumptions, which will be discussed in turn.

8.1. A non-zero rate of interest

Let $r$ be the rate of return on the safe asset. It can then be shown that positive holding of the risky asset is optimal if and only if its expected return exceeds $r$. This will be assumed in the following.

Suppose first that the tax is levied only on the excess return to the risky asset, so that the tax base is $a(x - r)$. The budget constraint (8.2) then becomes

$$Y = a(1 + x) + m(1 + r) - a(x - r)t = A(1 + r) + a(x - r)(1 - t). \quad (8.6)$$

The first-order condition is now

$$E[U'(Y)(x - r)(1 - t)] = 0, \quad (8.7)$$

\[\text{For an analysis of the case where } t \text{ is stochastic due to "political risk", see Ekern (1971). Of course the marginal tax rate could also be stochastic if it depended on } Y.\]
and it is easy to see that the analysis goes through as before with the result (8.5) still holding. We shall refer to this as the net taxation case.

An alternative assumption is that the returns from both assets are taxed at the rate $t$; this is the gross taxation case, where final wealth becomes

$$Y = a[1 + x(1 - t)] + m[1 + r(1 - t)]$$

$$= A[1 + r(1 - t)] + a(x - r)(1 - t). \quad (8.8)$$

The condition that at the optimum the expected marginal utility of a further increase in $a$ must be zero, has exactly the same form as (8.7). But since $Y$ is now defined differently, the comparative statics of this case differs from the previous one. Taking the derivative with respect to $t$ we obtain

$$\frac{\partial a}{\partial t} = \frac{\mathbb{E}[U''(Y)(x-r)]}{\mathbb{E}[U''(Y)(x-r)^2]} \cdot \frac{Ar}{1-t} + \frac{a}{1-t}.$$

It is also straightforward to compute

$$\frac{\partial a}{\partial A} = -\frac{1 + r(1-t)}{1-t} \frac{\mathbb{E}[U''(Y)(x-r)]}{\mathbb{E}[U''(Y)(x-r)^2]},$$

and substituting into the previous expression we can write

$$\frac{\partial a}{\partial t} = -\frac{Ar}{1 + r(1-t)} \frac{\partial a}{\partial A} + \frac{a}{1-t},$$

or, in elasticity form,

$$\frac{\partial a}{\partial t} \cdot \frac{t}{a} = -\frac{tr}{1 + r(1-t)} \left( \frac{\partial a}{\partial A} \frac{A}{a} \right) + \frac{t}{1-t}. \quad (8.9)$$

The last term is the same as in (8.5), while in addition we now have an income effect which is proportional to the wealth elasticity of the risky asset. If this is positive, which is a reasonable assumption, it tends to reduce the demand for the risky asset. We thus have conflicting wealth and substitution effects, and no firm conclusion can be drawn about the relationship between taxation and risk taking.

However, it may be tempting to argue that the substitution effect is likely to dominate the income effect. For “reasonable” parameter values like $t = 0.5$ and $r = 0.05$, the wealth elasticity must be in excess of 41 in order for the whole expression (8.9) to become negative. But this argument overlooks the fact that the
choice of a reasonable value for \( r \) depends crucially on the length of the time period. If the holding period is one month instead of one year, \( r = 0.004 \) would roughly correspond to an annual rate of 5 percent, while if the period were 23 years, one would as an approximation have \( r = 2 \). In the first case the crucial value of the wealth elasticity is 501, in the second case it is 2. The model does not explain the length of the holding period, and a realistic assumption about it should reflect the application that one has in mind. Perhaps in most cases of interest it would not be realistic to assume holding periods much in excess of one year. If so, there may indeed be reason to suspect the substitution effect of the simple model to be the dominating one.

8.2. Several risky assets

The two-asset model is attractive because it provides a well-defined measure of the degree of risk taking, which is simply measured by the share of the total portfolio going into the risky asset. The assumption that there exists an asset which has a certain return is of course an idealization which is meant to capture the point that some assets are less risky than others. Perhaps more serious is the assumption that there is only one risky asset. Suppose there are two assets whose returns are correlated in some way. The intuitive argument about the tax being a risk subsidy is hardly convincing any longer because one has to take into account the covariances of the returns. Thus, it is no longer clear that one can use the total amount invested in all risky assets as a meaningful measure of the degree of risk taking.

The extension of the analysis to the case of an arbitrary number of risky assets has been considered by Sandmo (1977). It is shown that in the net taxation case the results of the two-asset model carry over without modification, and that no assumption about the joint probability distribution of the rates of return is required. Thus, if the tax rate on the excess return of the \( j \)th risky asset is \( t_j \), then (8.5) becomes simply

\[
\frac{\partial a_j}{\partial t_j} = \frac{a_j}{1 - t_j} \quad \text{and} \quad \frac{\partial a_i}{\partial t_j} = 0 \quad \text{for} \quad i \neq j. \tag{8.10}
\]

A partial increase in the rate of tax on asset \( j \) increases the demand for that asset, while leaving the demand for all other assets unchanged. The result for the total share of risky assets when there is a single tax rate follows trivially by summing over \( j \) and taking \( t_j = t \) for all \( j \).

For the gross taxation case the analysis is more complicated. For a partial tax on the returns from the \( j \)th risky asset, it can be shown that the tax effect is a
linear combination of the effect of a decrease in expected return and a decrease in riskiness. Although suggestive, this result is somewhat inconclusive, since the theory does not predict how these two opposing effects are to be weighed together. In the case of a general tax which applies to all asset returns with the same rate, it turns out—perhaps surprisingly—that the result (8.9) holds for all assets, and the above comments on that result apply also to the case of many risky assets.

8.3. Imperfect loss offset

The assumption that there are perfect loss offset provisions is clearly a very strong one, and one might indeed suspect that the conclusions are very heavily dependent on this. For if it is true that the government does not take any part in a possible loss, the economic intuition behind the previous results can no longer be upheld. The tax reduces the expected rate of return as before. However, the government now shares in the risk only by taking part in the expected gain, while leaving the losses to be carried by the individual. One might conjecture on this basis that an increase in the tax rate would most likely lead to a reduction of the amount invested in the risky asset.

It was shown by Mossin (1968) and Stiglitz (1969) that the theoretical conclusions that can be drawn with respect to this question are ambiguous. With no loss offset, if the tax rate is sufficiently high, the demand for the risky asset must fall. More surprising is the insight that in general the tax effect on risk taking cannot be determined without fairly restrictive assumptions about the properties of the utility function. On reflection it is clear that even a pure reduction in the expected rate of return—with no further effects on the probability distribution—would have both substitution and income effects on the demand for the risky asset, and these could easily pull in opposite directions. A tax increase with partial or no loss offset would imply a similar ambiguity, to which we have to add the effect of the partial risk reduction associated with the distribution of the positive values of the rate of return. Thus, the ambiguity is not really surprising.18

8.4. Other tax forms in the pure portfolio model

The tax on returns from investment is only one of many taxes which could be considered from the viewpoint of its effects on risk taking. Following Stiglitz (1969) we could, e.g., imagine a wealth tax whose base is final wealth. We would

18Recently, Eeckhoudt and Hansen (1982) have argued that a tightening of the opportunity to write off losses does not necessarily lead to less risk taking.
then have

$$Y = (A(1 + r) + a(x - r))(1 - \tau), \quad (8.11)$$

where $\tau$ is the tax rate. The first-order condition for expected utility maximization is

$$E[U'(Y)(x - r)(1 - \tau)] = 0. \quad (8.12)$$

Differentiating with respect to $\tau$ we obtain

$$\frac{\partial a}{\partial \tau} = \frac{1}{1 - \tau} \frac{E[U''(Y)Y(x - r)]}{E[U''(Y)(x - r)^2(1 - \tau)]}. \quad (8.13)$$

The denominator of this expression is negative from the assumption of risk aversion. Suppose that the utility function has the property of constant elasticity of the marginal utility of wealth or constant relative risk aversion in the Arrow–Pratt sense. We then have that

$$- \frac{U''(Y)Y}{U'(Y)} = \alpha \quad \text{or} \quad U''(Y)Y = -\alpha U'(Y),$$

where $\alpha$ is a positive constant. Substituting into (8.13) we can write

$$\frac{\partial a}{\partial \tau} = \frac{1}{1 - \tau} \cdot -\alpha \frac{E[U'(Y)(x - r)]}{E[U''(Y)(x - r)^2(1 - \tau)]} = 0, \quad (8.14)$$

where the last equality follows from the first-order condition (8.12). Thus, in an interesting special case, the wealth tax has no effect on portfolio composition. In a more general analysis, it turns out that constant relative risk aversion is the borderline case, so that an increase in the tax rate will increase or decrease the amount of the risky asset held according to whether relative risk aversion is increasing or decreasing. It is difficult to say which of these possibilities is the empirically most relevant.

A purely lump sum tax is equivalent to a reduction of initial wealth, $A$. This too has ambiguous effects unless further assumptions are made; it can be shown that the wealth effect on the demand for the risky asset is positive if and only if the Arrow–Pratt measure of absolute risk aversion is decreasing. However, this result does not carry over to the case of many risky assets.
8.5. Interaction with saving and labour supply decisions

So far we have postulated a decision maker who is solely concerned with the level of his final wealth. This opens for a number of interpretations of the theory. The decision maker could, e.g., be a firm which undertakes investments to maximize the expected utility of its wealth, and this interpretation has been used by a number of writers; see, e.g., Mintz (1981). However, in the present context it is most natural to look at the decision maker who is concerned with the composition of his savings portfolio. But then we must take account of the fact that taxation affects not only the relative degrees of attractiveness of different assets, but also the relative price of present and future consumption. In other words, there is a need for an integrated model of saving and portfolio decisions. As shown in Drèze and Modigliani (1972), there exists a class of utility functions for which saving and portfolio decisions are separable in the sense that, loosely speaking, each can be analyzed taking the other as given. The combined effects of taxation on saving and portfolio decisions can then be studied by "adding up" the results. However, this class is fairly restrictive, and in any case it is not always easy to see how the adding up ought to be done.

It was shown in Sandmo (1969) that when the simple two-asset model with a zero rate of return was extended to account for consumption decisions, the simple result (8.5) carries over without change. The model has two periods, labour supply is fixed, and the consumer maximizes expected utility $E[U(C_1, C_2)]$. The budget constraint can be written as

$$C_2 = A - C_1 + ax(1 - t),$$ (8.15)

where $A$ is again initial wealth. Working through the comparative statics it is easy to see that

$$\frac{\partial a}{\partial t} = \frac{a}{1 - t} \quad \text{and} \quad \frac{\partial C_1}{\partial t} = 0.$$ (8.16)

The first part of this result is just (8.5), and the interpretation is the same. The interpretation of the second part follows as a corollary: If the consumer can achieve the same probability distribution of future consumption as before by a simple rearrangement of his portfolio, there is no reason why he should change his level of saving and thereby his present consumption. Of course, the result holds also for the net taxation case in which the tax base is $a(x - r)$.

The gross taxation case is more complicated, and the result (8.9) no longer holds; this is essentially because of the intertemporal substitution effect which changes the size of the savings portfolio. The problem has been analyzed by Ahsan (1976), who considers the class of additive utility functions in a two-period
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model. Within this framework he considers a number of special cases both with respect to utility functions and with respect to the type of tax in existence. While his results are clearly interesting, they are also quite complex and difficult to summarize in general terms. The only general conclusion one can draw from his analysis seems to be that there is hardly any good reason to believe that income or consumption taxes have any clear-cut effect on the degree of risk taking in the framework of intertemporal consumption decisions.\(^9\)

Given the lack of general results in the two-period model, it is hardly to be expected that multi-period models would have more to offer in this regard. Perhaps the most interesting insight that multi-period consumption portfolio models have to offer,\(^2\) is that there exist cases where the effects of taxation on portfolio composition are of exactly the same nature as in the one-period pure portfolio model which was discussed above. Hagen (1970) demonstrated this in a model where there is an infinite horizon utility function of the form

\[
U(C_1, C_2, C_3, \ldots) = \sum_{t=1}^{\infty} \alpha^{t-1}c_t^\rho,
\]

where \(\alpha\) is a parameter expressing the consumer's "impatience" \((0 < \alpha < 1)\), and the instantaneous utility function has the property that the elasticity of marginal utility or the coefficient of relative risk aversion is constant and equal to \(1 - \rho\). In this case the effects on portfolio allocation of investment income and consumption taxes are of exactly the same form as in the one-period model [a similar result is also in Atkinson and Stiglitz (1980, pp. 121–123)]. Hagen shows how tax effects in this model can be neatly separated into saving and portfolio effects. Even in this very special case, however, the combined income and substitution effects on investment in the risky asset are quite complex and do not allow of any unambiguous conclusion as to the total effect on risk taking. To some extent this may be a matter of interpretation, however. One could take the view that the problem of tax effects on risk taking should be seen as one which is conceptually separate from that of the effects on saving and thereby on the size of the portfolio. Seen from this angle one could then argue that Hagen's results confirm those of the pure portfolio models.

With the exception of the work of Drèze and Modigliani (1972), studies of consumption and portfolio behaviour usually ignore problems related to labour supply, occupational choice, etc. An extension to include labour supply in the manner of the models of Section 2 might be particularly interesting if the wage

\(^9\)Ahsan's paper considers both proportional and progressive taxes; the effect of progression on risk taking is explicitly considered in Ahsan (1974) and Cowell (1975).

\(^2\)For a few examples of a large number of contributions to this general topic, see Hakansson (1970) and Merton (1969).
rate were also assumed to be uncertain. The optimum degree of portfolio risk would then clearly be related to labour supply, and the individual would have to consider the riskiness of his portfolio with regard to the riskiness of his occupation. At the level of general equilibrium and welfare economics one could then study the overall effects of taxes on the allocation of resources, not only as a problem related to capital markets but to labour markets as well.

9. Empirical studies of taxation and portfolio choice

When one turns from the theory of the tax effects on risk taking via portfolio adjustment to the empirical study of portfolio composition, it is important to be aware of the fact that taxation may affect portfolio choices from a number of causes which have little or nothing to do with risk taking. In many countries it is, e.g., the case that income from investment in housing gets a more favourable tax treatment than income from common stock. With a progressive tax system we then have a complicated picture of a world in which the rate of tax on asset yield varies not only between assets but also among individuals. This fact emerges clearly from the pioneering econometric study of Feldstein (1976b) of the effects of taxes on the portfolio composition of private investors. His work is based on survey data for the U.S. from 1962, and the results reflect the special provisions of American tax laws and financial market structure; nevertheless, they are of considerable general interest. Feldstein concludes that the effect of the personal income tax on portfolio composition is very powerful. Within each income class the pattern of asset holdings depends on relative net yields. The fact that tax rates, and therefore net yields, vary across income classes explains the pattern of ownership for each particular class.

Higher-income individuals hold a larger proportion of their wealth in common stock; this appears to be largely due to the special treatment of capital gains. The total mean yield on the portfolio is increasing with income, while the variance is increasing in nominal terms and approximately constant in real terms. Due to the complexities of the tax system it is difficult to say whether the results of Feldstein can be said to support the (admittedly weak) theoretical presumption that taxation encourages risk taking; certainly they do not contradict it. It should be noted, however, that Feldstein’s study covers only investment in financial assets, excluding in particular investment in residential housing.

A recent article by Shorrocks (1982), using United Kingdom data for 1975–76 reports patterns of asset ownership which are in some ways similar to those found by Feldstein, in particular regarding the high wealth elasticity of common stock. However, Shorrocks makes no attempt to account explicitly for the influence of taxation, so that his results can at most be considered as suggestive of a similar effect of the tax system on portfolio composition.
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The study of the effects of taxation on portfolio composition is complicated, chiefly because of the many special provisions made in the tax laws of most countries. This makes it difficult to compare effective rates of tax on different assets. Moreover, since one of the most interesting sets of questions relates to the tax wedge between the real rate of return in production and the corresponding rate received by the saver, one has to study both the personal and corporate tax systems in order to arrive at meaningful conclusions. King and Fullerton (1984) report on an ambitious international comparative study, where they estimate effective rates of tax on investment in three alternative real assets (machinery, buildings, inventories), in three different industries (manufacturing, other industry, commerce), financed by three alternative sources (debt, new share issues, retained earnings), and three ownership categories (households, tax-exempt institutions, insurance companies). The general results of the study, which compares data for the United States, the United Kingdom, Sweden and West Germany, are that effective tax rates vary widely, both within each country (with respect to asset type, industry, source of finance and ownership category) and between countries. It is hardly possible to draw any conclusions from this study as to a possible bias in the tax system with respect to the encouragement of risk taking. It does suggest, however, that for the purpose of empirical application of the theory of taxation and risk taking, it is the model where tax rates are differentiated among assets which is the most relevant one.

Social security and pension wealth, the effects of which on saving behaviour were discussed above, could also be thought to have important effects on the composition of savings. To the extent, e.g., that pension wealth is seen as a close substitute for relatively safe assets, the growth of pension wealth could reasonably be thought to encourage the holding of more risky private portfolios. A recent study by Dicks-Mireaux and King (1982b) on Canadian cross-section data did not, however, find any significant effects of this kind.

10. Other dimensions of risk taking

Individuals' willingness to bear risk reveals itself not only in their choice of portfolio composition but also in other areas of economic decision making. Of such areas, some - like insurance - can easily be interpreted as coming under the portfolio choice framework, for an insurance policy can always be interpreted as an asset with a particular pattern of returns. But there are other areas where the analogy is less obvious and which call for separate analysis.

One of these is occupational choice, and the effects of taxation on the choice between safe and risky occupations have been explored by Kanbur (1981). His analysis provides some interesting contrasts with the portfolio choice framework. First, occupational choice is treated as one between mutually exclusive alterna-
tives; the individual is not allowed to choose a portfolio of occupations. Second, because of this assumption, an equilibrium distribution of agents among occupations cannot be defined in partial equilibrium marginal utility terms, but must be defined in terms of equality between total expected utilities of alternative occupations. Therefore, unlike in the portfolio choice framework, one cannot ask questions about the individual behavioural response to changes in the level of taxation; the effect has to be evaluated in terms of the equilibrium distribution of the population between occupations.

More specifically, Kanbur's model assumes a population consisting of identical individuals who can choose between two occupations: They can either become labourers, earning a certain wage rate, or they can become entrepreneurs, in which case they hire labourers and produce according to a production function which depends on a random variable, reflecting uncertainty about their entrepreneurial ability. Kanbur's main interest lies not in the positive problems of the effects on occupational distribution of changes in taxation but rather in the normative question of the optimal tax system according to some social welfare function. He finds, e.g., that an optimal linear tax should be a progressive one, but he is unable to derive any firm conclusion as to whether this tax system implies a reduction of the fraction of the population engaged in the risky activity.

A different framework for the study of occupational choice problems in this context has been chosen by Eaton and Rosen (1980a). In their model the individual's demand for leisure is constant. However, in the first period of their lives workers can use some of their time to acquire human capital; this means less income in the first period, but more in the second when the wage rate increases as a result of education. If the rate of return to human capital is uncertain, Eaton and Rosen show that the effect of a proportional income tax may be either to increase or decrease the investment in human capital. On the one hand there is a kind of risk substitution effect which tends to increase investment in human capital; the government bears some of the risk associated with uncertain future wages. But, on the other hand, taxation reduces overall income, and this effect depends on attitudes to risk in a way which makes it difficult to predict the total effect except in very special cases.

Another area in which taxation has an effect on risk taking behaviour is tax evasion. As first explored in the articles by Allingham and Sandmo (1972) and Srinivasan (1973), models of tax evasion have several features in common with analyses of portfolio choice. The taxpayer is supposed to have a given income, and he is deciding on the fraction of it to report to the tax authorities. On this fraction he will pay tax at the regular rate, while on the amount evaded he pays either nothing or, in case he is detected, at a penalty rate which is higher than the regular tax rate. The taxpayer is assumed to maximize expected utility (according to Allingham and Sandmo) or expected net income (according to Srinivasan). One can then derive comparative statics results for the effects of changes in the tax
parameters and the probability of detection. Even in these simple models it turns out that a number of the results are ambiguous and that the basic simplicity of the portfolio choice results is lost. When the model is extended to take account of variable labour supply, as in Sandmo (1981b) and Cowell (1985), this tendency naturally becomes more pronounced. So far, however, there has hardly been any theoretical work done on the interrelationship between tax evasion and saving decisions (including portfolio choice), although applied work in this area typically emphasizes the importance of the link between income tax evasion and the choice of more or less observable asset holdings.

11. General equilibrium and tax incidence

Returning to the portfolio choice framework for the analysis of risk taking, it should be observed that the partial equilibrium framework for the analysis of taxation is incomplete in several respects. A complete analysis of the effects of taxation on the degree of risk taking in the economy should take into account the supply of alternative investment opportunities and not only the demand. It should also model the link between financial and real investment decisions and provide a description of the connection between public expenditure and tax revenue.

Of course, even the simple model of Section 8 could be given a general equilibrium interpretation. One would then have to assume an economy with only one consumption good, identical individuals, one safe and one risky industry operating under constant (stochastic) returns to scale and a “neutral” disbursement of the tax revenue. But these are very special assumptions which it would be desirable to relax.

The first paper to tackle this set of problems in a systematic fashion was Stiglitz (1972). He postulated an economy with constant stochastic returns to scale, in which firms issue bonds (bearing a safe rate of return) and stocks and choose a policy of investment and financing so as to maximize their market value. Consumers are assumed to invest their wealth in bonds and stocks according to the portfolio model of Section 8 above. Among the results of the analysis is that, if tax revenue simply disappears from the economy, the results of the partial equilibrium analysis focusing on the demand side retain their validity. In general,

\[ X = G(\theta)I. \]

This type of model and the limitations of the market value maximization hypothesis are discussed in a separate chapter of the Handbook.

Stiglitz actually formulates his model in a mean–variance framework, but this is not an essential feature of his analysis.
however, the results are sensitive to the particular assumption made about the tax system and the distribution of the tax revenue. This is of course what we should expect. The tax on investment income which has received so much attention in partial equilibrium theory leads to strong conclusions only in a very special case. There is little reason to believe that the result will continue to hold in general equilibrium, or that similar results can be derived for a more general class of tax systems.\textsuperscript{24}

12. The optimum taxation of risky assets

The question of the appropriate tax treatment of assets of varying degree of riskiness has been a controversial one both in the debate on practical tax policy and in the theory of public finance. It has commonly been maintained that a market economy requires some stimulus to risk-taking activities either because individuals are too risk-averse from a social point of view or because of some market imperfection. On the other hand, it has also been argued that economic efficiency requires that taxation of assets be non-distortionary or at least “neutral” in the sense of taxing all assets at the same rate. It is evidently necessary to consider carefully both the criterion of welfare or efficiency and the modelling of market structure.

It may be useful to approach the study of this problem by means of a rather extreme case which has recently been studied by Auerbach (1981). He assumes an economy with a full set of Arrow–Debreu markets. In the context of the two-period consumption model this means that assets are state-contingent in the sense that each asset provides a claim to future consumption if and only if state \( s \) occurs. Assume that there are two states and that consumer preferences can be represented by the single utility function

\[
U = U(C_1, C_{21}, C_{22}) = \pi_1 U(C_1, C_{21}) + (1 - \pi_1) U(C_1, C_{22}), \tag{12.1}
\]

where it has been assumed that the expected utility theorem holds with \( \pi_1 \) being the probability of state 1 and \( C_{2s} \) the amount of consumption in state \( s \) (\( s = 1, 2 \)). Assume further that there is a constant coefficient technology such that produc-

\textsuperscript{24} Gordon (1981) shows that, where the taxation of corporate profits is on a net basis (in the sense of Section 8), the tax is basically neutral when tax revenue is distributed among investors as lump sum payments. Recent work in the general equilibrium framework by Kihlstrom and Laffont (1983) can be seen as combining elements from the work of Stiglitz (1972) and Kanbur (1981). From the point of view of the firm and its investment and financing decisions, there is now a large literature on the effects of taxation. This is surveyed in a separate chapter in the Handbook.
tion opportunities are given by

\[ C_1 + p_{21}C_{21} + p_{22}C_{22} = I. \] (12.2)

Assume now that the government wishes to raise a given amount of tax revenue by levying taxes on the claims to future consumption, raising their consumer prices to \( P_{2s} = p_{2s} + t_{2s} \). From a formal point of view, this is a standard problem of optimum tax theory of the form first studied by Corlett and Hague (1953-54), and in this case there exists an appealing characterization of the relative tax rates in terms of the compensated elasticities (see Chapter 2). By utilizing the special structure of preferences implied by the expected utility hypothesis, Auerbach is able to transform the Corlett–Hague characterization into one involving ordinal (the elasticity of substitution) and cardinal (relative risk aversion) properties of the utility function. Although interesting, this result does not in itself tell us much about the taxation of assets, since these do not in reality take the form of state-contingent claims. However, the analysis is easily extended to the more general case of asset markets where the number of assets equals the number of states; in the language of capital market theory we then have complete “spanning”. For this case Auerbach’s analysis leads him to conclude that differential taxation of asset returns is in general desirable, and further that apparently reasonable restrictions on preferences implies a heavier taxation of the more risky asset.

These results should be interpreted with care, particularly since the assumption of a complete set of asset markets is probably a crucial one. One of the complications which arises when this assumption is abandoned concerns the formulation of the government revenue constraint: Should there be one constraint for each state of nature, or should there be one constraint in terms of expected tax revenue? (The latter alternative clearly implies risk neutrality on the part of the government.) The importance of these considerations were one of the points brought out in the pioneering contribution of Stiglitz (1972), who studied the optimal taxation of assets in the context of the model described in the previous section.\(^25\) He shows, e.g., that if the government is risk-neutral while individuals are not, there is a case for taxing the safe asset (or industry) at a higher rate than the risky one.

It seems reasonable to conclude that the few studies which have been made of the optimum taxation of risky assets cannot provide any \textit{a priori} foundation for a recommendation that risky assets be taxed at either higher or lower rates than safe ones. From a practical point of view, tax policy with regard to income from assets should take into account the specific structure of risk markets as well as administrative and political concerns.

\(^25\) For a summary and simplified account of Stiglitz’ model, see Allingham (1972).
One important case where the structure of risk markets is such that the conditions for optimum risk sharing are clearly violated, is uncertainty with respect to future wage rates and labour income. Insurance markets are absent here, primarily because of moral hazard problems. Varian (1980) and Eaton and Rosen (1980b) have pointed out that under these conditions lump sum taxation is in general not optimal. Even if lump sum taxes were available, it would be desirable to have a positive marginal tax rate on labour income in order to decrease riskiness. If work effort were given exogenously, the optimum solution in a world of risk-averse individuals would be to have a marginal tax rate of 100% combined with some lump sum redistribution of the tax revenue; what prevents this solution is the effect of high marginal tax rates on labour supply. Thus, the optimal tax scheme represents a compromise between the concern for labour supply incentives on the one hand and the desire for risk diversification on the other. This type of reasoning would of course also be applicable to the case of imperfect capital markets with limited possibilities for portfolio diversification.

13. Concluding remarks

The amount of work done on the theory and econometrics of tax effects on saving and risk taking is impressive. No doubt this reflects the practical importance of the issues as well as the intellectual challenges in the area, and it is pleasant to think that in this area of economics at least the two sets of motivations for research have reinforced each other.

Are there any general lessons for economic policy which can be extracted from the work surveyed here? As far as positive economics is concerned perhaps the most important general lesson is that empirical work of high quality can be done on problems of central concern to policy makers. To some extent there is also valuable information in the numerical estimates which have been made; certainly this is true for the countries which have been studied intensively in empirical work. For other countries there may also be valuable information to be had from empirical results derived, e.g., from the U.S. data. On the other hand, there is hardly any strong reason to believe that empirical results are valid for all countries and periods. The institutions, market structure and tax system of each country must be expected to influence behaviour with respect to saving and portfolio choice.

On the normative side perhaps the most important insight derived from recent work is that there are no easy options in tax policy with respect to saving and risk taking. Feasible tax systems all involve distortions of the decisions made by consumers and firms, and one faces the now familiar second-best problem of designing tax systems which are welfare-maximizing subject to the constraints on the choice of tax instruments. Recent work has also emphasized that tax policy
towards saving and risk taking cannot be studied in isolation from the effects on other areas of the economy; thus, one is led to a general equilibrium approach to the issues. To paraphrase a remark by Robert Solow, this makes work in the area more difficult, but also more fun.

References