Tax Notches in Pakistan: Tax Evasion, Real Responses, and Income Shifting*

Henrik Jacobsen Kleven, London School of Economics
Mazhar Waseem, London School of Economics

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Abstract

Using administrative tax records from Pakistan, we investigate behavioral responses to income tax notches—discontinuous jumps in tax liability—offering an unusual and compelling source of identifying variation. Notches create regions of strictly dominated choice where the taxpayer can increase consumption and leisure by lowering earnings to the notch point, and therefore produce very strong incentives for bunching. We find evidence of large and sharp bunching at notches, which is used to estimate taxable income responses, real earnings responses, and income shifting. A recent tax reform facilitates a comparison between the response to notches and the response to kinks created by discontinuous jumps in the marginal tax rate, and we find that the effects of notches are much larger and clearer. However, while the overall response to notches is large, it is fairly small in elasticity terms as the tax-price changes created by notches are extremely strong. In fact, we show that elasticities are too small to be consistent with the standard frictionless economic model, pointing to the presence of important optimization frictions. We explore the nature of such frictions and argue that they are likely to reflect a low degree of “tax literacy”—such as misperception and unawareness of tax incentives—in Pakistan.

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1 Introduction

The historical expansion of fiscal capacity and government size in developed countries was facilitated by the ability to implement and enforce modern broad-based taxes such as income taxes and value-added taxes (e.g. Kleven, Kreiner, and Saez 2009). It is often argued that such taxes are central to the economic growth process, because they allow governments to provide basic public goods without destroying incentives and productive efficiency (e.g. Newbery and Stern 1987; Burgess and Stern 1993). Indeed, developing countries all over the world have started to replace traditional revenue sources by modern broad-based taxes (e.g. Keen and Lockwood 2010). Yet there is virtually no evidence, and in particular micro-based evidence, on the efficacy of such taxes within a less-developed country context. This paper takes an initial step to fill this gap by investigating behavioral responses to the income tax in Pakistan.

The context which we consider offers two key methodological advantages. First, the Pakistani income tax offers an unusual and compelling source of identifying variation. The system consists of a large number of tax brackets each of which imposes its own proportional tax rate and therefore produces discontinuous jumps in tax liability—notches—at bracket cutoffs. Such notches create regions of strictly dominated choice above cutoffs and provide strong incentives for bunching below cutoffs. By contrast, standard piecewise linear tax schedules are associated with discontinuous jumps in the marginal tax rate—kinks—which do not generate strictly dominated regions and provide weaker incentives for bunching than notches. A recent literature using bunching around kink points to identify behavioral responses (Saez 2010; Kleven et al. 2010; Chetty et al. 2011) finds that only very sharp and salient kinks create any bunching at all, and even there the response is modest. The likely explanation is the presence of optimization frictions such as adjustment costs, misperception and inattention combined with the fact that the utility gain of bunching in response to jumps in marginal tax rates is not very large (Chetty 2011). Notches are different from kinks, because they create much stronger and more salient incentives and are therefore more likely to overcome optimization frictions.1

Second, for this study, we have gained access to administrative records covering the universe

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1Chetty (2011) makes a similar argument. Slemrod (2010) provides an overview and conceptual discussion of notches in tax systems, arguing that they provide a promising opportunity to estimate behavioral responses to taxation. A few empirical studies have considered behavioral responses to notches in other contexts: the Medicaid notch in the U.S. (Yelowitz 1995), social security notches in different countries (Gruber and Wise 1999), and car taxation notches in the U.S. and Canada (Sallee and Slemrod 2010).
of personal income tax filers in Pakistan over the period 2006-2008. While the use of large administrative tax records is emerging as the norm for public finance research on developed countries, such data have so far been unavailable for research on developing countries. For the study of bunching, it is crucial to use administrative data as survey data have too much measurement error to detect bunching (Hausman 1983; Saez 2010). As our study includes only registered taxpayers, what we are able to estimate is the intensive margin of response (the response to taxation conditional on being in the tax net) as opposed to the extensive margin of response (movements in and out of the tax net).²

Our paper offers the following main contributions and findings. First, we present a method for estimating behavioral elasticities using bunching at notch points, extending the kink-based estimation technique developed by Saez (2010). While notches and kinks affect the income distribution in different ways, they both create excess clustering at cutoffs which can be used to identify the elasticity of reported taxable income. This elasticity concept has become the conventional measure of behavioral response to taxation in the public finance literature (see Slemrod 1998 and Saez, Slemrod, and Giertz 2010 for surveys) and encompasses the sum total of behavioral responses to taxation, including real labor supply and tax evasion. We consider two different versions of the elasticity of taxable income: (i) the elasticity with respect to the statutory (proportional) net-of-tax rate provides a measure of the bunching response in terms of the size of the notch, (ii) the elasticity with respect to the implicit marginal net-of-tax rate around the notch provides a measure of the underlying structural elasticity consistent with the observed bunching response.

Second, we provide clear graphical evidence of behavioral responses by showing income density distributions around notch points. The tax system involves a separate treatment of wage earners (a schedule with 20 notches) and self-employed individuals (a schedule with 13 notches), and each taxpayer is treated as one or the other according to another notch-based rule which we come back to below. For the self-employed, we find large and sharp bunching below every notch combined with sharp drops in the density above every notch. For wage earners, we find sharp but much smaller bunching below notch points. As wage earnings are subject to third-party reporting by employers (and therefore difficult to under-report) while self-employment income is not, the discrepancy in bunching between wage earners and the self-

²However, this does not imply that we exclude informality effects as there are important elements of informality even in this sample.
employed points to the importance of tax evasion in Pakistan. Following Chetty et al. (2011), we explore the presence of real responses by considering the distribution of third-party reported wage earnings (excluding deductions and any non-wage income) around statutory cutoffs defined with respect to taxable income. We find clear evidence of bunching in wage earnings, showing that the notches generate real responses in addition to evasion responses.

Third, we estimate behavioral elasticities at different notches in the income distribution. Elasticities with respect to the statutory net-of-tax rate are in general large (up to around 4) and strongly significant, consistent with the graphical evidence of large bunching at notches. By contrast, structural elasticities with respect to the implicit marginal net-of-tax rate are always very small (below 0.1) and precisely estimated. The combined findings of large bunching and small structural elasticities are not inconsistent: notches create extremely strong distortions and therefore induce strong behavioral responses even under small structural elasticities. From a mechanism design perspective, this is indeed one of the key problems with notches. At the same time, the presence of small taxable income elasticities with respect to the marginal net-of-tax rate is a priori surprising given that tax enforcement in Pakistan is notoriously weak. This points to the presence of important optimization frictions due to for example misperception or inattention, aspects that may be particularly important in under-developed countries with low “tax literacy”. We come back to this below.

Fourth, we provide evidence on shifting between wage income and self-employment income by considering a different type of notch. Each taxpayer is treated either as a wage earner or a self-employed person depending on whether the share of self-employment income in total income is below or above 50%, with tax rates on wage earners being substantially lower than on the self-employed. This income-composition notch creates a strong incentive to shift income between the two sources. We show that there is clear excess bunching in the self-employment income share at this notch, which provides compelling evidence of an income shifting response. While income shifting has been much discussed in the public finance literature (e.g. Slemrod 1995, 1998; Saez, Slemrod, and Giertz 2010), there is very little direct evidence on this type of response.

Fifth, we consider the effects of an unusual tax reform for wage earners in 2008. This

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3 Slemrod (1996) and Gordon and Slemrod (2000) provide evidence on shifting between personal and corporate tax bases, but we are not aware of previous evidence on shifting between wage income and self-employment income.
reform combined the existing notch-based tax schedule with an alternative kink-based schedule, and allowed taxpayers to choose between the two in order to minimize taxes. As we show in the paper, the reform effectively replaced each notch by two kinks, a convex kink and a non-convex kink, located at slightly different income levels than the pre-existing notch. Those kinks are very sharp in terms of marginal tax rate jumps and drops, but vary with respect to salience: the convex kinks are salient as they occur at statutory cutoffs in the tax code; the non-convex kinks are not salient because the cutoffs are implicit and require a tax-minimization calculation by the taxpayer. We find the following: (i) eliminating a notch kills bunching at the prior cutoff, (ii) introducing a (salient) convex kink sometimes create bunching, but much less than the corresponding notch the year before, and (iii) introducing a (non-salient) concave kink creates no hole in the distribution. Findings (i)-(iii) shows clearly the difference between notches and kinks in generating bunching, and the difference between salient and non-salient kinks in generating any response at all.

Sixth, we analyze the presence and determinants of suboptimal taxpayer behavior. A distinguishing feature of notches is that they create strictly dominated regions where the taxpayer can increase consumption and leisure by lowering earnings to the notch point. In a frictionless economic model, everyone in a dominated region moves to the notch and some individuals above this region move as well. This creates a theoretical lower bound on the response to the notch, and it implies that the earnings elasticity with respect to the statutory net-of-tax rate must be larger than one. Since we find that this elasticity is significantly smaller than one at many notches, behavioral elasticities are too small from the perspective of the frictionless model. To explore the nature of such optimization frictions, we analyze what determines strictly dominated behavior. Our findings suggest that under-responsiveness is driven by aspects of tax illiteracy such as misperception and unawareness of true tax incentives. These aspects appear to be of more central importance in a country as under-developed as Pakistan than in the advanced countries usually considered in the public finance literature.

The paper is organized as follows. Section 2 sets out the theoretical framework and empirical strategy. Section 3 describes the Pakistani income tax context and data. Section 4 presents the empirical results. Section 5 discusses policy implications and concludes.
2 Model and Empirical Methodology

2.1 Model and Small-Notch Analysis

Saez (2010) develops a method to estimate the elasticity of taxable income using the variation created by discontinuities in marginal tax rates (kinks). In this section, we extend this method to variation created by discontinuities in average tax rates (notches). We focus on the analysis of small notches, because the identifying variation in the empirical application is indeed created by small notches: most of the notches in the Pakistani income tax schedule create jumps in the average net-of-tax rate of about 0.25-2.5 percentage points. A small-notch analysis is analytically simpler than a large-notch analysis, and importantly it facilitates an estimation of behavioral responses without making any parametric assumptions about preferences.

Following the recent tax responsiveness literature, we consider a simple extension of the standard labor supply model. The model specifies taxpayer utility as $u (z - T (z), z, n)$ where $z$ is before-tax income, $T (z)$ is tax liability, $z - T (z)$ is consumption, and $n$ is earnings capacity (ability). Taxpayers choose before-tax income to maximize utility given the tax schedule. We may think of before-tax income as being generated by a number of underlying choices such as hours worked, tax avoidance and tax evasion. We will estimate the overall responsiveness of taxable income capturing the sum total of those behavioral margins. Clearly, the model does not provide a realistic description of tax evasion behavior, which would depend on aspects such as audit probabilities, penalties and side payments to tax inspectors. However, given that those variables are not observed in the data, considering a more realistic model of tax evasion complicates the analysis without adding much to the empirical analysis.\footnote{For realistic models of tax evasion behavior and empirical analyses of the link between tax evasion and tax enforcement parameters, see for example Slemrod et al. (2001) and Kleven et al. (2010).}

There is a smooth distribution of ability $n$ in the population. We impose the (weak) Spence-Mirrlees condition on preferences that the marginal rate of substitution between consumption and before-tax income is decreasing in ability $n$. Under this assumption and given a smooth tax system (no notches, no kinks), the smooth ability distribution converts into a smooth income distribution where higher-ability individuals always choose higher income. In particular, we start from a linear tax system $T (z) = t \cdot z$ and denote by $h_0 (z)$ the smooth density distribution of income associated with this baseline.

Suppose a small notch is introduced by raising the tax rate (average and marginal) from $t$ to
\[ t + dt \text{ at the income cutoff } z^*. \text{ This implies } T(z) = t \cdot z + dt \cdot z \cdot 1(z > z^*) \text{ where } 1() \text{ is an indicator for being above the notch. As illustrated in Figure 1 (Panel A), this tax change generates bunching at the notch point of all individuals who had incomes in the interval } [z^*, z^* + dz^*] \text{ before the introduction of the notch. Individual L is the lowest-income individual (before the notch) who locates at the notch point; this individual chooses earnings } z^* \text{ both before and after the tax change. Individual H is the highest-income individual (before the notch) who locates at the notch point; this individual chooses earnings } z^* + dz^* \text{ before the tax change and is exactly indifferent between the notch point } z^* \text{ and the best interior point in the top bracket after the tax change. Every individual between L and H locates at the notch point, and no individual locates in the interval } (z^*, z^* + dz^*). \]

The elasticity of taxable income can be inferred by considering the marginal bunching individual H, who reduces his income by \( dz^* \) in response to the notch. To understand how the method works, it is useful to compare the notch to a hypothetical kink that creates a response of the same size. This is illustrated in Figure 1 (Panel B) as a jump in the marginal tax rate from \( t \) to \( \bar{t} \equiv t + d\bar{t} \) at the income level \( z^* \). This kink induces the marginal bunching individual to reduce income by the same amount as the notch, but in this case the response reflects a movement between points of tangency with the pre-kink and post-kink budget sets, respectively. Hence, the response is qualitatively similar to an interior response, and we can relate the percentage change in taxable income \( dz^*/z^* \) to the percentage change in the marginal net-of-tax rate at the kink \( d\bar{t}/(1 - t) \) to measure the elasticity of taxable income. Using the Slutsky decomposition, it can be shown that the elasticity obtained in this way is a weighted average of the compensated elasticity and the uncompensated elasticity, where the weight is determined by the change in the average relative to the marginal tax rate. When the change in the average tax rate is small (as in the case of a small kink), the estimated elasticity equals the compensated elasticity.\(^5\)

\[^5\]The income supply function of a taxpayer in the top bracket can be written as \( z = z(1 - \bar{t}, y) \) where \( \bar{t} \) is the marginal tax rate in the top bracket and \( y \equiv tz - T(z) \) is virtual income. Denoting by \( \varepsilon^c \) and \( \varepsilon^u \) the compensated and uncompensated elasticities of \( z \) with respect to \( 1 - \bar{t} \), the Slutsky decomposition implies \( \varepsilon^u = \varepsilon^c + \eta \) where \( \eta \equiv (1 - \bar{t}) \frac{\partial y}{\partial y} \) is the income effect. Considering a small change in the top marginal tax rate \( d\bar{t} \), and using the Slutsky decomposition, we obtain

\[
\frac{dz/z}{d(1 - \bar{t})/ (1 - t)} = \left( 1 - \frac{da}{d\bar{t}} \right) \cdot \varepsilon^c + \frac{da}{d\bar{t}} \cdot \varepsilon^u,
\]

where \( da \equiv dT(z)/z \) is the change in the average tax rate at income level \( z \). For top-bracket taxpayers located close to the kink (\( z \) close to \( z^* \)), we have \( da \approx 0 \) and the right-hand side then equals \( \varepsilon^c \). In particular, if a small kink is introduced, the marginal bunching individual H comes from a point close to the kink (\( z^* + dz^* \) close to \( z^* \)), and his response \( dz^* \) therefore identifies \( \varepsilon^c \).
A notch is different from a kink, because the response to a notch is not like an interior response in the top bracket: the change in income of the marginal bunching individual (from A to B in the figure) is much larger than the interior change in income if he does not bunch (from A to C). This is because the interior response is driven by a small proportional tax change $dt$, whereas the bunching response is driven by the discontinuity in the budget set at $z^*$. How do we uncover the underlying income supply elasticity using the bunching response? What the kink analysis suggests is that the elasticity should be based on a comparison between the bunching response to the notch $dz^*$ and the hypothetical jump in the marginal tax rate $d\bar{t}$ creating the same response. But the hypothetical kink $d\bar{t}$ is not observed; it depends on the curvature of the indifference curve and can only be inferred by making strong parametric assumptions on utility. We therefore follow a more agnostic approach, which provides a good approximation of the underlying supply elasticity as long as notches are not too large. We first describe how the elasticity is measured under this approach, and then we discuss the relationship between the approximation and the true structural elasticity.

The elasticity concept we consider compares the bunching response to the change in the implicit marginal tax rate $t^*$ associated with moving between $z^*$ and $z^* + dz^*$. Before the introduction of the notch, we have $t^* = t$. After the introduction of the notch, we have

$$ t^* = \frac{T(z^* + dz^*) - T(z^*)}{dz^*} \approx t + \frac{dt \cdot z^*}{dz^*}, $$

where we use that $dz^*$ is small. Under the notched schedule, the implicit net-of-tax rate $1 - t^*$ is given by the slope of the linear segment connecting points B and D in Panel B of Figure 1. The elasticity of taxable income with respect to the implicit net-of-tax rate is given by

$$ \varepsilon = \frac{dz^*/z^*}{dt^*/(1 - t^*)} = \frac{(dz^*/z^*)^2}{dt/ (1 - t)}. $$

For a small notch, the number of bunchers is given by $B = h_0(z^*) dz^*$, and so the elasticity of elasticity of taxable income can be rewritten as

$$ \varepsilon = \frac{(B/ [h_0(z^*) z^*])^2}{dt/ (1 - t)}. $$

We use this formula to estimate the elasticity of taxable income using evidence on bunching around notch points.\footnote{Compared to the kink-based formula used by Saez (2010) and Chetty et al. (2011), our notch-based formula differs in two respects: (i) the denominator is the percentage change in the average tax rate as opposed to the marginal tax rate in the standard formula, and (ii) the numerator is squared as opposed to unsquared in the standard formula.} A few conceptual points are worth making about this elasticity formula.
First, the elasticity concept corresponds to how discrete labor supply elasticities are measured in other contexts, for example in the frameworks of Saez (2002) and Kleven and Kreiner (2004) in which intensive labor supply elasticities are defined with respect to the implicit marginal tax rate over the range between two discretely different earnings points. Similarly in equations (2)-(3), the elasticity is defined with respect to the implicit marginal tax rate over the range between the interior top-bracket location and the notch point.

Second, the elasticity reflects an upper bound on the underlying "structural" elasticity of the income supply function with respect to the marginal net-of-tax rate. This is because the notch-driven tax rate jump from $t$ to $t^*$ is smaller than the kink-based tax rate jump from $t$ to $\bar{t}$ that would generate the same response. This is illustrated in Panel B of Figure 1 and can be understood as follows. For a small notch $dt$, the best interior income level of individual H (at point C) is very close to the original income level (at point D), and so the slope of the line segment BD ($= 1 - t^*$) corresponds approximately to the average slope of the indifference curve over the range $(z^*, z^* + dz^*)$. With convex preferences, the average slope over $(z^*, z^* + dz^*)$ is larger than the slope at $z^*$ ($= 1 - \bar{t}$). This implies that the variation in $1 - t^*$ understates the variation in $1 - \bar{t}$ that would uncover the true structural elasticity, with the difference being determined by the curvature of preferences over the income range $(z^*, z^* + dz^*)$. When the bunching segment is small, such curvature effects will be small and the elasticity in (3) provides a good approximation of the underlying income supply elasticity.

Third, in our empirical application, the income supply elasticities given by (3) are very informative about structural elasticities, because they turn out to be very small. This is despite the fact that we find large and sharp bunching at all notches. Indeed, an empirical advantage of notches is that they can create strong bunching even under small structural elasticities, which makes them more useful than kinks to identify small elasticities. In the case of kinks, if the true structural elasticity is small, it will generally be difficult to detect any bunching in the data. We verify this last point empirically in section 4.2 by considering a recent tax reform that converted notches into kinks.

Finally, we also consider behavioral elasticities defined with respect to the statutory net-of-

\footnote{The term "structural" is here used somewhat loosely. In general, elasticities of reported taxable income are never truly structural, but depend among other things on the tax and enforcement system. We use the term "structural" to refer to the underlying (local) responsiveness of the income supply function with respect to the marginal net-of-tax rate implied by the observed bunching at the notch.}
tax rate $1 - t$, corresponding to equation (3) with the numerator unsquared. This alternative elasticity concept provides a useful measure of the size of the bunching response in terms of the size of the notch, but does not reveal the underlying structural supply elasticity. As we shall see in section 4.4, the elasticity with respect to the statutory tax-price is useful for evaluating potential under-responsiveness to notches.

2.2 Estimating Behavioral Elasticities using Bunching at Notches

From equation (3), the earnings elasticity with respect to the marginal net-of-tax rate can be estimated using information on (i) the notch threshold $z^*$, (ii) the percent-change in the statutory net-of-tax rate $\delta t / (1 - t)$, (iii) the earnings density $h_0 (z^*)$ in the absence of the notch, and (iv) the amount of bunching $B$ at the notch. Information on (i)-(ii) is obtained directly from the tax code, while (iii)-(iv) can be estimated using the observed earnings distribution. In the following, we describe this estimation procedure.

While the counterfactual density $h_0 (z^*)$ is smooth, the actual density will not be smooth. In principle, there would be a sharp spike exactly at $z^*$ along with a hole in the distribution on the segment $[z^*, z^* + dz^*]$. This is illustrated in Figure 2 (Panel A). The amount of bunching (excess mass at $z^*$ in the post-notch density compared to the pre-notch density) is exactly equal to the size of the hole (the total mass between $z^*$ and $z^* + dz^*$ in the pre-notch density). This implies that, in principle, behavioral responses can be identified either from bunching at the notch point or from the hole in the distribution above the notch; the two methods are equivalent.

Observed density distributions will never look as sharp as in Panel A of Figure 2 due to factors such as inability to perfectly control income (e.g., resulting from random income components), labor supply adjustment costs, inattention, misperception, and potential measurement error in the data.\footnote{See Saez (1999, 2010), Chetty et al. (2011), and Chetty (2011) for a discussion of such aspects in the context of kinks.} Hence, an empirical distribution is more likely to look like the one depicted in Panel B of Figure 2, with excess bunching of taxpayers in a small interval below the notch $[z^* - \delta, z^*]$ and a depressed density in a small interval above the notch $(z^*, z^* + \delta]$.\footnote{For simplicity of notation, we assume here that the bunching segment and “hole” segment have the same width $\delta$. However, the width of those segments will in general not be the same, and the empirical method and application do not rely on this assumption.} While the density is depressed above the notch, there is no hole per se and it would be very difficult to clearly identify the behavioral response by considering the segment above the notch. We therefore estimate the...
elasticity based on excess clustering below the notch threshold. Note that this is different from a kink-based method where the elasticity is estimated by considering excess clustering around the threshold.

To estimate the counterfactual density and the amount of bunching (excess density) in the interval \([z^* - \delta, z^*]\), we define two surrounding intervals, \([z^* - 2\delta, z^* - \delta]\) and \([z^* + \delta, z^* + 2\delta]\). We denote by \(\hat{H}_-, \hat{H}_+\) the fractions of individuals in the lower surrounding segment, bunching segment and upper surrounding segment, respectively. These fractions are obtained by regressing a dummy for belonging to each band on a constant in the sample of individuals belonging to any of those three bands. The estimated densities in the lower and upper surrounding bands are then given by \(\hat{\nu}_- = \hat{H}_- / \delta\) and \(\hat{\nu}_+ = \hat{H}_+ / \delta\). Assuming that without a notch the density would have evolved linearly between the upper and lower band (a reasonable first-order approximation when the bandwidth \(\delta\) is small), the counterfactual density on the bunching segment can be estimated as a weighted average of the surrounding densities, i.e. \(\hat{h}_0 (z^*) = \omega \hat{\nu}_- + (1 - \omega) \hat{\nu}_+\). Given linearity, the weight \(\omega\) is determined by the distance between the upper segment and the bunching segment relative to the total distance between the upper and lower segments. To estimate the excess mass \(B\), we compare the observed probability mass in the bunching interval \(\hat{H}_+\) to a weighted average of the probability masses in the two surrounding intervals \(\hat{H}_-\) and \(\hat{H}_+\). Given the local linearity assumption, we have \(\hat{B} = \hat{H}_+ - (\omega \hat{H}_- + (1 - \omega) \hat{H}_+) = \hat{H}_+ - \delta \hat{h}_0 (z^*)\). The estimated elasticity \(\hat{\varepsilon}\) is then given by equation (3) for \(B = \hat{B}\) and \(h_0 (z^*) = \hat{h}_0 (z^*)\). Finally, we estimate standard errors using the delta method as in Saez (2010).

The local linearity assumption made above will be associated with bias if there is strong curvature in the baseline density around the notch point (a similar issue arises in the kink-based bunching method of Saez, 2010). The direction of this bias will depend on whether the density is convex or concave in the vicinity of the notch. However, this issue will be negligible when bunching in the data is sharp, because in this case the bandwidth \(\delta\) can be allowed to be small and curvature effects will always be small in a sufficiently narrow interval. As we shall see, because notches generate much sharper bunching than kinks, this is not going to be an

10If we denote by \(z^*_-, z^*_0, z^*_+\) the midpoint income levels in the lower, bunching, and upper bands, we have \(\omega = \frac{z^*_0 - z^*_-}{z^*_+ - z^*_-}\). In Figure 2 where all bands have the same width \(\delta\), we have \(\omega = \frac{\delta}{\delta + \delta}\). In the empirical application, those bandwidths are in general not the same and the weight is then adjusted accordingly. Notice also that the unequal weighting of the lower and upper bands is a consequence of the gap between the bunching segment and the upper surrounding segment. In the case of bunching around kink points, one would not include such a gap and the weighting of the upper and lower bands would then be equal.
important issue in our application.

3 Context and Data

3.1 Pakistani Income Tax and Enforcement System

The personal income tax in Pakistan can be traced back to British India in the mid-19th century, making it one of the oldest in the world. The current federal income tax raises revenue of 1.1 percent of GDP, or 11 percent of total tax revenue, and the share of registered taxpayers in the working-age population is about 2 percent.\(^\text{11}\) This level of income tax is broadly consistent with the level in neighboring India (Piketty and Qian 2009) and in developing countries more generally (Gordon and Li 2009). The personal income tax applies to wage earners, self-employed individuals and unincorporated firms. There is an extensive list of exemptions from the income tax base, the most important of which are capital gains, agriculture income, and most pensions. The tax treatment of married couples is fully individual-based and the tax schedule differs slightly between men and women.

Two unusual features of the Pakistani income tax are exploited in the empirical analysis. First, the tax system consists of two separate schedules, one for wage earners and one for self-employed individuals and unincorporated firms, with substantially higher tax rates in the latter schedule than in the former. Each individual is classified either as a wage earner or a self-employed individual depending on whether self-employment income is below or above 50% of total income, and is then taxed according to the assigned schedule on the entire income. Because average tax rates are substantially different in the two schedules, this creates a strong discontinuity in tax liability at a self-employment income share equal to 50%. This *income-composition notch* allows us to estimate income shifting between wage income and self-employment income.

Second, the two income tax schedules consist of a large number brackets (21 for wage earners, 14 for the self-employed and firms) each of which is associated with a given proportional tax rate. Each bracket threshold therefore creates a discontinuity in the average tax rate. Figure 3 shows the average tax rate as a function of taxable income (in Pakistani Rupees, PKR) in each schedule over an income range that includes all 14 brackets for the self-employed and the first 14 of 21 brackets for wage earners.\(^\text{12}\) These are the schedules applying to male taxpayers

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\(^{11}\) World Bank (2009) and authors’ calculations.

\(^{12}\) We do not show empirical results for the upper 7 notches for wage earners, because there are too few
and cover years 2006-08 in the case of the self-employed and years 2006-07 in the case of wage earners.\textsuperscript{13} The tax rate on self-employed individuals ranges from 0 to 25 percent, and the tax rate jump at each bracket threshold is increasing in income (from 0.5\% at the bottom to 4\% at the top). The tax rate on wage earners ranges from 0 to 12.5 percent, and again the tax rate jump at each bracket threshold is increasing in income (from 0.25\% at the bottom to 1.5\% at the top).\textsuperscript{14} We use these \textit{income-level notches} to estimate the elasticity of earnings with respect to the net-of-tax rate.\textsuperscript{15}

The tax system in Pakistan is not indexed for inflation despite the fact that inflation is high in Pakistan (8-20\% annually over the period we consider). A nominal tax system in a high-inflation economy is associated with substantial "bracket creep" as nominal wage growth pushes taxpayers into higher tax brackets over time.\textsuperscript{16} To deal with this issue, the Pakistani income tax system has been subject to frequent tax reforms (around every third or fourth year) that increase bracket thresholds to compensate for inflation. While there was no such reform for the self-employed over the period considered here (2006-08), a tax reform was implemented for wage earners in 2008.

Importantly, besides increasing bracket thresholds, the 2008 reform involved a more fundamental change in the tax treatment of wage earners. In particular, it combined the notch-based schedule with an alternative kink-based schedule and allowed wage earners to choose between the two in order to minimize taxes. As we describe later, for a tax-minimizing individual, the reform effectively replaced each notch by two closely spaced kinks, a convex kink and a non-convex kink, located at a higher income level than the pre-existing notch. This unusual tax reform is very useful for two reasons: (i) it allows us to see what happens when a notch is eliminated, thereby providing a check on the bunching strategy used to identify behavioral responses, (ii) it allows us to explore the difference between notches and different types of kinks.

\textsuperscript{13}The key difference in the tax schedule for females is that the first bracket threshold (where the tax rate goes from zero to positive) is located at a higher income level. Above the first threshold, schedules are the same for males and females.

\textsuperscript{14}As income increases beyond the range in the figure, the tax rate on wage earners increases from 12.5 to 20 percent over the 7 additional brackets. Hence, the tax rate on wage earners is lower than on the self-employed at any income level.

\textsuperscript{15}The notches described above (income-level and income-composition notches) were introduced in Pakistan in 2004. Before that time, the income tax schedule featured standard kinks and the same schedule applied to wage earners and self-employed individuals. It is not feasible to directly analyze the impact of the 2004 reform, because no individual tax return data are available from that period.

\textsuperscript{16}Such bracket creep is in itself a potentially useful source of identifying variation (Saez 2003), but we leave this aspect aside in the present paper.
in creating responses.

Registered taxpayers are required to file income tax returns declaring taxable income, tax liability, and taxes owed net of withholding unless they meet certain filing exemption requirements.\textsuperscript{17} The system operates on a pure self-assessment basis whereby the tax return is considered final unless selected for audit. Moreover, the enforcement system involves some third-party information reporting and withholding, the extent and form of which vary across taxpayer types. For most wage earners, there is third-party reporting and withholding by employers. As shown by Kleven et al. (2010), the crucial element in ensuring effective income tax enforcement is precisely third-party reporting, suggesting that tax enforcement is quite strong for the population of wage earners in Pakistan. By contrast, self-employed individuals and firms face no third-party information reporting, although they are subject to various withholding schemes. These are schemes that withhold taxes in connection with certain transactions (e.g. electricity bills, phone bills, and cash withdrawals), which are credited against income tax liability at the time of filing. This unusual form of withholding comes with no third-party information on the tax base itself (taxable income), and is therefore not as powerful for enforcement as the more traditional system of third-party reporting and withholding in place for wage earners. The bottom line is that, for self-employed and firms, tax evasion is deterred primarily by the threat of audits and penalties. As tax audits in Pakistan are rare, ineffective, and associated with frequent allegations of corruption, tax enforcement is very weak for this population.

3.2 Tax Filing with Poor Income Information: Round-Number Bunching

Besides deliberate underreporting of taxable income in order to evade taxes, there will be some honest misreporting due to the fact that many taxpayers do not know themselves their exact income. Due to the prevalence of cash transactions and the absence of reliable record keeping in Pakistan, it is hard even for an honest taxpayer to reconstruct exact income at the time of filing. This is primarily the case for self-employed individuals, who do not have payment slips

\textsuperscript{17}In particular, the following filing exemption rule will play a role in the empirical analysis: a wage earner is exempt from filing if (i) wage income is below 500,000 rupees, (ii) the employer has filed a tax return (third-party report), and (iii) the taxpayer has no other non-wage income and do not wish to claim any deductions. For wage earners who do not file under this exemption, the third-party reported wage income declared by the employer is taken as the taxable income of the taxpayer. Notice that this creates a different form of notch at 500,000 rupees as wage earners below this threshold do not have to incur personal filing costs. We will be able to investigate the effects of this filing notch in the empirical analysis.
from employers to aid them. For taxpayers who do not know their exact income, it is natural to report income in salient round numbers. Indeed, we find that a large share of self-employed individuals report income in even thousands, and among those round-filers there is a preference for the more salient numbers such as 10s or 100s of thousands. This shows up in the density distribution of taxable income as excess bunching at salient round numbers. Since notch points are also located at salient round numbers, a failure to control for round-number bunching would lead to upward bias.

In the empirical analysis, we therefore split the population of tax filers into two: those who report income in even thousands (“basic filers”) and those who do not (“sophisticated filers”). For sophisticated filers, density distributions are clean and the bunching method described above can be applied directly. For basic filers, round-number bunching at notch points can be controlled for by using the amount of excess bunching at similarly round numbers that are not notches. This method is described in more detail in section 4.

3.3 Data

Our study uses administrative data from the Federal Board of Revenue (FBR) in Pakistan. The dataset includes the universe of personal income tax returns for tax years 2006/07, 2007/08 and 2008/09 (referred to as 2006-08), amounting to about 1 million tax returns per year.¹⁸ For this period, returns were filed either electronically through the FBR website or by hard-copy at designated bank branches and fed to computers using an IT firm distinct from FBR. This data collection process, introduced in 2006, ensures that the data has much less measurement error than what is typically the case for an under-developed country. Variables in the data include taxpayer type (wage earner, self-employed, unincorporated firm, etc.), a number of income variables, tax liability, taxes withheld, geographical area, and gender.

The universe of tax filers is not perfectly overlapping with the universe of registered taxpayers (national tax number holders) for two main reasons: (i) some registered taxpayers do not file returns either because of noncompliance or because of filing exemptions and (ii) a fraction of national tax numbers in Pakistan are invalid or inactive (World Bank 2009). Moreover, the

¹⁸As described in section 3.1, some wage earners are exempt from filing provided that the employer has filed a third-party information report (and some other conditions are met) in which cases gross wage income reported by the employer is the taxable income of the individual. For taxpayers who opt for non-filing under these rules, the data contains income information from the third-party reports. Hence, the dataset contains all taxpayers for whom returns have been filed either by the individual or by the employer.
population of tax filers is a select and perhaps relatively compliant subsample of the Pakistani population. It is also a higher-income subsample than the general population: average annual income in our data is 175,000 PKR for self-employed individuals and about 560,000 PKR for wage earners, both substantially higher than the current GDP per capita of 85,000 PKR.

Using this data and the exogenous variation created by notches, it is possible to identify behavioral responses to taxation among the population of income tax filers. That is, we estimate responses to taxation conditional on being in the tax net (intensive margin) rather than movements in and out of the tax net (extensive margin). This does not imply that we do not capture informality or evasion effects, because there are important elements of informality even for tax filers (in particular for the self-employed) due to the weak enforcement system described earlier.

4 Empirical Results

4.1 Self-Employed Individuals and Firms: Responses to Notches

We start by presenting bunching evidence for self-employed individuals and unincorporated firms. Almost half of all personal income tax filers in the data are self-employed individuals and about 1% are unincorporated firms.\(^{19}\) As discussed in section 3.2, because of a lack of accurate business records and the low tax literacy in Pakistan, many taxpayers will not be able to calculate their exact taxable income at the time of filing. This creates some amount of honest misreporting, and importantly leads to the presence of round-number bunching in the data. Almost 60\% of the self-employed report income in even thousands, and a disproportionate amount of those round-filers show a preference for the more salient round numbers such as 10s, 50s and 100s of thousands. Since notch points are located at salient round numbers, a failure to control for round-number bunching will lead to upward bias. We therefore split the sample by those who report income in even thousands ("basic filers") and those who do not ("sophisticated filers"), and analyze those two groups separately. For sophisticated filers, any excess bunching at notches can be directly interpreted as a behavioral response to the tax system. For basic filers, excess bunching at notches combines behavioral responses and round-number preferences, and we have to control for round-number bunching by using similarly round numbers that are

\(^{19}\)We always show results for self-employed individuals and firms together. Results for the two groups separately are very similar, with responses being slightly sharper for firms.
not notches as counterfactuals.

As described earlier, the tax schedule differs slightly between male and female taxpayers, and the two groups should therefore be analyzed separately. However, since almost all individual tax filers in Pakistan are male (more than 99% of all filers), the data contains too few observations of females to analyze this group alone (except around the mode of the distribution). This section therefore focuses on behavioral responses by males and firms.

The empirical density distribution of taxable income for sophisticated filers is presented in Figure 4 (six bottom notches), Figure 5 (four middle notches), and Figure 6 (three top notches). The density graphs plot the number of taxpayers in different income bins (represented by dots), using a bin size of 1000 PKR at the bottom, 2000 PKR at the middle, and 5000 PKR at the top. Each dot is located at the upper bound of a given bin, so that a dot located at a notch point represents the number of taxpayers at and below the notch. Each notch point is shown by a vertical line and is itself part of the tax-favored side of the notch.

Four main findings emerge from the density diagrams. First, every notch is associated with sharp bunching below the cutoff combined with a sharp drop in the density above the cutoff. This provides clear evidence of a response to the tax structure. Second, the amount of bunching increases in the size of the tax change, which can be seen by comparing adjacent notches associated with different tax rate jumps: bunching at the 125k notch is much stronger than at the preceding 110k notch (tax rate jumps of 1 and 0.5 percentage points, respectively), bunching at the 300k notch is much stronger than at the preceding 200k notch (tax rate jumps of 2.5 and 1 percentage points, respectively), and bunching at the 1000k notch is stronger than at the preceding 800k notch (tax rate jumps of 3.5 and 2.5 percentage points, respectively). Third, bunching at the top of the distribution is weaker than at the bottom and middle of the distribution, despite larger tax rate jumps at the top. That is, while bunching increases locally whenever the size of the notch increases, there is a gradual decline in responsiveness as we move towards the top of the distribution. A possible explanation is that self-employed individuals and firms at the very top are fully formal and therefore have little opportunity to evade. Fourth, although the density always drops immediately after a notch, there are no discernible holes.

20 Around notches where we have enough observations of women to analyze them separately, the behavioral responses of women are qualitatively similar to those of men, but somewhat smaller in magnitude.
21 For the analysis of wage earners in the next section, we focus on notches that apply to both men and women and the two groups are therefore analyzed together.
in the distribution. In particular, a non-negligible number of taxpayers are located in strictly
dominated ranges where they could increase consumption and leisure by reducing earnings to
the notch point, a point we analyze in more detail in section 4.4.

We turn to basic filers in Figure 7. Since basic filers tend to be concentrated at the bottom of
the distribution, there are very few observations around the top three notches and we therefore
do not show them. Panels A-E in the figure show the density distribution around all notches
up to 600k, while Panel F shows the density distribution around 700k (which is not a notch) as
a placebo.

To see how behavioral responses can be identified for this sample, consider for example the
notch at 400k. The following findings emerge. First, there is very strong excess bunching at
the 400k notch and a sharp drop in the density right above. Second, while there is also excess
bunching at the surrounding salient round numbers 350k and 450k, as well as at 360k which
is a salient number in monthly terms (30k per month), this is not as strong as at the notch
point. Third, the density distribution is asymmetric around the notch bin (a much lower density
above it than below), whereas the density distribution around the other salient round numbers
is symmetric. Fourth, if we consider the placebo threshold at 700k, there is no sign of an effect
there: excess bunching at 700k is no stronger than at the surrounding round numbers and the
density around 700k is symmetric. Taken together, these four points show clearly that there is a
behavioral response to the 400k notch by the basic filers. The same arguments show that there
is a response at the other notches as well. Note that, although the spike at the 500k notch is
not as strong as elsewhere, bunching at this notch occurs over several bins (so that there is in
fact excess mass compared to the surrounding round numbers) and that the density is strongly
asymmetric at this point, both of which suggest a behavioral response at 500k as well.

Applying the method from section 2, this bunching evidence can be used to estimate the
elasticity of taxable income at different notches. This is done in Table 1 for sophisticated filers
and in Table 2 for basic filers. As explained in section 2, we consider two different elasticity
concepts: the elasticity of taxable income with respect to the statutory average net-of-tax rate
$1 - \tau$ as well as the elasticity of taxable income with respect to the implicit marginal net-of-tax
rate $1 - \tau^*$. The former elasticity measures the size of the bunching response in terms of the
size of the notch $dt$, while the latter elasticity provides an approximation of the underlying

22 The elasticity with respect to $1 - \tau^*$ is given by equation (3), while the elasticity with respect to the $1 - t$
corresponds to equation (3) with the numerator unsquared.

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"structural" income supply elasticity consistent with the observed bunching response. Because of the strong incentives provided by notches, even a small structural elasticity can generate large bunching and hence a substantial elasticity with respect to the statutory tax change.

For sophisticated filers, three main findings emerge. First, the elasticity with respect to the statutory net-of-tax rate is generally large (up to around 4 at the bottom) and always strongly significant. This shows that the notched schedule generates substantial behavioral responses. Second, the elasticity with respect to the implicit marginal net-of-tax rate is very small (always below 0.1) and precisely estimated. Hence, while the notched schedule generates substantial behavioral responses, these responses are consistent with very modest structural income supply elasticities. This is perhaps a priori surprising given the weak tax enforcement system and low tax compliance in Pakistan, a question we come back to below. Third, elasticities are declining in income and are close to zero at the top of the distribution. This is consistent with the fact that self-employed individuals and firms at the top are fully formal and therefore display close to zero tax evasion. Finally, as in any bunching study, these elasticities are based on assumptions about the bandwidths of the bunching segment and surrounding segments used to estimate the counterfactual density and excess clustering. A sensitivity analysis shows that our results are very robust to changing the bandwidths of the different segments.

To estimate behavioral elasticities for basic filers, we control for round-number bunching at notches using the amount of bunching at similar round numbers that are not notches. For notch point $z^*$, if the total amount excess bunching is estimated to $\hat{B}$ and the baseline density is estimated to $\hat{h}_0 (z^*)$, then purely tax-driven bunching is estimated as $\hat{B} - \hat{R} \cdot \hat{h}_0 (z^*)$ where $\hat{R}$ is the ratio of excess bunching to baseline density at a counterfactual round number. To implement this method, we have to pick counterfactual round numbers allowing us to estimate $\hat{R}$ for each notch. This raises the issue that some notches do not have any very obvious counterfactuals in the vicinity of the notch, especially at the bottom of the distribution where notches are very closely spaced, and the estimation of elasticities for basic filers is therefore not always as robust as for sophisticated filers. Table 2 shows elasticity estimates only for those notches where relatively clean counterfactuals can be found.

In general, the findings for basic filers are consistent with the findings for sophisticated filers.

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23 Moreover, income points below the first notch (where tax liability is zero) do not provide good controls for round-number bunching as many taxpayers do not file returns in this range, which creates considerable noise in the distribution at the very bottom.
filers: elasticities with respect to the statutory net-of-tax rate are large and roughly similar in magnitude, elasticities with respect to the implicit marginal net-of-tax rate are very small, and elasticities are declining in income. In general, behavioral responses are slightly smaller for basic filers than for sophisticated filers. Given that basic filers are those who have no precise knowledge of their own taxable income, one might expect them to be either more or less responsive than sophisticated filers: the lower tax literacy among basic filers presumably makes them less able to respond to the incentives created by the notched schedule, but at the same time the lack of accurate income documentation implies less traceable evidence of use to tax authorities. Our results suggest that the former effect dominates.

Given that the elasticity of taxable income includes evasion responses, it is perhaps a priori surprising that the elasticity with respect to the marginal net-of-tax rate is so small. As described in section 3, the tax enforcement system in Pakistan is very weak (in particular, for the self-employed) and tax evasion is undoubtedly large even among the sample of registered self-employed taxpayers. However, when interpreting the results, it is important to distinguish between the evasion level and the evasion response to marginal tax rates. A weak enforcement system will be associated with large tax evasion, but not necessarily a large response of tax evasion to the marginal tax rate. For example, in a benchmark model with a linear income tax, a linear penalty in the evaded tax and an exogenous audit probability, the substitution effect of the marginal tax rate on evasion is always zero (e.g. Slemrod and Yitzhaki 2002). In the presence of nonlinear penalties (including side payments to inspectors), nonlinear taxes and endogenous audit probabilities, the substitution effect is non-zero but the sign is in general ambiguous (e.g. Kleven et al. 2010). Hence, there is no reason to believe that a weak enforcement system is necessarily associated with large evasion responses to notches. Kleven et al. (2010) study this question empirically using Danish audit data and a large kink in the income tax schedule, and finds that for self-employed individuals the response of evasion to the marginal tax rate is positive but very small. Hence, while small elasticities of taxable income in Pakistan are somewhat surprising, they are not inconsistent with the presence of weak enforcement.

At the same time, and as mentioned earlier, a more direct indication of under-responsiveness to the tax system is the presence of a non-negligible number of taxpayers in strictly dominated regions above notches. Notice that those regions are strictly dominated only in terms of real labor supply: consumption and leisure can be increased by reducing real labor supply. Condi-
tional on real income being located in a strictly dominated region, it is not necessarily optimal to reduce reported income to the notch threshold (indeed, this evasion response is theoretically ambiguous as just described). The question of strictly dominated choice and under-responsiveness to taxation is analyzed in detail in section 4.4.

4.2 Wage Earners: Responses to Notches and Kinks

As described earlier, the tax treatment of wage earners was changed quite fundamentally in 2008 from the notch-based schedule shown in Figure 3 to a complicated kink-based schedule. We start by presenting evidence on behavioral responses to the notched schedule in years 2006-07, and then turn to the effects of the 2008 tax reform.

The issue of round-number bunching discussed above is much less important for wage earners than for self-employed individuals, because wage earners have pay slips and third-party reports from their employers to aid them in their assessment of taxable income. Hence, most wage earners do know their exact earnings and a much smaller proportion of them (about 8%) report income in even thousands. Moreover, the small degree of round-number bunching that we do observe for wage earners takes a different form than for the self-employed. In particular, because job contracts are negotiated in monthly rather than annual wages, round-number bunching for wage earners takes place at annual incomes that are multiples of 12 and salient round numbers such as 10s and 100s of thousands (i.e., annual incomes of 120k, 240k, 360k, etc.). Those numbers coincide with notch points only at 600k and 1200k, where we see very strong bunching in the data. In order to not confound notch bunching with round-number bunching at those points, we drop round filers from the sample of wage earners and focus on sophisticated filers. As basic filers constitute a very small fraction of the sample, we do not analyze them separately as we did in the previous section. An alternative would be to consider sophisticated and basic filers together (showing density distributions with spikes at salient round numbers in monthly terms) and control for round-number bunching at notches using the method set out in the previous section. Since this strategy gives the same qualitative results as those we show below, we prefer to present and analyze the clean density distributions for sophisticated filers only.

The tax schedule for wage earners has 20 notches, but we do not show bunching evidence for all of them. In particular, the density distribution is too noisy around the bottom notches (in part because of filing exemptions at the bottom) and around the top notches (because of
very few observations at the extreme top) to precisely estimate bunching.\(^{24}\) Hence, we focus on the middle notches where the density distribution is sufficiently clean to identify bunching responses clearly. As those middle notches apply to both male and female taxpayers, we show evidence for males and females together.

The first set of evidence for wage earners is presented in Figure 8, which shows density distributions of taxable income (black graph connected by dots) and gross wage income (blue graph connected by triangles) for all wage earners in 2006-07 around six notches at the middle of the distribution. Taxable income equals gross wage income plus non-wage income (self-employment income, capital and property income) minus deductions. As wage income is third-party reported for this sample and therefore difficult to under-report without being detected (Kleven et al. 2009, 2010), tax evasion by wage earners will occur mostly in the non-wage and deduction component of taxable income.\(^{25}\) Hence, following Chetty et al. (2011), we consider bunching in gross wage income around statutory cutoffs (defined with respect to taxable income) to detect real responses. A bunching individual with any non-wage income or deductions will be mechanically displaced from the notch when considering the distribution of wage income alone, and hence bunching in this distribution is likely to reflect real responses. We can compare this to bunching in taxable income, which reflects the total behavioral response including evasion.

The following findings emerge from the figure. First, the distribution of taxable income features bunching below every notch along with a drop in the density above every notch. Moreover, unlike the findings for self-employed individuals, there is evidence of holes in the distribution above some of the notches (at 700k, 850k, and 950k). Second, as one would expect, bunching in taxable income is not as strong for wage earners as it is for self-employed individuals. It should be noted though that the notches for wage earners are associated with smaller tax rate jumps than for the self-employed. Third, there is clear evidence of bunching even in the distribution of third-party reported wage income, which shows that there is a real labor supply response to the tax structure. In fact, bunching in wage income is only slightly weaker than bunching in taxable income, suggesting that most of the response by wage earners may be a real response.

Fourth, the evidence of bunching at the 500k notch should be interpreted with some caution.

\(^{24}\)The tax schedule for wage earners contains notches all the way up to an annual income of 8.4 million rupees, which is extremely high in a Pakistani context.

\(^{25}\)We cannot completely rule out that there is some evasion in gross wage income even though it is third-party reported, for example because of collusion between the taxpayer and the third party (see Kleven et al. 2009 for a study of this question).
As described in section 3, wage earners are exempt from the filing requirement if wage income is below 500k and a third-party information report has been submitted by the employer. For taxpayers who opt for non-filing under these rules, taxable income is given by third-party reported gross wage income. Since income tax filing is not costless to the taxpayer (due to for example administrative hassle), these rules introduce a filing notch and create an additional incentive for bunching below 500k. Hence, bunching at this particular cutoff possibly combines responses to the tax rate notch and the filing notch. We will be able to explore this below, using that the 2008 tax reform removed the tax rate notch but not the filing notch at 500k.

Table 3 uses the bunching evidence for wage earners to present estimates of the elasticity of taxable income (total response) and the elasticity of third-party reported wage income (real response) with respect to the statutory net-of-tax rate and the implicit marginal net-of-tax rate (structural elasticity). Most of the elasticity estimates are statistically significant, but unsurprisingly elasticities for wage earners are much smaller in magnitude than for the self-employed. While elasticities with respect to the statutory net-of-tax rate are fairly substantial in size (mostly in the range 0.1-0.2 for the total response and in the range 0.07-0.12 for the real response), the structural elasticities consistent with those bunching responses are extremely small. The small responses by wage earners to the strong incentives created by notches suggest the presence of important optimization frictions such as adjustment costs, inattention, or misperception of true incentives (Chetty 2011; Chetty et al., 2011). We come back to the question of under-responsiveness later.

We now turn to the analysis of an unusual tax reform in 2008, which allows us to compare the effects of notches and kinks. Rather than replacing the notch schedule by a standard kink schedule, the reform introduced a kink-based option in addition to the existing notch schedule and allowed the taxpayer to self-assess tax liability in a tax-minimizing fashion. The effect of the reform on the budget set of taxpayers is illustrated if Figure 9. In Panel A of this figure, the solid (blue) curve shows the pre-reform budget set featuring a notch at \( z^* \). Following the reform, a kink option (shown by the dashed red curve) associated with a marginal tax rate of \( \bar{\tau} \) above the cutoff was introduced. A kink alternative of this sort was introduced at every notch, with the marginal tax rate \( \bar{\tau} \) varying across different ranges of the income distribution (from 20% at the bottom to 60% at the top). Because the marginal tax rate above the cutoff is much higher under the kink than under the notch (\( \bar{\tau} >> t + dt \)), it eventually becomes optimal to switch
back to the notch schedule as income increases sufficiently above the cutoff (where the solid and dashed curves intersect in Panel A). Therefore, and as shown in Panel B, the post-reform budget set for a tax-minimizing individual features two kinks (a convex kink at $z^*$ and a non-convex kink at the intersection of the two underlying schedules $z^{**}$) for every notch. Hence, the tax reform effectively replaced the twenty notches by forty kinks, half of which are convex (and should produce bunching) and half of which are non-convex (and should produce holes). At the same time, bracket cutoffs were increased such that each pair of kinks were located higher than the pre-existing notch.

Two factors are important to keep in mind when analyzing this unusual reform. First, the post-reform tax system is very complex, and it requires a high degree of taxpayer sophistication to take advantage of the incentives created by the system. While the reform replaces notches by kinks and hence eliminates strictly dominated regions in principle, it leaves this entirely to taxpayers’ self-assessment. Those self-assessments are corrected only in the case of tax audits (which are very rare), and it is therefore possible for taxpayers to be on the notch schedule when they should be on the kink schedule, and vice versa. The post-reform tax system therefore creates a different form of strictly dominated choice, namely to choose the wrong schedule at a given level of taxable income. We find evidence of such behavior in the data: among taxpayers with incomes in between the convex and non-convex kinks (all of whom should pay tax according to the kink schedule), about 20% self-assess their tax liability according to the notch schedule. Second, under the new tax schedule, the convex kink and non-convex kink are very different in terms of salience. The convex kinks are located at statutory cutoffs in the tax system (all of which are simple round numbers as in the pre-existing notch schedule) and are therefore quite salient. The non-convex kinks, on the other hand, are determined by intercepts between the two underlying schedules and requires a tax-minimization calculation by the taxpayer. The location of each non-convex kink depends on the tax rates in both schedules ($t, dt, \bar{t}$) as well as the statutory income cutoff ($z^*$), and the distance between the statutory cutoff and the non-convex kink therefore varies across cutoffs. Hence, the non-convex kinks are much less salient than the convex kinks.

We present graphical evidence on the effects of the reform in Figure 10, which shows density distributions of taxable income for wage earners before the reform (2006-07) and after the reform (2008) around notches that were eliminated and kinks that were introduced as part of
the reform. The before-reform distribution is depicted by the black graph connected by dots and shown on the left y-axis, while the after-reform distribution is depicted by the blue graph connected by triangles and shown on the right y-axis. Before-reform notches are shown by solid vertical lines, while after-reform kinks are shown by dashed lines (convex kinks) and dashed-dotted lines (non-convex kinks), respectively. While cutoffs were increased in every bracket, some income levels serve as bracket cutoffs both before and after the reform (in particular, 400k and 1050k in the figure) as the new higher cutoff came to coincide with a pre-existing cutoff associated with a higher bracket.\(^{26}\) Hence, the solid vertical lines at 400k and 1050k reflect both a notch in 2006-07 and a convex kink in 2008. In this way, the reform provides very rich variation by allowing us to evaluate the effects of (i) eliminating notches, (ii) replacing notches by kinks, and (iii) introducing kinks in places with no pre-existing notches.

Our findings are the following. First, at cutoffs where notches are eliminated and not replaced by kinks (at 500k, 600k, 700k, 850k, 950k), bunching disappears completely with the only exception being 500k. Interestingly and as discussed above, this specific cutoff represents both a tax rate notch (before the reform) and a filing notch (before and after the reform). The post-reform bunching at 500k therefore identifies a behavioral response to the filing exemption below 500k. For this cutoff, the elasticity estimates presented in Table 3 include the filing exemption response and are therefore upward biased. For all other cutoffs, the evidence in the figure lends strong support to the identifying assumption that, in the absence of discontinuities in tax rates at bracket thresholds, the income distribution would be smooth around those thresholds.

Second, at cutoffs where notches are replaced by convex kinks (at 400k and 1050k), most but not all of the bunching disappears. In particular, there is still some bunching in 2008 around the 1050k kink, which is a very sharp kink involving a marginal tax rate jump from 9% to 40%. There is also very slight excess bunching at the smaller 400k kink, which involves a jump in the marginal tax rate from 1.5% to 20%. Third, the introduction of convex kinks in places with no pre-existing notches creates bunching at most but not all cutoffs. In particular, the kinks introduced at 450k, 650k, and 750k are associated with bunching in the after-reform distribution (and no bunching in the before-reform distribution), which shows clearly an effect of the reform. Fourth, the introduction of non-convex kinks does not create any discernible

\(^{26}\)In particular, the 350k cutoff before the reform was increased to 400k after the reform, which was the before-reform cutoff associated with the next bracket. Likewise, the 950k cutoff before the reform was increased to 1050k after the reform, which had been the next-bracket cutoff before the reform.
holes in the distribution. While those non-convex kinks are very sharp in terms of the drop in the marginal tax rate, they are also very complex and non-salient as described above. Hence, it is not surprising that there is no response to those kinks.

Overall, the amount of bunching created by kinks is always very small compared to the bunching created by notches, despite the fact that all of the kinks are associated with very large marginal tax rate changes. This shows that notches offer more powerful identifying variation than kinks, because notches create much stronger incentives and perhaps also because those incentives are simpler and more salient. In particular, while notches present the taxpayer with two different linear tax systems on either side of a cutoff and a strictly dominated region above, kinks create incentives that are much more subtle. Moreover, it is worth noting that the small bunching responses observed at convex kinks are qualitatively similar to the bunching responses observed at notches: convex kinks are associated with excess bunching right below the cutoff but not above, whereas in principle we would expect kinks to create bunching around the cutoff. This suggests that many taxpayers respond to those kinks as if they were still notches, presumably because the kink schedule was introduced in addition to the notch schedule rather than replacing it entirely.

It is possible to estimate behavioral elasticities around convex kinks based on the method of Saez (2010). Using his method around kinks that produce bunching (400k, 450k, 650k, 750k, and 1050k), we find that elasticities with respect to the marginal net-of-tax rate tend to be slightly larger than the elasticities with respect to the implicit marginal net-of-tax rate estimated from notches in Table 3. Although bunching at kink points is always much smaller than at notch points, the effective tax rate changes are also much smaller and this produces larger elasticities. However, as just observed, taxpayers tend to respond to the newly introduced kinks as if they were still notches, which clearly invalidates the Saez method of estimating elasticities. If we instead estimate elasticities around kinks as if they were notches, they are much smaller than in Table 3 and mostly insignificant. Those two estimation techniques presumably provide upper and lower bounds for the true elasticity around kinks in the 2008 schedule.

4.3 Shifting Between Self-Employment and Wage Income

Having studied behavioral responses among wage earners and self-employed individuals separately, we now turn the question of shifting between the two. While income shifting has been
much discussed in the literature, there is relatively little direct evidence on this type of behavioral response. As identifying variation, we use the income-composition notch described earlier: each taxpayer is treated either as a wage earner or a self-employed individual depending on a cutoff rule applying to the composition of income, with tax rates on the latter being much higher than on the former. In particular, if the share of self-employment income in total income is greater than or equal to 50%, the individual is treated as self-employed. Otherwise, the individual is treated as a wage earner. This creates a large notch at an income share of 50% and provides very strong incentives to shift income from self-employment to wages in order to be treated as a wage earner.

We provide evidence on income shifting responses in Figure 11, which shows the density distribution of the self-employment income share between zero and one in bins of 0.025. Taxpayers are treated as wage earners and face a low tax rate on the left side of the vertical line at 0.50, and are treated as self-employed and face a high tax rate on the right side of the vertical line. Importantly, unlike the density diagrams shown earlier, the notch point itself belongs to the high-tax region and we would therefore expect to see bunching strictly below the notch rather than at the notch. To evaluate this, each bin excludes the upper bound of the interval such that the bin right below the notch spans \([0.475; 0.50)\) and the bin right above spans \([0.50; 0.525)\).

Panel A shows the raw distribution of the self-employment income share. While this panel shows a very clear effect on the density distribution at the notch, it is puzzling that bunching seems to occur around the notch rather than below it. Interestingly, it turns out that almost all of the taxpayers in the first bin above the notch have a self-employment income share exactly equal to the cutoff value of 0.50. This points to two possible explanations. The first possibility is that this is a form of “round-number bunching” as discussed and analyzed earlier. These taxpayers may not know the exact composition of their income into wage income and self-employment income and therefore naturally report the same amount in the two cells on the self-assessment. As discussed earlier, ignorance about one’s own taxable income is a signal of low tax literacy in general, and hence those taxpayers may also be unaware of the presence of the income-composition notch. A second possibility is that these taxpayers are actually trying to game the system by bunching at the notch point, but are unaware of the subtlety that the notch point itself (unlike all other notches in the tax system) belongs to the high-tax range.

Whatever the explanation for taxpayers locating at an income share of precisely 0.50, an
empirical test of income shifting responses to the notch can be obtained by slicing out those specific taxpayers from the distribution. That is, if there were no response to the notch, the density distribution would be smooth around the notch even if we drop observations at one specific value. Hence, Panel B shows the density distribution when dropping taxpayers with a self-employment income share equal to 0.50, and this diagram shows a very clear income shifting response to the notch: there is bunching below the notch along with a very sharp drop in the density above the notch. Arguably this diagram represents a lower bound for the shifting response as at least some of the taxpayers at the cutoff are in fact responding to the notch. An upper bound can be obtained by moving taxpayers at the cutoff to the bin below, which would imply very strong bunching below the notch.

While this graphical evidence shows a clear income shifting response to the tax system, the response is not very large in elasticity terms. This is because the income-composition notch considered in this section is much larger than the income-level notches considered earlier. The tax rate jump at the income-composition notch is given by the difference in tax rates between being treated as a wage earner and a self-employed person (see Figure 3). This tax rate jump depends on income level and is therefore heterogeneous across taxpayers, but is in general very large except at the very bottom of the distribution. Indeed, among the sample of filers in Figure 11, the tax rate jump between wage-earner and self-employment status (taking the level of total taxable income of each filer as given) is above 6 percentage points on average. This is a truly massive notch and therefore the observed bunching response is associated with a small shifting elasticity.

4.4 Strictly Dominated Choice and Under-Responsiveness to Taxation

A central feature of notches is that they create strictly dominated regions where the taxpayer can increase consumption and leisure by lowering earnings to the notch point. In theory, everyone in a dominated region moves to the notch and some taxpayers above this region move as well. This implies that excess bunching at the notch in proportion to the pre-notch mass in the strictly dominated range is always larger than one, which creates a theoretical lower bound on the behavioral response to the notch (in a frictionless model). We can evaluate this empirically by comparing our estimates of excess bunching to estimates of the mass in strictly dominated
ranges in absence of notches.

For this purpose, consider a notch point \( z^* \) as illustrated in Figure 1. Define the strictly dominated earnings range as \((z^*, z^* + d z^D)\) where the upper bound \( z^* + d z^D \) is the earnings level ensuring the same consumption as the notch point: \((1 - t - dt) \cdot (z^* + d z^D) = (1 - t) \cdot z^*\). This can be rewritten to \(d z^D = z^* \cdot dt / (1 - t - dt)\). Given that the notch \( dt \) and therefore \(d z^D\) are small, the density mass in the earnings range \((z^*, z^* + d z^D)\) had there been no notch is given by \(h_0(z^*) \cdot d z^D\) where \(h_0(z^*)\) is the counterfactual density at \(z^*\). As excess bunching in proportion to the pre-notch mass in the strictly dominated range is supposed to be larger than one, we have

\[
\frac{B}{h_0(z^*) \cdot d z^D} = \frac{d z^* / z^*}{dt / (1 - t - dt)} \approx \frac{d z^* / z^*}{dt / (1 - t)} > 1,
\]

(4)

where we have used \(B = h_0(z^*) \cdot d z^*\) and the expression for \(d z^D\) above. Hence, the theoretical lower bound on the behavioral response to a notch (assuming no frictions) implies that the elasticity of earnings with respect to the statutory net-of-tax rate is larger than one.

We may evaluate the theoretical bound in (4) empirically by considering the estimates in Tables 1-3 of earnings elasticities with respect to the statutory net-of-tax rate. Those results show that taxpayers under-respond to tax incentives at many notches as elasticities are significantly smaller than one at the middle and top notches for self-employed individuals and at all notches for wage earners. This points to the presence of important optimization frictions.

To explore the nature of such optimization frictions, we now investigate what determines locating in strictly dominated regions. Table 4 reports the results of OLS regressions of a dummy for locating in a 500 rupee range above a notch on various dummy covariates. The 500 rupee range is smaller than the strictly dominated range \(d z^D\) at every notch. By considering a constant range above each notch rather than the full dominated range, we avoid mechanical effects of the dominated range becoming larger as the cutoff \(z^*\) increases. The table shows results for self-employed individuals and firms in columns (1)-(2) for years 2007 and 2008, respectively, and for wage earners in columns (3)-(4) for the same years.\(^{27}\) The dummy covariates we consider

\(^{27}\)A caveat is in place with regards to the regression for wage earners in 2008 in column (4). As the tax reform in 2008 introduced a kink option in addition to the notch schedule, locating in a 500 rupee range above the cutoff is not necessarily strictly dominated. However, as analyzed in section 4.2, many taxpayers did not understand this complex reform and behave as under the notch schedule (e.g., a substantial fraction of taxpayers right above the cutoff self-assess tax liability according to the notch rather than the kink schedule and the effect of kinks on the density distribution takes the same form as the effect of notches). Hence, it makes sense to consider a regression for wage earners in 2008 that defines suboptimal choice as being located in a 500 rupee range above a cutoff.
include (i) dummies for being a registered taxpayer in prior years, (ii) dummies for being located in strictly dominated regions in prior years, (iii) income quantile dummies (percentiles 25-50, 50-75, 75-90, 90-95, 95-100), (iv) firm dummy, (v) capital and property income dummy, (vi) rural region dummy, and (vii) male gender dummy.

The two most interesting findings in the table are the following. First, strictly dominated choice in a given year is negatively associated with being a registered in previous years. Compared to the unconditional probability of strictly dominated choice (the mean of the dependent variable is shown at the bottom of the table), those point estimates are very large. Hence, taxpayers who have less experience in dealing with the tax code are much more likely to make mistakes, which suggests that individual tax literacy and learning are important for behavioral responses. Second, dominated choice in a given year is positively associated with dominated choice in previous years. Those effects are extremely strong compared to the baseline probability of dominated choice, especially for the self-employed. These effects are unlikely to be driven by serial correlation in income, because we are considering a very small earnings range above each notch and inflation in Pakistan is very high (the 500 rupee range is less than 0.5% of income at every notch, while inflation in Pakistan is 10-20% annually over the period considered). The fact that suboptimal behavior is strongly persistent over time suggests that certain taxpayers consistently misperceive tax incentives while others consistently do not, a finding that is consistent with the importance of tax literacy mentioned above. By contrast, if under-responsiveness was driven by adjustment costs in labor supply (as analyzed by Chetty 2011 and Chetty et al. 2011), it would be a short-run phenomenon and therefore less persistent over time. Moreover, under the adjustment cost story, the effects would not be stronger for the self-employed than for wage earners. Overall, our findings suggest that under-responsiveness in Pakistan is driven by misperception and tax illiteracy rather than real adjustment costs.

A number of additional findings in the table are worth noting. Taxpayers at the top of the income distribution are less likely to locate in strictly dominated ranges than taxpayers at the bottom, consistent with high-income individuals being more sophisticated and the incentives created by notches being stronger at the top. The firm dummy has a negative coefficient presumably because firms are more sophisticated and have larger capacity to optimize than individuals. The presence of capital and property income has a negative effect (but is not always significant), which is probably driven by the fact that taxpayers with such income forms
are relatively sophisticated. Finally, the coefficient on the rural dummy is negative, while gender is mostly insignificant.

To conclude, despite the finding of large and sharp bunching in earlier sections, the evidence implies that agents under-react to the strong incentives created by notches. This shows that there are important optimization frictions among taxpayers, and the evidence suggests that those frictions are driven by aspects of tax illiteracy such as misperception or unawareness of tax incentives. Mistakes driven by tax illiteracy may be of first-order importance in settings as under-developed as Pakistan.

5 Implications and Conclusions

Notches tend to have a bad reputation among economists, although there is very little empirical evidence on their effects. Indeed, one of the key results in the canonical optimal income tax framework by Mirrlees (1971) is that the marginal tax rate is everywhere below one, which rules out discontinuous jumps in tax liability as created by the notches analyzed in this paper. Such notches create regions of strictly dominated choice where the marginal tax rate is larger than one. In the standard economic model, since no taxpayer is willing to locate in a region of strictly dominated choice, the presence of such regions create strong distortions without collecting any revenue. This can never be socially optimal.28

This paper presents for the first time evidence on behavioral responses to income tax notches, allowing for an evaluation of these theoretical results. Considering a series of different notches, we consistently find large and sharp bunching below cutoffs combined with sharp falls in the density above cutoffs. We use this evidence to identify taxable income responses (real responses plus evasion), real responses, and income shifting responses. However, while behavioral responses to notches are always clear and significant, the incentives created by notches are extremely strong and responses are therefore small in elasticity terms. In fact, we show that elasticities are often smaller than a theoretical lower bound (in a frictionless economic model) due to the fact that a non-negligible amount of taxpayers locate in strictly dominated regions (with marginal tax

28 Exceptions to this negative view on notches are provided by Blinder and Rosen (1985) and Slemrod (2010). Both of these papers advocate a view that notches may be optimal in the presence of restrictions on policy instruments (e.g. due to administrative constraints). In particular, Blinder and Rosen show that a notched schedule may dominate a fully linear schedule under some conditions. By contrast, the Mirrleesian literature on optimal nonlinear incentive schemes shows that there always exists a better schedule without notches if we allow for enough flexibility in instruments.
rates above one). The evidence suggests that the under-responsiveness to notches is driven by misperception or inattention to tax incentives resulting from the low degree of tax literacy in Pakistan. These findings raise some doubt about the conclusion from the Mirrleesian literature that notches are always suboptimal, because this conclusion relies on the assumption of no misperception (or other optimization frictions) such that taxpayers never locate in strictly dominated regions. Future work will continue the exploration of taxpayer misperception and other behavioral aspects in economies with low tax literacy and investigate the implications of such behavior for optimal tax design in developing countries.

References


FIGURE 1
Effect of Notch on Taxpayer Behavior

Panel A: Bunching at the Notch

Panel B: Comparing the Notch to a Hypothetical Kink
FIGURE 2
Effect of Notch on Density Distribution

Panel A: Theoretical Density Distributions

Panel B: Empirical Density Distribution and Bunching Estimation
Notes: the figure shows the statutory (average) tax rate as a function of annual taxable income in the personal income tax schedules for wage earners (red dashed line) and self-employed individuals and unincorporated firms (blue solid line), respectively. Taxable income is shown in thousands of Pakistani Rupees (PKR), and the PKR-USD exchange rate is around 85 as of April 2011. The schedule for the self-employed applies to the full period of this study (2006-08), while the schedule for wage earners applies only to 2006-07 and was changed by a tax reform in 2008. The tax system classifies individuals as either wage earners or self-employed based on whether income from wages or self-employment constitute the larger share of total income, and then taxes total income according to the assigned schedule. The tax schedule for self-employed individuals and firms consists of 14 brackets, while the tax schedule for wage earners consists of 21 brackets (the first 14 of which are shown in the figure). Each bracket cutoff is associated with a notch, and the cutoff itself belong to the tax-favored side of the notch.
FIGURE 4
Density Distribution around Bottom Notches:
Self-Employed Individuals and Firms (Sophisticated Filers)

Panel A: First Three Notches

Panel B: Next Three Notches

Notes: the figure shows the density distribution of taxable income for male self-employed individuals and unincorporated firms in 2006-08 around the six bottom notches in the schedule applying to those taxpayers (shown in Figure 3). The densities include only “sophisticated filers” defined as those who do not report income in even thousands. Each dot represents the upper bound of a 1000 Rupee bin and thus shows the number of taxpayers located within a 1000 Rupee range below the dot. Notch points are shown by red vertical lines, and each notch point is itself part of the tax-favored side of the notch. The average tax rate jumps by 0.5%-point at the bottom two notches (100k, 110k) and by 1%-point at the following four notches (125k, 150k, 175k, 200k).
FIGURE 5
Density Distribution around Middle Notches: Self-Employed Individuals and Firms (Sophisticated Filers)

Panel A: Notch at 300k
Panel B: Notch at 400k
Panel C: Notch at 500k
Panel D: Notch at 600k

Notes: The figure shows the density distribution of taxable income for male self-employed individuals and unincorporated firms in 2006-08 around the four middle notches in the schedule applying to those taxpayers (shown in Figure 3). The densities include only “sophisticated filers” defined as those who do not report income in even thousands. Each dot represents the upper bound of a 2000 Rupee bin and thus shows the number of taxpayers located within a 2000 Rupee range below the dot. Notch points are shown by red vertical lines, and each notch point is itself part of the tax-favored side of the notch. The average tax rate jumps by 2.5%-points at all the middle notches.
FIGURE 6
Density Distribution around Top Notches:
Self-Employed Individuals and Firms (Sophisticated Filers)

Panel A: Notch at 800k

Panel B: Notch at 1000k

Panel C: Notch at 1300k

Notes: the figure shows the density distribution of taxable income for male self-employed individuals and unincorporated firms in 2006-08 around the three top notches in the schedule applying to those taxpayers (shown in Figure 3). The densities include only “sophisticated filers” defined as those who do not report income in even thousands. Each dot represents the upper bound of a 5000 Rupee bin and thus shows the number of taxpayers located within a 5000 Rupee range below the dot. Notch points are shown by red vertical lines, and each notch point is itself part of the tax-favored side of the notch. The average tax rate jumps by 2.5%-points at the 800k notch, by 3.5%-points at the 1000k notch, and by 4%-points at the 1300k notch.
FIGURE 7
Density Distribution around Bottom and Middle Notches:
Self-Employed Individuals and Firms (Basic Filers)

Panel A: Six Bottom Notches  Panel B: Notch at 300k

Panel C: Notch at 400k  Panel D: Notch at 500k

Panel E: Notch at 600k  Panel F: Placebo-Notch at 700k

Notes: the figure shows the density distribution of taxable income for male self-employed individuals and unincorporated firms in 2006-08 around all notches up to 600k in the schedule applying to those taxpayers (shown in Figure 3), and around 700k which is not a notch. The densities include only “basic filers” defined as those who report income in even thousands. Each dot represents the upper bound of a 5000 Rupee bin and thus shows the number of taxpayers located within a 5000 Rupee range below the dot. Notch points are shown by red vertical lines, and each notch point is itself part of the tax-favored side of the notch. The average tax rate jumps by 0.5%-point at the bottom two notches (100k, 110k), by 1%-point at the next four notches (125k,…, 200k), and by 2.5%-points at the last four notches (300k,…, 600k).
FIGURE 8
Density Distributions around Middle Notches for Wage Earners

Panel A: Notch at 400k

Panel B: Notch at 500k

Panel C: Notch at 600k

Panel D: Notch at 700k

Panel E: Notch at 850k

Panel F: Notch at 950k

Notes: the figure shows the density distributions of gross wage income and taxable income for all wage earners (male and female) in 2006-07 around six middle notches in the schedule applying to those taxpayers (shown in Figure 3). Taxable income equals gross wage income plus non-wage income (self-employment income, capital income and property income) minus deductions. Each dot represents the upper bound of a 2500 Rupee bin and thus shows the number of taxpayers located within a 2500 Rupee range below the dot. Notch points are shown by red vertical lines, and each notch point is itself part of the tax-favored side of the notch. The average tax rate jumps by 1%-point at the 400k, 500k and 950k notches, while it jumps by 1.5%-point at the 600k, 700k and 850k notches.
FIGURE 9
Tax Reform for Wage Earners in 2008

Panel A: Budget Sets under the Notch and Kink Options

Panel B: Budget Set of a Tax-Minimizing Individual
FIGURE 10
Effects of the 2008 Tax Reform for Wage Earners: Notches vs. Kinks

A: Statutory Cutoffs at 400k & 450k

B: Statutory Cutoffs at 500k & 550k

C: Statutory Cutoffs at 600k & 650k

D: Statutory Cutoffs at 700k & 750k

E: Statutory Cutoffs at 850k, 900k & 950k

F: Statutory Cutoff at 1050k

Notes: the figure shows the density distribution of taxable income for wage earners before the reform (2006-07) on the left y-axis and after the reform (2008) on the right y-axis around notches and kinks. Each dot represents the upper bound of a 2500 Rupee bin and thus shows the number of taxpayers located within a 2500 Rupee range below the dot. Notches apply to 2006-07 and are shown by solid vertical lines, while kinks apply to 2008 and are shown by dashed lines (convex kinks) and dashed-dotted lines (non-convex kinks). Notches and convex kinks occur at statutory cutoffs in the tax law, whereas non-convex kinks are implicitly determined by tax minimization. At two cutoffs (400k,1050k), the solid vertical line is both a notch in 2006-07 and a convex kink in 2008. The average tax rate jump at 2006-07 notches is shown in Figure 3. The marginal tax rate change (jumps and drops) at 2008 kinks varies from just below 20%-points (bottom kinks in Panel A) to just above 30%-points (top kink in Panel B).
FIGURE 11
Shifting between Self-Employment and Wage Income:
Density Distribution of Self-Employment Income Share around 50% Notch

Panel A: All Taxpayers with Self-Employment Income Share in (0,1) Range

Panel B: Dropping Taxpayers at the Cutoff Value of 0.5

Notes: the figure shows the density distribution of the share of self-employment income in total income from self-employment and wages in 2006-08. Taxpayers are treated as wage earners (associated with a low tax) on the left side of the vertical line at 0.50 and as self-employed (associated with a high tax) on the right side of the vertical line. The notch point at 0.50 is itself part of the high-tax region. The density distribution is shown in bins of 0.025, where each bin excludes the upper bound of the interval such that the bin right below the notch spans [0.475;0.50) and the bin right above spans [0.50;0.525). Panel A considers all taxpayers who report strictly positive income amounts from both wages and self-employment. Panel B considers taxpayers who report strictly positive income amounts from both wages and self-employment, except those who report the exact same amount from the two sources (i.e., a self-employment income share equal to the cutoff-value of 0.50, which belongs to the high-tax region).
<table>
<thead>
<tr>
<th>Notch point</th>
<th>Jump in statutory ATR (%-points)</th>
<th># obs.</th>
<th>Elasticity w.r.t. statutory 1-ATR</th>
<th>Elasticity w.r.t. implicit 1-MTR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>100k</td>
<td>0.5</td>
<td>24,669</td>
<td>3.92</td>
<td>0.077</td>
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<td></td>
<td></td>
<td></td>
<td>(0.11)</td>
<td>(0.004)</td>
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<tr>
<td>110k</td>
<td>0.5</td>
<td>36,425</td>
<td>4.39</td>
<td>0.097</td>
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<tr>
<td></td>
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<td></td>
<td>(0.09)</td>
<td>(0.004)</td>
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<tr>
<td>125k</td>
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<td></td>
<td></td>
<td></td>
<td>(0.05)</td>
<td>(0.003)</td>
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<tr>
<td>150k</td>
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<td>25,884</td>
<td>2.99</td>
<td>0.091</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.06)</td>
<td>(0.003)</td>
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<tr>
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<td>(0.06)</td>
<td>(0.003)</td>
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<tr>
<td>200k</td>
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<td>7,715</td>
<td>0.91</td>
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<td></td>
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<td>(0.04)</td>
<td>(0.001)</td>
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<td>0.37</td>
<td>0.004</td>
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<td></td>
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<td></td>
<td>(0.03)</td>
<td>(0.001)</td>
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<td>800k</td>
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<td>2,907</td>
<td>0.19</td>
<td>0.001</td>
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<td>(0.000)</td>
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<td>1,884</td>
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<td></td>
<td></td>
<td></td>
<td>(0.03)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>1300k</td>
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<td>947</td>
<td>0.09</td>
<td>0.0004</td>
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<td></td>
<td></td>
<td></td>
<td>(0.02)</td>
<td>(0.0002)</td>
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Notes: This table presents estimates of the elasticity of taxable income at each notch of the tax schedule for self-employed individuals and unincorporated firms. The sample is restricted to sophisticated filers and cover years 2006-2008. Column (1) shows the statutory cutoffs in the tax schedule (in thousands of Pakistani Rupees), while column (2) shows the percentage-point jumps in the statutory average tax rate (ATR) at those cutoffs. Column (3) shows the number of taxpayers located on the bunching segment and surrounding segments used to estimate the counterfactual density and excess clustering at the notch (as described in section 2.2). The width of those segments is chosen separately for each notch based on the density graphs. Column (4) shows elasticities with respect to the statutory average net-of-tax rate 1-ATR, while column (5) shows elasticities with respect to the implicit marginal net-of-tax rate 1-MTR. Those elasticity concepts are presented in section 2.1. Standard errors are shown in parentheses and have been computed using the delta method. Elasticity estimates emphasized in bold are significant at the standard 5% level.
## TABLE 2
Elasticity Estimates for Self-Employed Individuals and Firms: Basic Filers

<table>
<thead>
<tr>
<th>Notch point</th>
<th>Counterfactual round point</th>
<th>Jump in statutory ATR (%-points)</th>
<th># obs.</th>
<th>Elasticity w.r.t. statutory 1-ATR</th>
<th>Elasticity w.r.t. implicit 1-MTR</th>
</tr>
</thead>
<tbody>
<tr>
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<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>110k</td>
<td>210k</td>
<td>0.5</td>
<td>201,433</td>
<td>1.51 (0.08)</td>
<td>0.011 (0.001)</td>
</tr>
<tr>
<td>125k</td>
<td>225k</td>
<td>1.0</td>
<td>158,354</td>
<td>3.11 (0.04)</td>
<td>0.097 (0.002)</td>
</tr>
<tr>
<td>175k</td>
<td>275k</td>
<td>1.0</td>
<td>36,758</td>
<td>2.74 (0.05)</td>
<td>0.078 (0.003)</td>
</tr>
<tr>
<td>200k</td>
<td>250k</td>
<td>1.0</td>
<td>26,257</td>
<td>1.04 (0.10)</td>
<td>0.011 (0.002)</td>
</tr>
<tr>
<td>300k</td>
<td>350k</td>
<td>2.5</td>
<td>11,725</td>
<td>1.37 (0.07)</td>
<td>0.050 (0.005)</td>
</tr>
<tr>
<td>400k</td>
<td>450k</td>
<td>2.5</td>
<td>3,927</td>
<td>0.65 (0.10)</td>
<td>0.012 (0.003)</td>
</tr>
<tr>
<td>500k</td>
<td>550k</td>
<td>2.5</td>
<td>2,547</td>
<td>0.35 (0.10)</td>
<td>0.003 (0.002)</td>
</tr>
<tr>
<td>600k</td>
<td>900k</td>
<td>2.5</td>
<td>1,060</td>
<td>0.35 (0.28)</td>
<td>0.004 (0.006)</td>
</tr>
</tbody>
</table>

Notes: This table presents estimates of the elasticity of taxable income at selected notches in the tax schedule for self-employed individuals and unincorporated firms. The sample is restricted to basic filers (those who report income in even thousands) and cover years 2006-2008. Column (1) shows notch points (in thousands of Pakistani Rupees), while column (2) shows counterfactual round points used to control for round-number bunching at notches (as described in section 4.1). Column (3) shows the percentage-point jumps in the statutory average tax rate (ATR) at notch points. Column (4) shows the number of taxpayers located on the bunching segment and surrounding segments used to estimate the counterfactual density and excess clustering at the notch (as described in section 2.2). The width of those segments is chosen separately for each notch based on the density graphs. Column (5) shows elasticities with respect to the statutory average net-of-tax rate 1-ATR, while column (6) shows elasticities with respect to the implicit marginal net-of-tax rate 1-MTR. Those elasticity concepts are presented in section 2.1. Standard errors are shown in parentheses and have been computed using the delta method. Elasticity estimates emphasized in bold are significant at the standard 5% level.
<table>
<thead>
<tr>
<th>Notch point</th>
<th>ATR jump (%-points)</th>
<th># Obs.</th>
<th>Elasticity w.r.t. 1-ATR</th>
<th>Elasticity w.r.t. 1-MTR</th>
<th># Obs.</th>
<th>Elasticity w.r.t. 1-ATR</th>
<th>Elasticity w.r.t. 1-MTR</th>
</tr>
</thead>
<tbody>
<tr>
<td>400k</td>
<td>1.0</td>
<td>19,102</td>
<td>0.179</td>
<td>0.0003</td>
<td>19,084</td>
<td>0.118</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.024)</td>
<td>(0.0001)</td>
<td></td>
<td>(0.023)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>500k</td>
<td>1.0</td>
<td>12,952</td>
<td>0.240</td>
<td>0.0006</td>
<td>12,291</td>
<td>0.091</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.025)</td>
<td>(0.0001)</td>
<td></td>
<td>(0.023)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>600k</td>
<td>1.5</td>
<td>8,353</td>
<td>0.134</td>
<td>0.0003</td>
<td>7,972</td>
<td>0.073</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.017)</td>
<td>(0.0001)</td>
<td></td>
<td>(0.016)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>700k</td>
<td>1.5</td>
<td>5,081</td>
<td>0.110</td>
<td>0.0002</td>
<td>4,845</td>
<td>0.069</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.018)</td>
<td>(0.0001)</td>
<td></td>
<td>(0.017)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>850k</td>
<td>1.5</td>
<td>5,882</td>
<td>0.091</td>
<td>0.0001</td>
<td>5,527</td>
<td>0.075</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.026)</td>
<td>(0.0000)</td>
<td></td>
<td>(0.026)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>950k</td>
<td>1.0</td>
<td>4,214</td>
<td>0.070</td>
<td>0.0001</td>
<td>4,074</td>
<td>0.043</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.038)</td>
<td>(0.0001)</td>
<td></td>
<td>(0.038)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>1050k</td>
<td>1.0</td>
<td>3,103</td>
<td>0.093</td>
<td>0.0001</td>
<td>3,042</td>
<td>0.098</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.041)</td>
<td>(0.0001)</td>
<td></td>
<td>(0.042)</td>
<td>(0.0001)</td>
</tr>
</tbody>
</table>

Notes: This table presents estimates of behavioural elasticities at selected notches in the tax schedule for wage earners in 2006-07. Column (1) shows selected notch points (in thousands of Pakistani Rupees), while column (2) shows the percentage-point jumps in the statutory average tax rate (ATR) at those notch points. Columns (3)-(5) consider taxable income responses (total responses) based on the density distribution of reported taxable income for all wage earners. Columns (6)-(8) consider gross wage income responses (real responses) based on the density distribution of (third-party reported) gross wage income for all wage earners. Taxable income equals gross wage income plus non-wage income (self-employment income, capital income and property income) minus deductions. Number of observations equals the number of taxpayers located on the bunching segment and surrounding segments used to estimate the counterfactual density and excess clustering underlying the elasticity estimates (as described in section 2.2). The width of those segments is chosen separately for each notch based on the density graphs. The table shows both elasticities with respect to the statutory average net-of-tax rate 1-ATR and with respect to the implicit marginal net-of-tax rate 1-MTR (as described in section 2.1). Standard errors are shown in parentheses and have been computed using the delta method. Elasticity estimates emphasized in bold are significant at the standard 5% level.
## TABLE 4
Determinants of Strictly Dominated Choice

<table>
<thead>
<tr>
<th></th>
<th>Self-employed &amp; firms</th>
<th></th>
<th>Wage earners</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.0361***</td>
<td>(0.0022)</td>
<td>0.0301***</td>
<td>(0.0021)</td>
</tr>
<tr>
<td>Registered 2006</td>
<td>-0.0064***</td>
<td>(0.0004)</td>
<td>-0.0020***</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>Registered 2007</td>
<td>-0.0050***</td>
<td>(0.0005)</td>
<td>-0.0089***</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>Suboptimal 2006</td>
<td>0.1301***</td>
<td>(0.0022)</td>
<td>0.0392***</td>
<td>(0.0023)</td>
</tr>
<tr>
<td>Suboptimal 2007</td>
<td></td>
<td></td>
<td>0.1127***</td>
<td>(0.0024)</td>
</tr>
<tr>
<td>Income quantile 25-50</td>
<td>-0.0283***</td>
<td>(0.0006)</td>
<td>-0.0271***</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>Income quantile 50-75</td>
<td>-0.0192***</td>
<td>(0.0006)</td>
<td>-0.0185***</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Income quantile 75-90</td>
<td>-0.0233***</td>
<td>(0.0007)</td>
<td>-0.0219***</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Income quantile 90-95</td>
<td>-0.0225***</td>
<td>(0.0010)</td>
<td>-0.0190***</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>Income quantile 95-100</td>
<td>-0.0231***</td>
<td>(0.0010)</td>
<td>-0.0214***</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>Firm</td>
<td>-0.0026***</td>
<td>(0.0008)</td>
<td>-0.0035***</td>
<td>(0.0007)</td>
</tr>
<tr>
<td>Capital &amp; property income</td>
<td>-0.0020</td>
<td>(0.0015)</td>
<td>-0.0061***</td>
<td>(0.0016)</td>
</tr>
<tr>
<td>Rural</td>
<td>-0.0043***</td>
<td>(0.0004)</td>
<td>-0.0030***</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>Male</td>
<td>-0.0027</td>
<td>(0.0022)</td>
<td>0.0022</td>
<td>(0.0021)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0276</td>
<td>0.0229</td>
<td>0.0128</td>
<td>0.0031</td>
</tr>
<tr>
<td>Mean of dependent variable</td>
<td>0.0102</td>
<td>0.0090</td>
<td>0.0139</td>
<td>0.0080</td>
</tr>
<tr>
<td># Obs.</td>
<td>241,525</td>
<td>259,299</td>
<td>360,677</td>
<td>376,747</td>
</tr>
</tbody>
</table>

Notes: the table reports coefficients of OLS regressions of a dummy for locating in a 500 rupee range above a notch on dummy covariates. The dummy covariates are (i) dummies for being a registered taxpayer in prior years, (ii) dummies for strictly dominated choice in prior years, (iii) income percentile dummies, (iv) firm dummy, (v) capital & property income dummy, (vi) rural region dummy, and (vii) male gender dummy. Standard errors are shown in parentheses. Bottom rows show R-squared, mean of dependent variable (unconditional probability of strictly dominated choice), and the number of observations.