Source: IRS, Statistics of Income Division, Historical Table 23
Source: statistics computed by the author
above $\bar{z}$ as depicted on Figure 1. This tax change has two effects on tax revenue. First, there is a mechanical effect, which is the change in tax revenue if there were no behavioural responses, and second, there is a reduction in tax revenue due to reduced earnings through behavioural responses. Let us examine these two effects successively.

- **Mechanical effect.**

  The mechanical effect (denoted by $M$) represents the increase in tax receipts if there were no behavioural responses. A taxpayer with income $z$ (above $\bar{z}$) would pay $(z - \bar{z})d\tau$ additional taxes. Therefore, summing over the population above $\bar{z}$ and denoting the mean of incomes above $\bar{z}$ by $z_m$, the total mechanical effect $M$ is equal to

  \[
  M = [z_m - \bar{z}]d\tau. \tag{5}
  \]

- **Behavioural responses.**

  As shown in Figure 1, the tax change can be decomposed into two parts; first, an overall *uncompensated* increase $d\tau$ in marginal rates (starting from 0 and not just from $\bar{z}$), second, an overall increase in virtual income $dR = zd\prime$. Therefore, an individual with income $z$ changes its earnings by

  \[
  dz = -\frac{\partial z}{\partial (1 - \tau)}d\tau + \frac{\partial z}{\partial R}dR = -(\xi z - \eta z)\frac{d\tau}{1 - \tau}, \tag{6}
  \]

  where we have used definitions (1) and (2). The reduction in income $dz$ displayed in equation (6) implies a reduction in tax receipts equal to $\tau dz$. The total reduction in tax receipts due to the behavioural responses is simply the sum of the terms $\tau dz$ over all
FIGURE 2 – Ratio mean income above z divided by z, \( \frac{z_m}{z} \), years 1992 and 1993

Source: Saez (2001), p. 211
Obtaining (15) in the context of the Mirrlees model is possible using the Mirrlees first-order condition. This derivation is presented in the Appendix. This rearrangement of terms of the Mirrlees formula is a generalization of the one developed by Diamond (1998) in the case of quasi-linear utility functions. This method, however, does not show the economic effects which lead to formula (14). Formula (14) can, however, be fruitfully derived directly in terms of elasticities using the same method as in Section 3. The formula is commented in the light of this direct derivation just after the proof.

**Direct proof of Proposition 1.** I consider the effect of the following small tax reform perturbation around the optimal tax schedule. As depicted on Figure 3, marginal rates are increased by an amount $d\tau$ for incomes between $z^*$ and $z^* + dz^*$. I also assume that $d\tau$ is second order compared to $dz^*$ so that bunching (and inversely gaps in the income distribution) around $z^*$ or $z^* + dz^*$ induced by the discontinuous change in marginal rates are negligible. This tax reform has three effects on tax receipts: a mechanical effect, an elasticity effect for taxpayers with income between $z^*$ and $z^* + dz^*$, and an income effect for taxpayers with income above $z^*$.

- **Mechanical effect net of welfare loss.**

As shown in Figure 3, every taxpayer with income $z$ above $z^*$ pays $d\tau dz^*$ additional taxes which are valued $(1 - g(z))d\tau dz^*$ by the government therefore the overall mechanical

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17. Revesz (1989) has also attempted to express the optimal non-linear tax formula of Mirrlees in terms of elasticities. His derivation is similar in spirit to the one presented in the Appendix.
FIGURE 4 – Hazard Ratio \((1-H(z))/(zh(z))\), years 1992 and 1993

Source: Saez (2001), p. 219
FIGURE 5 – Optimal Tax Simulations

Utilitarian Criterion, Utility type I

[Graph showing marginal tax rate for different utility types and income levels.]

Utilitarian Criterion, Utility type II

[Graph showing marginal tax rate for different utility types and income levels.]
Reform: Increase $\tau_1$ by $d\tau_1$ and $c_0$ by $dc_0= z_1 d\tau_1$

1) Mechanical fiscal cost: $dM= -H_0 dc_0 = -H_0 z_1 d\tau_1$
2) Welfare effect: $dW= g_0 H_0 dc_0 = g_0 H_0 z_1 d\tau_1$
3) Fiscal cost due to behavioral responses:

$$dB= -dH_0 \tau_1 z_1 = d\tau_1 e_0 H_0 \tau_1 / (1-\tau_1) z_1$$

Optimal phase-out rate $\tau_1$:

$$dM+dW+dB=0 \Rightarrow \tau_1 / (1-\tau_1) = (g_0-1)/e_0$$
Starting from a Means-Tested Program

Source: revised version of Saez (2002), p. 1050
Introducing a small EITC is desirable for redistribution.

Starting from a Means-Tested Program

Source: revised version of Saez (2002), p. 1050
Introducing a small EITC is desirable for redistribution.

Starting from a Means-Tested Program

Participation response saves government revenue

Source: revised version of Saez (2002), p. 1050
Figure 3a: Optimal Tax/Transfer Derivation

Consumption $c$

Wage $w$

$45^\circ$

Source: revised version of Saez (2002), p. 1052
Figure 3a: Optimal Tax/Transfer Derivation (assuming $g_1 > 1$)

Consumption $c$

Wage $w$

Welfare Effect: $h_1 g_1 dc_1 > 0$

Fiscal Effect: $-h_1 dc_1 < 0$

Source: revised version of Saez (2002), p. 1052
Figure 3a: Optimal Tax/Transfer Derivation (assuming $g_1 > 1$)

Net Welfare effect: $h_1 dc_1 (g_1 - 1) > 0$

Labor Supply: $dh_1 w_1 \tau_1 < 0$

Source: revised version of Saez (2002), p. 1052
Figure 3a: Optimal Tax/Transfer Derivation (assuming $g_1 > 1$)

At the optimum:
\[ dh_1 w_1 \tau_1 + h_1 dc_1 (g_1 - 1) = 0 \]
implies
\[ \tau_1 / (1 - \tau_1) = (1 - g_1) / e_1 < 0 \]

Net Welfare effect: $h_1 dc_1 (g_1 - 1) > 0$

Source: revised version of Saez (2002), p. 1052
2. Optimal Tax/Transfer System (no min wage)

Source: Lee and Saez (2008)
2. Set Min wage $\bar{w}=w_1$ and increase $c_1$ by $dc_1$

Welfare Effect > Direct Fiscal Effect
if govt values redistribution to low skill workers

Source: Lee and Saez (2008)
2. Desirability of Min Wage with Optimal Taxes

Consumption $c$

Wage $w$

$45^\circ$

$w=w_1$

$c_0$

$c_1$

$c_2$

$c_1 + dc_1$

Welfare Effect > Direct Fiscal Effect if govt values redistribution to low skill workers

dc$_1$>0 makes low skilled job $w_1$ more attractive → would reduce $w_1$ through demand effects

Source: Lee and Saez (2008)
2. Desirability of Min Wage with Optimal Taxes

Consumption \( c \)

Wage \( w \)

\( c_0 \) \( c_1 \) \( c_2 \)

With min wage set at \( w_1 \), \( dc_1 > 0 \) does not affect labor supply because \( w_1 \) cannot go down

\( \rightarrow \) No indirect fiscal effect

\( \rightarrow \) Welfare increases

Welfare Effect > Direct Fiscal Effect

if govt values redistribution to low skill workers

Source: Lee and Saez (2008)
3. Pareto Improving Policy when $\tau_1 > 0$ and min wage binds

$\tau_1 > 0 = $ Tax on low skilled work: $c_1 - c_0 < \bar{w}$

Source: Lee and Saez (2008)
3. Pareto Improving Policy when $\tau_1 > 0$ and min wage binds

Reduce $\bar{w}$ while keeping $c_1$, $c_2$ constant:

No direct fiscal effect of $d\bar{w}$, $dw_2$ as $h_1 d\bar{w} + h_2 dw_2 = 0$ (no profits) and tax = $\bar{w} - c_1$ $h_1 + (w_2 - c_2) h_2$

Source: Lee and Saez (2008)
3. Pareto Improving Policy when $\tau_1>0$ and min wage binds

Consumption $c$

Wage $w$

Unemployment decreases $\rightarrow$
New Workers better off and pay more taxes $\rightarrow$ Pareto Improvement

Reduce $\bar{w}$ while keeping $c_1$, $c_2$ constant:
No direct fiscal effect of $d\bar{w}$, $dw_2$ as $h_1d\bar{w}+h_2dw_2=0$ (no profits)
and tax=$(\bar{w}-c_1)h_1+(w_2-c_2)h_2$

Source: Lee and Saez (2008)
Optimal Top Income Tax Rate (Mirrlees ’71 model)

Disposable Income $c = z - T(z)$

Top bracket: Slope $1 - \tau$

Reform: Slope $1 - \tau - d\tau$

Source: Diamond and Saez JEP’11
Optimal Top Income Tax Rate (Mirrlees ’71 model)

Disposable Income
\[ c = z - T(z) \]
Market income \( z \)
\[ z^* \]
\[ z^* - T(z^*) \]

Mechanical tax increase:
\[ d\tau[z-z^*] \]

Behavioral Response tax loss:
\[ \tau \ dz = - \ d\tau \ e^{z \ \tau/(1-\tau)} \]

Source: Diamond and Saez JEP'11
Empirical Pareto Coefficient $\alpha = \frac{z^* h(z^*)}{1 - H(z^*)}$

$z^* = \text{Adjusted Gross Income (current 2005 $)}$

$a = \frac{zm}{zm - z^*}$ with $zm = E(z|z > z^*)$

Source: Diamond and Saez JEP'11
A. Top 1% Income Shares and Top MTR

Source: Piketty, Saez, and Stantcheva NBER'11
B. Top 1% Income Shares and Top MTR

Source: Piketty, Saez, and Stantcheva NBER'11
C. Top 1% and Bottom 99% Income Growth

Source: Piketty, Saez, and Stantcheva NBER'11
A. Top 1% Share and Top Marginal Tax Rate in 1975–9

Source: Piketty, Saez, and Stantcheva NBER'11
B. Top 1% Share and Top Marginal Tax Rate in 2004–8

Source: Piketty, Saez, and Stantcheva NBER'11
A. Changes Top 1% Share and Top Marginal Tax Rate

Source: Piketty, Saez, and Stantcheva NBER'11
B. Growth and Change in Top Marginal Tax Rate

Source: Piketty, Saez, and Stantcheva NBER'11
Disposable Income: $c = z - T(z)$

Pre-tax income: $z$

Mechanical tax increase: $d\tau dz [1-H(z)]$

Social welfare effect: $-d\tau dz [1-H(z)] G(z)$

Behavioral response: $\delta z = -d\tau e^{z/(1-T'(z))}$

$\rightarrow$ Tax loss: $T'(z) \delta z h(z) dz$

$= -h(z) e^{z} T'(z)/(1-T'(z))$ dzd$\tau$

Small band $(z, z+dz)$: slope $1 - T'(z)$

Reform: slope $1 - T'(z) - d\tau$

Source: Diamond and Saez JEP'11
Reform: Increase $\tau_1$ by $d\tau_1$ and $c_0$ by $dc_0 = z_1 d\tau_1$

$g_0 >> 1 \rightarrow$ welfare effect $>>$ mechanical fiscal cost

Source: Diamond and Saez JEP'11
Reform: Increase $\tau_1$ by $d\tau_1$ and $c_0$ by $dc_0 = z_1 d\tau_1$

$g_0 >> 1 \implies$ welfare effect $>>$ mechanical fiscal cost

Fiscal cost due to behavioral responses proportional to $\tau_1/(1-\tau_1)$ and elasticity $e_0 = (1-\tau_1)/H_0 \frac{dH_0}{d(1-\tau_1)}$

Optimal phase-out rate $\tau_1$:

$\tau_1 = \frac{g_0-1}{g_0-1+ e_0}$

Example: if $g_0=3$ and $e_0=0.5$, $\tau_1=80\%$

Source: Diamond and Saez JEP'11
Reform: Increase $\tau_1$ by $d\tau_1$ and $c_0$ by $dc_0 = z_1 d\tau_1$

1) Mechanical fiscal cost: $dM = -H_0 dc_1 = -H_0 z_1 d\tau_1$
2) Welfare effect: $dW = g_0 H_0 dc_1 = g_0 H_0 z_1 d\tau_1$
3) Fiscal cost due to behavioral responses:
   
   $$dB = -dH_0 \tau_1 z_1 = d\tau_1 e_0 H_0 \tau_1 / (1-\tau_1) z_1$$

Optimal phase-out rate $\tau_1$:

$$dM + dW + dB = 0 \Rightarrow \frac{\tau_1}{1-\tau_1} = \frac{(g_0-1)}{e_0}$$
Starting from a positive phasing-out rate $\tau_1 > 0$:

1) Increasing transfers by $dc_1$ at $z_1$ is desirable for redistribution: net effect $(g_1-1)h_1 \, dc_1 > 0$ if $g_1 > 1$

2) Participation response saves government revenue

$$\tau_1 \, z_1 \, dh_1 = e_1 \, \tau_1/(1-\tau_1) \, h_1 \, dc_1 > 0$$

→ Win-win reform … if intensive response is small

Optimal phase-out rate $\tau_1$:

$$(g_1-1)h_1 \, dc_1 + e_1 \, \tau_1/(1-\tau_1) \, h_1 \, dc_1 = 0$$

→ $\tau_1/(1-\tau_1) = (1-g_1)/e_1 < 0$ if $g_1 > 1$
EITC Amount as a Function of Earnings

Earnings ($)
0 5000 10000 15000 20000 25000 30000 35000 40000

Subsidy: 34%
Phase-out tax: 16%
Single, 2+ kids
Married, 2+ kids
Single, 1 kid
Married, 1 kid
No kids

EITC Amount ($)
0 1000 2000 3000 4000 5000

Source: Federal Govt
Source: Piketty, Thomas, and Emmanuel Saez (2012)
Table 2: Equality of Opportunity vs. Utilitarian Optimal Tax Rates

<table>
<thead>
<tr>
<th>Income percentile</th>
<th>Equality of Opportunity</th>
<th>Utilitarian (log-utility)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction from low background (parents below median) above each percentile</td>
<td>Implied social welfare weight G(z) above each percentile</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
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<tr>
<td>z= 25th percentile</td>
<td>44.3%</td>
<td>0.886</td>
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<tr>
<td>z= 50th percentile</td>
<td>37.3%</td>
<td>0.746</td>
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<tr>
<td>z= 75th percentile</td>
<td>30.3%</td>
<td>0.606</td>
</tr>
<tr>
<td>z= 90th percentile</td>
<td>23.6%</td>
<td>0.472</td>
</tr>
<tr>
<td>z= 99th percentile</td>
<td>17.0%</td>
<td>0.340</td>
</tr>
<tr>
<td>z= 99.9th percentile</td>
<td>16.5%</td>
<td>0.330</td>
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Notes: This table compares optimal marginal tax rates at various percentiles of the distribution (listed by row) using an equality of opportunity criterion (in column (3)) and a standard utilitarian criterion (in column (5)). Both columns use the optimal tax formula $T'(z) = \frac{1-G(z)}{1-G(z)+\alpha(z)\cdot e}$ discussed in the text where $G(z)$ is the average social marginal welfare weight above income level $z$, $\alpha(z) = \frac{zh(z)}{1-H(z)}$ is the local Pareto parameter (with $h(z)$ the density of income at $z$, and $H(z)$ the cumulative distribution), and $e$ the elasticity of reported income with respect to $1-T'(z)$. We assume $e=0.5$. We calibrate $\alpha(z)$ using the actual distribution of income based on 2008 income tax return data. For the equality of opportunity criterion, $G(z)$ is the representation index of individuals with income above $z$ who come from a disadvantaged background (defined as having a parent with income below the median). This representation index is estimated using the national intergenerational mobility statistics of Chetty et al. (2013) based on all US individuals born in 1980-1 with their income measured at age 30-31. For the utilitarian criterion, we assume a log-utility so that the social welfare weight $g(z)$ at income level $z$ is proportional to $\frac{1}{(z-T(z))}$.

Source: Saez and Stantcheva (2014)
Figure 1
EITC refunds by family size and income (CBPP 2013)

Source: Center on Budget and Policy Priorities.
1. Optimal Labor Income Taxation
T(z) is continuous in z.
Marginal Income Tax

T'(z) is a step function

10%  15%  39.6%
taxable income z
Budget Set

c = z - T(z)

after-tax and transfer income

slope = 1 - T'(z)

pre-tax income z
\[ c = z - T(z) \]

\[ \tau_p = \text{participation tax rate} \]

\[ (1 - \tau_p)z \]

\[ -T(0) \]

45°
Marshallian Labor Supply

\[ l(w, R) \]

Indifference Curve

Consumption

\[ c = c = wl + R \]

Labor Supply Theory

\[ \text{slope} = w \]

\[ 0 \]

\[ l^* \]

\[ \text{labor supply } l \]
Hicksian Labor Supply $l^c(w, u)$
Labor Supply Income Effect

\[ \eta = w \frac{\partial l}{\partial R} \leq 0 \]
Labor Supply Substitution Effect

\[ \varepsilon^c = \frac{w \partial l^c}{l \partial w} > 0 \]
Uncompensated Labor Supply Effect

\[ \varepsilon^u = \varepsilon^c + \eta \]

- slope = \( w + \Delta w \)
- income effect: \( \eta \leq 0 \)
- substitution effect: \( \varepsilon^c > 0 \)

\[ l(w, R) \]

\[ l(w + \Delta w, R) \]

labor supply \( l \)
Effect of Tax on Labor Supply

\[ c = z - T(z) \]

- \( T(z) < 0: \) income effect \( l \downarrow \)
- \( T'(z) > 0: \) substitution effect \( l \downarrow \)
- \( T(z) > 0: \) income effect \( l \uparrow \)
- \( T'(z) > 0: \) substitution effect \( l \downarrow \)

Slope = 1 - \( T'(z) \)

45°
Laffer Curve

\[ R = \tau \cdot Z(1 - \tau) \]

\[ \tau^* = \frac{1}{1 + e} \text{ with } e = \frac{1 - \tau}{Z} \cdot \frac{dZ}{d(1 - \tau)} \]
Utilitarianism and Redistribution

\[ u \left( \frac{c_1 + c_2}{2} \right) \]

\[ u(c_1) + u(c_2) \]

\[ \frac{c_1 + c_2}{2} \]
Labor Supply Theory

\[ c = (1-t)z + R \]

Marshallian Labor Supply
\[ z(1-\tau, R) \]

Indifference Curves

\[ \text{Slope} = 1-\tau \]

Consumption
\[ c = \text{consumption} \]
Labor Supply Theory

c = consumption

utility \( u \)

Slope = 1 - \( \tau \)

Hicksian Labor Supply

\( z^c(1-\tau,u) \)

earnings supply \( z \)
Labor Supply Income Effect

\[ \eta = (1 - t) \frac{\partial z}{\partial R} \leq 0 \]
Labor Supply Substitution Effect

utility \( u \)

slope = 1 - \( \tau \)

\[ \varepsilon^c = \frac{(1-\tau)}{z} \frac{\partial z^c}{\partial (1-\tau)} > 0 \]

\[ z^c(1-\tau, u) \]

\[ z^c(1-\tau + d\tau, u) \]
Uncompensated Labor Supply Effect

Slutsky equation: $\varepsilon^u = \varepsilon^c + \eta$

- Income effect: $\eta \leq 0$
- Substitution effect: $\varepsilon^c > 0$

Earnings $z$

$\tau$, $\omega$, $\varepsilon$, $\eta$, $\gamma$, $\Omega$, $\varphi$, and $\mu$ are symbols used in the equation.
Effect of Tax on Labor Supply

\[ \tau = z - T(z) \]

- **T(z) < 0:** income effect \( z \downarrow \)
- **T'(z) > 0:** substitution effect \( z \downarrow \)
- **T(z) > 0:** income effect \( z \uparrow \)
- **T'(z) > 0:** substitution effect \( z \downarrow \)

Slope: \( 1 - T'(z) \)

Pre-tax income \( z \)

Income effect: \( z \downarrow \)
Substitution effect: \( z \downarrow \)

45° line