Graduate Public Economics
Optimal Labor Income Taxes/Transfers

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TAXATION AND REDISTRIBUTION

Key question: Should government reduce inequality using taxes and transfers?

1) Governments use taxes to raise revenue

2) This revenue funds transfer programs:

a) Universal Transfers: Education, Health Care (only 65+ in the US), Retirement and Disability

b) Means-tested Transfers: In-kind (e.g., public housing, nutrition, Medicaid in the US) and cash (direct welfare and refundable tax credits)

Means-tested transfers relatively small relative to universal transfers

This lecture follows Piketty and Saez ’13 handbook chapter
FACTS ON US TAXES AND TRANSFERS

References: Comprehensive description in Gruber undergrad textbook (taxes/transfers) and Slemrod-Bakija (taxes)

http://www.taxpolicycenter.org/taxfacts/

A) Taxes: (1) individual income tax (fed+state), (2) payroll taxes on earnings (fed, funds Social Security+Medicare), (3) corporate income tax (fed+state), (4) sales taxes (state)+excise taxes (state+fed), (5) property taxes (state)

B) Means-tested Transfers: (1) refundable tax credits (fed), (2) in-kind transfers (fed+state): Medicaid, public housing, nutrition (SNAP), education (3) cash welfare: TANF for single parents (fed+state), SSI for old/disabled (fed)
FEDERAL US INCOME TAX

US income tax assessed on annual family income (not individual) [most other OECD countries have shifted to individual assessment]

Sum all cash income sources from family members (both from labor and capital income sources) = called Adjusted Gross Income (AGI)

Main exclusions: fringe benefits (health insurance, pension contributions and returns), imputed rent of homeowners, undistributed corporate profits, unrealized capital gains, interest from state+local bonds

⇒ AGI base is only 70% of factor national income
FEDERAL US INCOME TAX

Taxable income = AGI - deductions

deduction is max of standard deduction or itemized deductions

Standard deduction is a fixed amount ($12K for singles, $24K for married couple)

Itemized deductions: (a) state and local taxes paid (only up to $10K since 2018), (b) mortgage interest payments (capped), (c) charitable giving, various small other items

[about 10% of AGI lost through itemized deductions, called tax expenditures]

In 2018+, only 10% of tax filers itemize (30% before 2018)
FEDERAL US INCOME TAX: TAX BRACKETS

Tax $T(z)$ is piecewise linear and continuous function of taxable income $z$ with constant marginal tax rates (MTR) $T'(z)$ by brackets.

In 2018+, 6 brackets with MTR 10%, 12%, 22%, 24%, 32%, 35%, 37% (top bracket for $z$ above $600K), indexed on price inflation.

Lower preferential rates (up to a max of 20%) apply to dividends (since 2003), realized capital gains [in part to offset double taxation of corporate profits].

20% of business profits are exempt since 2018.

Tax rates change frequently over time. Top MTRs have declined drastically since 1960s (as in many OECD countries).
T(z) is continuous in z

slope 10%
slope 12%
slope 37%
Marginal Income Tax

$T'(z)$ is a step function

0  taxable income $z$

$10\%$

$12\%$

$37\%$
Historically, a 70 percent marginal tax rate is not unusual

The top marginal income tax rates from 1913 to 2018

1981
Reagan took office
**FEDERAL US INCOME TAX: AMT AND CREDITS**

**Alternative minimum tax (AMT)** is a parallel tax system (quasi flat tax at 28%) with fewer deductions: actual tax $= \max(T(z), AMT)$ (hits < 1% of taxpayers in 2018+)

**Tax credits:** Additional reduction in taxes

(1) **Non refundable** (cannot reduce taxes below zero): foreign tax credit, child care expenses, education credits, energy credits

(2) **Refundable** (can reduce taxes below zero, i.e., be net transfers): EITC (earned income tax credit, up to $3.5K, $5.7K, $6.5K for working families with 1, 2, 3+ kids), Child Tax Credit ($2K per kid, partly refundable)

Refundable credits have become the largest means-tested cash transfer in the US
FEDERAL US INCOME TAX: TAX FILING

Taxes on year $t$ earnings are withheld on paychecks during year $t$ (pay-as-you-earn)

Income tax return filed in late January-April 15th, year $t + 1$ [filers use either software or tax preparers, huge private industry]

Most tax filers get a tax refund as withholdings $>\text{net taxes owed}$

Payers (employers, banks, etc.) send income information to IRS (US tax administration) (3rd party reporting)

Third party reporting + withholding at source is key for successful enforcement
MAIN MEANS-TESTED TRANSFER PROGRAMS

1) **Traditional transfers:** managed by welfare agencies, paid on monthly basis, high stigma and take-up costs ⇒ low take-up rates

Main programs: Medicaid (health insurance for low incomes), SNAP (former food stamps), public housing, TANF (traditional welfare), SSI (aged+disabled)

2) **Refundable income tax credits:** managed by tax administration, paid as an annual lumpsum in year $t+1$, low stigma and take-up cost ⇒ high take-up rates

Main programs: EITC and Child Tax Credit [large expansion since the 1990s] for low income working families with children
The Phase-In and Phaseout of the EITC

Credit Amount by Marital Status and Number of Children

BOTTOM LINE ON ACTUAL TAXES/TRANSFERS

1) Based on current income, family situation, and disability (retirement) status ⇒ Strong link with **current ability to pay**

2) Some allowances made to reward / encourage certain behaviors: charitable giving, home ownership, savings, energy conservation, and more recently work (refundable tax credits such as EITC)

3) Provisions pile up overtime making tax/transfer system more and more complex until significant simplifying reform happens (such as US Tax Reform Act of 1986, or TCJA 2018)
**KEY CONCEPTS FOR TAXES/TRANSFERS**

1) Transfer benefit with zero earnings $-T(0)$ [sometimes called demogrant or lumpsum grant]

2) Marginal tax rate (or phasing-out rate) $T'(z)$: individual keeps $1 - T'(z)$ for an additional $1$ of earnings (intensive labor supply response)

3) Participation tax rate $\tau_p = \frac{T(z) - T(0)}{z}$: individual keeps fraction $1 - \tau_p$ of earnings when moving from zero earnings to earnings $z$ (extensive labor supply response):

$$z - T(z) = -T(0) + z - [T(z) - T(0)] = -T(0) + z \cdot (1 - \tau_p)$$

4) Break-even earnings point $z^*$: point at which $T(z^*) = 0$
$c = z - T(z)$

after-tax and transfer income

Budget Set

slope = $1 - T'(z)$
\[ c = z - T(z) \]

\[ \tau_p = \text{participation tax rate} \]

\[ (1 - \tau_p)z \]

- \[ T(0) \]

\[ 45^\circ \]

pre-tax income \( z \)
OPTIMAL TAXATION: SIMPLE MODEL WITH NO BEHAVIORAL RESPONSES

Utility $u(c)$ strictly increasing and concave

Same for everybody where $c$ is after tax income.

Income is $z$ and is fixed for each individual, $c = z - T(z)$ where $T(z)$ is tax on $z$. $z$ has density distribution $h(z)$

Government maximizes Utilitarian objective:

$$\int_0^\infty u(z - T(z))h(z)dz$$

subject to budget constraint $\int T(z)h(z)dz \geq E$ (multiplier $\lambda$)
SIMPLE MODEL WITH NO BEHAVIORAL RESPONSES

Form lagrangian: \( L = [u(z - T(z)) + \lambda \cdot T(z)] \cdot h(z) \)

First order condition (FOC) in \( T(z) \):
\[
0 = \frac{\partial L}{\partial T(z)} = [-u'(z - T(z)) + \lambda] \cdot h(z) \Rightarrow u'(z - T(z)) = \lambda
\]
\( \Rightarrow z - T(z) = \text{constant for all } z. \)

\( \Rightarrow c = \bar{z} - E \) where \( \bar{z} = \int zh(z)dz \) average income.

100% marginal tax rate. Perfect equalization of after-tax income.

Utilitarianism with decreasing marginal utility leads to perfect egalitarianism [Edgeworth, 1897]
Utilitarianism and Redistribution

\[
\text{Utility} = u(c_{1} + c_{2})
\]

\[
\frac{u(c_{1}) + u(c_{2})}{2}
\]

\[
\mathcal{U}(c_{1} + c_{2})
\]

\[
\mathcal{U}(c_{1}) + \mathcal{U}(c_{2})
\]

\[
0 \quad c_{1} \quad \frac{c_{1} + c_{2}}{2} \quad c_{2}
\]

\[
\text{Consumption } c
\]
ISSUES WITH SIMPLE MODEL

1) **No behavioral responses:** Obvious missing piece: 100% redistribution would destroy incentives to work and thus the assumption that $z$ is exogenous is unrealistic.

   $\Rightarrow$ Optimal income tax theory incorporates behavioral responses $u(c,z)$ (Mirylls REStud ’71): **equity-efficiency trade-off**

2) **Issue with Utilitarianism:** Even absent behavioral responses, many people would object to 100% redistribution [perceived as confiscatory]

   $\Rightarrow$ Citizens’ views on fairness impose **bounds** on redistribution

We will discuss at the end of the lecture alternatives to utilitarianism (and be agnostic for now on how society’s preferences for redistribution are shaped)
1) **Standard labor supply model**: Individual maximizes \( u(c, l) \) subject to \( c = wl - T(wl) \) where \( c \) consumption, \( l \) labor supply, \( w \) wage rate, \( T(.) \) nonlinear income tax \( \Rightarrow \) taxes affect labor supply

2) **Individuals differ in ability** \( w \): \( w \) distributed with density \( f(w) \).

3) **Govt social welfare maximization**: Govt maximizes

\[
SWF = \int G(u(c, l))f(w)\,dw
\]

\((G(.) \uparrow \text{concave})\) subject to

(a) budget constraint \( \int T(wl)f(w)\,dw \geq E \) (multiplier \( \lambda \))

(b) individuals’ labor supply \( l \) depends on \( T(.) \)
MIRRLEES MODEL RESULTS (skip)

Optimal income tax trades-off redistribution and efficiency (as tax based on $w$ only not feasible)

$\Rightarrow T(.) < 0$ at bottom (transfer) and $T(.) > 0$ further up (tax)
[full integration of taxes/transfers]

MIRRLEES formulas complex, only a couple fairly general results:

1) $0 \leq T'(.) \leq 1$, $T'(.) \geq 0$ is non-trivial (rules out EITC) [Seade '77]

2) Marginal tax rate $T'(.)$ should be zero at the top (if skill distribution bounded) [Sadka '76-Seade '77]

3) If everybody works and lowest $wl > 0$, $T'(.) = 0$ at bottom...
MIRRLEES '71 had a huge impact on information economics: models with asymmetric information in contract theory.

Discrete 2-type version of Mirrlees model developed by Stiglitz JpubE ’82 with individual FOC replaced by Incentive Compatibility constraint [high type should not mimic low type]

Till late 1990s, Mirrlees results not closely connected to empirical tax studies and little impact on tax policy recommendations

Since late 1990s, Diamond AER’98, Piketty ’97, Saez ReStud ’01 have connected Mirrlees model to practical tax policy / empirical tax studies

[new approach summarized in Diamond-Saez JEP’11 and Piketty-Saez Handbook’13]
max \_{c,z} u(c, z) \text{ subject to } c = z \cdot (1 - \tau) + R

R \text{ is virtual income and } \tau \text{ marginal tax rate. FOC in } c, z \Rightarrow (1 - \tau)u_c + u_z = 0 \Rightarrow \text{Marshallian labor supply } z = z(1 - \tau, R)

Uncompensated elasticity \quad \varepsilon^u = \frac{(1 - \tau) \partial z}{z \partial (1 - \tau)}

Income effects \quad \eta = (1 - \tau) \frac{\partial z}{\partial R} \leq 0

Substitution effects: Hicksian labor supply: \quad z^c(1 - \tau, u) \text{ minimizes cost needed to reach } u \text{ given slope } 1 - \tau \Rightarrow

Compensated elasticity \quad \varepsilon^c = \frac{(1 - \tau) \partial z^c}{z \partial (1 - \tau)} > 0

Slutsky equation \quad \frac{\partial z}{\partial (1 - \tau)} = \frac{\partial z^c}{\partial (1 - \tau)} + z \frac{\partial z}{\partial R} \Rightarrow \varepsilon^u = \varepsilon^c + \eta
Labor Supply Theory

\[ R \]
\[ z(1-\tau, R) \]

Indifference Curves

Marshallian Labor Supply

\[ c = (1-t)z + R \]

Slope = 1 - \( \tau \)

consumption

0
earnings supply \( z \)
Labor Supply Theory

\[ c = \text{consumption} \]

utility \( u \)

Slope = 1 - \( \tau \)

Hicksian Labor Supply

\[ z^c(1-\tau,u) \]

earnings supply \( z \)
Labor Supply Income Effect

\[ \eta = (1-t) \frac{\partial z}{\partial R} \leq 0 \]

\[ z(1-\tau, R) \]

\[ z(1-\tau, R+\Delta R) \]
Labor Supply Substitution Effect

utility $u$

slope $= 1 - \tau + d\tau$

$\varepsilon^c = \frac{(1-\tau)}{z} \partial z^c / \partial (1-\tau) > 0$

slope $= 1 - \tau$

$z^c(1-\tau,u)$

$z^c(1-\tau+d\tau,u)$

Earnings $z$

$c$
Uncompensated Labor Supply Effect

Slutsky equation: $\varepsilon^u = \varepsilon^c + \eta$

slope = $1 - \tau + d\tau$

income effect $\eta \leq 0$

substitution effect: $\varepsilon^c > 0$

Earnings $z$

$c$

$R$

$0$
Labor Supply Effects of Taxes and Transfers

Taxes and transfers change the slope $1 - T'(z)$ of the budget constraint and net disposable income $z - T(z)$ (relative to the no tax situation where $c = z$)

Positive MTR $T'(z) > 0$ reduces labor supply through substitution effects

Net transfer ($T(z) < 0$) reduces labor supply through income effects

Net tax ($T(z) > 0$) increases labor supply through income effects
Effect of Tax on Labor Supply

\[ c = z - T(z) \]

- \( T(z) < 0 \): income effect, \( z \downarrow \)
- \( T'(z) > 0 \): substitution effect, \( z \downarrow \)
- \( T(z) > 0 \): income effect, \( z \uparrow \)
- \( T'(z) > 0 \): substitution effect, \( z \downarrow \)

Slope: \( 1 - T'(z) \)
WELFARE EFFECT OF SMALL TAX REFORM

Indirect utility: \( V(1 - \tau, R) = \max_z u((1 - \tau)z + R, z) \) where \( R \) is virtual income intercept

Small tax reform: \( d\tau \) and \( dR \):

\[
dV = u_c \cdot [-zd\tau + dR] + dz \cdot [(1 - \tau)u_c + u_z] = u_c \cdot [-zd\tau + dR]
\]

**Envelope theorem:** no effect of \( dz \) on \( V \) because \( z \) is already chosen to maximize utility \(((1 - \tau)u_c + u_z = 0)\)

\([-zd\tau + dR]\) is the **mechanical** change in disposable income due to tax reform

Welfare impact of a small tax reform is given by \( u_c \) times the money metric mechanical change in tax

Remains true for any nonlinear tax system \( T(z) \) (just need to look at \( dT(z) \), mechanical change in taxes)
SOCIAL WELFARE FUNCTIONS (SWF)

Welfarism = social welfare based solely on individual utilities

Any other social objective will lead to Pareto dominated outcomes in some circumstances (Kaplow and Shavell JPE’01)

Most widely used welfarist SWF:

1) Utilitarian: \( SWF = \int_i u^i \)

2) Rawlsian (also called Maxi-Min): \( SWF = \min_i u^i \)

3) \( SWF = \int_i G(u^i) \) with \( G(.) \uparrow \) and concave, e.g., \( G(u) = u^{1-\gamma}/(1 - \gamma) \) (Utilitarian is \( \gamma = 0 \), Rawlsian is \( \gamma = \infty \))

4) General Pareto weights: \( SWF = \int_i \mu_i \cdot u^i \) with \( \mu_i \geq 0 \) exogenously given
SOCIAL MARGINAL WELFARE WEIGHTS

Key sufficient statistics in optimal tax formulas are **Social Marginal Welfare Weights** for each individual:

Social Marginal Welfare Weight on individual $i$ is $g_i = \mu_i u_c^i / \lambda$ ($\lambda$ multiplier of govt budget constraint) measures $\$\text{ value for govt of giving}$ $\$1\text{ extra to person }i$

No income effects $\Rightarrow \int_i g_i = 1$: giving $\$1\text{ to all costs }$ $\$1\text{ (pop. has measure 1)}$ and increases SWF (in $\$\text{ terms})\text{ by } \int_i g_i$

$g_i$ typically depend on tax system (endogenous variable)

Utilitarian case: $g_i$ decreases with $z_i$ due to decreasing marginal utility of consumption

Rawlsian case: $g_i$ concentrated on most disadvantaged (typically those with $z_i = 0$)
OPTIMAL LINEAR TAX RATE: LAFFER CURVE

\( c = (1 - \tau) \cdot z + R \) with \( \tau \) linear tax rate and \( R \) demogrant funded by taxes \( \tau Z \) with \( Z \) aggregate earnings

Population of size one (continuum) with heterogeneous preferences \( u^i(c, z) \) [differences in earnings ability are built in utility function]

Individual \( i \) chooses \( z \) to maximize \( u^i((1 - \tau) \cdot z + R, z) \) labor supply choices \( z^i(1 - \tau, R) \) aggregate to economy wide earnings \( Z(1 - \tau) = \int z^i \)

Tax Revenue \( R(\tau) = \tau \cdot Z(1 - \tau) \) is inversely U-shaped with \( \tau \): \( R(\tau = 0) = 0 \) (no taxes) and \( R(\tau = 1) = 0 \) (nobody works): called the Laffer Curve
OPTIMAL LINEAR TAX RATE: LAFFER RATE

Top of the Laffer Curve corresponds to tax rate $\tau^*$ maximizing tax revenue: inefficient to have $\tau > \tau^*$

$$R'(\tau^*) = Z - \tau^* dZ/d(1 - \tau) = 0 \Rightarrow$$

Revenue maximizing tax rate (Laffer rate):

$$\tau^* = \frac{1}{1 + e} \quad \text{with} \quad e = \frac{1 - \tau}{Z} \frac{dZ}{d(1 - \tau)}$$

elasticity of earnings with respect to the net-of-tax rate

$\tau > \tau^*$ is (second-best) Pareto inefficient: cutting $\tau$ increases individuals welfare and government revenue (and hence $R$)

$\tau = \tau^*$ is the optimum in Rawlsian case (if some people have zero earnings)
R = \tau \cdot Z(1 - \tau)

\tau^* = \frac{1}{1 + e} \quad \text{with} \quad e = \frac{1 - \tau}{Z} \cdot \frac{dZ}{d(1 - \tau)}
Government chooses $\tau$ to maximize
\[
\int_i G[u^i((1 - \tau)z^i + \tau Z(1 - \tau), z^i)]
\]
Govt FOC (using the envelope theorem as $z^i$ maximizes $u^i$):
\[
0 = \int_i G'(u^i) u^i_c \cdot \left[-z^i + Z - \frac{dZ}{d(1 - \tau)} \right],
\]
\[
0 = \int_i G'(u^i) u^i_c \cdot \left[(Z - z^i) - \frac{\tau}{1 - \tau} eZ \right],
\]
First term $(Z - z^i)$ is mechanical redistributive effect of $d\tau$,
second term is efficiency cost due to behavioral response of $Z$
⇒ we obtain the following optimal linear income tax formula
\[
\tau = \frac{1 - \bar{g}}{1 - \bar{g} + e} \quad \text{with} \quad \bar{g} = \frac{\int g_i \cdot z_i}{Z \cdot \int g_i}, \quad g_i = G'(u^i) u^i_c
\]
OPTIMAL LINEAR TAX RATE: FORMULA

\[ \tau = \frac{1 - \bar{g}}{1 - \bar{g} + e} \quad \text{with} \quad \bar{g} = \frac{\int g_i \cdot z_i}{Z \cdot \int g_i}, \quad g_i = G'(u^i)u^i_c \]

\(0 \leq \bar{g} < 1\) if \(g_i\) is decreasing with \(z_i\) (social marginal welfare weights fall with \(z_i\)).

\(\bar{g}\) low when (a) inequality is high, (b) \(g^i \downarrow\) sharply with \(z^i\).

Formula captures the equity-efficiency trade-off **robustly** \((\tau \downarrow \bar{g}, \tau \downarrow e)\)

Rawlsian case: \(g_i \equiv 0\) for all \(z_i > 0\) so \(\bar{g} = 0\) and \(\tau = 1/(1 + e)\)
OPTIMAL TOP INCOME TAX RATE (SAEZ '01)

Consider constant MTR $\tau$ above fixed $z^*$. Goal is to derive optimal $\tau$

Assume w.l.o.g there is a continuum of measure one of individuals above $z^*$

Let $z(1 - \tau)$ be their average income [depends on net-of-tax rate $1 - \tau$], with elasticity $e = [(1 - \tau)/z] \cdot dz/d(1 - \tau)$

Note that $e$ is a mix of income and substitution effects (see Saez '01)
Optimal Top Income Tax Rate (Mirrlees ’71 model)

Disposable Income
\( c = z - T(z) \)

Market income \( z \)

Top bracket:
Slope \( 1 - \tau \)

Reform:
Slope \( 1 - \tau - d\tau \)

Source: Diamond and Saez JEP'11
Optimal Top Income Tax Rate (Mirrlees ’71 model)

Disposable Income
\[ c = z - T(z) \]

Mechanical tax increase:
\[ d\tau [z - z^*] \]

Behavioral Response tax loss:
\[ \tau \, dz = - \frac{d\tau \, e \, z \, \tau}{1 - \tau} \]

Source: Diamond and Saez JEP’11
OPTIMAL TOP INCOME TAX RATE

Consider small $d\tau > 0$ reform above $z^*$. 

1) **Mechanical increase** in tax revenue:

$$dM = [z - z^*]d\tau$$

2) **Welfare effect**:

$$dW = -\bar{g} \cdot dM = -\bar{g} \cdot [z - z^*]d\tau$$

where $\bar{g}$ is the social marginal welfare weight for top earners

3) **Behavioral response** reduces tax revenue:

$$dB = \tau \cdot dz = -\tau \frac{dz}{d(1 - \tau)}d\tau = -\tau \cdot \frac{1 - \tau}{z} \cdot \frac{dz}{d(1 - \tau)} \cdot zd\tau$$

$$\Rightarrow dB = -\frac{\tau}{1 - \tau} \cdot e \cdot zd\tau$$
**OPTIMAL TOP INCOME TAX RATE**

\[
dM + dW + dB = d\tau \left[ (1 - \bar{g})[z - z^*] - e \frac{\tau}{1 - \tau} z \right]
\]

Optimal \( \tau \) such that \( dM + dW + dB = 0 \) \( \Rightarrow \)

\[
\frac{\tau}{1 - \tau} = \frac{(1 - \bar{g})[z - z^*]}{e \cdot z}
\]

\[
\tau = \frac{1 - \bar{g}}{1 - \bar{g} + a \cdot e} \quad \text{with} \quad a = \frac{z}{z - z^*}
\]

Optimal \( \tau \downarrow \bar{g} \) [redistributive tastes]

Optimal \( \tau \downarrow \) with \( e \) [efficiency]

Optimal \( \tau \downarrow a \) [thinness of top tail]
Suppose top earner earns $z^T$

When $z^* \rightarrow z^T \Rightarrow z \rightarrow z^T$

\[
dM = d\tau [z - z^*] \quad dB = d\tau \cdot e \cdot \frac{\tau}{1 - \tau} z \quad \text{when} \quad z^* \rightarrow z^T
\]

Intuition: extra tax applies only to earnings above $z^*$ but behavioral response applies to full $z \Rightarrow$

Optimal $\tau$ should be zero when $z^*$ close to $z^T$ (Sadka-Seade zero top rate result) but result applies only to top earner

Top is uncertain: If actual distribution is finite draw from an underlying Pareto distribution then expected revenue maximizing rate is $1/(1 + a \cdot e)$ (Diamond and Saez JEP’11)
Empirical Pareto Coefficient

\[ z^* = \text{Adjusted Gross Income (current 2005 $)} \]

\[ a = zm/(zm-z^*) \text{ with } zm=E(z|z>z^*) \]

\[ \alpha = z^*h(z^*)/(1-H(z^*)) \]

Source: Diamond and Saez JEP'11
Empirically: \( a = z/(z - z^*) \) very stable above \( z^* = 400K \)

Pareto distribution \( 1 - F(z) = (k/z)^\alpha, f(z) = \alpha \cdot k^\alpha / z^{1+\alpha} \), with \( \alpha \) Pareto parameter

\[
z(z^*) = \frac{\int_{z^*}^{\infty} s f(s) ds}{\int_{z^*}^{\infty} f(s) ds} = \frac{\int_{z^*}^{\infty} s^{-\alpha} ds}{\int_{z^*}^{\infty} s^{-\alpha-1} ds} = \frac{\alpha}{\alpha - 1} \cdot z^*
\]

\( \alpha = z/(z - z^*) = a \) measures thinness of top tail of the distribution

Empirically \( a \in (1.5, 3) \), US has \( a = 1.5 \), Denmark has \( a = 3 \)

\[
\tau = \frac{1 - \bar{g}}{1 - \bar{g} + a \cdot e}
\]

Only difficult parameter to estimate is \( e \)
TAX REVENUE MAXIMIZING TAX RATE (skip)

Utilitarian criterion with $u_c \to 0$ when $c \to \infty \Rightarrow \bar{g} \to 0$ when $z^* \to \infty$

Rawlsian criterion (maximize utility of worst off person) $\Rightarrow \bar{g} = 0$ for any $z^* > \min(z)$

In the end, $\bar{g}$ reflects the value that society puts on marginal consumption of the rich

$\bar{g} = 0 \Rightarrow$ Tax Revenue Maximizing Rate $\tau = 1/(1+a \cdot e)$ (upper bound on top tax rate)

Example: $a = 2$ and $e = 0.25 \Rightarrow \tau = 2/3 = 66.7\%$

Laffer linear rate is a special case with $z^* = 0$, $z^m/z^* = \infty = a/(a - 1)$ and hence $a = 1$, $\tau = 1/(1 + e)$
EXTENSION: MIGRATION EFFECTS (skip)

Migration issues may be particularly important at the top end (brain drain). Some theory papers (Mirrlees ’82, Lehmann-Simula QJE’14).

Migration depends on average tax rate. Define $P(z - T(z) | z)$ fraction of $z$ earners in the country: Elasticity

$$\eta^m = \frac{z - T(z)}{P} \cdot \frac{\partial P}{\partial (z - T(z))}$$

Tax revenue maximizing formula (Brewer-Saez-Shepard ’10):

$$\tau = \frac{1}{1 + a \cdot e + \bar{\eta}^m}$$

Note: $\bar{\eta}^m$ depends on size of jurisdiction: large for cities, zero worldwide $\Rightarrow$ (1) Redistribution easier in large jurisdictions, (2) Tax coordination across countries increases ability to redistribute (big issue currently in EU)
Behavioral response to income tax comes not only from reduced labor supply but also shifts to other forms of income or activities: (untaxed fringe benefits, deferred compensation, shift to corporate income tax base, shift toward tax favored capital gains, etc.)

Real responses vs. tax avoidance responses is critical for 2 reasons:

1) Govt can control tax avoidance through other tools: closing loopholes, broadening the tax base ⇒ Elasticity $e$ is endogenous to tax system design (Slemrod)

2) Most tax avoidance responses create “fiscal externalities” in the sense that tax revenue increases at other time periods or in other tax bases (Saez-Slemrod-Giertz JEL’ 12)
REAL VS. AVOIDANCE RESPONSES (skip)

**Key policy question:** Is it possible to eliminate avoidance elasticity using base broadening, etc.? or would new avoidance schemes keep popping up?

a) Some forms of tax avoidance are due to **poorly designed tax codes** (preferential treatment for some income forms, deductions)

b) Some forms of tax avoidance/evasion can only be addressed with **international cooperation** (off-shore tax evasion in tax heavens)

c) Some forms of tax avoidance/evasion are due to technological limitations of tax collection (impossible to tax informal cash businesses)
GENERAL NON-LINEAR INCOME TAX $T(z)$

(1) Lumpsum grant given to everybody equal to $-T(0)$

(2) Marginal tax rate schedule $T'(z)$ describing how (a) lump-sum grant is taxed away, (b) how tax liability increases with income

Let $H(z)$ be the income CDF [population normalized to 1] and $h(z)$ its density [endogenous to $T(.)$]

Let $g(z)$ be the social marginal value of consumption for taxpayers with income $z$ in terms of public funds. With no income effects $\Rightarrow \int g(z)h(z)dz = 1$

Redistibution valued $\Rightarrow g(z)$ decreases with $z$

Let $G(z)$ the average social marginal value of $c$ for taxpayers with income above $z$ [$G(z) = \int_z^{\infty} g(s)h(s)ds/(1 - H(z))$]
Disposable Income $c = z - T(z)$

Pre-tax income $z$

Mechanical tax increase: $d\tau dz [1-H(z)]$

Social welfare effect: $-d\tau dz [1-H(z)] G(z)$

Behavioral response: $\delta z = -d\tau e_z / (1-T'(z))$

$\rightarrow$ Tax loss: $T'(z) \delta z h(z) dz$

$= -h(z) e_z T'(z)/(1-T'(z)) dzd\tau$

Small band $(z,z+dz)$: slope $1- T'(z)$

Reform: slope $1- T'(z) - d\tau$

Source: Diamond and Saez JEP'11
GENERAL NON-LINEAR INCOME TAX (skip)

Assume away income effects $\varepsilon^c = \varepsilon^u = e$ [Diamond AER’98 shows this is the key theoretical simplification]

Consider small reform: increase $T'$ by $d\tau$ in small band $z$ and $z + dz$

Mechanical effect $dM = dzd\tau[1 - H(z)]$

Welfare effect $dW = -dzd\tau[1 - H(z)]G(z)$

Behavioral effect: substitution effect $\delta z$ inside small band $[z, z + dz]$: $dB = h(z)dz \cdot T' \cdot \delta z = -h(z)dz \cdot T' \cdot d\tau \cdot z \cdot e(z)/(1 - T')$

Optimum $dM + dW + dB = 0$
GENERAL NON-LINEAR INCOME TAX

\[ T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e(z)} \]

1) \( T'(z) \) decreases with \( e(z) \) (elasticity efficiency effects)

2) \( T'(z) \) decreases with \( \alpha(z) = (zh(z))/(1 - H(z)) \) (local Pareto parameter)

3) \( T'(z) \) decreases with \( G(z) \) (social weight above \( z \))

Asymptotics: \( G(z) \to \bar{g}, \ \alpha(z) \to a, \ e(z) \to e \ \Rightarrow \ \text{Recover top rate formula} \ \tau = (1 - \bar{g})/(1 - \bar{g} + a \cdot e) \)
Empirical Pareto Coefficient

\[ a = \frac{z_m}{z_m - z^*} \text{ with } z_m = E(z|z > z^*) \]
\[ \alpha = z^* h(z^*) / (1 - H(z^*)) \]

Source: Diamond and Saez JEP'11
Negative Marginal Tax Rates Never Optimal (skip)

Suppose $T' < 0$ in band $[z, z + dz]$

Increase $T'$ by $d\tau > 0$ in band $[z, z + dz]$: $dM + dW > 0$ and $dB > 0$ because $T'(z) < 0$

$\Rightarrow$ Desirable reform

$\Rightarrow T'(z) < 0$ cannot be optimal

EITC schemes are not desirable in Mirrlees ’71 model
NUMERICAL SIMULATIONS (skip)

\(H(z)\) [and also \(G(z)\)] endogenous to \(T(\cdot)\). Calibration method (Saez Restud ’01):

Specify utility function (e.g. constant elasticity):

\[
  u(c, z) = c - \frac{1}{1 + \frac{1}{e}} \cdot \left( \frac{z}{n} \right)^{1+\frac{1}{e}}
\]

Individual FOC \(\Rightarrow z = n^{1+e}(1 - T')^e\)

Calibrate the exogenous skill distribution \(F(n)\) so that, using actual \(T'(\cdot)\), you recover empirical \(H(z)\)

Use Mirrlees ’71 tax formula (expressed in terms of \(F(n)\)) to obtain the optimal tax rate schedule \(T'\).
NUMERICAL SIMULATIONS (skip)

\[
\frac{T'(z(n))}{1 - T'(z(n))} = \left(1 + \frac{1}{e}\right) \left(\frac{1}{nf(n)}\right) \int_0^\infty \left[1 - \frac{G'(u(m))}{\lambda}\right] f(m) dm,
\]

Iterative Fixed Point method: start with \( T'_0 \), compute \( z^0(n) \) using individual FOC, get \( T^0(0) \) using govt budget, compute \( u^0(n) \), get \( \lambda \) using \( \lambda = \int G'(u)f \), use formula to estimate \( T'_1 \), iterate till convergence.

Fast and effective method (Brewer-Saez-Shepard '10)
NUMERICAL SIMULATION RESULTS (skip)

\[ T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e(z)} \]

Take utility function with \( e \) constant

2) \( \alpha(z) = (zh(z))/(1 - H(z)) \) is inversely U-shaped empirically

3) \( 1 - G(z) \) increases with \( z \) from 0 to 1 (\( \bar{g} = 0 \))

⇒ Numerical optimal \( T'(z) \) is U-shaped with \( z \): reverse of the general results \( T' = 0 \) at top and bottom [Diamond AER’98 gives theoretical conditions to get U-shape]
FIGURE 5 – Optimal Tax Simulations

Utilitarian Criterion, Utility type I

\[ \zeta^c = 0.25 \]

\[ \zeta^c = 0.5 \]

Rawlsian Criterion, Utility type I

\[ \zeta^c = 0.25 \]

\[ \zeta^c = 0.5 \]

Utilitarian Criterion, Utility type II

\[ \zeta^c = 0.25 \]

\[ \zeta^c = 0.5 \]

Rawlsian Criterion, Utility type II

\[ \zeta^c = 0.25 \]

\[ \zeta^c = 0.5 \]

Source: Saez (2001), p. 224
Mirrlees model predicts that optimal transfer at bottom takes the form of a “Negative Income Tax”:

1) Lumpsum grant $-T(0)$ for those with no earnings

2) High MTRs $T'(z)$ at the bottom to phase-out the lumpsum grant quickly: intuition

Intuition: high MTRs at bottom are efficient because:

(a) they target transfers to the most needy

(b) earnings at the bottom are low to start with so intensive response does not generate large output losses
OPTIMAL TRANSFERS: MIRRLEES MODEL

Diamond-Saez JEP’11:

\[ T'(0) = \frac{g_0 - 1}{g_0 - 1 + e_0} \]

\( e_0 \) elasticity of the fraction non-working wrt to \( 1 - T'(0) \)

\( g_0 \) social marginal welfare weight on non workers

\( \Rightarrow T'(0) \) large: e.g. \( g_0 = 3 \) and \( e_0 = .5 \) \( \Rightarrow T'(0) = 80\% \)

However if \( g_0 < 1 \) then \( T'(0) < 0 \) \( \Rightarrow \) EITC is optimal
Disposable income \( c = z - T(z) \)

Starting from a means-tested program

\[ G \]

45°

0

\( z^* \)

Pre-tax earnings \( z \)
Reducing generosity of $G$ and phase-out rate disposable income $c = z - T(z)$ is desirable if society puts low weight on zero earners. $G - dG = $1 to zero earners less valued than $1 distributed to all.
Starting from a means-tested program
Reducing generosity of $G$ and phase-out rate
is desirable if society puts low weight on zero earners

Labor supply response saves government revenue
Win-Win reform

Disposable income $c = z - T(z)$
Pre-tax earnings $z$

$G - dG$

$45^\circ$

$z^*$

Pre-tax earnings $z$
Optimal Transfers: Participation Responses

Empirical literature shows that participation labor supply responses [due to fixed costs of working] are large at the bottom [much larger and clearer than intensive responses]

Diamond JpubE’80, Saez QJE’02, Laroque EMA’05 incorporate such extensive labor supply responses in the optimal income tax model

Participation depends on participation tax rate: 

\[ \tau_p = \frac{T(z) - T(0)}{z} \]

individual keeps fraction \(1 - \tau_p\) of earnings when moving from zero earnings to earnings \(z\):

\[ z - T(z) = -T(0) + z - [T(z) - T(0)] = -T(0) + z \cdot (1 - \tau_p) \]

Key result: in-work subsidies with \(T'(z) < 0\) (such as EITC) become optimal when labor supply responses are concentrated along extensive margin and social marginal welfare weight on low skilled workers > 1.
Introducing a small EITC is desirable for redistribution. If $1 to low paid workers more valued than $1 distributed to all starting from a Means-Tested Program.
Introducing a small EITC is desirable for redistribution

Participation response saves government revenue

Disposable income
\[ c = z - T(z) \]

Pre-tax earnings \( z \)

Starting from a Means-Tested Program

\[ 45^\circ \]
Introducing a small EITC is desirable for redistribution.

Starting from a Means-Tested Program

Participation response saves government revenue

Win-Win reform

Disposable income $c = z - T(z)$
Introducing a small EITC is desirable for redistribution. Participation response saves government revenue. Win-Win reform. If intensive response is small.
ACTUAL TAX/TRANSFER SYSTEMS

1) Transfer programs used to be of the traditional form with high phasing-out rates (sometimes above 100%) ⇒ No incentives to work (even with modest elasticities)

Initially designed for groups not expected to work [widows in the US] but later attracting groups who could potentially work [single mothers]

2) In-work benefits have been introduced and expanded in OECD countries since 1980s (US EITC, UK Family Credit, etc.) and have been politically successful

⇒ (a) Redistribute to low income workers, (b) improve incentives to work
LIMITS OF WELFARIST APPROACH

Welfarism is the dominant approach in optimal taxation

Welfarism: social objective is a sole function of individual utilities

Tractable and coherent framework that captures the equity-efficiency trade-off but generates puzzles:
(a) 100% taxation absent behavioral responses
(b) Whether income is deserved or due to luck is irrelevant
(c) What transfer recipients would have done absent transfers is irrelevant

A number of alternatives to welfarism have been proposed

Saez-Stantcheva '16 (Piketty-Saez '13, section 6 summary) propose a new generalized framework nesting welfarism and many alternatives which can resolve those puzzles
Saez-Stantcheva ’16 survey people online (using Amazon MTurk) by asking hypothetical questions to elicit social preferences:

1) People typically do not have “utilitarian” social justice principles (consumption lover not seen as more deserving than frugal person)

2) People put weight on whether income has been earned through effort vs. not (hard working vs. leisure lover)

3) People put weight of what people would have done absent the government intervention (deserving poor vs. free loaders)

4) People put weight on taxes paid conditional on $c$: person $z=$50K, $T=$15K, $c=$35K more deserving than $z=$40K, $T=$5K, $c=$35K

⇒ Aversion for free loaders and liking tax contributors is like an intuitive or evolutionary optimal tax tool (needed to sustain cooperative societies)
Which of the following two individuals do you think is most deserving of a $1,000 tax break?

Individual A earns $50,000 per year, pays $10,000 in taxes and hence nets out $40,000. She greatly enjoys spending money, going out to expensive restaurants, or traveling to fancy destinations. She always feels that she has too little money to spend.

Individual B earns the same amount, $50,000 per year, also pays $10,000 in taxes and hence also nets out $40,000. However, she is a very frugal person who feels that her current income is sufficient to satisfy her needs.

- Individual A is most deserving of the $1,000 tax break
- Individual B is most deserving of the $1,000 tax break
- Both individuals are exactly equally deserving of the tax $1,000 break

Source: survey in Saez and Stantcheva (2013)
Which of the following two individuals is most deserving of a $1,000 tax break?

Individual A earns $30,000 per year, by working in two different jobs, 60 hours per week at $10/hour. She pays $6,000 in taxes and nets out $24,000. She is very hard-working but she does not have high-paying jobs so that her wage is low.

Individual B also earns the same amount, $30,000 per year, by working part-time for 20 hours per week at $30/hour. She also pays $6,000 in taxes and hence nets out $24,000. She has a good wage rate per hour, but she prefers working less and earning less to enjoy other, non-work activities.

- Individual A is most deserving of the $1,000 tax break
- Individual B is most deserving of the $1,000 tax break
- Both individuals are exactly equally deserving of the $1,000 tax break

Source: survey in Saez and Stantcheva (2013)
We assume now that the government can increase benefits by $1,000 for some recipients of government benefits.

Which of the following four individuals is most deserving of the $1,000 increase in benefits?

Please drag and drop the four individuals into the appropriate boxes on the left. The upper box, marked 1 should contain the individual you think is most deserving. The box labeled "2" should contain the second most-deserving individual, etc.. Please note that you can put two individuals in the same box if you think that they are equally deserving.

Individual A gets $15,000 per year in Disability Benefits because she cannot work due to a disability and has no other resources.

Individual B gets $15,000 per year in Unemployment Benefits and has no other resources. She lost her job and has not been able to find a new job even though she has been actively looking for one.

Individual C gets $15,000 per year in Unemployment Benefits and has no other resources. She lost her job but has not been looking actively for a new job, because she prefers getting less but not having to work.

Individual D gets $15,000 per year in Welfare Benefits and Food Stamps and has no other resources. She is not looking for a job actively because she can get by living off those government provided benefits.

Source: survey in Saez and Stantcheva (2013)
### Table 2: Revealed Social Preferences

<table>
<thead>
<tr>
<th>Panel</th>
<th>Comparison</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Consumption lover vs. Frugal</strong></td>
<td>Consumption lover vs. Frugal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Consumption lover &gt; Frugal</td>
<td>4.1%</td>
<td>74.4%</td>
<td>21.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Consumption lover = Frugal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Consumption lover &lt; Frugal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td># obs. = 1,125</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>B. Hardworking vs. leisure lover</strong></td>
<td>Hardworking vs. leisure lover</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hardworking &gt; Leisure lover</td>
<td>42.7%</td>
<td>54.4%</td>
<td>2.9%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Leisure lover = Leisure lover</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Leisure lover &lt; Leisure lover</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td># obs. = 1,121</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>C. Transfer Recipients and free loaders</strong></td>
<td>Disabled person unable to work</td>
<td>1.4</td>
<td>1.6</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>Unemployed looking for work</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unemployed not looking for work</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Welfare recipient not looking for work</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td># obs. = 1,098</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports preferences for giving a tax break and or a benefit increase across individuals in various scenarios. Panel A considers two individuals with the same earnings, same taxes, and same disposable income but high marginal utility of income (consumption lover) vs. low marginal utility of income (frugal). In contrast to utilitarianism, 74% of people report that consumption loving is irrelevant and 21.5% think the frugal person is most deserving. Panel B considers two individuals with the same earnings, same taxes, and same disposable income but different wage rates and hence different work hours. 54.4% think hours of work is irrelevant and 42.7% think the hardworking low wage person is more deserving. Panel C considers transfer recipients receiving the same benefit levels. Subjects find the disabled person unable to work and the unemployed person looking for work much more deserving than the abled bodied unemployed or welfare recipient not looking for work.
Generalized Social Marginal Welfare Weights (skip)

Social planner uses \textit{generalized social marginal welfare weights} $g_i \geq 0$ to value marginal consumption of individual $i$

Standard utilitarian case $SWF = \int_i u^i$ has $g_i = u^i_c$

But we can define generalized $g_i$ that might depend on fairness judgements as well

\textbf{Optimal tax criterion:} $T(z)$ is optimal if for any budget neutral small tax reform $dT(z)$, $\int_i g_i \cdot dT(z_i) = 0$ with $g_i \geq 0$

1) Generates same optimal tax formulas as welfarist approach

2) Respects (local) constrained Pareto efficiency ($g_i \geq 0$)

3) No social objective is maximized
Application 1: Optimal Tax with Fixed Incomes (skip)

Utilitarian approach has degenerate solution with 100% taxation when $u'(c)$ decreases with $c$

Public may not support confiscatory taxation even absent behavioral responses

Generalized social marginal welfare weights: $g_i = g(c_i, T_i)$

$g(c, T)$ decreases with $c$ (ability to pay)

$g(c, T)$ increases with $T$ (contribution to society)

Optimum: $g(z - T(z), T(z))$ equalized across $z$

$\Rightarrow T'(z) = 1/(1 - g_T/g_c)$ and $0 \leq T'(z) \leq 1$
Application 1: Optimal Tax with Fixed Incomes (skip)

Preferences for redistributions embodied in \( g(c, T') \)

Polar cases:

1) Utilitarian case: \( g(c, T) = u'(c) \Rightarrow T'(z) \equiv 1 \)

2) Libertarian case: \( g(c, T) = g(T) \Rightarrow T'(z) \equiv 0 \)

We use Amazon mTurk online survey to estimate \( g(c, T) \)

We find that revealed preferences depend on both \( c \) and \( T \)

- \( z = \$40K, T = \$10K, c = \$30K \) more deserving than \( z = \$50K, T = \$10K, c = \$40K \)
- \( z = \$50K, T = \$15K, c = \$35K \) more deserving than \( z = \$40K, T = \$5K, c = \$35K \)
Saez-Stantcheva ’16 online survey shows strong public preference for redistributing toward “deserving poor” (unable to work or trying hard to work) rather than “undeserving poor” (who would work absent transfers)

Generalized social welfare weights can capture this by setting $g_i = 0$ on free loaders (transfer recipients who would have worked absent the transfer) ⇒

1) Behavioral responses reduce desirability of transfers (over and above standard budgetary effect)

2) In-work benefit \(T'(0) = \frac{(g_0 - 1)}{(g_0 - 1 + e_0)} < 0\) at bottom becomes optimal in Mirrlees (1971) optimal tax model if \(g_0 < 1\)
Various alternatives to welfarism have been proposed (survey Fleurbaey-Maniquet '11)

Each alternative can be recast in terms of implied generalized social marginal welfare weights (as long as it generates constrained Pareto efficient optima)

In all cases, we can use simple and tractable optimal income tax formula for heterogeneous population from Saez Restud'01 (case with no income effects):

\[ T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e(z)} \]

with \( G(z) \) average of \( g_i \) above \( z \)

\( g_i \) average to one in the full population and hence \( G(0) = 1 \)
1) **Rawlsian:** \( g_i \) concentrated on worst-off individual \( \Rightarrow G(z) = 0 \) for \( z > 0 \) and \( T'(z) = \frac{1}{1 + \alpha(z) \cdot e} \) revenue maximizing

2) **Libertarian:** \( g_i \equiv 1 \) \( \Rightarrow G(z) \equiv 1 \) and \( T'(z) \equiv 0 \)

3) **Equality of Opportunity:** (Roemer '98) \( g_i \) concentrated on those coming from disadvantaged background. \( G(z) = \) relative fraction of individuals above \( z \) coming from disadvantaged background

\( \Rightarrow G(z) \) decreases with \( z \) for reasons unrelated to decreasing marginal utility
### Table 2: Equality of Opportunity vs. Utilitarian Optimal Tax Rates

<table>
<thead>
<tr>
<th>Income Percentile</th>
<th>Equality of Opportunity</th>
<th>Utilitarian (log-utility)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction from low background (=parents below median) above each percentile</td>
<td>Implied social welfare weight G(z) above each percentile</td>
</tr>
<tr>
<td>z= 25th percentile</td>
<td>44.3%</td>
<td>0.886</td>
</tr>
<tr>
<td>z= 50th percentile</td>
<td>37.3%</td>
<td>0.746</td>
</tr>
<tr>
<td>z= 75th percentile</td>
<td>30.3%</td>
<td>0.606</td>
</tr>
<tr>
<td>z= 90th percentile</td>
<td>23.6%</td>
<td>0.472</td>
</tr>
<tr>
<td>z= 99th percentile</td>
<td>17.0%</td>
<td>0.340</td>
</tr>
<tr>
<td>z= 99.9th percentile</td>
<td>16.5%</td>
<td>0.330</td>
</tr>
</tbody>
</table>

Notes: This table compares optimal marginal tax rates at various percentiles of the distribution (listed by row) using an equality of opportunity criterion (in column (3)) and a standard utilitarian criterion (in column (5)). Both columns use the optimal tax formula $T'(z)=\frac{1-G(z)}{1-G(z)+\alpha(z)\cdot e}$ discussed in the text where $G(z)$ is the average social marginal welfare weight above income level $z$, $\alpha(z)=\frac{zh(z)}{1-H(z)}$ is the local Pareto parameter (with $h(z)$ the density of income at $z$, and $H(z)$ the cumulative distribution), and $e$ the elasticity of reported income with respect to $1-T'(z)$. We assume $e=0.5$. We calibrate $\alpha(z)$ using the actual distribution of income based on 2008 income tax return data. For the equality of opportunity criterion, $G(z)$ is the representation index of individuals with income above $z$ who come from a disadvantaged background (defined as having a parent with income below the median). This representation index is estimated using the national intergenerational mobility statistics of Chetty et al. (2013) based on all US individuals born in 1980-1 with their income measured at age 30-31. For the utilitarian criterion, we assume a log-utility so that the social welfare weight $g(z)$ at income level $z$ is proportional to $1/(z-T(z))$.

Source: Saez and Stantcheva (2014)
COMMODITY VS. INCOME TAXATION

Suppose we have $K$ consumption goods $c = (c_1, .., c_K)$ with pre-tax price $p = (p_1, .., p_K)$. Individual $h$ has utility $u^h(c_1, .., c_K, z)$

Key question: Can government increase $SWF$ using differentiated commodity taxation $t = (t_1, .., t_K)$ (after tax price $q = p + t$) in addition to nonlinear Mirrlees income tax on earnings $z$?

In practice, govt (a) exempts some goods (food, education, health) from sales tax or value-added-tax, (b) imposes additional excise taxes on some goods (cars, gasoline, luxury goods)

$$\max_{t, T(.)} SWF \geq \max_{t=0, T(.)} SWF$$ because more instruments cannot hurt
Famous Atkinson-Stiglitz JpubE’ 76 shows that

$$\max_{t,T(.)} SWF = \max_{t=0,T(.)} SWF$$

(i.e., commodity taxes not useful over and above $T(z)$) under two assumptions on utility functions $u^h(c_1, .., c_K, z)$

1) Weak separability between $(c_1, .., c_K)$ and $z$ in utility

2) Homogeneity across individuals in the sub-utility of consumption $v(c_1, .., c_K)$ [does not vary with $h$]

$$(1) \text{ and } (2): \quad u^h(c_1, .., c_K, z) = U^h(v(c_1, .., c_K), z)$$

Original proof was based on optimum conditions, new straightforward proof by Laroque EL ’05, and Kaplow JpubE ’06.
ATKINSON-STIGLITZ THEOREM PROOF

Let $V(y, p+t) = \max_c v(c_1, .., c_K) \text{ st } (p+t) \cdot c \leq y$ be the indirect utility of consumption $c$ [common to all individuals]

Start with $(T(\cdot), t)$. Let $c(t)$ be consumer choice.

Replace $(T(\cdot), t)$ with $(\bar{T}(\cdot), t = 0)$ where $\bar{T}(z)$ such that $V(z-T(z), p+t) = V(z-\bar{T}(z), p) \Rightarrow$ Utility $U^h(V, z)$ and labor supply choices $z$ unchanged for all individuals.

Attaining $V(z-\bar{T}(z), p)$ at price $p$ costs at least $z-\bar{T}(z)$

Consumer also attains $V(z-\bar{T}(z), p) = V(z-T(z), p+t)$ when choosing $c(t) \Rightarrow z-\bar{T}(z) \leq p \cdot c(t) = z-T(z) - t \cdot c(t)$

$\Rightarrow \bar{T}(z) \geq T(z) + t \cdot c(t)$: the government collects more taxes with $(\bar{T}(\cdot), t = 0)$


**ATKINSON-STIGLITZ INTUITION**

With separability and homogeneity, conditional on earnings $z$, consumption choices $c = (c_1, .., c_K)$ do not provide any information on ability

⇒ Differentiated commodity taxes $t_1, .., t_K$ create a tax distortion with no benefit ⇒ Better to do all the redistribution with the individual income tax

Note: With weaker linear income taxation tool (Diamond-Mirrlees AER ’71, Diamond JpubE ’75), need $v(c_1, .., c_K)$ homothetic (linear Engel curves, Deaton EMA ’81) to obtain no commodity tax result

[Unless Engel curves are linear, commodity taxation can be useful to “non-linearize” the tax system]

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WHEN ATKINSON-STIGLITZ ASSUMPTIONS FAIL

Thought experiment: force high ability people to work less and earn only as much as low ability people: if higher ability consume more of good $k$ than lower ability people, then taxing good $k$ is desirable. Happens when:

1) High ability people have a relatively higher taste for good $k$ (independently of income) [indirect tagging]

2) Good $k$ is positively related to leisure (consumption of $k$ increases when leisure increases keeping after-tax income constant) [tax on holiday trips, subsidy on work related expenses such as child care]

In general Atkison-Stiglitz assumption is a good starting place for most goods $\Rightarrow$ Zero-rating on some goods under VAT for redistribution is inefficient and administratively burdensome [Mirrlees 2010 review]
Standard two period model ($w =$ wage rate in period 1, retired in period 2)

$$u^h(c_1, c_2, z) = u(c_1) + \frac{u(c_2)}{1 + \delta} - b(z/w)$$

$\delta$ is the discount rate, $b(.)$ is the disutility of effort, budget $c_1 + c_2/(1 + r(1 - t_K)) \leq z - T(z)$

Aktinson-Stiglitz implies that savings taxation $t_K$ (equivalent to tax on $c_2$) is useless in the presence of an optimal income tax if $\delta$ is the same for everybody

If low ability people have higher $\delta$ [empirically plausible] then savings tax $t_K > 0$ is desirable (Saez JpubE ’02)
REFERENCES


**Laroque, G. “Indirect Taxation is Superfluous under Separability and Taste Homogeneity: A Simple Proof”, Economic Letters, Vol. 87, 2005, 141-144.** (web)


