

Econ 230B
Spring 2017
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Problem Set 1

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1. Lorenz Curve and Gini Coefficient

The IRS posts online tabulations of the distribution of annual individual incomes based on Federal Individual Income Tax data. We will focus on statistics for years 2004 and 2014 available online in Table 1.1 posted at ([link here](#)).

a) Using excel or STATA, draw the empirical Lorenz curve for the Adjusted Gross Income (AGI) distribution for all returns (but excluding returns with no AGI). For this, use columns (1) and (3) of the excel Table 1.1 for the two years 2004 and 2014.

Compute the Gini coefficient from the Lorenz curve. Has inequality increased or decreased from year 2004 to year 2014?

b) Using realistic interpolation, compute the following inequality statistics: top 10% income share, top 1% income share, percentile 90 to percentile 50 ratio (P90/P50). Has inequality increased or decreased from 2004 to 2014?

c) Use a Pareto interpolation to compute the top 10% income share, top 1% income share, percentile 90 to percentile 50 ratio (P90/P50). How different are your results compare to part (b)? (See [Atkinson \(2005\)](#) for a description of the Pareto interpolation procedure with tabulated data)

d) Table 1.1 also provides in column (14) the income tax paid by bracket (non-taxable returns pay by definition zero tax). Assume that the ranking of individual tax filers is the same when using AGI (col. (3)) and after-tax income (col. (3)-(14)). Redo a) and b) for after-tax income. Can you conclusively say that the Federal income tax reduces inequality?

Has tax progressivity increased or decreased from 2004 to 2014?

e) In reality, the ranking of individuals by after-tax incomes is not strictly the same as the ranking by pre-tax income. In that case, did you over-estimate or under-estimate the Gini coefficients in question c)?

2. Optimal Income Taxation

An economy is populated by individuals with preferences over consumption and labor. They have utility $u_i(c, y)$ where y is income, $u_c(c, y) > 0$ and $u_y(c, y) < 0$. Suppose the tax schedule

in place has a constant marginal tax rate τ above a fixed threshold y^* . The government wants to choose τ to maximize the tax revenue raised from top earners.

- (a) As we saw in class, the tax rate that maximizes revenues depends on a Pareto parameter a and the elasticity of total income of the top earners who are in the top bracket, ε . Provide intuition about why ε is a mix of substitution and income effects.
- (b) The individual solves the following utility maximization problem:

$$\max_{c,y} u_i(c, y)$$

subject to:

$$c = (1 - \tau)y + I$$

Denote by $y_i(1 - \tau, I)$ the Marshallian income supply. The uncompensated elasticity of labor supply with respect to $1 - \tau$ is $\varepsilon_i^u = (\partial y_i / \partial (1 - \tau))((1 - \tau) / y_i)$. We denote by $\eta_i = (1 - \tau) \partial y_i / \partial I$ the income parameter.

Suppose a government advisor suggests to run an experiment where the top tax rate τ (above y^*) is raised by $d\tau$. The advisor claims that the response dy_i can be rewritten as a function of ε_i^u and η_i . Is the advisor right? If yes, show how dy_i depends on ε_i^u and η_i .

- (c) Using the expression derived in point b) write ε as a function of the Pareto parameter $a = y^m / (y^m - y^*)$ and a weighted average of the uncompensated elasticities and income effect parameters. Why are uncompensated elasticities weighted by incomes y_i , while the η_i s are not?
- (d) Now suppose the utility is logarithmic in consumption and exponential in income. It takes the following form:

$$u_i(c, y) = \log c - \phi_i y^{1 + \frac{1}{\varepsilon}}$$

where ϕ_i can vary across individuals and captures heterogeneity in the disutility from labor. Derive the uncompensated elasticity, income, and compensated elasticity parameters (i.e., ε_i^u , η_i , ε_i^c) by solving the utility maximization problem of the individual under the linear tax and the same budget constraint as above.

- (e) Study what happens to η_i and ε_i^u when ϕ_i becomes small (find their limits). Using the relation found previously, write the optimal top tax rate formula as a function of ε and the Pareto parameter a when ϕ_i is small.

The parameter ε is the Frisch elasticity of labor supply for this class of utility functions. Suppose we calibrate the parameters a and ε such that $a = 1.5$ and $\varepsilon = 1$. What is the optimal τ ? What is the optimal τ when ε is very large? Discuss why the optimal tax rate is high even with a large Frisch elasticity.

3. Optimal Linear Income Taxation

Suppose that utility is quasi-linear and takes the form: $u(c, l) = c - \frac{l^{1+\epsilon}}{1+\epsilon}$ with $\epsilon > 0$. Each individual earns income $y = wl$ and consumes $c = y - T(y)$. The wage rate w can be interpreted as a measure of skills and is distributed with density $f(w) > 0$ over $[0, \infty)$. The total population is normalized to one so that $\int_0^\infty f(w) dw = 1$

- (a) Suppose the tax schedule is linear with a flat tax rate τ . The tax is hence $T(y) = -S + \tau y$ where $S > 0$ is the transfer that the individual receives when labor supply is zero ($T(0) = -S$). Find the optimal labor supply choice as a function of the parameters S and $w(1 - \tau)$. Also, derive the uncompensated and compensated elasticities of labor supply as a function of ϵ and find the income effect parameter.
- (b) Assume that taxes are entirely rebated to the individuals in the economy. We have that $S = \tau Y$, where Y is average earnings in the economy. Find the optimal tax rate τ in the case where the government only cares about the worst-off individual (i.e. the government is Rawlsian) and in the case where the government maximizes the sum of utilities (i.e. the government is utilitarian). Always explain the intuition behind your results.
- (c) Do points (a)-(b) again using utility function $u(c, l) = \log(c) - l$. If exact analytical expressions are not possible to derive, just provide implicit formulas with economic explanation. Is this utility function more or less realistic than the one used in questions (a)-(b)?

Go back to utility function $u(c, l) = c - \frac{l^{1+\epsilon}}{1+\epsilon}$. We now study an economy with two tax brackets such that:

$$T(y) = \begin{cases} -S + \tau_1 y & \text{if } y \leq \hat{y} \\ -S + \tau_1 \hat{y} + \tau_2 (y - \hat{y}) & \text{if } y > \hat{y} \end{cases}$$

$-S$ is the transfer to non-working individuals.

- (d) Plot the budget constraint on a graph with axes (l, c) .
- (e) Suppose that $\tau_1 < \tau_2$. Find the optimal labor supply and earnings for an individual with wage w . Consider the three cases where the individual is in the bottom bracket, the top bracket, or exactly at \hat{y} .

Suppose that there are 3 types of individuals: disabled individuals unable to work $w_0 = 0$, low skilled individuals with wage rate w_1 , and skilled individuals with wage rate w_2 . We assume that $w_1 < w_2$. The fractions of disabled, low skilled, and high skilled in the population are respectively λ_0 , λ_1 and λ_2 such that $\lambda_0 + \lambda_1 + \lambda_2 = 1$. Further assume that low skilled workers are always in the bottom bracket and that high skilled workers are always in the top bracket.

- (f) Find the tax rate τ_2^* that maximizes taxes collected from the high skilled, assuming that S , τ_1 , and \hat{y} are given. Express it as a function of ϵ and \hat{y} .

- (g) Compute the tax rate τ_1 that maximizes total taxes collected taking S and \hat{y} as given and setting $\tau_2 = \tau_2^*$ (the optimal tax rate you found in the previous question). Explain why (intuitively) $\tau_2^* < \tau^* < \tau_1^*$, where τ^* is the one computed in question (b).