1. Optimal Commodity Taxation with Linear Income Tax

Consider a population of individuals with the utility of individual $i$ given by $u^i(v(c_1, .., c_K), z)$ where $v(c_1, .., c_K)$ is a sub-utility of consumption for goods $c_1, .., c_K$ and $z$ is earnings. Assume that $v(c_1, .., c_K)$ is the same across all individuals and homogeneous of degree one, i.e. $v(\lambda c_1, .., \lambda c_K) = \lambda v(c_1, .., c_K)$ for any $\lambda > 0$. Let $p$ be the pre-tax vector of prices of goods $1, .., K$ and $q = p + t$ the post-tax vector of prices of goods $1, .., K$. We assume that the government uses a linear income tax on earnings with constant marginal tax rate $\tau$ and uniform lumpsum grant $R$. The individual budget constraint is $q \cdot c \leq (1 - \tau)z + R$.

a) Let us denote by $V(y, q) = \max_{c_1, .., c_K} v(c_1, .., c_K)$ st $q \cdot c \leq y$ the indirect sub-utility of consumption. Show that $V(y, q)$ is linear in $y$, i.e., takes the form $\phi(q) \cdot y$ and that the demand for each good $c_k(y, q)$ is also linear in $y$.

b) Adapt the proof of the Atkinson-Stiglitz theorem to show that any joint commodity, linear earnings tax system $(t, \tau, R)$ can be replaced by a pure linear earnings tax system with no commodity taxation $(\bar{t} = 0, \bar{\tau}, \bar{R})$ that leaves all individuals indifferent and that raises at least as much government revenue.

c) If $v(c_1, .., c_K)$ is not homogeneous of degree one, can commodity taxation be useful to increase social welfare when using a linear income tax? Explain intuitively why this is the case or not and how this relates to the Atkinson-Stiglitz theorem discussed in class.

2. Chasing Natural Experiments within a Country

As seen in class, many of the best papers on labor supply responses to taxes and transfers exploit a policy change (a so-called “Natural Experiment”) in order to obtain convincing estimates. This exercise asks you to find a Natural Experiment and propose an estimation methodology.

Download the pdf copy of the OECD annual publication Taxing Wages for years 2017 and 2022. Those publications are available online [link here] in pdf format (when connected through UC Berkeley). Part II of this publication describes the tax/benefits systems (including payroll
taxes, income taxes, and various benefits) faced by wage income earners for each OECD country. Note that recent changes in the tax/benefit system are explicitly described in Section 4 for each country.

a) Find one reform in one country which took place between 2017 and 2022 that could be used to estimate labor supply responses to taxes or transfers for some group of interest in the population. Make sure the reform is large enough to be useable for compelling identification. Describe the reform you have picked.

b) Describe the methodology you would use to estimate such labor supply responses. In particular, make sure to be fully explicit about the assumptions you need to identify the labor supply response parameters. Try to explain whether your estimates capture participation versus intensive elasticities, uncompensated versus compensated elasticities, income effects, etc.

c) Describe the data you would need to carry out the analysis. Survey or administrative data, variables, realistic sample size, time period, panel or repeated cross section, etc. Search online to investigate whether such data exist and how they could be obtained for the research analysis you are proposing.

d) (FOR FUTURE WORK): If you find a really promising Natural Experiment, the next step is to look for the related literature (you want to be the first to analyze this change!) and then try and get the data to carry out the research project.

3. Optimal Top Income Tax Rate

Consider a population of individuals with individual $i$ utility equal to $u^i(c, z)$ increasing in consumption $c$ and decreasing in earnings $z$. We consider a nonlinear tax system $T(z)$ with a constant marginal tax rate $\tau$ above a fixed threshold $z^*$ (the “top bracket”). The tax system below $z^*$ is irrelevant in what follows and is kept constant. We assume that the government wants to choose $\tau$ to maximize tax revenue raised from top bracket earners.

a) We saw in class that the tax rate $\tau$ maximizing tax revenue takes the simple form $\tau^* = 1/(1 + a \cdot e)$ with $a = z^m/(z^m - z^*)$ the Pareto parameter ($z^m$ is the average income in the top bracket) and $e$ the elasticity of top bracket incomes. Explain intuitively why $e$ is a mix of substitution effects and income effects.

b) Let $z^i(1 - \tau, R)$ be the earnings supply function obtained from solving the individual utility maximization problem under a linear tax:

$$\max_{c,z} u^i(c, z) \text{ st } c = (1 - \tau)z + R$$

We denote by $e^i_u$ the uncompensated elasticity of $z^i$ with respect to $1 - \tau$ and by $\eta^i = (1 - \tau)\partial z^i/\partial R$ the income effect parameter.

Considering the small tax reform $d\tau$ in the top bracket (keeping $z^*$ fixed) as we saw in class, show that the response $dz^i$ to the small reform can be expressed in terms of $e^i_u$ and $\eta^i$. 

2
c) Use b) to express $e$ in function of the average of the uncompensated elasticities and income effect parameters among top bracket earners, along with the Pareto parameter $a = z^m / (z^m - z^*)$. Explain intuitively why uncompensated elasticities are weighted by incomes $z^i$ while the income effect parameters are not.

d) Suppose, each individual utility takes the form $u^i(c, z) = c - a^i \cdot z^{1+1/\varepsilon}$ with $a^i$ is a parameter (that can vary across individuals) and $\varepsilon$ is a fixed positive parameter. Solve the individual maximization problem under a linear tax

$$\max_{c,z} u^i(c, z) \text{ st } c = (1 - \tau)z + R$$

to obtain the uncompensated elasticity, income effect, and compensated elasticity parameters $e^i_u, \eta^i, e^i_c$ for such utility functions expressed as a function of $\varepsilon$. Use c) to express the optimal top tax rate formula as a function of $\varepsilon$ and the Pareto parameter $a$ in that case.

e) Suppose, each individual utility takes the form $u^i(c, z) = \log c - a^i \cdot z^{1+1/\varepsilon}$ with $a^i$ is a parameter (that can vary across individuals). Solve the individual maximization problem under a linear tax

$$\max_{c,z} u^i(c, z) \text{ st } c = (1 - \tau)z + R$$

to obtain the uncompensated elasticity, income effect, and compensated elasticity parameters $e^i_u, \eta^i, e^i_c$ for such utility functions.

f) Show that, when $a^i$ becomes small (i.e. when $z^i$ is large), we have $\eta^i \simeq -\varepsilon/(\varepsilon + 1)$ and $e^i_u \simeq 0$. Use c) to obtain an optimal top tax rate formula as a function of $\varepsilon$ and the Pareto parameter $a$ in that case.

This class of utility functions is widely used in macro-economics and $\varepsilon$ is known as the Frisch elasticity. Suppose that $a = 1.5$ and $\varepsilon = 1$ (macro models use in general large Frisch elasticities). How large is the optimal $\tau$ in that case? What is the optimal $\tau$ when $\varepsilon$ is very large? Explain why the optimal tax rate is high even with such a large Frisch elasticity?