1. **Optimal Linear Income Taxation**

Suppose that utility is quasi-linear and takes the form: \( u(c, l) = c - \frac{t_1 l}{1 + \epsilon} \) with \( \epsilon > 0 \). Each individual earns income \( y = w l \) and consumes \( c = y - T(y) \). The wage rate \( w \) can be interpreted as a measure of skills and is distributed with density \( f(w) > 0 \) over \([0, \infty)\). The total population is normalized to one so that \( \int_0^\infty f(w) dw = 1 \).

(a) Suppose the tax schedule is linear with a flat tax rate \( \tau \). The tax is hence \( T(y) = -S + \tau y \) where \( S > 0 \) is the transfer that the individual receives when labor supply is zero \( (T(0) = -S) \). Find the optimal labor supply choice as a function of the parameters \( S \) and \( w(1 - \tau) \). Also, derive the uncompensated and compensated elasticities of labor supply as a function of \( \epsilon \) and find the income effect parameter.

(b) Assume that taxes are entirely rebated to the individuals in the economy. We have that \( S = \tau Y \), where \( Y \) is average earnings in the economy. Find the optimal tax rate \( \tau \) in the case where the government only cares about the worst-off individual (i.e. the government is Rawlsian) and in the case where the government maximizes the sum of utilities (i.e. the government is utilitarian). Always explain the intuition behind your results.

(c) Do points (a)-(b) again using utility function \( u(c, l) = \log(c) - l \). If exact analytical expressions are not possible to derive, just provide implicit formulas with economic explanation. Is this utility function more or less realistic than the one used in questions (a)-(b)?

Go back to utility function \( u(c, l) = c - \frac{t_1 l}{1 + \epsilon} \). We now study an economy with two tax brackets such that:

\[
T(y) = \begin{cases} 
-S + \tau_1 y & \text{if } y \leq \hat{y} \\
-S + \tau_1 \hat{y} + \tau_2 (y - \hat{y}) & \text{if } y > \hat{y}
\end{cases}
\]

\(-S\) is the transfer to non-working individuals.

(d) Plot the budget constraint on a graph with axes \((l, c)\).

(e) Suppose that \( \tau_1 < \tau_2 \). Find the optimal labor supply and earnings for an individual with wage \( w \). Consider the three cases where the individual is in the bottom bracket, the top bracket, or exactly at \( \hat{y} \).
Suppose that there are 3 types of individuals: disabled individuals unable to work \( w_0 = 0 \), low skilled individuals with wage rate \( w_1 \), and skilled individuals with wage rate \( w_2 \). We assume that \( w_1 < w_2 \). The fractions of disabled, low skilled, and high skilled in the population are respectively \( \lambda_0 \), \( \lambda_1 \) and \( \lambda_2 \) such that \( \lambda_0 + \lambda_1 + \lambda_2 = 1 \). Further assume that low skilled workers are always in the bottom bracket and that high skilled workers are always in the top bracket.

(f) Find the tax rate \( \tau^*_2 \) that maximizes taxes collected from the high skilled, assuming that \( S \), \( \tau_1 \), and \( \hat{y} \) are given. Express it as a function of \( \epsilon \) and \( \hat{y} \).

(g) Compute the tax rate \( \tau_1 \) that maximizes total taxes collected taking \( S \) and \( \hat{y} \) as given and setting \( \tau_2 = \tau^*_2 \) (the optimal tax rate you found in the previous question). Explain why (intuitively) \( \tau^*_2 < \tau^* < \tau^*_1 \), where \( \tau^* \) is the one computed in question (b).

2. Chasing Natural Experiments within a Country

As seen in class, many of the best papers on labor supply responses to taxes and transfers exploit a policy change (a so-called “Natural Experiment”) in order to obtain convincing estimates. This exercise asks you to find a Natural Experiment and propose an estimation methodology.

Download the pdf copy of the OECD annual publication *Taxing Wages* for years 2011 and 2021. Those publications are available online [link here] in pdf format (when connected through UC Berkeley). Part II of this publication describes the tax/benefits systems (including payroll taxes, income taxes, and various benefits) faced by wage income earners for each OECD country. Note that recent changes in the tax/benefit system are explicitly described in Section 4 for each country.

a) Find one reform in one country which took place between 2011 and 2021 that could be used to estimate labor supply responses to taxes or transfers for some group of interest in the population. Make sure the reform is large enough to be useable for compelling identification. Describe the reform you have picked.

b) Describe the methodology you would use to estimate such labor supply responses. In particular, make sure to be fully explicit about the assumptions you need to identify the labor supply response parameters. Try to explain whether your estimates capture participation versus intensive elasticities, uncompensated versus compensated elasticities, income effects, etc.

c) Describe the data you would need to carry out the analysis. Survey or administrative data, variables, realistic sample size, time period, panel or repeated cross section, etc. Search online to investigate whether such data exist and how they could be obtained for the research analysis you are proposing.

d) (FOR FUTURE WORK): If you find a really promising Natural Experiment, the next step is to look for the related literature (you want to be the first to analyze this change!) and then try and get the data to carry out the research project.