

Econ 230B  
 Spring 2022  
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**Problem Set 1**

**1. Optimal linear income tax**

Suppose that utility is quasi-linear and takes the form:  $u(c, l) = c - \frac{l^{1+\epsilon}}{1+\epsilon}$  with  $\epsilon > 0$ . Each individual earns income  $y = wl$  and consumes  $c = y - T(y)$ . The wage rate  $w$  can be interpreted as a measure of skills and is distributed with density  $f(w) > 0$  over  $[0, \infty)$ . The total population is normalized to one so that  $\int_0^\infty f(w)dw = 1$

- (a) Suppose the tax schedule is linear with a flat tax rate  $\tau$ . The tax is hence  $T(y) = -S + \tau y$  where  $S > 0$  is the transfer that the individual receives when labor supply is zero ( $T(0) = -S$ ). Find the optimal labor supply choice as a function of the parameters  $S$  and  $w(1 - \tau)$ . Also, derive the uncompensated and compensated elasticities of labor supply as a function of  $\epsilon$  and find the income effect parameter.

$\max_l wl(1 - \tau) + S - l^{1+\epsilon}/(1 + \epsilon) \implies w(1 - \tau) = l^\epsilon \implies l = [w(1 - \tau)]^{1/\epsilon}$  so  $\epsilon^u = \epsilon^c = 1/\epsilon$  and  $\eta = 0$ . Let us denote by  $\epsilon = 1/\epsilon$  the common compensated and uncompensated elasticity.

- (b) Assume that taxes are entirely rebated to the individuals in the economy. We have that  $S = \tau Y$ , where  $Y$  is average earnings in the economy. Find the optimal tax rate  $\tau$  in the case where the government only cares about the worst-off individual (i.e. the government is Rawlsian) and in the case where the government maximizes the sum of utilities (i.e. the government is utilitarian). Always explain the intuition behind your results.

$$\max_{\tau} wl(1 - \tau)^{\frac{1}{\epsilon}} \int w^{1+\frac{1}{\epsilon}} f(w)dw \implies \tau^* = \frac{\epsilon}{\epsilon + 1} = \frac{1}{1 + \epsilon}$$

Worst off individual has  $w = 0$  and hence  $l = 0$  and utility  $u = S = \tau Y$  so Rawlsian optimal rate maximizes tax revenue (to maximize  $S$ ) and is set at  $\tau^* = \frac{\epsilon}{\epsilon+1}$  from (b). Given that all utilities are linear, there is no concern for redistribution and hence the optimal utilitarian tax rate is zero.

- (c) Do points (a)-(b) again using utility function  $u(c, l) = \log(c) - l$ . If exact analytical expressions are not possible to derive, just provide implicit formulas with economic explanation. Is this utility function more or less realistic than the one used in questions (a)-(b)?

Go back to utility function  $u(c, l) = c - \frac{l^{1+\epsilon}}{1+\epsilon}$ . We now study an economy with two tax brackets such that:

$$T(y) = \begin{cases} -S + \tau_1 y & \text{if } y \leq \hat{y} \\ -S + \tau_1 \hat{y} + \tau_2 (y - \hat{y}) & \text{if } y > \hat{y} \end{cases}$$

$-S$  is the transfer to non-working individuals.

With utility  $\log(c) - l$ , we have  $\max_l \log(wl(1-\tau) + S) - l \implies w(1-\tau)/[w(1-\tau)l + S] = 1$  so that  $l = 1 - S/[w(1-\tau)]$ . Note that  $l = 0$  when  $w(1-\tau) \leq S$ . Income effect  $\eta = -1$ ,  $\epsilon^u = S/[S - w(1-\tau)]$ ,  $\epsilon^c = \epsilon^u - \eta = w(1-\tau)/[S - w(1-\tau)]$ .  $\tau^* = 1/(1 + \hat{\epsilon}^u)$  where  $\hat{\epsilon}^u$  is the (income weighted) average uncompensated elasticity.  $\hat{\epsilon}^u$  does not have a simple analytic expression. Worst-off individual has  $l = 0$  and utility  $u = \log(S)$  so government wants to maximize  $S$  which is done by maximizing tax revenue with  $\tau^* = 1/(1 + \hat{\epsilon}^u)$ . In utilitarian case, the optimal  $\tau$  is given by  $\tau = (1 - \hat{g})/(1 - \hat{g} + \epsilon)$  as seen in class notes with  $\epsilon$  a mix of uncompensated and income effects (see Piketty-Saez handbook chapter for details). There is no simple analytic expression.

- (d) Plot the budget constraint on a graph with axes  $(l, c)$ .
- (e) Suppose that  $\tau_1 < \tau_2$ . Find the optimal labor supply and earnings for an individual with wage  $w$ . Consider the three cases where the individual is in the bottom bracket, the top bracket, or exactly at  $\hat{y}$ .

The budget has a kink generating bunching at  $\hat{y}$ .

Case 1 (first bracket):  $w \leq \underline{w}$ :  $l = [w(1-\tau_1)]^{\frac{1}{\epsilon}}$  with  $\underline{w}$  s.t.  $\underline{w}^{1+\frac{1}{\epsilon}}(1-\tau_1)^{\frac{1}{\epsilon}} = \hat{y}$

Case 2 (bunching at  $\hat{y}$ ):  $\underline{w} \leq w \leq \hat{w}$ :  $l = \hat{y}/w$

Case 3 (top bracket):  $w \leq \bar{w}$ :  $l = [w(1-\tau_2)]^{\frac{1}{\epsilon}}$  with  $\bar{w}$  s.t.  $\bar{w}^{1+\frac{1}{\epsilon}}(1-\tau_2)^{\frac{1}{\epsilon}} = \hat{y}$

Suppose that there are 3 types of individuals: disabled individuals unable to work  $w_0 = 0$ , low skilled individuals with wage rate  $w_1$ , and skilled individuals with wage rate  $w_2$ . We assume that  $w_1 < w_2$ . The fractions of disabled, low skilled, and high skilled in the population are respectively  $\lambda_0$ ,  $\lambda_1$  and  $\lambda_2$  such that  $\lambda_0 + \lambda_1 + \lambda_2 = 1$ . Further assume that low skilled workers are always in the bottom bracket and that high skilled workers are always in the top bracket.

- (f) Find the tax rate  $\tau_2^*$  that maximizes taxes collected from the high skilled, assuming that  $S$ ,  $\tau_1$ , and  $\hat{y}$  are given. Express it as a function of  $\epsilon$  and  $\hat{y}$ .

Amount of bunching is proportional to  $\epsilon$  (see Saez AEJ:EP10 for details):

$$\frac{\bar{w}^{1+\epsilon}}{\underline{w}^{1+\epsilon}} = \left( \frac{1-\tau_1}{1-\tau_2} \right)^\epsilon$$

$$T = \lambda_1 \tau_1 (1-\tau_1)^{\frac{1}{\epsilon}} w_1^{1+\frac{1}{\epsilon}} + \lambda_2 \left\{ \tau_1 \hat{y} + \tau_2 \left[ (1-\tau_2)^{\frac{1}{\epsilon}} w_2^{1+\frac{1}{\epsilon}} - \hat{y} \right] \right\}$$

Take the FOC wrt to  $\tau_2$  to get:

$$\tau_2^*/(1 - \tau_2^*) = \varepsilon[y_2 - \bar{y}]/y_2$$

- (g) Compute the tax rate  $\tau_1$  that maximizes total taxes collected taking  $S$  and  $\hat{y}$  as given and setting  $\tau_2 = \tau_2^*$  (the optimal tax rate you found in the previous question). Explain why (intuitively)  $\tau_2^* < \tau^* < \tau_1^*$ , where  $\tau^*$  is the one computed in question (b).

Take the FOC wrt to  $\tau_1$  to get:

$$\tau_1^*/(1 - \tau_1^*) = \varepsilon[1 + \lambda_2\bar{y}/(\lambda_1 y_1)]$$

In order to explain  $\tau_2^* < \tau^* < \tau_1^*$ :

1. Increasing the flat tax rate  $\tau$  creates a mechanical increase in revenue proportional to average earnings and creates a negative behavioral response proportional to average earnings as well.
2. Increasing the tax rate  $\tau_2$  in the top bracket creates a mechanical increase in revenue proportional to  $(y_2 - \bar{y})$  but creates a negative behavioral response proportional to  $y_2$ .
3. Increasing the tax rate  $\tau_1$  in the bottom bracket creates a mechanical increase in revenue proportional to  $y_1$  and creates a negative behavioral response proportional to  $y_1$ . However, the tax rate increase also raises more tax from high skilled worker with no negative behavioral response (inframarginal tax).