Econ 131
Spring 2022
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Final Exam

May 13

Exam Instructions:

- **Explanation should be written using pens (we recommend black or blue ink, as these often scan the best).** No pencils, except for graphs.
- **Show your work**. Credit will only be awarded on the basis of what is written on the exam.
- **Sign the academic honesty pledge**. Cheating will be dealt with harshly.

Student Name:

Student ID Number:

Affirm the academic honesty pledge below. For those writing on a non-printed copy, please just write "Academic Honesty Pledge as on exam", and sign your name.
If you do not affirm this pledge, your exam will be marked invalid.

0. ACADEMIC HONESTY PLEDGE

I confirm that I have abided by all academic honesty rules for UC Berkeley and Economics 131. I confirm that I did not see this exam before my official exam start time. I confirm that I have not shared and will not share this exam with anyone else. I confirm that I haven't copied from anybody else's exam.

Signature: _____

1. True/False/Uncertain (20 points, 2 points per question.)

Explain your answer fully based on what was discussed in class, since all the credit is based on the explanation. Your grade depends entirely on the substance of your justification, not on whether you are correct in writing "True" or "False". Note that it is possible to answer each question for full credit with three sentences or fewer, and answers longer than ten lines long will not be graded.

(a) Suppose there three options for combatting climate change are put to a vote: L=low spending, M=medium spending, H=high spending. Suppose that M beats L in majority voting and that H beats M in majority voting. Does this imply that H beats L in majority voting?

SOLUTION: No, majority voting can create cycles $H > M > L$ but $L > H$ (see the example in seen in class for public school spending). If every voter has single peaked preferences, then such cycles cannot happen (median voter theorem). In the case of climate change spending, single peaked preferences make sense.

(b) A tax of \$1 per unit imposed on producers of good B has exactly the same economic impact as a tax of \$1 per unit imposed on consumers of good B.

SOLUTION: Yes, this is a prediction that holds in the standard competitive economic model.

(c) There is no need for government to provide retirement benefits as rational individuals can save for retirement themselves.

SOLUTION: True in the theoretical life-cyle model of savings (Modigliani model). In practice however, most individuals are not able to save for retirement as predicted by the life-cycle model and would have to keep working into very old age, be destitute, or get support from children (as happened before government social security programs were created). Therefore, in practice, government provided retirement benefits are needed and are indeed ubiquitous.

(d) More generous unemployment insurance benefits lead to longer unemployment spells, therefore the government should not provide any unemployment insurance benefits.

SOLUTION: False. It is true that more generous unemployment insurance benefits lead to longer unemployment spells (studies for the US or Austria seen in class) and therefore unemployment insurance benefits create moral hazard. However, unemployment insurance benefits also provide consumption smoothing benefits. Hence, there is a trade-off and the optimal unemployment insurance replacement rate should be positive but less than 100%.

(e) Thanks to Obamacare, everybody in the United States can get affordable health care insurance.

SOLUTION: This was the aim of Obamacare through a combination of Medicaid expansion for people below 138% of the poverty line and then subsidies on Obamacare exchanges for people above. In practice, this is not perfectly true because a number of states (including Texas and Florida) refused to do the Medicaid expansion so that low income people in these states can find themselves in a gap: not eligible for Medicaid and not eligible for Obamacare exchange subsidies. Undocumented immigrants are not eligible for Obamacare and also can't get affordable health care insurance.

(f) In the life-cycle model where people work and save when young and life off their savings and returns on savings when old, the government should not tax capital income.

SOLUTION: True, this is the Atkinson-Stiglitz 1976 theorem discussed in class. It says that the the optimal tax rate on capital income should be zero. Using a labor tax on earnings is sufficient.

(g) Majoring in economics in college has a large positive causal impact on future earnings.

SOLUTION: True based on the study for UC Santa Cruz that carried out a regression discontinuity design using a GPA requirement in econ 1-2 to be able to become econ major. There is a jump in likelihood of being an econ major at the GPA requirement threshold which then translates into a jump in earnings after graduation. This is very compelling evidence that majoring in economics in college has a large positive causal impact on future earnings.

(h) The spike in retirement hazard at the Early Retirement Age of 62 in the US Social Security system is evidence that many individuals do not follow the rational model of life-cycle savings.

SOLUTION: True: There is indeed a spike in retirement hazard at the ERA of 62. A rational person is not affected by the ERA because somebody who wants to retire earlier than the ERA would save in advance and live off savings before getting the benefits at ERA.

(i) It is rational for someone to want the government to tax everyone in order to fund a public good and then for that person to try to avoid paying the tax herself.

SOLUTION: True. This is an example of the free-rider problem. Public goods are under-supplied privately and hence it is desirable to have a system (a government in practice) that forces everybody to contribute to public goods through taxes and government provision of the public good. From an individual selfish perspective, it makes sense to try and avoid paying taxes.

(j) The optimal linear tax rate is never above the revenue maximizing tax rate.

SOLUTION: True. If the tax rate were above the maximizing revenue rate, decreasing the tax rate would make taxpayers happier and increase tax revenue, a win-win situation [a Pareto improvement].

2. Local Public Goods (20 points)

Destruction of wildlife due to wildfires is a serious problem in the state of Califirenia. The cities of Animaley (A) and Birdeley (B) are trying to decide how much they want to contribute to a joint fire department to provide fire protection services for both cities. The two cities ($i = A,\ B$) have identical preferences:

$$U_i(x,\ F) = \log(x) + 3\log(F),$$

where log denotes the natural logarithm, x is each city's private consumption, and F is the sum of total contributions to the joint fire department. Animaley contributes f_A and Birdeley contributes f_B to finance fire services, so total spending on the joint fire department is $F = f_A + f_B$. Assume that $p_x = p_F = 1$, where p_x is the price of one unit of private consumption and p_F is the price of one unit of public good. Finally, assume that Animaley's budget is \$100 and Birdeley's budget is \$50.

(a) What are the characteristics of a pure public good? Do fire departments and fire protection services fit these characteristics? Please explain why or why not **in no more than five sentences.** *(3 points)*

Solution: The two characteristics are non-rivalry and non-excludability. Putting out fires can be considered non-excludable, since putting out a house fire not only benefits the directly affected residence, but all neighboring residences threatened by the fire. Also, it can be non-rival, since one resident's fire protection doesn't generally impact a neighbor's fire protection. However, it may be considered rival in consumption if constant emergencies in a particular neighborhood constrain the fire department's ability to put out new fires. Also, there are a few cases of fire departments refusing to assist residents who do not pay required service fees (excludability).

(b) For the rest of the problem, let's assume that the fire department is a pure public good. Write down the budget constraint and determine the private equilibrium level of contributions for each city, f_A and f_B. Show that total fire services $F_{private}$ are equal to 60% of the total budget \$150 (\$150 is the sum of \$100 from town A and \$50 from town B). *(3 points)*

Solution: The two cities have identical preferences but different budgets, so their best-response functions are going to be slightly different. Under private provision, each city solves $MRS = MRT$. In addition, note that $MRT = 1$ because the price of the private good and an unit of fire protection services are both equal to \$1. Animaley's budget constraint is $100 = x_A + f_A$. Therefore,

$$MRS_A = \frac{\partial U_A/\partial F}{\partial U_A/\partial x_A} = \frac{3x_A}{F} = \frac{3(100 - f_A)}{f_A + f_B} = 1 = MRT$$

$$\Rightarrow \quad 300 - 3f_A = f_A + f_B \quad \Rightarrow \quad f_A = \frac{300 - f_B}{4}.$$

Birdeley's budget constraint is $50 = x_B + f_B$. Hence, by the same token,

$$MRS_B = \frac{\partial U_B/\partial F}{\partial U_B/\partial x_B} = \frac{3x_B}{F} = \frac{3(50 - f_B)}{f_A + f_B} = 1 = MRT$$

$$\Rightarrow \quad 150 - 3f_B = f_A + f_B \quad \Rightarrow \quad f_B = \frac{150 - f_A}{4}.$$

Substituting the expression for Birdeley's best response function into Animaly's best response function and solving for f_A, we get $f_A = \frac{300}{4} - \frac{1}{4}\left[\frac{150}{4} - \frac{1}{4}f_A\right] \Rightarrow \frac{15}{16}f_A = \frac{1050}{16} \Rightarrow f_A^* = \frac{1050}{16} \times \frac{16}{15} = 70$. Plugging $f_A^* = 70$ back into Animaley's best response function yields $f_B^* = \frac{150-70}{4} = 20$. Therefore, $F_{private} = f_A + f_B = 90$ which is 60% of \$150.

(c) Derive the socially optimal contribution to fire services F^*_{social} and express it as a percentage of the total budget \$150. Why is this percentage higher than in question (b)? *Hint: you don't need to derive socially optimal f^*_A and f^*_B separately. (2 points)*

Solution: The social planner sets F^*_{social} such that $\sum MRS = MRT$. Recall that $MRS_A = \frac{3x_A}{F}$ and $MRS_B = \frac{3x_B}{F}$. Therefore, F^*_{social} is pinned down by the following equation:

$$\frac{3x_A}{F} + \frac{3x_B}{F} = 1.$$

Substituting the budget constraints $x_A = 100 - f_A$ and $x_B = 50 - f_B$, we get:

$$\frac{300 - 3f_A + 150 - 3f_B}{F} = \frac{450 - 3(f_A + f_B)}{f_A + f_B} = 1 \quad \Rightarrow$$

$$\Rightarrow 3 \cdot 150 = 4(f_A + f_B) = 4F \quad \Rightarrow \quad F^*_{social} = 150 \cdot \frac{3}{4} = 112.5.$$

This is 75% of \$150. The percentage is higher than in (b) because the private solution to a public good problem leads to under-provision.

(d) Governor of Califirenia Gabby Oldsom is aware of the difference in private equilibrium contributions versus the social optimum level of fire protection for the two cities, and she proposes a block grant of \$30 that is to be distributed **proportionally** to cities' existing budgets, i.e., \$20 to town A and \$10 to town B. Derive the new levels of private contributions f_A and f_B in equilibrium. Discuss whether this block grant would help to achieve socially-optimal level of fire services spending under the free market allocation. *Hint: this last part does not require math and can be done with reasoning.*

(3 points)

Solution: The new budget constraints under the grant are $x_A = 120 - f_A$ and $x_B = 60 - f_B$. Paralleling the derivations from (a), you will find that everything is 20% higher so that $f_A = 84$ and $f_B = 24$ so that $F = 108$.

$$f_A = \frac{360 - f_B}{4} = 90 - \frac{1}{4}f_B$$
$$f_B = \frac{180 - f_A}{4} = 45 - \frac{1}{4}f_A$$

Substituting Animaley's best response into Birdeley's best response and solving for f_B yields:

$$f_B = 45 - \frac{1}{4}\left[90 - \frac{1}{4}f_B\right] \quad \Rightarrow \quad \frac{15}{16}f_B = \frac{90}{4} \quad \Rightarrow \quad f^*_B = \frac{90}{4} \times \frac{16}{15} = 24.$$

Substituting $f_B^* = 24$ into Animaley's best response function yields $f_A^* = 84$. Therefore, after the block grant is disbursed, the new private equilibrium level of fire services spending is $F_{private}^{block} = 84 + 24 = 108$, which is still less than the socially-optimal level of 112.5 found in (c). Neither of the two cities increase their fire services spending by the full amount of the grant. The reason is because the block grant can be interpreted as an upward shift in the budget constraint equal to the size of the grant. Therefore, we should expect an increase in consumption of both x and F.

(e) Suppose that Governor Oldsom decides to drop the block grant idea. However, she is still determined to bring spending on fire services in both cities to the social optimum level. She approaches a prominent economist at the University of Califirenia at Birdeley (you) and asks them to design a matching grant proposal that would achieve this goal. A matching grant at rate g adds g dollars to any dollar of private contributions by Animaley and Birdeley. Therefore, we now have $F = (1 + g) \cdot (f_A + f_B)$. Derive the private equilibrium level of contributions for each city, f_A and f_B when the matching grant at rate g is in place. Derive the matching grant rate g that can deliver the social optimal level of fire services. *(4 points)*

Solution: Given the log additivity, $\log(F) = \log(1 + g) + \log(f_A + f_B)$ so that each town maximization is exactly identical as in (a) so we will get the same solutions $f_A = 70$ and $f_B = 20$, but now $F = (1 + g) \cdot 90$. So the subsidy at rate g simply increases the public good by a factor $1 + g$. The optimum level of fire remains 75% of the total budget \$150 while private contributions are 60% of the total budget \$150. So the matching grant that delivers the social optimum is $1 + g = 75/60$ or $g = 15/60 = 25\%$.

For the rest of the problem, suppose that Animaley and Birdeley do not share their fire departments. Each city has 2 types of individuals, M and W. Type Ms want fire department contributions in their city to equal $F = 60$, while type Ws want contributions to equal $F = 20$. The population of Animaley has 60% type Ms, whereas the population of Birdeley has 60% type Ws.

(f) Explain (without maths) what mechanism for local public good provision could generate the socially optimal level of fire department services being provided in both cities and such that **all** type M and W individuals will be satisfied. *(3 points)*

Solution: The median voter theorem predicts $F = 60$ in Animaley, and $F = 20$ in Birdeley. Tiebout sorting then predicts that type Ws in Animaley and type Ms in Birdeley who want their preferred level of re protection can choose to move to the other city.

(g) Why might the mechanism in (f) not work in the real world? **In no more than three sentences**, discuss all possible reasons you can think of. *(2 points)*

Solution: Moving costs, imperfect information about benefits and taxes in each city, possible spillovers of public services provided.

3. Disability Insurance (20 points)

In the country called Communalia, society consists of identical people each earning a wage W ($W > \$10$) when they are employed. Each worker faces a probability q of sustaining a workplace injury. If injured, they become disabled, they cannot work and their total income goes down to \$10. All people have the same utility function of consumption C:

$$U(C) = \log(C),$$

where C equals to worker's total income.

(a) What is the expression for the expected utility of each worker? *(2 points)*

Solution:

$$EU^{uninsured} = (1-q)\log(W) + q\log(10).$$

Now, suppose that the government of Communalia introduces a disability compensation program. Under this system, individuals pay a tax t when they are employed (but not when they are injured as they can't work then) and get a benefit b when they are injured. The system must break even *in expectation*, that is, the benefits paid to injured workers must be equal to the taxes collected from employed workers.

(b) Write down the government's break-even constraint and the expression for each worker's expected utility under this program. *Hint: t is a dollar tax, not a tax rate. (2 points)*

Solution: The government break-even constraint implies that

$$(1-q)t = qb \quad \Rightarrow \quad t = \frac{qb}{1-q}$$

Each worker's expected utility under this program is

$$EU^{program} = (1-q)\log(W-t) + q\log(10+b).$$

(c) What is the optimal worker compensation system? That is, what is the system that, subject to the government's break-even constraint, maximizes worker utility? Derive both the optimal benefit level b^* and the optimal tax t^*. *(2 points)*

Solution: Substituting the government break-even constraint into the expected utility function and maximizing the resulting objective function with respect to b yields:

$$EU(b) = (1-q)\log(W - \frac{qb}{1-q}) + q\log(10+b) \to \max_b$$

The first order condition is:

$$\frac{-q}{W-\frac{qb}{1-q}}+\frac{q}{10+b}=0 \quad \Rightarrow \quad \left(1+\frac{q}{1-q}\right)b=W-10 \quad \Rightarrow \quad b^*=(1-q)\big(W-10\big)$$

Therefore, the optimal tax is:

$$t^*=\frac{q}{1-g}\times(1-q)\big(W-10\big)=q\big(W-10\big)$$

(d) Without any calculations, can you say if the government compensation program from (c) beneficial for the workers? Would your answer change if worker preferences were of the $U(C)=C$ form? If preferences were $U(C)=\sqrt{C}$ instead, would the optimal program derived in (c) be the same or different? *(3 points)*

Solution: Yes, there are welfare gains because the agent is risk averse. If $U(C)=C$, the workers are risk-neutral, so there will be no welfare gains from the program. The optimal program derived in (c) provides perfect insurance because the utility $\log(C)$ is concave. Any concave utility, including $U(C)=\sqrt{C}$ would also lead to an optimum with perfect insurance (as we derived in class).

(e) Suppose now that the government of Communalia wants to maximize its *expected* profit from the program instead of just breaking-even. Assuming the government decides to keep the benefits at the same level as b^* you found in (c) and assuming the workers would choose to stay in the program if they are indifferent, what is the highest possible amount of the tax, t^{max}, the government can charge? Simplify the resulting expression as much as you can. *Hint: you are asked to derive the worker individual rationality aka participation constraint. (4 points)*

Solution: Assuming that $b^* = (1-q)\big(W-10\big)$, we want to find such level of t for which workers would be indifferent between staying in the program and remaining uninsured. Specifically, we need to find all t for which the following inequality holds (and turns into equality at $t = t^{max}$):

$$\begin{aligned}
EU^{program} &\geq EU^{uninsured} \\
(1-q)\log(W-t) + q\log\big(10 + (1-q)\big(W-10\big)\big) &\geq (1-q)\log(W) + q\log(10) \\
(1-q)\log\left(\frac{W-t}{W}\right) &\geq q\log\left(\frac{10}{10+(1-q)\big(W-10\big)}\right) \\
\left(\frac{1-q}{q}\right)\log\left(\frac{W-t}{W}\right) &\geq \log\left(\frac{10}{10+(1-q)\big(W-10\big)}\right) \\
\log\left(\left(\frac{W-t}{W}\right)^{\frac{1-q}{q}}\right) &\geq \log\left(\frac{10}{10+(1-q)\big(W-10\big)}\right) \\
\left(\frac{W-t}{W}\right)^{\frac{1-q}{q}} &\geq \frac{10}{10+(1-q)\big(W-10\big)} \\
1-\frac{t}{W} &\geq \left(\frac{10}{10+(1-q)\big(W-10\big)}\right)^{\frac{q}{1-q}}
\end{aligned}$$

$$\Rightarrow \quad t^{max} = W \times \left[1 - \left(\frac{10}{10+(1-q)\big(W-10\big)}\right)^{\frac{q}{1-q}}\right].$$

(f) **For this question only**, assume that the wage is $W = \$90$ and the probability of becoming disabled is $q = \frac{1}{2}$. Using your answer in (e), calculate the t^{max} that a profit-maximizing government would set. What would be its *expected* per-worker profit from this program assuming it still pays out the disability benefit b^* that you found in (c)? *(2 points)*

Solution: Plugging the numbers into the expressions for b^* and t^{max} we found in (c) and (e), respectively, yields:

$$b^* = \frac{1}{2} \times (90-10) = 40,$$

$$t^{max} = 90 \times \left[1 - \left(\frac{10}{10 + \frac{1}{2} \times 80}\right)\right] = 90 \times \left(1 - \frac{1}{5}\right) = 90 \times \frac{4}{5} = 72.$$

Therefore, the government's expected per-worker profit is:

$$E\pi = (1-q)t^{max} - qb^* = \frac{1}{2} \times 72 - \frac{1}{2} \times 40 = 36 - 20 = 16.$$

(g) The workers of Communalia, astonished by the overt rent-seeking behavior of their government, overthrow it. The new government lifts the ban on private disability insurance and soon the country's insurance market becomes perfectly competitive, with private insurers offering disability plans at actuarially fair price. Assuming that workers cannot simultaneously own private insurance and participate in the government program, will the new government be able to continue running its worker compensation program at a positive profit? Explain using **no more than three sentences**. *(2 points)*

Solution: No. Under the profit-maximizing government's regime, workers enrolled in the program were receiving the same utility as in the case of no insurance. Since workers are risk-averse and private insurance premiums are actuarially fair, workers will be able to receive higher expected utility if they buy private insurance. Therefore, everyone will leave the government disability compensation program.

(h) Suppose that the new government of Communalia decides to keep its worker compensation program and switches back to the break-even regime. Show that under actuarially fair price of private insurance plans, the workers will be indifferent between staying in the government program and buying private insurance. *Hint: under actuarially fair price, the expected utility of being privately insured is $EU^{insurance} = U(W-p)$, where p is the price of insurance, and the optimal private insurance benefit is $b^* = W - 10$ (full insurance). (3 points)*

Solution: We want to show that under actuarially fair price of private insurance and break-even tax level of the government program, $EU^{program} = EU^{insurance}$. Recall that the break-even level of tax is $t^* = q(W-10)$ and $b^* = (1-q)(W-10)$. Therefore, we want to show that:

$$\begin{aligned}
EU^{program} &\stackrel{?}{=} EU^{insurance} \\
(1-q)\log(W - q(W-10)) + q\log(10 + (1-q)(W-10)) &\stackrel{?}{=} \log(W - q(W-10)) \\
q\log(10 + (1-q)(W-10)) &\stackrel{?}{=} q\log(W - q(W-10)) \\
\log(10 + (1-q)(W-10)) &\stackrel{?}{=} \log(W - q(W-10)) \\
10 + (1-q)(W-10) &\stackrel{?}{=} W - q(W-10) \\
W - 10 + 10 - q(W-10) &= W - q(W-10)
\end{aligned}$$

Indeed, given our assumptions on t and p, the workers will be indifferent between the two options for all possible values of W and q.