Econ 131
Spring 2020
Emmanuel Saez

Problem Set 3 Solution

DUE DATE: April 22

Student Name:
Student ID:
GSI Name:

- You must submit your solutions using this template.
- Although you may work in groups, each student must submit individual sets of solutions. You must note the names other students that you worked with. Write their names here:
1. Essay

Read the following recent Washington Post article on the response to the pandemic in developing countries. Write a short essay [the essay has to fit in the page below] on the extra difficulties (relative to richer countries) of fighting the pandemic and alleviating the resulting economic hardship in developing countries relative to richer countries. In light of this, should developing countries should follow different policies than richer countries?

Washington Post link:

https://eml.berkeley.edu/~saez/course131/coronadevo.pdf
2. True/False Statements

Determine whether each statement is true, false, or uncertain and explain why. Answers with no explanation will receive no points.

(a) Suppose that candidates X and Z run for president. Candidate X is elected president after winning 51% of the vote. Then once in office, he appoints more conservative members to the Supreme Court than candidate Z would have. This means that a majority of American voters preferred more conservative Supreme Court members. (Assume that everyone is fully informed about the candidates’ plans and the President does not need Senate approval to appoint Supreme Court members.)

UNCERTAIN. This is possibly true. It’s also possible that voters are single-issue voters on several topics and that X assembles a coalition large enough to win and then does things that only a minority of voters support. For example, X could get elected by 26% of voters who care only about conservative Supreme Court members and another 25% of voters who care only about low tax rates on high-earners (which X supports but Z does not). Then it’s possible that only 26% of voters support conservative Supreme Court member appointments.

(b) There is no reason for the government to impose social distancing to fight the epidemic because private agents can create markets to price the corresponding externality.

False: This is theoretically true based on the famous Coase theorem we discussed in class. However, in practice, this is an externality involving many agents and there is no way in practice that such a market could develop. In practice, the only way to get the proper level of social distancing is through mandated government orders.

(c) As CO2 emissions create a classic externality, the only policy needed to solve the problem is a tax on carbon that would apply in all countries at the same rate and set equal to the marginal damage created by CO2 emissions.

FALSE. This is the classic view of economists derived from the standard model of externalities. However, as we discussed in class, there are great difficulties in pricing the cost of the externality. Furthermore, carbon taxes might suffer from popular opposition as they are regressive (e.g., yellow vests in France, developing countries that want cheap sources
of energy). Instead, the key goal should be decarbonization which can happen through a combination of policies: government mandated transition, subsidies to innovation in renewables, development of renewable infrastructure, and possibly a path of increased carbon taxes.

(d) The returns of education accrue primarily to the person receiving the education and hence the government should not be heavily involved in the provision of education.

FALSE: While it is true that the returns of education accrue primarily to the person receiving the education, there are compelling reasons why the government should be involved (and that explains why the government is actually involved in practice). Most important ones: 0) everybody should have access to education to have a chance of succeeding economically, 1) parents with low incomes may not afford education for their kids, 2) kids cannot borrow on their own to get an education, 3) people are not able to navigate well the education “market” (lack of good information, susceptible to false advertisement, etc.)

(e) According to the Tiebout model, local public good provision is efficient and tailored to the tastes of local residents. Hence, it is better to have a fully decentralized government.

FALSE: True if all public goods are local and society does not care about redistribution. False if there are global public goods (like national defense and expertise in solving problems that require coordination such as pandemic fighting). False if society cares about redistribution as local governments cannot do as much for redistribution given mobility threats (see class notes).
3. Externalities
US drivers consume millions of gallons of gas daily, and with every gallon consumed harmful emissions are released into the atmosphere. The aggregate demand function for gasoline is given by $P = 5 - 4Q^D$, where $Q$ is the quantity of gallons of gas per day in billions and $P$ reflects the price per gallon in dollars. The aggregate supply function is given by $P = 2 + 2Q^S$.

IMF economists have estimated a substantial external marginal damage of gas consumption, which we’ll round to $1.2$ per gallon.

(a) Solve for the equilibrium private market price and quantity that will be generated without any government intervention.

**Solution:**
To determine the free market result, we want to set the private marginal benefit equal to the private marginal cost:

$P_{MB} = P_{MC}$

$5 - 4Q = 2 + 2Q$

$P_{FM} = \$3/gallon$

$Q_{FM} = 0.5$bn gallons

(b) What is the socially optimal demand function taking into account externalities?

**Solution:**
We must subtract $MD = 1.2$ from PMB so that:

$P = 5 - 4Q^D - MD$

$P = 3.8 - 4Q^D$

(c) Solve for the socially optimal equilibrium price and quantity.

**Solution:**

$SMB = SMC$

$3.8 - 4Q = 2 + 2Q$

$Q_{SO} = 0.3$bn gallons

$P_{SO} = \$2.6/gallon$
(d) Graph the market for US gasoline with a supply/demand graph. Be sure to label 8 things: PMC, SMC, PMB, SMB curves, the Marginal Damage, the DWL, the private market equilibrium and the socially optimal equilibrium. (Also pay attention to the slopes – they certainly don’t need to be exact, but try to consider who is the more elastic/inelastic side of the market.)

Solution:

![Graph of the market for US gasoline with supply and demand curves, showing PMC, SMC, PMB, SMB curves, the Marginal Damage, the DWL, the private market equilibrium and the socially optimal equilibrium.](image)

(e) Calculate the dead-weight loss from the externality.

Solution:

\[ DWL = \frac{1}{2} (Q_M - Q_S)(MD) = \frac{1}{2} (0.5 - 0.3)(1.2) = 0.12 \]

i.e., about $120m dollars/day

(f) If the government uses a gas tax to address this externality, what is the optimal tax to offset the externality?

Solution:

The optimal tax for the externality is equal to the marginal damage at the socially optimal, i.e. $1.2 per gallon of gas.
(g) Calculate the revenue that would be raised by this tax.

**Solution:**

Revenue = \( \tau \cdot Q_S = \$1.2/\text{gallon} \times 0.3\text{bn gallons per day} \)
Revenue = \$360m dollars per day

(h) Will there be deadweight loss associated with this tax? If yes, how much? If no, why not?

**Solution:**

No, there will be no DWL because this is a Pigouvian tax designed to offset a negative consumption externality. In contrast, there will be a net welfare benefit of the tax (rather than a deadweight loss) equal to the DWL of the externality that is eliminated by the tax.

(i) What are the distributional consequences of the gas tax, is it a regressive, progressive or neutral tax?

**Solution:**

The answer to this question depends on how the use of cars is distributed among individuals. In a society where cars are luxury goods, the gas tax paid as a share of income would probably increase with income, so a gas tax would be progressive; while in a society where car use is more generalized, the amount that each individual would pay in gas tax would be somewhat constant across income brackets, so the gas tax paid as a proportion of income would be decreasing with income, and the gas tax would be regressive.
4. Public Goods

Arlen and Michael are the two GSIs for Econ 131. After eating, sleeping, socializing and doing research both of them have 42 hours a week to devote to Econ 131 related activities. There are two course related activities: Preparing their own sections (\(P_i\)) and Grading (\(G_i\)). Arlen’s utility over the time spent preparing his own sections and the time spent grading is given by \(U_A = 2 \ln P_A + \ln G\) while Michael’s utility over the time spent preparing his own sections and the time spent grading is by \(U_M = 2 \ln P_M + 2 \ln G\), where \(G\) is the total amount of time spent by both on grading, given by the sum of each individual’s contribution: \(G = G_A + G_M\). For this problem we are assuming that both Arlen and Michael benefit from the increases in the total amount of time they both spend on grading \(G\) but they don’t derive utility from the time the other spends preparing his own sections.

(a) Write down Arlen’s utility maximization problem.

**Solution:**

Arlen’s personal section prep time, \(P_A\), can be rewritten as \(42 - G_A\) because all the time not spent on grading \((G_A)\) can be spent on preparing his own sections. The public good enjoyed by Arlen can be rewritten as \(G_A + G_M\) because the grading done by either benefits both of them. If Arlen optimizes his own function, he will choose the amount of grading that maximizes his own utility, taking into consideration the amount of grading done by the other GSI. Therefore Arlen’s utility maximization problem is given by:

\[
\max_{P_A, S_A} 2 \cdot \ln(P_A) + \ln(G_A + G_M) \\
s.t. \quad P_A = 42 - S_A
\]

Which can be rewritten as:

\[
\max_{S_A} U_A = 2 \cdot \ln(42 - G_A) + \ln(G_A + G_M)
\]

(b) Find Arlen’s optimal number of hours devoted to grading \((G_A)\) as a function of the time on the same task spent by Michael \((G_M)\).

**Solution:**

Set \(\partial U_A/\partial G_A\) equal to zero:

\[
-2/(42 - G_A) + 1/(G_A + G_M) = 0 \\
2/(42 - G_A) = 1/(G_A + G_M)
\]
Cross-multiply:

\[ 2(G_A + G_M) = 42 - G_A \]

Solving for \( G_A \) yields:

\[ G_A = \frac{(42 - 2G_M)}{3} = 14 - \frac{2}{3}G_M \]

This is a response function which allows Arlen to calculate his optimal \( G_A \) as a function of the contribution to \( G \) made by Michael.

(c) Now, write down Michael’s utility maximization problem.

**Solution:**

Michael’s utility maximization problem is given by:

\[
\max_{P_M, G_M} 2 \cdot \ln(P_M) + 2 \cdot \ln(G_A + G_M) \\
\text{s.t. } P_M = 42 - G_M
\]

Which can be rewritten as:

\[
\max_{G_M} U_M = 2 \cdot \ln(42 - G_M) + 2 \cdot \ln(G_A + G_M)
\]

(d) Find Michael’s optimal number of hours devoted to grading \((G_M)\) as a function of the time on the same task spent by Arlen \((G_A)\).

**Solution:**

Set \( \partial U_M / \partial G_M \) equal to zero:

\[
-2/(42 - G_M) + 2/(G_A + G_M) = 0 \\
2/(42 - G_M) = 2/(G_A + G_M)
\]

Cross-multiply:
\[ G_A + G_M = 42 - G_M \]

Solving for \( G_M \) yields:

\[ G_M = \frac{(42 - G_A)}{2} = 21 - \frac{1}{2}G_A \]

This is a response function which allows Michael to calculate his optimal \( G_M \) as a function of the contribution to \( G \) made by Arlen.

(e) Use the response functions found in (b) and (d) to find the amount of time Michael and Arlen spend on grading and on the preparation of their own sections if they optimize their own functions.

**Solution:**

Plugging Michael’s response functions into Arlen’s response function yields

\[
\begin{align*}
G_A &= 14 - \frac{2}{3}G_M \\
G_A &= 14 - \frac{2}{3} \cdot \left( 21 - \frac{1}{2}G_A \right) \\
G_A &= 14 - 14 + \frac{1}{3}G_A \\
\frac{2}{3}G_A &= 0 \\
G_A &= 0
\end{align*}
\]

Then Michael will spend \( G_M = 21 - \frac{1}{2}(0) = 21 \) hours grading and \( P_M = 42 - 21 = 21 \) hours preparing his own sections, while Arlen will spend 42 hours doing sections prep and won’t contribute to the grading at all.

(f) From a utilitarian perspective (maximizing aggregate utility), what is the socially optimal amount of time they should spend on each task?

**Solution:**

The social planner maximizes \( U_A + U_M \) by choosing \( \{P_M, P_A, G_M, G_A\} \) subject to the budget constraints \( 42 = P_A + G_A \) and \( 42 = P_M + G_M \). This is equivalent to maximizing the following Lagrangian:
\[
max\{P_M, P_A, G_A, G_M\} L = [2\ln(P_A) + \ln(G_M + G_A)] + [2\ln(P_M) + 2\ln(G_M + G_A)] \\
+ \lambda_1(42 - P_A - G_A) + \lambda_2(42 - P_M - G_M)
\]

or

\[
max\{P_M, P_A, G_A, G_M\} L = 2\ln(P_A) + 2\ln(P_M) + 3\ln(G_M + G_A) \\
+ \lambda_1(42 - P_A - G_A) + \lambda_2(42 - P_M - G_M)
\]

Which gives first-order conditions:

(i) \[\frac{2}{P_A} - \lambda_1 = 0\]
(ii) \[\frac{3}{G_M + G_A} - \lambda_1 = 0\]
(iii) \[\frac{2}{P_M} - \lambda_2 = 0\]
(iv) \[\frac{3}{G_M + G_A} - \lambda_2 = 0\]
(v) \[42 - P_A - G_A = 0\]
(vi) \[42 - P_M - G_M = 0\]

There are a bunch of constraints, but they simplify quickly. Notice from (ii) and (iv) that \(\lambda_1 = \lambda_2 = \frac{3}{G_M + G_A}\). Then using \(\lambda_1 = \lambda_2\), we know from (i) and (iii) that \(\frac{2}{P_A} = \frac{2}{P_M}\) or \(P_A = P_M\) which given the time constraints in (v) and (vi) imply in turn that \(G_A = G_M\) or \(G = 2G_A = 2G_M\). Then, from (i) and things we have derived we know that \(\frac{3}{G_M + G_A} = \frac{2}{P_A}\), or \(\frac{3}{2G_A} = \frac{2}{42 - G_A}\) which delivers \(4G_A = 126 - 3G_A\). Solving, we get \(P_I = 18\) or \(G = G_A + G_M = 18 + 18 = 36\) and \(P_A = P_M = 24\).

(g) Is the answer for (f) different than (e)? If so, why? If not, why not?

**Solution:** Intuitively, in the computation in part (e), we set the marginal utility of the last hour of grading to each GSI equal to the marginal utility of preparing his own sections for that GSI. In part (f), we set the sum of the marginal utilities of the last hour of grading - the social marginal utility - equal to the marginal utility of preparing sections for either GSI. Since the social marginal utility of grading exceeds the individual marginal utilities of that hour of grading, a central planner optimally chooses more time on grading than individuals would if they were acting alone.