

Econ 131  
Spring 2017  
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Problem Set 3

**DUE DATE: April 5**

Student Name:

Student ID:

GSI Name:

- **You must submit your solutions using this template.**
- **Although you may work in groups**, each student must submit individual sets of solutions. You must note the names other students that you worked with. Write their names here:

## 1. Essay

A group of “conservative statesmen” issued in mid February a climate action proposal with a short summary in the NYTimes:

<https://www.nytimes.com/2017/02/08/opinion/a-conservative-case-for-climate-action.html>

Read this article and their full proposal at

<https://www.clcouncil.org/wp-content/uploads/2017/02/TheConservativeCaseforCarbonDividends.pdf>

Based on what you learned from the lecture on externalities, explain whether this proposal is a sound economic idea or not. In particular, would there be losers if such a proposal would be enacted? If there are losers, would it be possible to modify the proposal to compensate them for their losses? Answer this question by distinguishing theory from practice. Based on your reading of the news since February, has this proposal had any traction with the Trump administration and the Republican Congress?

## 2. True/False Statements

Determine whether each statement is true, false, or uncertain and explain why. Answers with no explanation will receive no points.

- (a) If bequests are accidental, then taxing inheritances is undesirable.

FALSE. With accidental bequests, inheritance tax does not affect donors (see class notes for details)

- (b) In a small open economy with perfect international mobility of capital, taxing corporate profits ends up hurting the workers and not the capitalists residing in the small economy.

TRUE. In small open economy, net-of-tax rate of return  $r(1 - \tau_{corp})$  is fixed by worldwide return  $r^*$ . Hence, increasing the local  $\tau_{corp}$  must leave  $r(1 - \tau_{corp})$  unchanged so that capitalists get the same return as before. Capital flows abroad until  $r$  has increased enough. This hurts local workers and reduced their wage.

- (c) Estate taxation is not popular in the United States in part because the public does not realize that the estate tax hits only the very rich.

TRUE. See the recent randomized experiment by Kuziemko et al. 2013 .

- (d) Suppose that candidates X and Z run for president. Candidate X is elected president after winning 51% of the vote. Then once in office, he appoints more conservative members to the Supreme Court than candidate Z would have. This means that a majority of American voters preferred more conservative Supreme Court members. (Assume that everyone is fully informed about the candidates plans and the President does not need Senate approval to appoint Supreme Court members.)

UNCERTAIN. This is possibly true. Its also possible that voters are single-issue voters on several topics and that X assembles a coalition large enough to win and then does things that only a minority of voters support. For example, X could get elected by 26% of voters who care only about conservative Supreme Court members and another 25% of voters who care only about low tax rates on high-earners (which X supports but Z does not). Then its possible that only 26% of voters support conservative Supreme Court member appointments.

- (e) Parks are an example of a pure public good.

UNCERTAIN. Most parks are large enough so that they are non-rival, and few public parks exclude people. It is, however, hypothetically possible to build a fence around a park to exclude people (think of a botanical garden).

### 3. Public Goods

San Francisco is considering building a new concert venue. Assume the city has two residents: Lakisha and Jamal. San Francisco will fund the concert venue solely from the individual contributions of these residents. Each of the two residents has a utility function over private goods ( $x_i$ ) and total venue size ( $S$ ), of the form:

$$U_i(x_i, S) = \frac{1}{2} \ln(x_i) + \frac{1}{2} \ln(S)$$

The total size of the venue is determined by the total number of seats built,  $S$ , and is the sum of the number of seats paid for by Lakisha and Jamal:  $S = s_L + s_J$ . Lakisha has an income of \$200 and Jamal has an income of \$100. Both the private good and a venue seat have a price of \$1.

- a) How many seats will be built if the government does not intervene? How many are paid for by Lakisha? By Jamal?

**Solution:**

For Lakisha, we can substitute  $x_L = 200 - s_L$  and  $S = s_L + s_J$ , so  $U_L = \frac{1}{2} \ln(200 - s_L) + \frac{1}{2} \ln(s_L + s_J)$ . Now we can attempt to maximize Lakisha's utility to find an interior solution:

$$\begin{aligned} \frac{\partial U_L}{\partial s_L} = 0 &= -\frac{1}{2(200 - s_L)} + \frac{1}{2(s_L + s_J)} \\ \Rightarrow s_L^* &= 100 - s_J/2 \end{aligned}$$

This is Lakisha's best response function for  $s_L$  when Jamal spends  $s_J$  on the public good. We can similarly find Jamal's best response function:  $s_J^* = \frac{100 - s_L}{2}$ . Since their incomes are different, the best response functions are not symmetric.

By solving this system of two equations, we have:  $s_L^* = 100$ ,  $s_J^* = 0$ , so  $S^* = s_L^* + s_J^* = 100$ .

- b) What is the socially optimal number of seats? If your answer differs from (a), explain why.

**Solution:**

Using  $MRS_L + MRS_J = MRT$ , we see that  $MRS_L = \frac{\partial U_L / \partial s_L}{\partial U_L / \partial x_L} = \frac{x_L}{s}$  and  $MRS_J = \frac{\partial U_J / \partial s_J}{\partial U_J / \partial x_J} = \frac{x_J}{s}$ . In addition,  $MRT = 1$ , because the price of the private good and a seat are both equal to \$1. When we also substitute  $x_L = 200 - s_L$  and  $x_J = 100 - s_J$ , and solve for  $S$ , we have  $S_{opt}^* = 150$ .

Now, imagine that starting from a price on seats of \$1, the price changes to  $p_S$ . The price on X is still \$1. As the price changes, Lakisha's and Jamal's incomes are increased in the following way: as the price changes from 1 to  $p_S$ , the terms  $C_L \equiv (p_S - 1)s_L$ , and  $C_J \equiv (p_S - 1)s_J$ , are added to the incomes of Lakisha's and Jamal's usual budget constraint, respectively. The resulting budget constraint is called the *compensated budget constraint*.

- c) Write up expressions for Lakisha's and Jamal's compensated budget constraints. Why do you think the budget constraints are called "compensated"?

**Solution:** The budget constraints are

$$\begin{aligned} p_S s_L + x_L &= 200 + C_L && \text{for Lakisha} \\ p_S s_J + x_J &= 100 + C_J && \text{for Jamal} \end{aligned}$$

Substituting in  $C_L$  and  $C_J$ , respectively, we get the exact same budget constraint as in parts a) and b). This way they can afford the exact same bundle of goods as before, thereby the word "compensation".

- d) Find the social optimum through vertical summation of demand curves:

i) Derive the inverse *compensated* demand curve for S in the following way:

- Maximize Lakisha's and Jamal's utility functions subject to their compensated budget constraints. Be careful not to plug in  $C_L$  and  $C_J$ , respectively, until *after* you've taken derivatives.
- Solve for  $p_S$  as a function of  $s_L$  and  $s_J$  for both Lakisha and Jamal. (*See Gruber 4th edition, ch. 2 (Theoretical Tools of Public Finance), p. 44 for intuition, though this reference is not necessary to solve the problem*)

ii) Using your result in i) derive the social demand curve.

iii) Return to a setup with  $p_S = 1, p_X = 1$ . Find the social equilibrium by equalizing the social demand curve with the supply curve for venue seats (i.e. the marginal cost of venue seats). Does this differ from what you found in (b)?

*Note: Lakisha's demand curve depends on Jamal's choice, and vice versa. This implies that we can't draw the demand curves in the usual way, as we simultaneously need to determine  $s_L, s_J$  and  $p_S$ , i.e. we have a 3-dimensional problem instead of our usual 2-dimensional problem (determining  $Q$  and  $p$ ). For illustration (**not for credit**), draw Lakisha's demand curve by fixing some value of  $s_J$  and Jamal's demand curve by fixing some value of  $s_L$ , and from that draw the social demand curve. However, you won't be able to graphically get the social optimum, only analytically as we did above.*

**Solution:**

- i) We find the demand curve by maximizing the utility function subject to the budget constraints  $x_L + p_S s_L = 200 + C_L$  for Lakisha and  $x_J + p_S s_J = 100 + C_J$  for Jamal. Setting  $MRS_i(s_i, p_s) = \frac{p_s}{p_x}$ :

$$MRS_L = \frac{200 + C - p_S s_L}{s_L + s_J} = \frac{200 + (p_S - 1)s_L - p_S s_L}{s_L + s_J} = \frac{200 - s_L}{s_L + s_J} = p_S \text{ and equivalently for Jamal, so the inverse demand curves are}$$

$$p_S = \frac{200 - s_L}{s_L + s_J} \quad \text{for Lakisha}$$

$$p_S = \frac{100 - s_J}{s_L + s_J} \quad \text{for Jamal}$$

- ii) Since the social demand curve is found by vertical summation, it is  $SMB = \frac{300-S}{S}$

- iii) Setting  $SMB = MC$ , we find that  $\frac{300-S}{S} = 1$  so  $S = 150$  in optimum. This is the same as in (b).

- e) Suppose, an anonymous fan pays for 60 seats. What is the new total number of seats? How many are provided by Lakisha? By Jamal? How does this compare to the level of provision in (d)?

**Solution:**

The anonymous gift increases the utility from the arena, and there is no tax to change Lakisha or Jamal's budget constraint. For example, Jamal's utility function would be:  $U_J = \ln(100 - s_J) + \ln(s_L + s_J + 60)$ . Solving again like in part (a) above, we get the best response functions

$$s_L^* = \frac{140 - s_J}{2}$$

$$s_J^* = \frac{40 - s_L}{2}.$$

Solving this, we get:  $s_L^* = 80$ ,  $s_J^* = -20$ . Of course, Jamal cannot purchase -20 seats, so instead, we assume Jamal purchases 0. Plugging this back into Lakisha's best response function, we see that she purchases 70. This is an equilibrium, since Jamal still optimizes by purchasing 0. Thus, the total amount of  $S$  is now 130.

- f) Propose a mechanism the government could use to achieve the socially optimal amount of seat provision.

**Solution:**

One possibility is for the government to provide 150 seats, and pay for them by applying lump-sum taxes to Lakisha and Jamal.