An Economical Business-Cycle Model

Pascal Michaillat (LSE) & Emmanuel Saez (Berkeley)

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Slack and inflation in the US since 1994

idle capacity (Census)

idle labor (ISM)
Slack and inflation in the US since 1994
Objective of the paper

- develop a tractable business-cycle model in which fluctuations in supply and demand lead to
  - some fluctuations in slack—unemployment, idle labor, and idle capacity
  - no fluctuations in inflation
- use the model to analyze monetary and fiscal policies
The model
Overview

- start from money-in-the-utility-function model of Sidrauski [AER 1967]
- add matching frictions on market for labor services as in Michaillat & Saez [QJE 2015]
  \[\Rightarrow\] generate slack
  \[\Rightarrow\] accommodate fixed inflation in general equilibrium
- add utility for wealth as in Kurz [IER 1968]
  \[\Rightarrow\] enrich aggregate demand structure
  \[\Rightarrow\] allow for permanent liquidity traps
Money and bonds

- Households hold $B$ bonds at nominal interest rate $i$
- Government circulates money $M$
- Open market operations impose $M(t) = -B(t)$
- Nominal financial wealth: $A = M + B$
- Price of labor services is $p$
- Inflation rate is $\pi = \dot{p}/p$
- Real variables: $m = M/p$, $a = A/p$, $r = i - \pi$
Behavior of representative household

- supply \( k \) labor services
- choose consumption \( c \), real money \( m \), real wealth \( a \)
- to maximize utility

\[
\int_{0}^{+\infty} e^{-\delta t} \cdot \left[ \frac{\varepsilon}{\varepsilon - 1} \cdot c^{\frac{\varepsilon - 1}{\varepsilon}} + \phi(m) + \omega(a) \right] dt
\]

- subject to law of motion of real wealth

\[
\frac{da}{dt} = f(x) \cdot k - \left[ 1 + \tau(x) \right] \cdot c - i \cdot m + r \cdot a + \text{seigniorage}
\]
Utility for real money

utility $\phi(m)$

real money $m$

money bliss point
Utility for real wealth

\[ a = m + b = 0 \]
Matching function and market tightness

\[ k \text{ units of labor services} \]

\[ \nu \text{ help-wanted ads} \]
Matching function and market tightness

sales = $k \cdot h(1, x) = k \cdot f(x)$

output: $y = h(k, v)$

purchases = $v \cdot h\left(\frac{1}{x}, 1\right) = v \cdot q(x)$

capacity $k$

tightness: $x = \frac{v}{k}$

help-wanted ads $v$
Matching cost: \( \rho \) services per ad

- output = \( \left[ 1 + \tau(x) \right] \cdot \text{consumption} \)

- proof:

\[
\begin{align*}
\sqrt[\text{output}]{y} &= \sqrt[\text{consumption}]{c} + \sqrt[\text{matching cost}]{\rho \cdot v} = c + \rho \cdot \frac{y}{q(x)} \\
\Rightarrow y \cdot \left[ 1 - \frac{\rho}{q(x)} \right] &= c \\
\Rightarrow y &= \left[ 1 + \frac{\rho}{q(x) - \rho} \right] \cdot c \equiv \left[ 1 + \tau(x) \right] \cdot c
\end{align*}
\]
Consumer’s first-order conditions

- costate variable:

\[ \lambda = \frac{c^{-1/\varepsilon}}{1 + \tau(x)} \]

- demand for real money balances:

\[ \phi'(m) = i \cdot \frac{c^{-1/\varepsilon}}{1 + \tau(x)} \]

- consumption Euler equation:

\[ \frac{d\lambda}{dt} \cdot \frac{\lambda}{\lambda} = 1 + \tau(x) \cdot \omega'(a) + i - \pi - \delta \]
Equilibrium: 6 variables, 5 equations

\[ [c(t), m(t), a(t), i(t), p(t), x(t)]_{t=0}^{+\infty} \text{ satisfy} \]

- consumption Euler equation
- demand for real money balances
- no wealth in aggregate: \( a(t) = 0 \)
- matching process: \( (1 + \tau(x(t))) \cdot c(t) = f(x(t)) \cdot k \)
- \( m(t) = M(t)/p(t) \) and monetary policy sets \( M(t) \)
Equilibrium selection: fixed inflation

- price $p(t)$ is a state variable with law of motion:
  \[ \dot{p}(t) = \pi \cdot p(t) \]

- $p(0)$ and $\pi$ are fixed parameters

- given $p(t)$, tightness $x(t)$ equalizes supply to demand
Steady-state equilibrium:
IS, LM, AD, and AS curves
IS curve (from consumption Euler equation)

\[ c^{IS}(i, \pi, x) = \left[ \frac{\delta + \pi - i}{(1 + \tau(x)) \cdot \omega'(0)} \right]^\varepsilon \]
IS curve without utility of wealth

\[ i_{IS}(x, \pi) = \pi + \delta \]
LM curve (from demand for real money balances)

\[ c^{LM}(i, m, x) = \left[ \frac{i}{(1 + \tau(x)) \cdot \phi'(m)} \right]^\epsilon \]
LM curve with money > bliss point (liquidity trap)

\[ i^{LM}(x, m) = 0 \]
IS & LM determine interest rate and AD

\[
i = \frac{\phi'(m)}{\phi'(m) + \omega'(0)} \cdot (\delta + \pi)
\]

\(c^{AD}(x, \pi, m)\)
IS & LM determine interest rate and AD

\[ c^{AD}(x' < x, \pi, m) \]
AD curve

\[ c^{AD}(x, \pi, m) = \left[ \frac{\delta + \pi}{(1 + \tau(x)) \cdot (\phi'(m) + \omega'(0))} \right]^{\varepsilon} \]
AS curve

market tightness $x$

quantity of labor services

capacity: $k$
AS curve

output: \( y = f(x) \ k \)

market tightness \( x \)

quantity of labor services

capacity \( k \)
AS curve

consumption: $c = \frac{f(x) \cdot k}{1 + \tau(x)} = \left[f(x) - \rho \cdot x\right] \cdot k$

output $y$

capacity $k$

market tightness $x$

quantity of labor services
AS curve

market tightness $x$

quantity of labor services

consumption recruiting slack

output capacity
$c^{AS}(x) = (f(x) - \rho \cdot x) \cdot k$
AS curve and state of the economy

- Overheating economy
- Efficient economy
- Slack economy
General equilibrium

market tightness

quantity of labor services

$\text{AD}$

$\text{AS}$

output

capacity

general equilibrium

slack

$\text{x}$

$\text{c}$

$\text{y}$

$\text{k}$
Dynamical system is a source

\[ \dot{\lambda} = (\delta + \pi) \cdot \lambda - \omega'(0) - \phi'(m) \]
Immediate adjustment to shock
Macroeconomic shocks
Increase in AD: fall in MU of wealth

- AD increases
- Nominal interest rate
- Consumption
Increase in AD: fall in MU of wealth

- Increase in AD leads to a fall in the marginal utility of wealth.
- This affects the supply curve (AS) as it moves to the left, indicating a decrease in supply given the price level.
- The intersection of the demand curve (AD) and the supply curve (AS) determines the new equilibrium point, leading to lower output and capacity utilization.

The diagram illustrates the relationship between labor market tightness and the quantity of labor services, with the supply curve (AS) shifting due to changes in demand (AD).
Increase in AS: rise in capacity

![Graph showing the relationship between labor market tightness, quantity of labor services, and output and capacity. The graph includes the schedules of AS and AD, with arrows indicating the movement from a lower to a higher output and capacity due to an increase in the supply of labor services.]
Monetary and fiscal policies
Increase in money supply

market tightness $x$  

output capacity

low tightness and output

depressed AD
Increase in money supply

AD increases

nominal interest rate $i$

consumption $c$

IS

LM
Increase in money supply
Money supply in a liquidity trap

very low tightness and output

very depressed AD
Money supply in a liquidity trap

IS

LM in liquidity trap

nominal interest rate $i$

consumption $c$
Money supply in a liquidity trap

inefficiently low tightness

AD in liquidity trap
Alternative policy: helicopter money

- government prints and distributes $M^h > 0$
- aggregate wealth is positive: $a = m^h > 0$
- IS curve depends on helicopter money:

$$c^{IS} = \left[ \frac{\delta + \pi - i}{(1 + \tau(x)) \cdot \omega'(m^h)} \right]^\varepsilon$$
Helicopter money always stimulates AD

AD increases

IS
LM
consumption
nominal interest rate

AD increases

IS
LM
consumption
nominal interest rate

Helicopter money always stimulates AD
Helicopter money always stimulates AD

IS
LM in liquidity trap
consumption
nominal interest rate

AD increases

LM in liquidity trap
Alternative policy: tax on wealth

- government taxes wealth at rate $\tau^a > 0$
- IS curve depends on wealth tax:

$$c^{IS} = \left[ \frac{\delta + \tau^a + \pi - i}{(1 + \tau(x)) \cdot \omega'(0)} \right]^\varepsilon$$
Tax on wealth always stimulates AD
Tax on wealth always stimulates AD
Alternative policy: government purchases

- government purchases $g(t)$ units of labor services
- $g(t)$ enters separately in utility function
- $g(t)$ financed by lump-sum taxes
- AD curve depends on government purchases:

$$c_{AD} = \left[ \frac{\delta + \pi}{(1 + \tau(x)) \cdot (\phi'(m) + \omega'(0))} \right]^\varepsilon + \frac{g}{1 + \tau(x)}$$
Government purchases stimulate AD

Labor market tightness

output

capacity

quantity of labor services

AS

AD
Summary of policies

- conventional monetary policy sets money supply $M$

- $M$ stabilizes economy out of liquidity trap
  - $M \rightarrow LM \text{ curve} \rightarrow AD \text{ curve}$

- $M$ is ineffective in liquidity trap
  - LM curve is stuck

- alternative policies work in liquidity trap
  - helicopter money / wealth tax $\rightarrow IS \text{ curve} \rightarrow AD \text{ curve}$
  - government purchases $\rightarrow AD \text{ curve}$
Inflation and slack dynamics in the medium run
Simplifying assumptions

1. no money growth
2. no liquidity trap
Directed search [Moen, JPE 1997]

- buyers search for best price/tightness compromise
- in equilibrium, buyers are indifferent across markets:
  \[(1 + \tau(x)) \cdot p = e\] in any market \((x, p)\)
- seller chooses \(p\) to maximize \(p \cdot f(x)\) subject to
  \[(1 + \tau(x)) \cdot p = e\]

\[\Leftrightarrow\] seller chooses \(x\) to maximize \(f(x)/(1 + \tau(x))\)

\[\Leftrightarrow\] seller chooses efficient tightness \(x^*\)

- if \(x < x^*\), seller wants to lower \(p\) and conversely
Price-adjustment cost [Rotemberg, REStud 1982]

- seller chooses $p$, $\pi$, and $x$
- to maximize the discounted sum of nominal profits

$$\int_{0}^{+\infty} e^{-I(t)} \cdot \left( p \cdot f(x) \cdot k \cdot \frac{\kappa}{2} \cdot \pi^2 \right) dt$$

- subject to

$$\dot{p} = \pi \cdot p$$

$$1 + \tau(x) = \frac{e}{p}$$

- solution yields Phillips curve
Dynamical system describing equilibrium

- system of 3 ODEs: law of motion of price ($\dot{p}$), Phillips curve ($\dot{\pi}$), consumption Euler equation ($\dot{x}$)
- state variable: $p$
- jump variables: $\pi$, $x$
- the unique steady state has $x = x^*$ and $\pi = 0$
- system is a saddle around steady state
- stable manifold is a line: dynamic determinacy
Short-run/long-run effects of shocks

<table>
<thead>
<tr>
<th>Increase in:</th>
<th>$x$</th>
<th>$\pi$</th>
<th>$p$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate demand</td>
<td>$+$</td>
<td>$+$</td>
<td>$0$</td>
<td>$+$</td>
</tr>
<tr>
<td>Money supply</td>
<td>$+$</td>
<td>$+$</td>
<td>$0$</td>
<td>$+$</td>
</tr>
<tr>
<td>Aggregate supply</td>
<td>$-$</td>
<td>$-$</td>
<td>$0$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Note: $\pi$ is inflation, $p$ is price, $y$ is output.