

PROBLEM SET 2 SOLUTION

DUE ON GRADESCOPE BY 11:59PM ON MONDAY OCTOBER 14

Student name:

Student ID:

You may work together on the problems, but your answers must be *in your own words* and *handwritten*. You also must *list the other students with whom you worked* here:

For all questions be sure to explain your answers and to use graphs whenever asked to. Write your answers in the spaces below.

1. (1 point) Consider Alice and Jeremy. They both have the same utility of income $\log(c)$ where c is their disposable income. Suppose Alice has \$10 while Jeremy has \$4.
 - a. What is the marginal utility of income for Alice and Jeremy? Explain why they are different.

$mc = u'(c) = 1/c = 1/10$ for Alice and $1/4$ for Jeremy. Lower for Alice because mc decreases with disposable income.

- b.** Suppose that Alice gives \$1 to Jeremy. Does this increase or decrease their utility sum? Why?

Yes, it increases total utility because Jeremy has higher mc than Alice. You can check directly that $\log(10)+\log(4)=\log(40)<\log(9)+\log(5)=\log(45)$

- c.** How much should Alice give to Jeremy to maximize their utility sum? Why?

As long as Alice has more than Jeremy, her mc is higher than his and hence extra transfers from Alice to Jeremy further increase total utility. So utility is maximized when their incomes are equal: 7 each (Alice has given 3 to Jeremy).

Mathematically: $\log(10-g)+\log(4+g)$ is maximized when $1/(10-g)=1/(4+g)$ ie when $10-g=4+g$ or $6=2g$ or $g=3$.

- d.** Can we conclude from this exercise that maximizing utility means that disposable incomes should be equated across individuals?

This result does generalize. If everybody has the same utility, what maximizes total utility is sharing the total pie equally. However, this assumes that the size of the total pie does not change with transfers. In reality, perfect equalization would reduce incentives to work and produce in the first place and hence reduce the pie. Therefore, there would be a trade-off between equality and efficiency.

- 2.** (1 point) Consider a household that buys two things: electricity and everything else.
- a.** What is the condition for the household to be maximizing its utility? Explain intuitively what this condition means.

For the household to be maximizing its utility, the quantities of electricity and everything else must satisfy the rational spending rule. In symbols, we can write this as:

$$\frac{mu_E}{P_E} = \frac{mu_{ee}}{P_{ee}},$$

where the P's are the market prices and the mu's are the household's marginal utilities, and where "E" denotes electricity and "ee" denotes everything else. Intuitively, this condition means that the extra utility the household gets from the last dollar it spends on electricity must equal the extra utility it gets from the last dollar it spends on everything else. (For example, consider the last dollar it spends on electricity. The number of units of electricity it gets is $1/P_E$, and the marginal utility of one unit of electricity is mu_E . So the additional utility it gets from that dollar is mu_E/P_E . If the rational spending condition didn't hold, the household could increase its utility by rearranging its spending. For example, if mu_E/P_E were greater than mu_{ee}/P_{ee} , it could increase its utility by spending more on electricity and less on everything else.)

Also, note that the utility-maximizing household will be on its budget constraint. That is, the quantities of the two goods it chooses will satisfy

$$P_E q_E + P_{ee} q_{ee} = \text{Income},$$

where the q's are the quantities and Income is the household's income. If the household's total spending were less than Income, it could increase its utility by increasing its spending on one or both goods. And it's not feasible for it to spend more than Income.

- b.** Suppose that the price of electricity rises. How will the household need to modify its consumption of electricity and everything else to continue maximizing its utility?

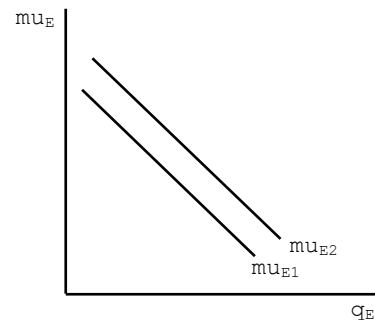
As described in part a, before the rise in the price of electricity, the household's choices satisfied

$$\frac{mu_E}{P_{E1}} = \frac{mu_{ee}}{P_{ee1}},$$

where 1's denote the prices before the change. The rise in the price of electricity causes the left-hand side of the expression to fall. Thus if the household didn't change its purchases, it would no longer be maximizing its utility. What it needs to do is consume less electricity and more of everything else. As the household consumes less electricity, this drives up the marginal utility of another unit of electricity. This process continues until the marginal utility of another dollar spent is once again the same for both types of goods.

- c. Suppose the area where the household lives experiences an intense heatwave (which can be counteracted by using electric air conditioners). Compared to its consumption before the heatwave hit, how, if at all, will the household need to modify its consumption of electricity and everything else to continue maximizing its utility? [assume that prices remain constant here]

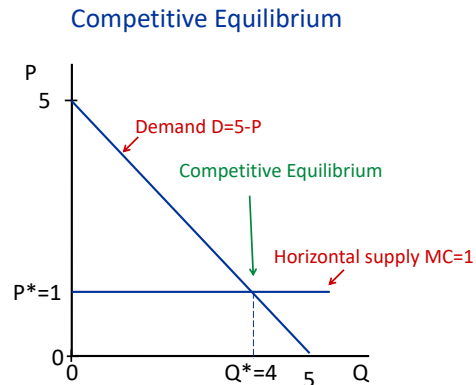
Before the heatwave, the household was on its budget constraint and was choosing the quantities of electricity and everything else such that $mu_E/P_E = mu_{ee}/P_{ee}$. Since the effects of the heat can be mitigated by air conditioning, the heatwave likely raises the marginal utility of another unit of electricity at a given level of electricity consumption. This corresponds to an upward (or rightward) shift of the marginal utility curve for electricity (from mu_{E1} to mu_{E2}).



Initially, the household was choosing its consumption of the two goods so that $mu_E/P_E = mu_{ee}/P_{ee}$. Thus if the household didn't change its consumption of the two goods, mu_E/P_E would be greater than mu_{ee}/P_{ee} . The household will need to increase its consumption of electricity and decrease its consumption of everything else to return the optimization condition to equality.

3. (2 points) Consider an industry producing a specific good (e.g. whistles) with constant marginal costs of production $MC=\$1$ per unit (and zero fixed costs) with a demand curve $D(P)=5-P$.

- a. Draw the supply and demand curves and find the competitive equilibrium. What is the producer surplus and what is the consumer surplus?



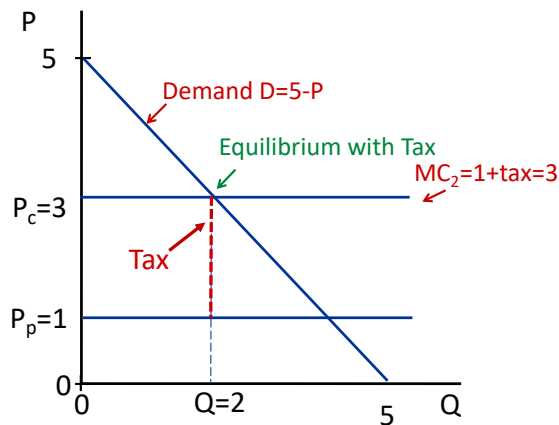
Producer surplus is zero. Consumer surplus is triangle below D and above price line: area $\$8$.

- b. Compute the elasticities of supply and demand at the competitive equilibrium from a.

Supply elasticity is infinite as supply curve is horizontal. Demand elasticity is: $PD'(P)/D=-P/(5-P)=-1/4$ at $P^*=1$.

- c. Suppose a $\$2$ per unit tax is introduced that is formally paid by producers. Figure out the new equilibrium, consumer and producer prices, consumer and producer surplus, tax revenue, and deadweight loss. Who ends up paying the tax in this case? Why?

Competitive Equilibrium with tax



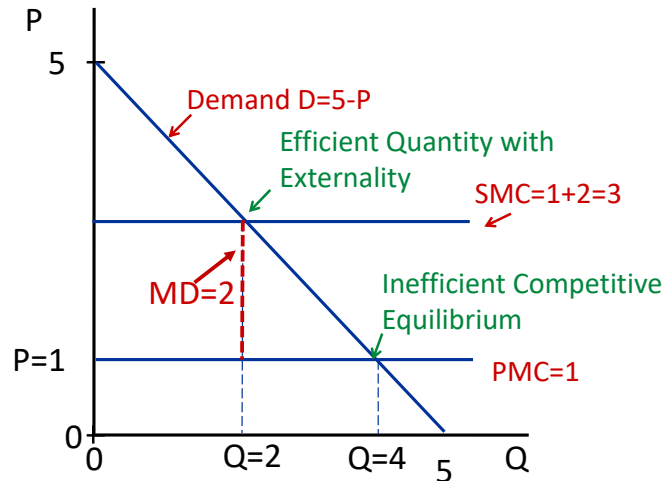
Tax on producers shifts MC curve up by \$2. Consumer price goes up to \$3. Producers receive \$1 per unit (net of the tax they pay). Hence, all the effect is on consumers. That's because supply is infinitely elastic and inelastic factors bear the tax. Consumer surplus shrinks to \$2. Tax Revenue is \$4. DWL is \$2.

- d. Suppose the production of the good generates a negative externality equal to \$2 per unit produced. Figure out the equilibrium, producer surplus, consumer surplus, total external cost, and deadweight loss without a tax. Figure out the tax that could restore efficiency.

With the externality with constant MD=\$2 per unit, the social marginal cost of production (SMC) is 3 ($MC+MD=1+2=3$). The competitive equilibrium described in a is no longer efficient. Efficient point is where $SMB=SMC$. Here $SMB=MB=D$ (no externality on consumption side) so $MB=5-Q=3$ is efficient. So $Q=2$. The inefficient equilibrium has zero producer surplus, \$8 of consumer surplus (as in a.) but an external damage of $4*2=\$8$. It has zero net social economic surplus and a deadweight loss triangle of \$2 (relative to social efficiency).

The efficient equilibrium can be obtained by putting a tax of $\$2=MD$ per unit as in c. In which case, we get $Q=2$ with zero producer surplus and consumer surplus of \$2, tax revenue of \$4, and external damage of \$4. This is a net social economic surplus of \$2.

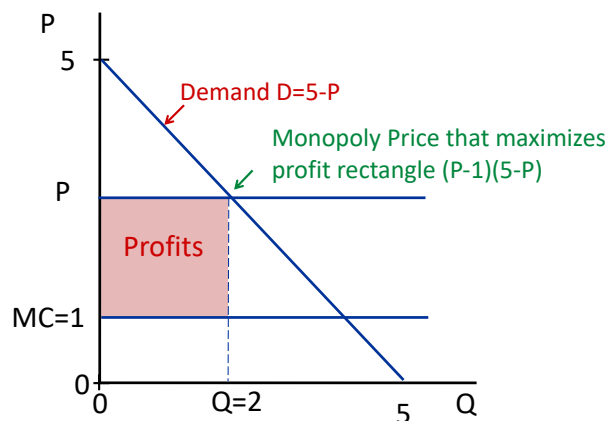
Competitive Equilibrium with externality



- e. Suppose now that the industry becomes a monopoly. With no taxes and no externalities as in a., what is the price that the monopoly charges? What is the resulting quantity? What are the producer surplus, consumer surplus, and deadweight losses? (draw a graph to explain your answer)

The monopoly sets the price P to maximize profits = sales – costs of production = $P \cdot D - D = (P-1) \cdot D = (P-1) \cdot (5-P)$. This is maximized when $P=3$ (this creates the largest profit rectangle). Then $Q=5-P=2$. The equilibrium is the same as in c, except that tax revenue is now the monopoly profit of \$4. $DWL=\$2$. Consumer surplus= $\$2$.

Monopoly Equilibrium



- f. Suppose that we now combine the monopoly from e. with the \$2 per unit externality from d. Figure out the equilibrium price, quantity, consumer and producer surplus, external cost, and deadweight loss? Can we conclude from this example that it was good that the Standard Oil has a monopoly on US oil in the 1890-1910 period?

With the externality from d., the monopoly equilibrium from e. is actually efficient. It has zero deadweight burden. Profits of \$4. Consumer surplus of \$2, and external damage of \$4, for a net social economic surplus of \$2. It turns out in this example that the higher price charged by the monopoly is exactly the MD which restore efficiency. In the 1900s, Standard oil probably kept oil prices above the competitive equilibrium, a good thing with the negative externality of oil in the basic economic model. However, in the real world, what would have been needed is an earlier shift to adopt clean electricity rather than oil for energy use. Standard oil probably would have used its power to make sure politicians would favor oil based development.

4. (1 point) Read the following recent article on the recent lawsuit the Federal government won against Google:

<https://www.nytimes.com/2024/08/13/technology/google-monopoly-antitrust-justice-department.html>

What does the article teach you about the recent shift in US anti-trust policy from only worrying about prices consumers pay toward worrying about firms' dominance over an industry?

Write your answer clearly and concisely in 10-15 lines below.

Key point is that case against Google is not driven by concerns about prices consumers pay but rather about Google's dominance over an industry. A big piece of Google dominance is that it negotiates to get the google search engine as the default on most systems, which gives it an enormous advantage (as most people stick with defaults).